Comments on nonfactorizable effects in $\bar{B}_s \to D_s^+ \pi^-$

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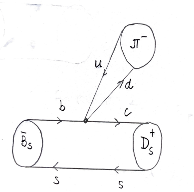
Particle Physics Phenomenology after the Higgs Discovery

Collaborative Research Center TRR 257

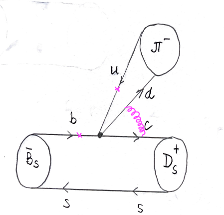
Mini-Workshop on non-leptonic tree-level decays,
March 25, 2021

An example of colour-allowed decay: $B_s \to D_s^+ \pi^-$

- penguin, annihilation topology absent; only emission topology:
- QCDF, leading power at $m_b \to \infty$

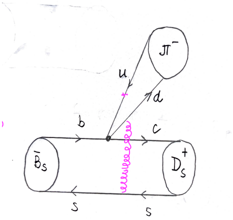


LO

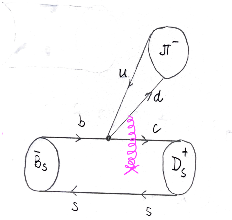


vertex, NNLO [T.Huber et al,1606.02888]

- nonfactorizable effects, $O(\Lambda_{QCD}/m_b)$ in QCDF



hard spectator



soft gluon

Confronting experiment

- Branching fraction (in units 10^{-3}):

QCDF NNLO	$4.39^{+1.36}_{-1.19}$	[T.Huber et al, 1606.02888]
QCDF$\oplus$power corr $\oplus$updated FFs	4.42 ± 0.21	[M. Bordone et al., 2007.10338] [see also the talk by Nico Gubernari]
Exp.	3.00 ± 0.23	[PDG]
	$3.20 \pm 0.10 \pm 0.16$	[LHCb, 2103.06810]

- ratio of semileptonic and nonleptonic widths ($\bar{B}_d \to D^+K^-$)

$$\frac{\Gamma(\bar{B}_d \to D^+K^-)}{d\Gamma(\bar{B}_d \to D^+\ell^-\bar{\nu}_\ell)/dq^2|_{q^2=m_K^2}} \sim |V_{us}|^2 f_K^2 |a_1(D^+K^-)|^2$$

$$a_1(D^+K^-)_{NNLO} = 1.07 \pm 0.01, \quad a_1(D^+K^-)_{exp} = 0.87 \pm 0.06$$

[T.Huber et al,1606.02888]

Nonfactorizable effects within QCDF

[M.Beneke et al, hep-ph/0006124]

- Hard spectator mechanism: the sum of two diagrams ($\mathcal{A} \sim$ HME)

$$\mathcal{A}(B_s \to D_s^+ \pi^-)_{HS} \sim f_\pi f_{D_s} f_{B_s} \alpha_s \int_0^1 \frac{d\xi}{\xi} \Phi_{B_s}(\xi) \int_0^1 \frac{d\eta}{\eta} \Phi_{D_s}(\eta) \int_0^1 \frac{du}{u} \Phi_\pi(u)$$
$$\sim \alpha_s m_b \Lambda_{QCD}^2$$

- Soft nonfactorizable gluon: interaction of a soft gluon with the collinear energetic light-quark pair forming the emitted pion

$$\mathcal{A}(B_s \to D_s^+ \pi^-)_{SNF} \sim \int_0^\infty ds \langle D_s | \bar{c} \Gamma^\mu g_s \tilde{G}_{\mu\nu}(-sn) n^\nu b | \bar{B}_s \rangle f_\pi \int_0^1 \frac{du}{u\bar{u}} \varphi_\pi(u)$$
$$\sim f_\pi m_b \Lambda_{QCD}$$

- in QCDF both effects definitely $\sim 1/m_b$ suppressed vs

$$\mathcal{A}(B_s \to D_s^+ \pi^-)_{fact} \sim m_b^2 f_\pi f_{B_s \to D_s}(0) \sim m_b^2 \Lambda_{QCD}$$

- additional suppression due to the ratio of Wilson coeffs c_1/a_1

The soft nonfactorizable gluon effect

- starting from the perturbative hard-scattering diagram [M.Beneke et al, hep-ph/0006124]

We have

$$\langle D^+\pi^-|O_8|\bar{B}_d\rangle_{1\text{-gluon}} = i g_s^2 \frac{C_F}{2}\int \frac{d^4k}{(2\pi)^4}\langle D^+|\bar{c}A_1(k)b|\bar{B}_d\rangle \times \frac{1}{k^2}\int_0^1 du \frac{d^2 l_\perp}{16\pi^3}\frac{\Psi^*(u,\vec{l}_\perp)}{\sqrt{2N_c}}\mathrm{tr}[\gamma_5 \not{q} A_2(l_q,l_{\bar{q}},k)], \qquad (55)$$

where

$$A_1(k) = \frac{\gamma^\lambda(\not{p}_c - \not{k} + m_c)\Gamma}{2p_c\cdot k - k^2} - \frac{\Gamma(\not{p}_b + \not{k} + m_b)\gamma^\lambda}{2p_b\cdot k + k^2}, \qquad (56)$$

$$A_2(l_q,l_{\bar{q}},k) = \frac{\Gamma(\not{l}_{\bar{q}} + \not{k})\gamma_\lambda}{2l_{\bar{q}}\cdot k + k^2} - \frac{\gamma_\lambda(\not{l}_q + \not{k})\Gamma}{2l_q\cdot k + k^2}. \qquad (57)$$

Here $\Gamma = \gamma^\mu(1-\gamma_5)$ and p_b, p_c are the momenta of the b quark and the c quark,

- switch to "external" gluon field with $k^2 = 0$,

a collinear, energetic gluon, whereas the spectator cloud is always soft.

We start from (57), take $l_q = uq$ and $l_{\bar{q}} = \bar{u}q$, and put the gluon on-shell (since now we are interested in an external gluon field). The resulting expression describes the interaction of a soft gluon with the collinear light-quark pair, since both quarks are energetic. Reintroducing colour, the coupling constant g_s and the gluon polarization vector ε^λ, the expression (57) projected onto the pion state becomes

$$\bar{d}A_2 u \to -\frac{q^\kappa k^\alpha \varepsilon^\lambda \epsilon_{\kappa\alpha\mu\lambda}}{2q\cdot k}\frac{f_\pi \Phi_\pi(u)}{u\bar{u}}\frac{g_s\,\mathrm{Tr}(T^A T)}{N_c}, \qquad (69)$$

where T denotes the colour matrix at the weak vertex ($T = 1$ for O_0, $T = T^B$ for O_8). We also used the ϵ-tensor with $\epsilon^{0123} = -1$ and

$$\bar{d}\gamma_\mu(1-\gamma_5)u \to i f_\pi \Phi_\pi(u) q_\mu \qquad (70)$$

for projecting the current on the pion wave function, see (9). To simplify the result, we have used the symmetry of $\Phi_\pi(u)$ under $u \leftrightarrow \bar{u}$. The dependence on the gluon momentum k in (69) involves the eikonal propagator $i/(q\cdot k)$, which has the Fourier-decomposition

$$\frac{i}{q\cdot k + i\epsilon} = \int d^4x\, e^{ik\cdot x}\int_0^\infty d\tau\, \delta^{(4)}(x - \tau q). \qquad (71)$$

Hence we see that in configuration space the right-hand side of (69) corresponds to the operator expression (at space–time point $x = 0$)

$$-\frac{f_\pi \Phi_\pi(u)}{4N_c u\bar{u}}\int_0^\infty ds\, g_s\,\mathrm{Tr}[\tilde{G}_{\mu\nu}(-sn)T]n^\nu, \qquad (72)$$

gluon "detached" from the pion DA, not a "regular" light-cone OPE, a hard-scattering kernel missing

Recent estimate of power suppressed effects in $B_{(s)} \to D_{(s)}K(\pi)$

[M. Bordone et al., 2007.10338]

- Hard scattering effect: not inlcluded?

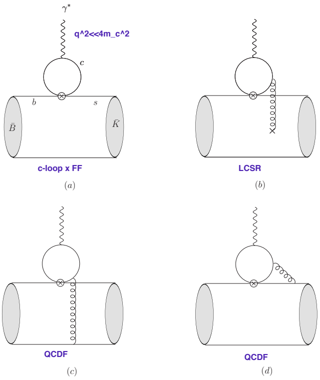
> (ii) The exchange of a hard-collinear gluon from the spectator quark is incompatible with the physical picture of the spectator having a soft momentum inside both the $\bar{B}_q^0$ and the D_q meson. The exchange of a hard-collinear gluon from the b or c quark is possible, and contributes through a quark–antiquark–gluon Fock state of the light meson through three-particle LCDAs. Due to the $V-A$ structure of the weak inter-

- Soft nonfactorizable gluon effect: included !
  - the $B \to D$ matrix element with nonlocal quark-antiquark-gluon
    taken from QCDF analysis
  - calculated using LCSR with B-meson DAs
    "in a similar fashion" as the charm loop effect in $B \to K\ell\ell$
  - a numerically tiny effect even after inflating the parameters ten times

The nonlocal charm loop and soft gluon in $B \to K\ell\ell$

[AK, Th.Mannel, A.Pivovarov, Y.-M.Wang 1006.4945]

[AK, Th.Mannel, Y.-M.Wang 1211.0234]



- Starting object: a correlation function of the weak operator and c-quark e.m. current taken far off-shell $q^2 \ll 4m_c^2$
- a light-cone expansion $\rightarrow$ the nonlocal quark-antiquark gluon operator convoluted with a c-loop kernel
- the h.m.e. is calculated from LCSRs
- the other contributions reduced to QCDF [M.Beneke,Th.Feldmann,D.Seidel,hep-ph/0106067]
- hadronic dispersion relation in q^2 links the OPE result with nonleptonic amplitudes (residues at the poles at $q^2 = m^2_{J/\psi}, \ldots$)

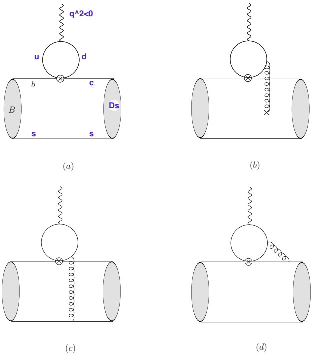
Proposal of a similar method for $B_{(s)} \to D_{(s)} K(\pi)$

- interpolate the pion with a current $j^\pi = \bar{u}\gamma_\alpha\gamma_5 d$:

see also [B.Blok, M.Shifman,hep-ph/9205221]

$$F_\alpha = \int d^4x e^{iqx} \langle D_s | T\{j^\pi_\alpha(x), c_1 O_1(0) + c_2 O_2(0)\} | B_s \rangle$$

- a three-step procedure:



- light-cone OPE valid at $q^2 \ll 0$
- hadronic $\bar{B}_s \to D_s$ matrix elements
  applying LCSR,QCDF, HQET, lattice QCD
- hadronic dispersion relation in q^2,
  isolating pion contribution
  quark-hadron duality in the pion channel

Instead of Conclusion

'We emphasize that without a rigorous treatment of power corrections in the QCDF approach nothing more can be said at the present stage.” from [T.Huber et al, 1606.02888]