Spacetime Topology and CPT violation

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Outline

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Lorentz and CPT invariance, CPT theorem

- Lorentz invariance is described by the Lorentz transformation

$$t' = \gamma(t - \frac{vx}{c^2}),\ x' = \gamma(x - vt),\ y' = y,\ z' = z \quad (1)$$

with $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$

- CPT invariance consists of CPT transformation, where C: Charge conjugation, P: Parity, and T: Time reversal.

CPT theorem (Lüders, Pauli, in the 1950-s)

Every local relativistic quantum field theory is invariant under the combined operation of C, P and T, even if some of the separate invariances do not hold.

- Some important references:
[1] G. Lüders, Ann. Phys. (N. Y.) 2 (1957) (1).
[2] J. Sakurai, Invariance Principles and Elementary Particles, Princeton Univ. Press, Princeton, 1964.
[3] R. Streater, A. Wightman, PCT, Spin and Statistics, and All That, Benjamin, New York, 1964.

Lorentz and CPT invariance, CPT theorem

- A consequence of CPT theorem is that a CPT violation automatically indicates a Lorentz violation.
- In order to preserve this symmetry, every violation of the combined symmetry of two of its components (such as CP) must have a corresponding violation in the third component (such as T). Thus violations in T symmetry are often referred to as CP violations.
- Thus any CPT violation gives a hint of fundamentally new physics at different scales for example quantum gravity or strings.
- The CPT violation may have been important in the very early universe.

- Introduce a gauge invariant Chern-Simons-like term into the Abelian gauge field Lagrangian, which violates CPT and Lorentz invariance. For example

$$\mathcal{L}_{CS-like} = \frac{1}{4} m \epsilon^{\mu\nu\rho\sigma} k_\mu A_\nu F_{\rho\sigma} \tag{2}$$

with real parameter m and fixed real symmetry breaking "vector" k_μ of unit length. [Adam and Klinkhamer, Nucl. Phys. B 607, 247 (2001)]
[Carroll, Field, Jackiw, Phys. Rev. D 41 (1990) 1231].

Considering a classical gauge field interacting with fermions, a CPT and Lorentz violating term can be introduced into the fermionic part of the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - e\not{A} - m - \gamma_5 \not{b})\psi. \tag{3}$$

The Lorentz and CPT breaking term is proportional to the constant "four vector" b_μ [Adam and Klinkhamer, Phys. Lett. B 513, 245 (2001)].

Different origins of CPT violation

The CPT and Lorentz violation occur in certain chiral gauge theories defined over a manifold having non trivial topology ($M = \mathbb{R}^3 \times S^1$) with one compactified coordinate. Perturbative expansion of the effective gauge field action gives rise to some CPT odd local Chern-Simons-like terms. The CPT and Lorentz violation occurs in the effective action of gauge field due to quantum effects of the chiral fermions.

The existence of a CPT anomaly is established for a particular four-dimensional Abelian lattice gauge theory with Ginsparg-Wilson fermions.
[Klinkhamer, Schimmel, Nuclear Physics B 639 (2002) 241].

Conventions

- We consider chiral gauge theories which are anomaly free. For example chiral Yang-Mills with gauge group $G = SO(10)$.
- In our calculation four-dimensional spacetime manifold ($M = \mathbb{R}^3 \times S^1$) is considered, with noncompact coordinates $x^1, x^2, x^3 \in \mathbb{R}$ and compact coordinate $x^0 \in [0, L]$.
- Natural units are used, that is: $\hbar = c = 1$.
- Metric is taken to be Euclidean flat metric $g_{ab} = \text{diag}(1, 1, 1, 1)$.
- The gauge fields and fermion fields are periodic with respect to x^0 coordinate.
- Another assumption is about the gauge field: $A_0(x) = 0$.

- The Euclidean Weyl action is given by

$$S[\bar{\psi}, \psi, A] = \int_M d^4x \; \psi_L^\dagger \; i\sigma_-^\mu (\partial_\mu + A_\mu) \; \psi_L . \tag{4}$$

  with $\sigma_-^\mu = (\mathbb{I}, -i\sigma^m)$, where σ^m are the 2×2 Pauli matrices and $\mathbb{I}$ is the 2×2 identity matrix.

- We shall look at the effective action of the gauge fields. In vacuum the virtual creation and annihilation of fermion-antifermion pairs interact with the classical gauge fields. The effective action $\Gamma[A]$ is a functional which takes these interactions into account.

- In path integral formalism the functional $\Gamma[A]$ is obtained by integrating out the fermionic degrees of freedom

$$\exp(\Gamma[A]) = \int \mathcal{D}\psi_L^\dagger(x) \mathcal{D}\psi_L(x) \; \exp\left(\int_M d^4x \; \mathcal{L}[\psi_L^\dagger, \psi_L, A] \right), \tag{5}$$

  which is formally equal to the determinant of the operator $[\sigma_-^a (\partial_a + A_a)]$.

Calculation

- First consider the case gauge fields are independent of x^0 [F. R. Klinkhamer, Nucl. Phys. **B578** 277 (2000).]
- We decompose the fermion fields into Fourier modes

$$\psi_L(\vec{x}, x^0) = \sum_{n=-\infty}^{\infty} e^{2\pi i n x^0/L} \xi_n(\vec{x}), \text{ and } \psi_L^\dagger(\vec{x}, x^0) = \sum_{n=-\infty}^{\infty} e^{-2\pi i n x^0/L} \xi_n^\dagger(\vec{x}). \quad (6)$$

- Then the Weyl action can be rewritten as

$$I = \sum_{n=-\infty}^{\infty} I_3[\chi_n^\dagger, \chi_n, A], \quad (7)$$

where $I_3[\chi_n^\dagger, \chi_n, A] = \int \ d^3x \ \chi_n^\dagger \ \left(i\sigma^m(\partial_m + A_m) + i\frac{2\pi n}{L} \ \right) \chi_n$

- Finally the effective action can be factorized as:

$$\exp(\Gamma[A]) = \prod_{n=-\infty}^{\infty} \int \mathcal{D}\chi_n^\dagger \mathcal{D}\chi_n \ \exp\left(-I_3[\chi_n^\dagger, \chi_n, A]\right) \quad (8)$$

- For $n = 0$ factor, we get the one loop result

$$\Gamma^{n=0}[A] \supseteq \frac{1}{L}\int_0^L dx^0 \int d^3x \, \frac{M}{|M|} \, \omega_{CS}, \tag{9}$$

where

$$\omega_{CS} \simeq \epsilon^{ijk} tr(A_i \partial_j A_k - \frac{2}{3} A_i A_j A_k) \tag{10}$$

is the Chern-Simons density.

- Now we consider the more general case where the gauge fields can depend upon the compactified coordinate x^0.

- Unlike the previous case we have to use the four-dimensional integration instead of three dimensional integration.
- Like the previous case we rewrite our Weyl action as:

$$\mathcal{I} = \int d^4x \; \psi_L^\dagger(x) \; [\gamma^\mu(\partial_\mu + A_\mu)] \; \psi_L, \tag{11}$$

where

$$\gamma^0 = \sigma^0, \;\; \gamma^i = i\sigma^i, \tag{12}$$

- The physically relevant factor in the effective action can be computed as

$$\Gamma[A] = \frac{g^2}{2} tr[T^a T^b] \int \frac{d^4p}{(2\pi)^4} A_i^a(-p) A_j^b(p) \pi^{ij}(p)$$
$$-\frac{g^3}{3} tr[T^a T^b T^c] \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} A_i^a(-p-q) A_j^b(p) A_k^c(q) \pi^{ijk}(p,q) + O(g^4),$$

with

$$\pi^{ij}(p) = \int \frac{d^4k}{(2\pi)^3} tr[\gamma^i S(k) \gamma^j S(k+p)], \qquad (13)$$

$$\pi^{ijk}(p,q) = \int \frac{d^4k}{(2\pi)^3} tr[\gamma^i S(k) \gamma^j S(k+p) \gamma^k S(k+p+q)] \qquad (14)$$

and

$$S(k) = \frac{1}{\gamma^i k_i - \gamma^0 k_0} = \frac{\gamma^i k_i + k_0}{(\gamma^i k_i)^2 - k_0^2} \qquad (15)$$

Calculation

- From the equation (13) we have

$$\pi^{ij}(p) = \int_0^1 dx \int \frac{d^4 l}{(2\pi)^4} \frac{\mathrm{tr}[\gamma^i(\not{l} - x\not{p})\gamma^j(\not{l} + (1-x)\not{p})]}{(\vec{l}^2 + \Delta)^2} \tag{16}$$

where $\Delta = x(1-x)p^2 + l_0^2$ and x is the Feynman parameter.

- We divide the domain of integration in two parts, namely, $l_0 = 0$ and $l_0 \neq 0$.
- For $l_0 = 0$ part (this is similar to the case when gauge fields does not depend upon the x^0 coordinate) we can introduce Pauli-Villars regularization scheme and get the usual Chern-Simons like term.

$$T_0 \sim \frac{M_0}{|M_0|} \epsilon^{ijk} p_k. \tag{17}$$

- The infrared divergences can be regularized by imposing antiperiodic boundary conditions for the Dirac (and Pauli-Villars) fields on the surface of a large ball B^3, where the gauge potentials vanish.

- We consider the case when $l_0 \neq 0$. Since l_0 is discrete So we shall do the following replacements with respect to the previous expressions:

$$\int \frac{dl_0}{2\pi} \longrightarrow \frac{1}{L} \sum_{m=-\infty}^{\infty}, \quad \text{where } m \neq 0 \tag{18}$$

- For a particular 'm',

$$T(p_n) = \frac{1}{L} \sum_{m=-\infty}^{\infty} \omega_m \int \frac{d^3 l}{(2\pi)^3} \int_0^1 dx \, \frac{\text{tr}[\gamma^i \gamma^j \gamma^k] p_k + (1-2x) \text{tr}[\gamma^i \gamma^j \gamma^0] \ \rho_n}{(\vec{l}^2 + \Delta)^2} \tag{19}$$

and

$$T_{div}(p_n) = \frac{1}{L} \sum_{m=-\infty}^{\infty} \omega_m \int \frac{d^3 l}{(2\pi)^3} \int_0^1 dx \, \frac{\text{tr}[\gamma^i \gamma^j \gamma^k \gamma^l] l_k l_l}{(l^2 + \Delta)^2}. \tag{20}$$

where,
$p_n = (\rho_n, \vec{p}), \ \rho_n = \frac{2\pi n}{L}$,
$l_n = (\omega_m, \vec{l}), \ \omega_m = \frac{2\pi m}{L}$,
$\Delta = x(1-x)|p_n|^2 + \omega_m^2$.

Calculation

- We shall concentrate on the term $T(p_n)$, which eventually gives rise to the CPT violating term.
- From equation (19) we calculate

$$\int \frac{d^3 l}{(2\pi)^3} \frac{1}{(l^2+\Delta)^2} = \frac{1}{8\pi\sqrt{\Delta}} \tag{21}$$

- So we have terms like

$$T_1(p_n) \sim \frac{1}{L} \sum_{m=-\infty}^{\infty} \omega_m \int_0^1 dx \frac{1}{\sqrt{\Delta}}, \tag{22}$$

$$T_2(p_n) \sim \frac{1}{L} \sum_{m=-\infty}^{\infty} \omega_m \int_0^1 dx \frac{(1-2x)}{\sqrt{\Delta}}, \tag{23}$$

- To remove the UV divergent term (20) we introduce Pauli-Villars fields having large positive mass M_n for a particular p_n. After subtracting it from $T_1(p_n)$ we get the regularized result:

$$T_r = -\lambda + \lambda^2 \arcsin\left(\frac{1}{\sqrt{1+\lambda^2}}\right) - \arctan(\lambda), \tag{24}$$

  where λ is the upper limit of $2\frac{|\omega_m|}{|p_n|}$. Now T_r converges with respect to λ with a limiting value $-\frac{\pi}{2}$.

- Finally from two point functions, we get the following contribution to the effective action:

$$T = F\,\frac{1}{L}\int d^4x\ \mathrm{tr}\ \left[\epsilon^{ijk} A_i \partial_j A_k\right] \tag{25}$$

  where, $F \sim \left((\frac{M_0}{|M_0|} + T_r\right)$, with M_0 is the Pauli-Villars mass, and $T_r = -\frac{\pi}{2}$.

- The three point function $\pi^{ijk}(p,q)$ is defined by

$$\begin{aligned}\pi^{ijk}(p,q) &= \int \frac{d^4k}{(2\pi)^4} tr[\gamma^i S(k)\gamma^j S(k+p)\gamma^k S(k+p+q)] \\ &= 2\int_0^1 dx \int_0^{1-x} dy \int \frac{d^4l}{(2\pi)^4} \quad \frac{N}{(\vec{l}^2+\Delta)^3},\end{aligned} \tag{26}$$

where

$$N = \mathrm{tr}[\gamma^i(\not{l} - x\not{p} - y\not{r} + M)\gamma^j(\not{l} + (1-x)\not{p} - y\not{r})\gamma^k(\not{l} - x\not{p} + (1-y)\not{r})], \tag{27}$$

$$l = k + xp + y(p+q) = k + xp + yr, \quad \text{and} \tag{28}$$

$$\Delta = x(1-x)p^2 + y(1-y)r^2 - 2prxy + l_0^2. \tag{29}$$

- Like the previous section we can divide the domain of l_0 in to two parts that is $l_0 = 0$ and $l_0 \neq 0$ part. For $l_0 = 0$ part from Pauli-Villars regularization we get the following term

$$T_3 \sim \frac{2}{3}\frac{M_0}{|M_0|}\epsilon^{ijk} \tag{30}$$

For $l_0 \neq 0$ case we look at the terms like

$$T_4 = \int \frac{d^3 l}{(2\pi)^3} \; \frac{\text{tr}[\gamma^i\gamma^l\gamma^j\gamma^m\gamma^k + \gamma^i\gamma^l\gamma^j\gamma^k\gamma^m + \gamma^i\gamma^j\gamma^l\gamma^k\gamma^m] l_l l_m}{(\vec{l}^2 + \Delta)^3} \sim \epsilon^{ijk} \frac{1}{\sqrt{\Delta}} \tag{31}$$

and

$$T_5 = \int \frac{d^3 l}{(2\pi)^3} \; \frac{\text{tr}[\gamma^i\gamma^j\gamma^k]}{(\vec{l}^2 + \Delta)^3} \sim \epsilon^{ijk} \frac{1}{\Delta^{3/2}} \tag{32}$$

The above term gives rise to the CPT violation in the effective action, which looks like.

$$T_6 = F \frac{1}{L} \epsilon^{ijk} \int d^4 x \; \text{tr}[A_i A_j A_k]. \tag{33}$$

where, $F \sim \frac{2}{3} \left(\frac{M_0}{|M_0|} + T_4 + T_5 \right)$, Where the limiting value of $(T_4 + T_5)$ is $-\frac{\pi}{2}$, and M_0 is the Pauli-Villars mass.

CPT and Lorentz violations

- The CPT transformation of the anti-Hermitian gauge field is given by

$$A_\mu(x) \rightarrow A_\mu^T(-x). \tag{34}$$

  For the Hermitian electromagnetic vector potential $A_\mu(x)$ CPT transformation is given by $A_\mu(x) \rightarrow -A_\mu(-x)$.

- Under CPT transformation the terms like $\epsilon^{ijk}(A_i \partial_j A_k)$ and $\epsilon^{ijk}(A_i A_j A_k)$ change sign. So they are CPT-odd terms. In the above terms (9), (25), (33) not every Lorentz index is contracted with a four-vector. So these terms are obviously not Lorentz invariant.

Discussion

- The CPT violating local terms are invariant under infinitesimal gauge transformations but not invariant large gauge transformation. For non-Abelian gauge group there are some non-local terms in perturbation theory of the effective gauge field action which restore the gauge invariance under the large gauge transformations.These non-local terms drop out for Abelian gauge fields with local support.
  [L. Alvarez-Gaume, S. Della Pietra, G. Moore, Ann. Phys. (N. Y.) 163 (1985) 288.]
- The mass scale of the CPT violating terms of the gauge fields is of the order $\frac{1}{L}$, which depends upon the range of the compactified coordinate.
- The amount of CPT violation changes when the gauge fields depend upon the compactified coordinate.
- If gauge fields depend upon the compactified coordinate x^0, they should not oscillate very rapidly with respect to x^0 coordinate.
- In future we are trying to establish the CPT anomaly nonperturbatively (more rigorously) also by choosing the lattice regularization of chiral gauge theories.

THANK YOU!