

# Transport of magnetic turbulence in supernova remnants

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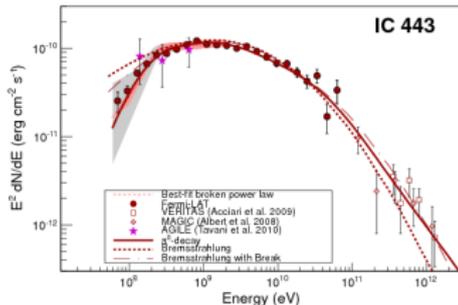
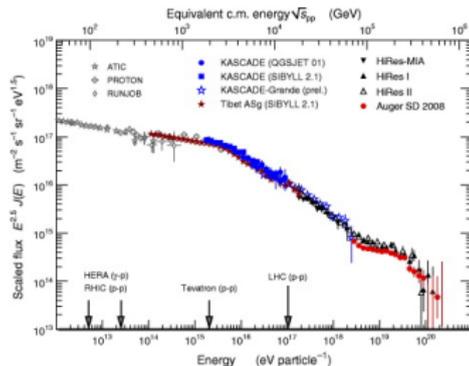


# Outline

- 1 Introduction
- 2 Cosmic ray acceleration
- 3 Transport of magnetic turbulence
- 4 First results and outlook



## Introduction



top-left: Cosmic ray spectrum  
at earth

bottom-left: IC443  
gamma-ray spectrum

right: IC443:  
multi-wavelength image

# Cosmic ray acceleration

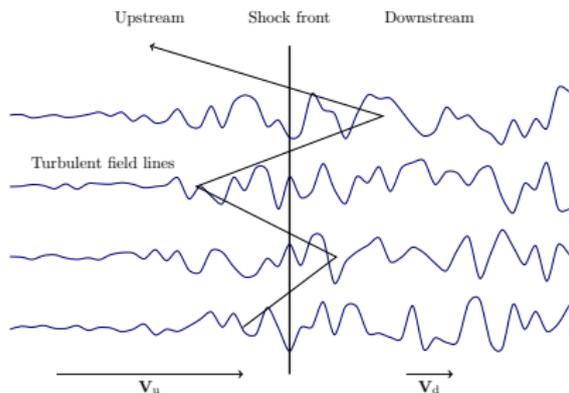


Figure: First order Fermi acceleration [Lee 2002]

There are three coupled sets of equations to solve:

- The MHD-equations for the SNR
- The cosmic ray transport equation
- The transport equation for the magnetic turbulence

# Cosmic ray acceleration

Transport equation for cosmic rays:

$$\frac{\partial N}{\partial t} = \nabla(D_r \nabla N - \vec{v}N) - \frac{\partial}{\partial p} \left( (N\dot{p}) - \frac{\nabla \vec{v}}{3} Np \right) + Q$$

Solving this equation:

- One-dimensional
- Spherically symmetric
- In a shock-centered coordinate system
- With a fine resolution near the shock and up to distance of several shock radii



# Ansatz

Consider isotropic Alfvénic turbulence and account for

- Advection and compression
- Resonant amplification
- Damping due to cn-collisions and IC-damping
- Spectral energy transfer through cascading

Used Quantity:  $E_w$  is the **energy density per unit logarithmic bandwidth**.

The RMS-field associated with the Alfvén-waves is given by:

$$\langle \delta b^2 \rangle = 4\pi \int \frac{E_w(k)}{k} dk$$



# Transport equation

Transport equation:

$$\begin{aligned} \frac{\partial E_w}{\partial t} + \mathbf{u} \cdot (\nabla E_w) + C_w (\nabla \cdot \mathbf{u}) E_w + k \frac{\partial}{\partial k} \left( k^2 D_k \frac{\partial E_w}{\partial k} \frac{1}{k^3} \right) &= \\ &= 2(\Gamma_g - \Gamma_d) E_w \end{aligned}$$

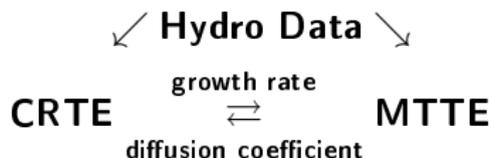
Using Bells resonant growth-rate and Kolmogorov-cascading.  
 Initial diffusion coefficient:

$$\begin{aligned} E_w &= \frac{4}{3\pi} \frac{v U_M}{k D_r} \\ D_{r,ism} &= \left( 10^{27} \frac{\text{cm}^2}{\text{s}} \right) \cdot \left( \frac{E}{10 \text{ GeV}} \right)^{\frac{1}{3}} \left( \frac{B_0}{3 \mu\text{G}} \right)^{-\frac{1}{3}} \end{aligned}$$

⇒ Diffusion coefficient decreased by a factor of 100.



# Properties



- **Time-dependend** treatment of cosmic ray acceleration and turbulence transport
- Hydrodynamic data from separate calculations: **Test particle approximation**
- Simulations in 1-d and spherically symmetric
- Cooling for electrons through synchrotron radiation
- Frozen in magnetic field
- No free-escape-boundary



# Results I: Sedov Phase

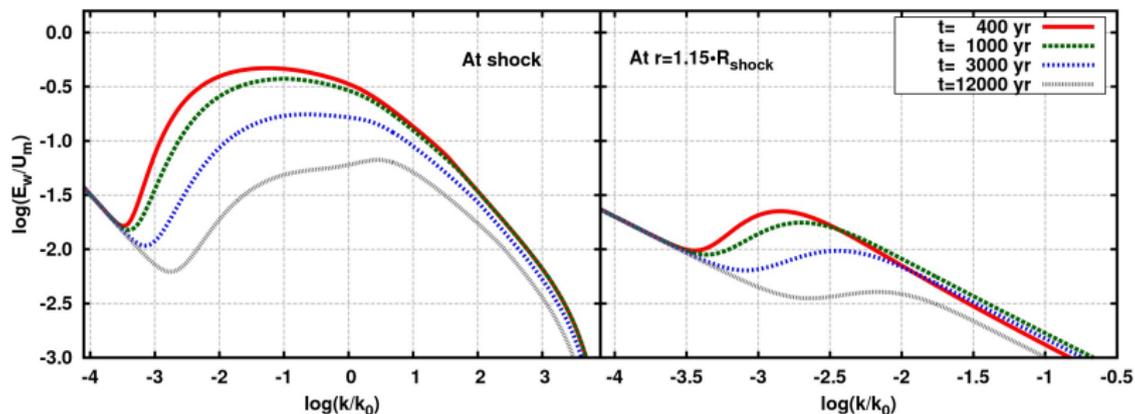


Figure: Turbulence spectra

# Results I: Sedov Phase

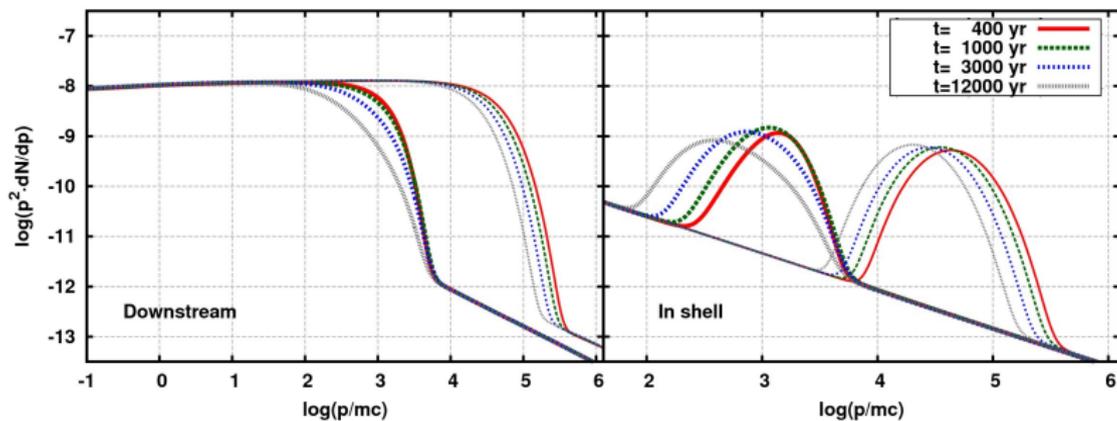


Figure: Cosmic ray spectra

## Results II: Free expansion Phase

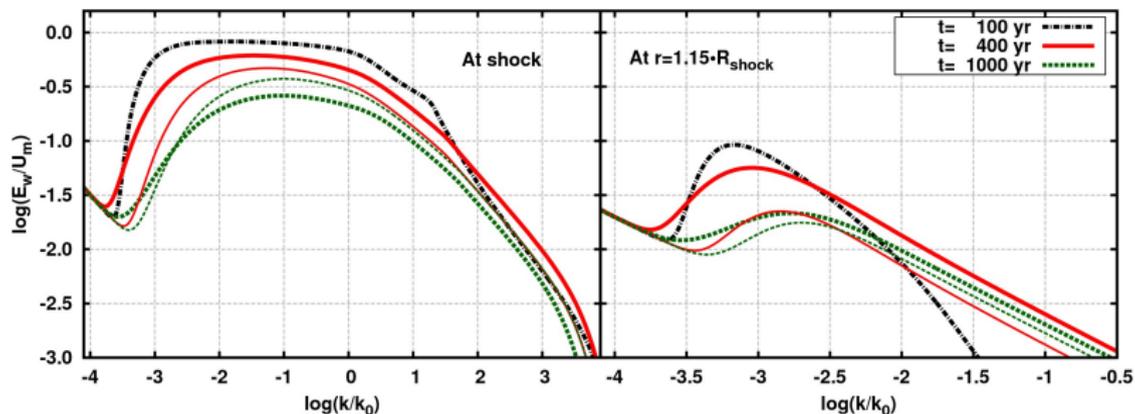


Figure: Turbulence spectra

## Results II: Free expansion Phase

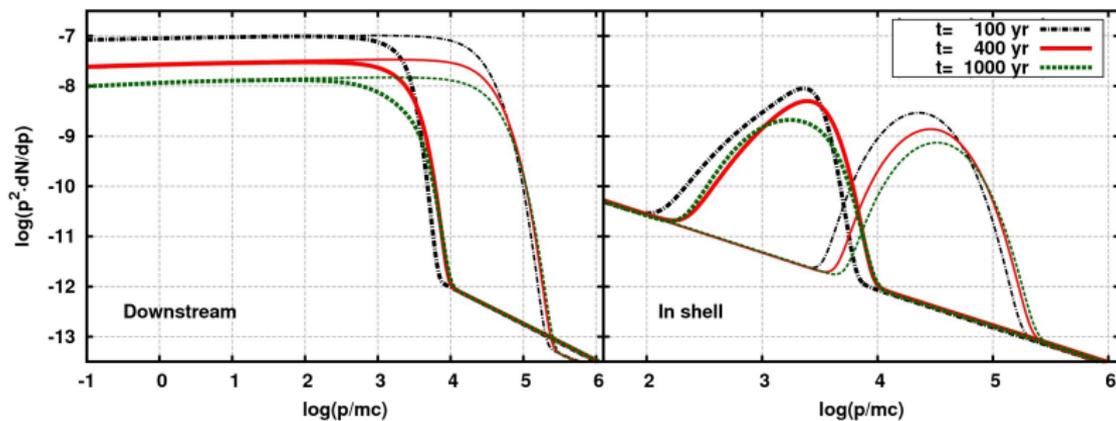


Figure: Cosmic ray spectra

## Conclusion

- Diffusion coefficient **not** uniformly **Bohm-like**
- Even for older SNR: **No steady-state** reached
- **Softer CR-spectra for old remnants** due to cosmic-ray escape
- Decrease in maximum cosmic-ray energy faster than predicted in steady-state models

See also: [arXiv:1606.04477](https://arxiv.org/abs/1606.04477)



# Thank You for your attention!



# Sources



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Cosmic Ray Streaming from Supernova Remnants and Gamma Ray Emission from nearby Molecular

Used **growth rate** [Bell 1978] and  
**cascading coefficient** [Schlickeiser 2002]:

$$D_k = kv_a \sqrt{\frac{E_w}{2U_M}}$$

$$\Gamma_{cr} = \frac{v_a q B_0}{3kU_M} \frac{\partial N}{\partial x}$$



# Numerical stability

High Resolution near shock constrains time steps.  
Compromise between **numerical stability** and **physical situation** needed.

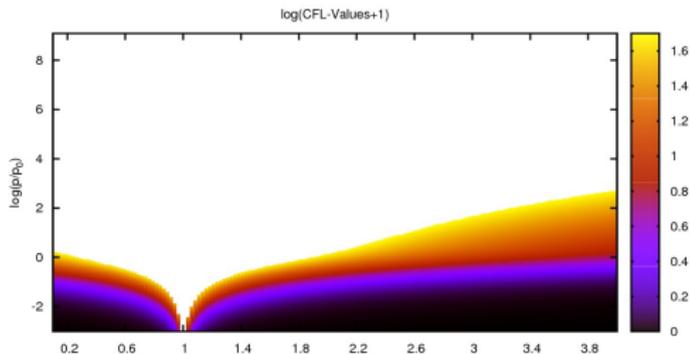


Figure: stability criteria



# Numerical stability

Compromise: Decreased diffusion-coefficient and accordingly decreased time-steps.

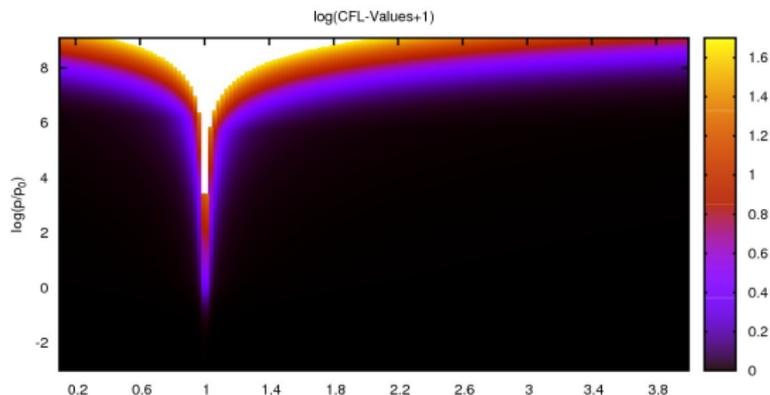


Figure: stability criteria