Flavour physics in the Standard Model

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TRR 257 - Particle Physics Phenomenology
after the Higgs Discovery

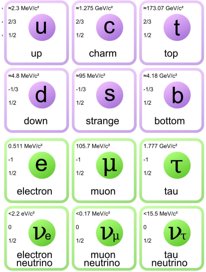
CRC Annual Meeting, May 26th, 2021

Outline

- Flavour: What? Why? How?
- Inclusive b decays
- Exclusive b decays
- Other exclusive channels and quantities
- Conclusion

Introduction

- The majority of the SM parameters resides in the Yukawa sector
  - Quarks and leptons are the principal actors of flavour physics
- Many aspects of flavour physics
  - Heavy (top, bottom, charm) and light quarks
  - Mesons and baryons
  - Charged leptons, neutrinos



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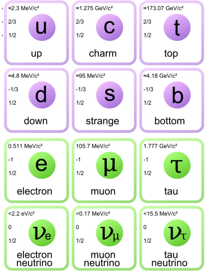

- Many aspects of flavour physics
  - Heavy (top, bottom, charm) and light quarks
  - Mesons and baryons
  - Charged leptons, neutrinos

- Motivation to study flavour physics
  - Huge amount of experimental data (B-factories, Tevatron, LHC, Belle II, . . .)
  - Numerous channels and observables
  - Synergy and complementarity between direct and indirect searches for NP
  - Precise measurements and predictions of SM parameters possible!
  - Probe our understanding of the strong interaction
  - CP-Violation, matter-antimatter asymmetry, . . .

Direct and indirect searches

- So far, no signal for NP in direct searches ☹
- Lack of direct NP signal necessitates precision studies in collider physics, flavour physics, low-energy observables
- Puzzling patterns in flavour data: hints for BSM physics from the flavour scale?

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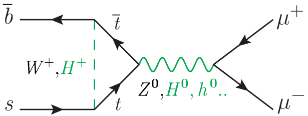
  - Look for virtual effects of new phenomena
  - Mostly looked for in rare processes
  - Requires precision in theory and experiment
  - Synergy and complementarity to direct searches



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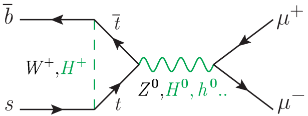


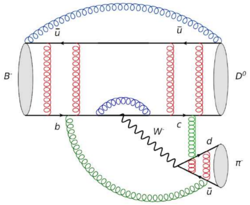
- Even if NP is found in direct searches, want to know the BSM flavour structure

Per aspera ad astra (rocky path to the stars)

- Problem: confinement of quarks into hadrons
- For instance, look at generic structure of amplitude for B decays

$$\mathcal{A}(\bar{B} \to f) = \lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED}$$

- Computation of hadronic matrix elements highly non-trivial
- Effects from many different scales
- QCD effects could overshadow the interesting fundamental dynamics



- Even if a separation (factorization) is achieved, power corrections of $\mathcal{O}\left(\Lambda_{\rm QCD}/m_Q\right)$ are often the limiting factor
  - Compare to a typical correction of $\mathcal{O}\left(\Lambda_{\rm QCD}/\sqrt{s}\right)$ to collider observables, also $\alpha_s(m_b) > \alpha_s(Q^2)$.

Tools for precision

- To get control over QCD effects, sophisticated tools have been developed
  - Effective field theories (HQET, SCET, SMEFT, . . .)
  - Heavy-Quark Expansion
  - Factorization (also at subleading power!)
  - Perturbative calculations: Loops, . . .
  - Non-perturbative techniques: Lattice, Sum rules, . . .
- Applications also in Higgs, Collider, Dark Matter, . . .

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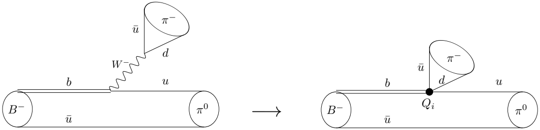
- Applications also in Higgs, Collider, Dark Matter, . . .
- Other interesting aspects
  - Understanding the general properties of power expansions in EFTs
  - Understand strong-interaction dynamics of heavy quark decays
  - Interplay between different QCD techniques (Lattice, Sum Rules, perturbation theory,. . .)

Effective theory for B decays



- $M_W, M_Z, m_t, m_H \gg m_b$: integrate out heavy gauge bosons, t-quark, Higgs
- Effective Weak Hamiltonian: [Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{10} C_k Q_k \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum\nolimits_q (\bar{q} \gamma_\mu T^a q)$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum\nolimits_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum\nolimits_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum\nolimits_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q)$$

$$Q_7 = \frac{e}{16\pi^2} m_b \, \bar{s}_L \, \sigma_{\mu\nu} F^{\mu\nu} b_R \qquad Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_9 = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \ell) \qquad Q_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \gamma_5 \ell) \qquad \lambda_p = V_{pb} V_{pd}^*$$

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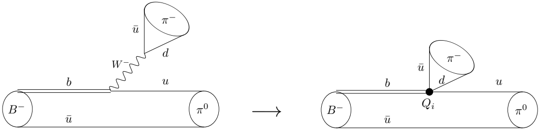

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- Size of Wilson coeffcients

$C_1 = -0.25$	$\lvert C_{3,5,6} \rvert < 0.01$	$C_7 = -0.30$	$C_9 = 4.06$
$C_2 = 1.01$	$C_4 = -0.08$	$C_8 = -0.15$	$C_{10} = -4.29$

Inclusive b decays

Inclusive B decays, generalities

- Main tool for inclusive decays: Heavy Quark Expansion (HQE)

[Khoze,Shifman,Voloshin,Bigi,Uraltsev,Vainshtein,Blok,Chay,Georgi,Grinstein,Luke,Neubert,. . . '80s and '90s]

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_{B_q} - p_X) |\langle X|\hat{\mathcal{H}}_{eff}|B_q\rangle|^2$$

- Use optical theorem

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \langle B_q|\hat{\mathcal{T}}|B_q\rangle \quad \text{with} \quad \hat{\mathcal{T}} = \text{Im}\, i \int d^4x \hat{T} \left[\hat{\mathcal{H}}_{eff}(x)\hat{\mathcal{H}}_{eff}(0)\right]$$

- Expand non-local double insertion of effective Hamiltonian in local operators

$$\begin{aligned}\Gamma \quad &= \quad \Gamma_0 \langle O_{D=3}\rangle + \Gamma_2 \frac{\langle O_{D=5}\rangle}{m_b^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6}\rangle}{m_b^3} + ... \\ &\quad +16\pi^2 \left[\Gamma_3 \frac{\langle O_{D=6}\rangle}{m_b^3} + \Gamma_4 \frac{\langle O_{D=7}\rangle}{m_b^4} + \Gamma_5 \frac{\langle O_{D=8}\rangle}{m_b^5} + ...\right]\end{aligned}$$

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- Γ_0: Decay of a free quark, known to $\mathcal{O}(\alpha_s^3)$
- Γ_1: Vanishes due to Heavy Quark Symmetry
- Two terms in Γ_2: Kinetic energy μ_π^2, Chromomagnetic moment μ_G^2
- Two more terms in Γ_3: Darwin term ρ_D^3, Spin-orbit term ρ_{LS}^3

Background effects in the inclusive V_{cb} determination

[Mannel,Rahimi,Vos'21]

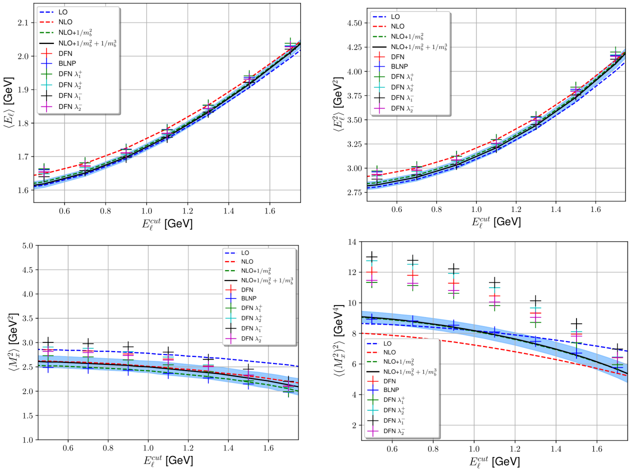
- Measurement at Belle II: $B \to X\ell$, $\ell = e, \mu$
- $\bar{B} \to X\ell$ consists of different components:
  - $\bar{B} \to X_c \ell \bar{\nu}$ (which is the process of interest and is $\propto |V_{cb}|^2$)
  - $\bar{B} \to X_u \ell \bar{\nu}$
  - $\bar{B} \to X_{c,u} \left(\tau \to \ell \bar{\nu}_\ell \nu_\tau\right) \bar{\nu}_\tau$
- Subtraction of the unwanted components by Monte Carlo
- Compare theoretical HQE studies to Monte-Carlo simulations of $b \to u \ell \bar{\nu}$
- Define moments of the spectrum for any observable $\mathcal{O}$

$$\left\langle \mathcal{O}^n \right\rangle_{E_\ell > E_\ell^{\text{cut}}} = \frac{\int_{E_\ell > E_\ell^{\text{cut}}} \mathrm{d}\mathcal{O}\, \mathcal{O}^n \frac{\mathrm{d}\Gamma}{\mathrm{d}\mathcal{O}}}{\int_{E_\ell > E_\ell^{\text{cut}}} \mathrm{d}\mathcal{O}\, \frac{\mathrm{d}\Gamma}{\mathrm{d}\mathcal{O}}}$$

  - Choose lepton energy moments $\langle E_\ell^n \rangle$,
    hadronic mass moments $\langle M_x^n \rangle$,
    lepton-invariant mass moments $\langle \left(q^2\right)^n \rangle$.

Background effects in the inclusive V_{cb} determination

- Comparison of the $b \to u\ell\bar{\nu}$ MC data used for the subtraction with HQE



- HQE expressions can help minimising the exptl. uncertainties from $b \to u\ell\bar{\nu}$ and $b \to c\,(\tau \to \ell\bar{\nu}_\ell\nu_\tau)\,\bar{\nu}_\tau$

Further topics and activities

- Miniworkshop on Quark Masses in October
- Third order corrections to the semi-leptonic $b \to c$ and muon decays

  [Fael,Schönwald,Steinhauser'20; Czakon,Czarnecki,Dowling'21]

  $\Longrightarrow$ see Kay Schönwald's talk in YSF

- NNLO QCD corrections to the B-meson mixing [Nierste,Shtabovenko,Steinhauser,'20 and w.i.p.]
  - n_f-terms of penguin contributions are not the dominant ones, need full NNLO $\Longrightarrow$ see Vlad Shtabovenko's talk in YSF
- HQE for charm $\Longrightarrow$ see Daniel Moreno's talk at annual meeting 2020

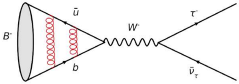
  [Mannel,Pivovarov,Moreno'21]

- Master Integrals for Inclusive Weak Decays at NLO [Mannel,Pivovarov,Moreno'21]
- Further ongoing projects
  - $\bar{B} \to X_s\gamma$ (KA+AC+SI)
  - $\bar{B} \to X_s\ell^+\ell^-$ with an M_{X_s}-cut
  - Improvement of inclusive determination of $|V_{cb}|$ and $|V_{ub}|$

Exclusive b decays

Exclusive B decays, generalities

- Leptonic decays



$$\langle 0|\bar{u}\gamma^{\mu}\gamma_5 b|B^-(p)\rangle = i\, f_B\, p^{\mu}$$

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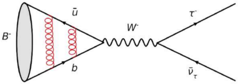

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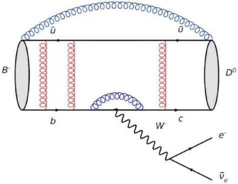
- Semi-leptonic decays



$$\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle$$
$$= F_+(q^2)\left[(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2}\, q^\mu\right]$$
$$+ F_0(q^2)\, \frac{m_B^2 - m_D^2}{q^2}\, q^\mu$$

Exclusive B decays, generalities

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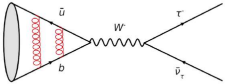

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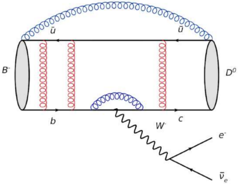

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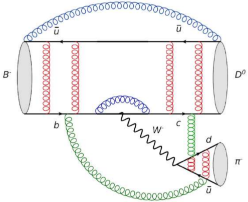
- Non-leptonic decays



$$\langle \pi^- D^+|Q_i|\bar{B}\rangle \simeq m_B^2\, f_{M_2}\, F_+^{B\to D}(m_\pi^2)$$

$$\times \int_0^1 du\ \ T_i^I(u)\ \phi_\pi(u)$$

Rare decays

Rare Semileptonic Decays of Λ_b Baryons

- Increasing information for b-baryon decays from experiment (LHCb,. . .)
- Interesting due to half-integer spin d. o. f.
- Matrix elements of weak effective Hamiltonian between fermionic states yield complementary phenomenological observables for NP studies
- But: theory predictions for exclusive baryon decays more challenging than for mesons (two valence spectators, . . .) [see e.g. Feldmann'21]

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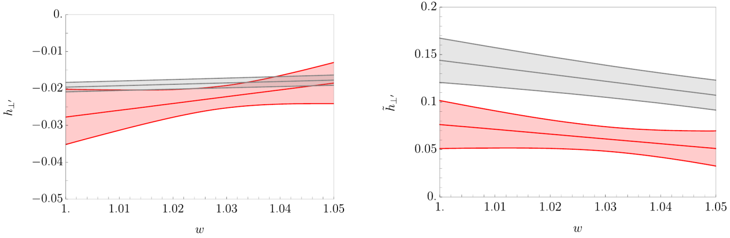
- Need transition form factors $\Lambda_b \to p$, $\Lambda_b \to \Lambda$, $\Lambda_b \to \Lambda^*$, ...
  - from lattice-QCD (small and moderate recoil energy)
  - light-cone sum rules (large recoil)

- Non-factorizable contributions
  - not accessible in lattice-QCD
  - relevant for radiative and non-leptonic decays
  - systematic sum-rule or factorization studies still missing

$\Lambda_b \to \Lambda^*(1520)$ form factors

[Bordone'21]

- Consider full set of $\Lambda_b \to \Lambda^*(1520)$ form factors
- Apply HQE at low recoil, include $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(1/m_b)$ corrections
- Obtain unknown hadronic parameters from a fit to recent lattice data
  - Use data on vector and axial-vector FFs, predict (pseudo)-tensor ones

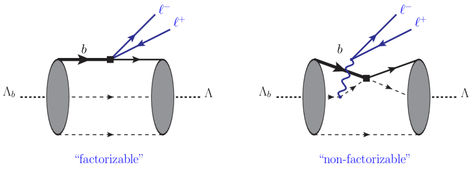


- Find certain tensions between lattice and HQE in (pseudo)-tensor case
  - Lattice uncertainties underestimated?
  - Higher order terms in the HQE?

Example: 4-Quark operators in $\Lambda_b \to \Lambda \ell^+ \ell^-$

Light-cone sum rule analysis:

[Bordone,Gubernari,Feldmann, w.i.p.]



- replace final-state hadron (Λ) by interpolating current
- perturbative calculation of a correlation function in the Euclidean

$$\Pi_\mu(p', q) \equiv \int d^4x\, e^{iq\cdot x} \int d^4y\, e^{ip'\cdot y} \langle 0 | \mathrm{T}\left\{ J_\Lambda(y), O_{3-6}(0), j_\mu^{\mathrm{em}}(x) \right\} | \Lambda_b(p) \rangle$$

- requires Λ_b distribution amplitudes (LCDAs) as hadronic input
- contribution to $\Lambda_b \to \Lambda \ell^+ \ell^-$ amplitude by dispersion relations
- numerical comparison with factorizable contributions, using same method and same hadronic input

[Feldmann,Yip'12]

Nonleptonic decays

Two-body heavy-light final states

[Bordone,Gubernari,Jung,van Dyk,TH'20]

- Determine b-quark fragmentation fractions f_s/f_d from hadronic two-body decays into heavy-light final states
- Requires ratio $$\mathcal{R}_{s/d}^{P(V)} \equiv \frac{\mathcal{B}(\bar{B}_s^0 \to D_s^{(*)+}\pi^-)}{\mathcal{B}(\bar{B}^0 \to D^{(*)+}K^-)}$$

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- QCD factorization for non-leptonic decays

[Beneke,Buchalla,Neubert,Sachrajda'99-'04]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,|\bar{B}_q^0\rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

- Particularly clean for heavy-light final states: Only colour-allowed tree amplitude
  - No colour-suppressed tree amplitude, no penguins
  - Spectator scattering and weak annihilation power suppressed
  - Weak annihilation absent if all final-state flavours distinct
    - as in $\bar{B}_s^0 \to D_s^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ but not in $\bar{B}^0 \to D^+\pi^-$

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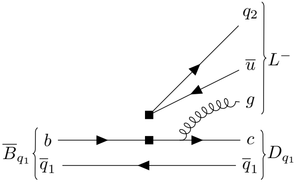
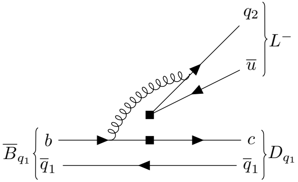
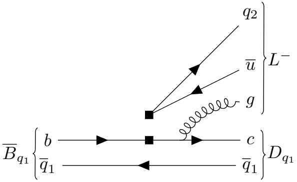
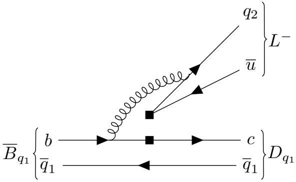
    - as in $\bar{B}_s^0 \to D_s^+\pi^-$ and $\bar{B}^0 \to D^+K^-$ but not in $\bar{B}^0 \to D^+\pi^-$
- Hard function known to $\mathcal{O}\left(\alpha_s^2\right)$ [Kränkl,Li,TH'16]
- Form factors from recent precision study [Bordone,Gubernari,Jung,van Dyk'19]





- Power corrections arise from several effects
  - Higher twist effects to the light-meson LCDA
  - Hard-collinear gluon emission from the spectator quark q
  - Hard-collinear gluon emission from the heavy quarks b and c
  - Soft-gluon exchange between $B \to D$ and light-meson system





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Estimate of total size of power corrections

$$\mathcal{R}^P_{s/d}\big|_{\text{NLP}}/\mathcal{R}^P_{s/d}\big|_{\text{LP}} - 1 \approx -1.7‰ \qquad \mathcal{R}^V_{s/d}\big|_{\text{NLP}}/\mathcal{R}^V_{s/d}\big|_{\text{LP}} - 1 \approx -1.7‰$$

- Supports the picture of these decays being very clean

Results

source	PDG	our fit (w/ QCDF, no f_s/f_d)		QCDF prediction
scenario	—	ratios only	~~$SU(3)$~~	—
χ^2/dof	—	4.6/6	3.7/4	—
$\mathcal{B}(\bar{B}_s^0 \to D_s^+\pi^-)$	3.00 ± 0.23	$3.11^{+0.21}_{-0.19}$	$3.20^{+0.20}_{-0.26}$ *	4.42 ± 0.21
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	0.186 ± 0.020	0.227 ± 0.012	0.226 ± 0.012	0.326 ± 0.015
$\mathcal{B}(\bar{B}^0 \to D^+\pi^-)$	2.52 ± 0.13	2.74 ± 0.12	$2.73^{+0.12}_{-0.11}$	—
$\mathcal{B}(\bar{B}_s^0 \to D_s^{*+}\pi^-)$	2.0 ± 0.5	$2.46^{+0.37}_{-0.32}$	$2.43^{+0.39}_{-0.32}$	$4.3^{+0.9}_{-0.8}$
$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$	0.212 ± 0.015	$0.213^{+0.014}_{-0.013}$	$0.213^{+0.014}_{-0.013}$	$0.327^{+0.039}_{-0.034}$
$\mathcal{B}(\bar{B}^0 \to D^{*+}\pi^-)$	2.74 ± 0.13	$2.76^{+0.15}_{-0.14}$	$2.76^{+0.15}_{-0.14}$	—
$\mathcal{R}_{s/d}^P$	16.1 ± 2.1	13.6 ± 0.6	$14.2^{+0.6}_{-1.1}$ *	$13.5^{+0.6}_{-0.5}$
$\mathcal{R}_{s/d}^V$	9.4 ± 2.5	$11.4^{+1.7}_{-1.6}$	$11.4^{+1.7}_{-1.5}$ *	$13.1^{+2.3}_{-2.0}$
$\mathcal{R}_s^{V/P}$	0.66 ± 0.16	$0.81^{+0.12}_{-0.11}$	$0.76^{+0.11}_{-0.10}$	$0.97^{+0.20}_{-0.17}$
$\mathcal{R}_d^{V/P}$	1.14 ± 0.15	0.97 ± 0.06	0.95 ± 0.07	1.01 ± 0.11
$(f_s/f_d)_{\rm LHCb}^{7\,\rm TeV}$	—	$0.261^{+0.018}_{-0.016}$	$0.252^{+0.023}_{-0.015}$ *	—
$(f_s/f_d)_{\rm Tev}$	—	$0.244^{+0.026}_{-0.023}$	$0.236^{+0.026}_{-0.022}$ *	—

- BR discrepancies

$$\bar{B}_s^0 \to D_s^+\pi^- \quad \to 4\sigma$$
$$\bar{B}^0 \to D^+K^- \quad \to 5\sigma$$
$$\bar{B}_s^0 \to D_s^{*+}\pi^- \quad \to 2\sigma$$
$$\bar{B}^0 \to D^{*+}K^- \quad \to 3\sigma$$

- Ratios OK

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- BR discrepancies

$$\bar{B}_s^0 \to D_s^+\pi^- \to 4\sigma$$
$$\bar{B}^0 \to D^+K^- \to 5\sigma$$
$$\bar{B}_s^0 \to D_s^{*+}\pi^- \to 2\sigma$$
$$\bar{B}^0 \to D^{*+}K^- \to 3\sigma$$

- Ratios OK

- Potential explanations
  - Universal non-factorizable contributions of $\mathcal{O}\left(-15 - 20\%\right)$ to amplitude?
  - Experimental issues?
  - Shift or larger uncertainties in the input (CKM) parameters?
  - BSM physics?
  - Combination thereof?
  - All not really satisfactory!

Further developments

- Result triggered quite some interest
  - New-physics interpretations [Iguro,Kitahara'20]
  - New tensor structures [Cai,Deng,Li,Yang'21]
  - Collider bounds on BSM explanations of the discrepancy [Bordone,Greljo,Marzocca'20]

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- Put under scrutiny SM prediction
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- The journey goes on . . .

Power corrections and endpoint divergences

[Bell,Böer,Feldmann,'21]

Factorization at subleading power is spoilt by endpoint divergences

$$\int_0^\infty \mathrm{d}\omega\, \frac{\phi_B^+(\omega)}{\omega^2}\,, \quad \int_0^1 \mathrm{d}u\, \frac{\phi_\pi(u)}{\bar{u}^2}\,, \quad \ldots \qquad \text{log-divergent for } \omega, \bar{u} \to 0$$

$\Rightarrow$ main limitation for precision phenomenology in exclusive B decays

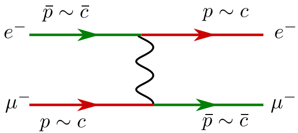
Idea: Study problem in a perturbative set-up

$$\boxed{B_c \to \eta_c \text{ form factors} \quad \text{for} \quad m_b \gg m_c \gg \Lambda_{\mathrm{QCD}}}$$

- consider B_c and η_c as non-relativistic bound states
- form factors calculable order-by-order in α_s
- soft-collinear factorization requires analytic rapidity regulator
- operator analysis rather involved (operator mixing, 3-particle Fock states, . . .)

$e^-\mu^-$ backward scattering

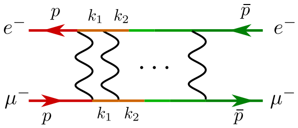
Cleaner laboratory to study the endpoint dynamics



$$s \sim -t \gg m_\mu^2 \sim m_e^2$$

$$u \approx 0 \quad (\theta \simeq \pi)$$

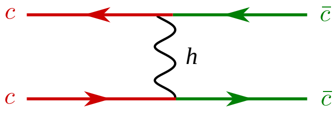
- count $\log m_e^2/s \sim \log m_\mu^2/s \sim 1/\alpha_{\rm em}$ and $\log m_e/m_\mu \sim \mathcal{O}(1)$
- focus on resummation of double logs (set $m_e = m_\mu$)
- double logs arise from (twisted) ladder diagrams in specific configuration

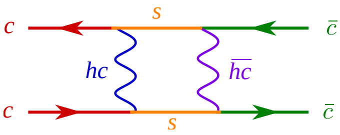


- all photon-propagators eikonal
- all lepton-propagators on-shell and ordered in rapidity

Bare factorization theorem

$$\mathcal{A} \simeq \int_0^1 \frac{\mathrm{d}u}{u}\frac{\mathrm{d}v}{v}\,\phi_c(u)\,\phi_{\bar{c}}(v)\left\{H(uv) + \int_0^\infty \frac{\mathrm{d}k_+}{k_+}\frac{\mathrm{d}k_-}{k_-}\,J_{hc}(uk_+)\,S(k_+k_-)\,J_{\overline{hc}}(k_-v)\right\}$$





- operator definitions of collinear and soft functions
- $H(uv)$, $J_{hc}(uk_+)$ and $J_{\overline{hc}}(k_-v)$ arise from matching QED onto SCET

$\Rightarrow$ bare factorization theorem is spoilt by endpoint divergences

$$u \to 0,\ v \to 0,\ k_+ \to 0,\ k_- \to 0,\ k_+ \to \infty,\ k_- \to \infty$$

Resummation of double logs

So far no renormalized factorization theorem

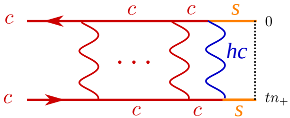
$\Rightarrow$ cannot use RG techniques to resum logarithmic corrections

Instead use bare factorization theorem in conjunction with

- refactorization constraints

[Böer'18; Liu,Neubert'19; Bell,Böer,Feldmann in preparation]

$$\phi_c(u \to 0) \simeq \int \frac{dv}{v} \, \phi_c(v) \int \frac{dk_+}{k_+} \, J_{hc}(vk_+) \, S(k_+ u)$$



- pole cancellation

rapidity divergences generate an infinite tower of collinear anomalies

$$\frac{\mathcal{A}}{\mathcal{A}_0} = r_0(\mu/m) \left(\frac{s}{m^2}\right)^{f_0(\mu/m)} + \frac{\hat{\alpha}}{\epsilon^2} \, h_1 \left(\frac{\mu^2}{s}\right)^{\epsilon} r_1(\mu/m) \left(\frac{s}{m^2}\right)^{f_1(\mu/m)} + \ldots$$

complicated cross-talk of $1/\epsilon$-poles, which must cancel in the sum

Resummation of double logs

Structure of double logs

$$\frac{\mathcal{A}}{\mathcal{A}_0} = 1 + \frac{\hat{\alpha}}{2} L^2 + \frac{\hat{\alpha}^2}{12} L^4 + \cdots = \sum_{n=0}^{\infty} \frac{\hat{\alpha}^n}{n!(n+1)!} L^{2n} = \frac{I_1\left(2\sqrt{\hat{\alpha}L^2}\right)}{\sqrt{\hat{\alpha}L^2}}$$

$$\hat{\alpha} = \alpha/2\pi \qquad L = \log m^2/s$$

- logs do not exponentiate, but resum to a modified Bessel function
- classical textbook result in QED [Gorshkov,Gribov,Lipatov,Frolov'67]
- highly non-trivial example of endpoint dynamics in SCET!

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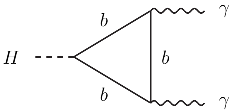
First NLL resummation in the presence of endpoint divergences [Neubert et al.'19'20]



- single rapidity divergence to all orders
- renormalized factorization theorem after endpoint subtractions (cutoff-dependent)

[Tetlalmatzi-Xolocotzi,TH'21]

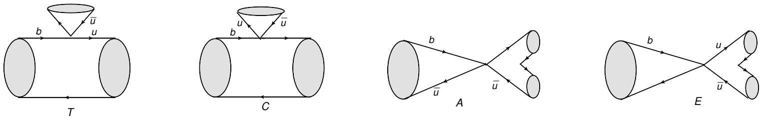
- The amplitudes for $B \to PP$ (P a pseudoscalar meson) can be expressed as

$$\mathcal{A} = \mathcal{T} + \mathcal{P}$$

$\mathcal{T}$: Tree sub-amplitudes. $\mathcal{P}$: Penguin sub-amplitudes.

- Topological decomposition of the sub-amplitudes [He,Wang'18]

$$\begin{aligned}\mathcal{T}^{TDA} = \; & T\, B_i (M)^i_j \bar{H}^{jl}_k (M)^k_l + C\, B_i (M)^i_j \bar{H}^{lj}_k (M)^k_l + A\, B_i \bar{H}^{il}_j (M)^j_k (M)^k_l \\ & + E\, B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + T_{ES} B_i \bar{H}^{ij}_l (M)^l_j (M)^k_k + T_{AS} B_i \bar{H}^{ji}_l (M)^l_j (M)^k_k \\ & + T_S B_i (M)^i_j \bar{H}^{lj}_l (M)^k_k + T_{PA} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j + T_P B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l \\ & + T_{SS} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k\end{aligned}$$



QCD-factorization and flavour symmetries

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& + T_{SS} B_i \bar{H}^{li}_l (M)^j_j (M)^k_k
\end{aligned}$$

$$(B_i) = (B^+, B^0, B_s)$$

$$(M^i_j) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K}^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}$$

QCD-factorization and flavour symmetries

- The non-zero elements of $\tilde{H}_k^{ij}$ and $\bar{H}_k^{ij}$ are

$$\begin{aligned} b \to d: \quad & \bar{H}_1^{12} = \lambda_u^{(d)}, \quad \tilde{H}_1^{12} = \lambda_t^{(d)}, \quad \bar{H}^2 = \lambda_u^{(d)}, \quad \tilde{H}^2 = \lambda_t^{(d)} \\ b \to s: \quad & \bar{H}_1^{13} = \lambda_u^{(s)}, \quad \tilde{H}_1^{13} = \lambda_t^{(s)}, \quad \bar{H}^3 = \lambda_u^{(s)}, \quad \tilde{H}^3 = \lambda_t^{(s)}. \end{aligned}$$

- $SU(3)$ decomposition:

$$H_k^{ij} = \frac{1}{8}\left(H_{\overline{15}}\right)_k^{ij} + \frac{1}{4}\left(H_6\right)_k^{ij} - \frac{1}{8}\left(H_{\bar{3}}\right)^i \delta_k^j + \frac{3}{8}\left(H_{\bar{3}'}\right)^j \delta_k^i$$

- $SU(3)$-invariant amplitudes (analogous for penguins)

$$A_3^T,\ B_3^T,\ C_3^T,\ D_3^T,\ A_6^T,\ B_6^T,\ C_6^T,\ A_{15}^T,\ B_{15}^T,\ C_{15}^T$$

- Linear relations between topological and $SU(3)$-invariant amplitudes , e.g.

$$\begin{aligned} A_3^T &= -\frac{A}{8} + \frac{3E}{8} + T_{PA}, \qquad & B_3^T &= T_{SS} + \frac{3T_{AS} - T_{ES}}{8}, \\ A_6^T &= \frac{1}{4}(A - E), \qquad & B_6^T &= \frac{1}{4}(T_{ES} - T_{AS}) \end{aligned}$$

QCD-factorization and flavour symmetries

- Determine the SU(3)-invariant amplitudes through a χ^2-fit.
  - 20 complex amplitudes (10 for trees, 10 for penguins)
  - One overall phase and the complex amplitudes A_6^T and A_6^P can be absorbed

    $\Longrightarrow$ 35 real parameters.
- Use the following experimental input for branching fractions and CP asymmetries
  - Branching fractions : 23 measurements plus 6-upper bounds
  - CP Asymmetries: 17 measurements plus 1-upper bound
- Implement η-η' mixing in the FKS scheme (a single mixing angle) [Feldmann,Kroll,Stech'98]
- The χ^2-fit results allow us to predict observables not measured so far

$$\mathcal{B}(B_s \to \pi^0 K^0), \quad \mathcal{B}(B_s \to \eta^0 K^0), \quad A_{\rm CP}(B_s \to \pi^0\pi^0), \quad A_{\rm CP}(B_s \to \eta'\eta), \text{ etc.}$$

QCD-factorization and flavour symmetries

- Sample results (preliminary). $\chi^2_\nu = 0.27$

Observable	Experiment (10^{-6})	χ^2-fit (10^{-6}) (Central value only)
$\mathcal{B}(B^- \to \pi^0\pi^-)$	5.5 ± 0.4	5.6
$\mathcal{B}(B^- \to K^0K^-)$	1.31 ± 0.17	1.19
$\mathcal{B}(B^- \to \pi^+\pi^-)$	5.12 ± 0.19	5.29
$\mathcal{B}(B^- \to \pi^0\pi^0)$	1.59 ± 0.26	1.53
$\mathcal{B}(B^- \to K^+K^-)$	0.078 ± 0.015	0.087
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$\mathcal{B}(B_s \to \pi^-K^+)$	5.80 ± 0.7	5.93

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- Annihilation contributions at most 10%

$$|A_3^T| = 0.039, \quad |A_{15}^T| = 0.007, \quad |B_3^T| = 0.023, \quad |B_6^T| = 0.123, \quad |B_{15}^T| = 0.045$$

$$|A_3^P| = 0.019, \quad |A_{15}^P| = 0.011, \quad |B_3^P| = 0.037, \quad |B_6^P| = 0.099, \quad |B_{15}^P| = 0.022$$

QCD-factorization and flavour symmetries

- Investigate connection to QCD factorization (QCDF)
- Amplitudes for two body non-leptonic B-meson decays in QCDF [Beneke,Neubert'03]

$$\mathcal{A}^{\rm QCDF} = \sum_{p=u,c} A_{M_1 M_2} \Big\{ B M_1 \left(\alpha_1 \delta_{pu} \hat{U} + \alpha_4^p \hat{I} + \alpha_{4,\rm EW}^p \hat{Q}\right) M_2 \, \Lambda_p$$
$$+ B M_1 \Lambda_p \cdot {\rm Tr}\left[\left(\alpha_2 \delta_{pu} \hat{U} + \alpha_3^p \hat{I} + \alpha_{3,\rm EW}^p \hat{Q}\right) M_2\right]$$
$$+ B \left(\beta_2 \delta_{pu} \hat{U} + \beta_3^p \hat{I} + \beta_{3,\rm EW}^p \hat{Q}\right) M_1 M_2 \Lambda_p$$
$$+ B \Lambda_p \cdot {\rm Tr}\left[\left(\beta_1 \delta_{pu} \hat{U} + \beta_4^p \hat{I} + b_{4,\rm EW}^p \hat{Q}\right) M_1 M_2\right]$$
$$+ B \left(\beta_{S2} \delta_{pu} \hat{U} + \beta_{S3}^p \hat{I} + \beta_{S3,\rm EW}^p \hat{Q}\right) M_1 \Lambda_p \cdot {\rm Tr} M_2$$
$$+ B \Lambda_p \cdot {\rm Tr}\left[\left(\beta_{S1} \delta_{pu} \hat{U} + \beta_{S4}^p \hat{I} + b_{S4,\rm EW}^p \hat{Q}\right) M_1\right] \cdot {\rm Tr} M_2 \Big\}$$

- Establish transformation rules between the QCDF amplitudes $\{\alpha_i, \beta_i, b_i\}$ and the $SU(3)$ ones
  - Quantify the size of the annihilation amplitudes β_i and b_i as dictated by data
- Quantify $SU(3)$-breaking (may introduce extra fit parameters)

Other exclusive channels and quantities

The pion-photon transition form factor at two loops

- Pion-photon transition form factor: theoretically (one of) the simplest hadronic matrix elements

$$\langle \pi(p) | j_\mu^{\rm em} | \gamma(p') \rangle = g_{\rm em}^2 \, \epsilon_{\mu\nu\alpha\beta} \, q^\alpha \, p^\beta \, \epsilon^\nu (p') F_{\gamma^* \gamma \to \pi^0}(Q^2)$$

- Ideally suited for
  - precision studies of the partonic landscape of composite hadrons
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- Ideally suited for
  - precision studies of the partonic landscape of composite hadrons
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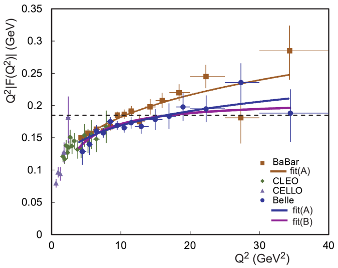
- Status of experimental measurements

[figures from Wang'18]



- Asymptotic limit (dashed line)

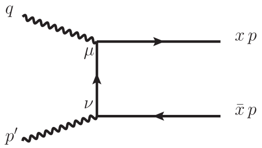
$$\lim_{Q^2\to\infty} Q^2 F_{\gamma^*\gamma\to\pi^0}(Q^2) = \sqrt{2}\, f_\pi$$

[Brodsky,Lepage'80]

- Scaling violation?

The pion-photon transition form factor

[Gao,Ji,Wang,TH'21]



- Factorization formula for $F_{\gamma^*\gamma\to\pi^0}$ at leading power

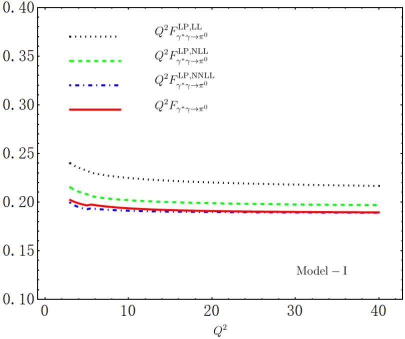
$$F^{\rm LP}_{\gamma^*\gamma\to\pi^0}(Q^2) = \frac{(Q_u^2 - Q_d^2)\, f_\pi}{\sqrt{2}\, Q^2} \int_0^1 dx\, T_2(x)\, \phi_\pi(x,\mu)$$

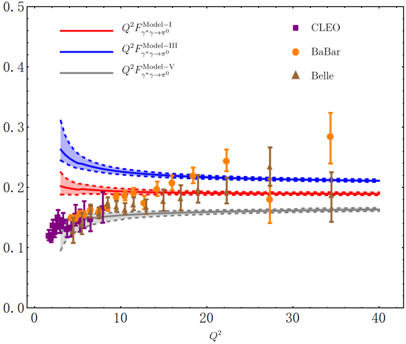
- $T_2(x)$: hard function, computable in perturbation theory
- $\phi_\pi(x,\mu)$: leading-twist pion light-cone distribution amplitude (LCDA), universal
- Recently, computed hard function $T_2(x)$ at two loops
  - Involves standard multi-loop techniques, analytic result in terms of HPLs

    [agrees with Braun,Manashov,Moch,Schoenleber'21]
  - Subtle point: Mixing of evanescent into physical operators at two loops

Numerical results

- Need to model the pion LCDA, choose five models
- Use three-loop evolution of pion LCDA, expand to first 12 Gegenbauer moments



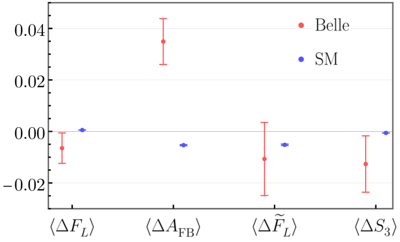


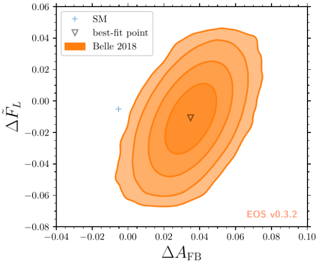
- Red line includes subleading power corrections (twist 4, hadronic photon effect) [Shen,Wang'17]
- Only perturbative uncertainties are shown
- Belle II data will allow to distinguish between LCDA models

LFU in $\bar{B} \to D^* \ell \nu$

[Bobeth,Bordone,Gubernari,Jung,van Dyk'21]

- Analysis of angular observables in $\bar{B} \to D^* \ell \nu$ decays
- Focus on $\mu - e$ lepton-flavour non-universality
- Include LFU-violating mass effects
- Explore BSM sensitivity of observables model-independently in EFT
- Compare SM predictions to 2018 Belle dataset
- Observe a 4σ tension between data and predictions in observables that probe $\mu - e$ LFU





LFU in $\bar{B} \to D^* \ell \nu$

- However, inconsistencies in Belle data found
  - Only 37 out of 40 bins linearly independent, but covariance matrix non-singular
- Considered BSM scenarios (despite above caveat). BSM contributions to ...
  - right-handed vector operators
  - left-handed vector operators
  - both pseudoscalar and tensor operators
- Findings
  - To accommodate $\Delta A_{\rm FB}$, contributions from RH vector operators or from both pseudoscalar and tensor operators are necessary
  - To describe the dataset well with only real BSM WCs, need LFUV contributions to both the RH and LH vector operators

$H^*H\pi$ couplings from LCSR

[Khodjamirian,Melic,Wang,Wei'20]

- Consider strong $H^*H\pi$ coupling $g_{H^*H\pi}$, where $H = B,\ D$
- Defined via the hadronic matrix element

$$\langle H^*(q)\pi(p)|H(p+q)\rangle = -g_{H^*H\pi}\, p^\mu \epsilon_\mu^{(H^*)}$$

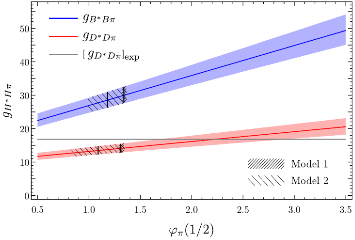
- Obtaining the Light-cone sum rule
  - Start with vacuum-to-pion correlation function

$$F_\mu(q,p) = i\int d^4x e^{iqx}\langle\pi(p)|T\{j_\mu(x), j_5(0)\}|0\rangle = F(q^2,(p+q)^2)\,p_\mu + \dots$$

    with two interpolating currents j_μ for H^* and j_5 for H
  - insert complete set of intermediate states with H and H^* quantum numbers
  - employ analyticity, resulting in double dispersion relation
  - match on light-cone OPE in terms of pion DAs
  - further steps involve quark-hadron duality approximation and double Borel transformation

$H^*H\pi$ couplings from LCSR

- LCSR predictions $g_{H^*H\pi}$ are sensitive to $\phi_\pi(u = 1/2)$



Method	$g_{D^*D\pi}$	$g_{B^*B\pi}$
LQCD, $N_f = 2$ [8]	$15.9 \pm 0.7^{+0.2}_{-0.4}$	–
LQCD, $N_f = 2+1$ [9]	16.23 ± 1.71	–
LQCD, $N_f = 2+1$ [12]	–	$\frac{2m_B}{f_\pi}(0.56 \pm 0.03 \pm 0.07)$ $= 45.3 \pm 6.0$
LQCD, $N_f = 2$ [7]	–	–
LQCD, $N_f = 2+1$ [10]	–	–
LQCD, $N_f = 2$ [11]	–	–
LCSR (this work)	$14.1^{+1.3}_{-1.2}$	$30.0^{+2.6}_{-2.4}$

- Decay constants of heavy mesons are taken from lattice QCD

B-meson DA parameters from QCD Sum Rules

[Rahimi,Wald'20]

- Definition of the HQET parameters $\lambda_E^2(\mu)$, $\lambda_H^2(\mu)$ [Grozin,Neubert'96]

$$\langle 0|\, g_s \bar{q}\, \vec{\alpha} \cdot \vec{E}\, \gamma_5 h_v \,|\bar{B}(v)\rangle = F(\mu)\, \lambda_E^2$$

$$\langle 0|\, g_s \bar{q}\, \vec{\sigma} \cdot \vec{H}\, \gamma_5 h_v \,|\bar{B}(v)\rangle = iF(\mu)\, \lambda_H^2$$

  - Dirac matrices $\alpha^i = \gamma^0 \gamma^i$ and $\sigma^i = \gamma^i \gamma_5$, HQET decay constant $F(\mu)$
  - Chromoelectric and chromomagnetic fields $E^i = G^{0i}$ and $H_i = -\frac{1}{2}\, \epsilon^{ijk}\, G^{jk}$
- Appear in the second moments of the B-meson LCDA defined in HQET
- Computation is based on two-point QCD sum rules
  - Derive sum rules for the diagonal $q\, \bar{q}\, g$ three-particle correlation function
  - All contributions up to mass-dimension seven in the OPE are included

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- Results for $\lambda_E^2(\mu)$, $\lambda_H^2(\mu)$ and their ratio $\mathcal{R}(\mu) = \lambda_E^2(\mu)/\lambda_H^2(\mu)$

Parameters	Grozin and Neubert	Nishikawa and Tanaka	this work
$\mathcal{R}(1\text{ GeV})$	(0.6 ± 0.4)	(0.5 ± 0.4)	(0.1 ± 0.1)
$\lambda_H^2(1\text{ GeV})$	$(0.18 \pm 0.07)\text{ GeV}^2$	$(0.06 \pm 0.03)\text{ GeV}^2$	$(0.11 \pm 0.05)\text{ GeV}^2$
$\lambda_E^2(1\text{ GeV})$	$(0.11 \pm 0.06)\text{ GeV}^2$	$(0.03 \pm 0.02)\text{ GeV}^2$	$(0.01 \pm 0.01)\text{ GeV}^2$

Conclusion

- The CRC explores the flavour sector of the SM to high precision
- We benefit from the interplay of many sophisticated tools, which we also further develop

Effective theories	QCD & QED corrections	Lattice, Sum rules
Factorisation	Experiment	BSM

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- Many of these tools and aspects are also important in other projects of the CRC
- Many more interesting results are underway and expected till the end of FP 1.