Adventures in the ALPs

Effective Lagrangians, Flavor Observables and Indirect Searches for Axion-Like Particles

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based on work with:
Martin Bauer, Anne Galda, Sophie Renner, Marvin Schnubel & Andrea Thamm
2012.12272, 2102.13112, 2105.01078 and in preparation

Annual Meeting of the CRC TRR 257: *Particle Physics Phenomenology after the Higgs Discovery*
(May 26, 2021)


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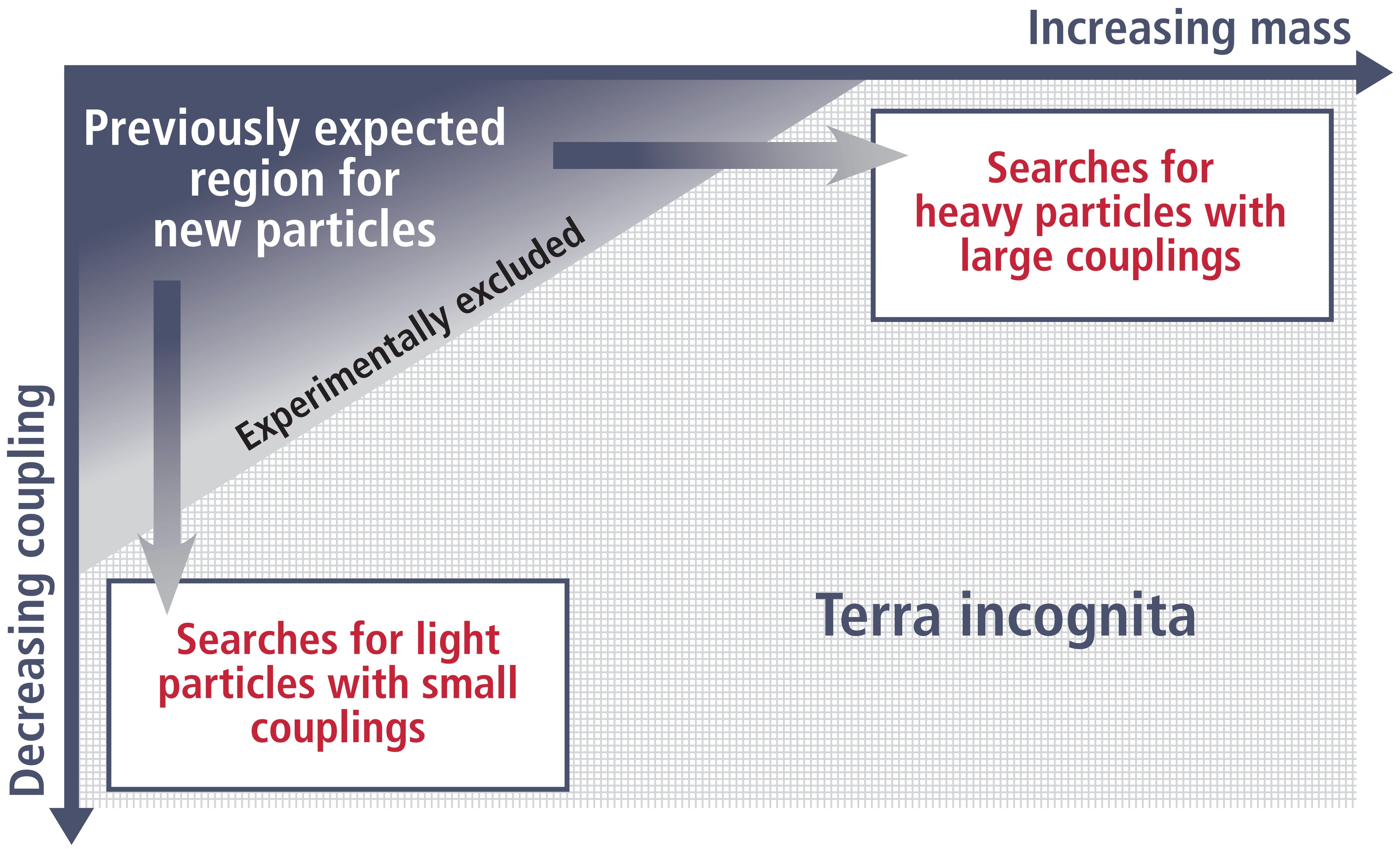
Outline:

- Matching and running for the ALP effective Lagrangian [2012.12272]
- Amusing facts about the rare decay $K \to \pi a$ [2102.13112]
- Flavor observables in some benchmark scenarios [work in preparation]
- ALP-SMEFT interference [2105.01078]

Motivation

Axions and axion-like particles (ALPs) are well motivated theoretically:

- Peccei-Quinn solution to strong CP problem
  [Peccei, Quinn (1977); Weinberg (1978); Wilczek (1978)]
- ALPs as pseudo Nambu-Goldstone bosons
- Light but weakly-coupled new particles are an interesting alternative to heavy new particles and might provide hints about physics at energies scales out of the reach for direct searches at the LHC
- Importance of low-energy processes in constraining ALP couplings



Effective Lagrangian in the UV

Assume the scale of global symmetry breaking $\Lambda = 4\pi f$ is above the weak scale, and consider the most general effective Lagrangian for a pseudoscalar boson a coupled to the SM via classically shift-invariant interactions, broken only by a soft mass term: [Georgi, Kaplan, Randall (1986)]

hermitian matrices

$$\mathcal{L}_{\text{eff}}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \frac{\partial^\mu a}{f}\sum_F \bar\psi_F\,\boldsymbol{c}_F\,\gamma_\mu\psi_F$$

$$+\,c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde G^{\mu\nu,a} + c_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde W^{\mu\nu,A} + c_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde B^{\mu\nu}$$

Couplings to Higgs bosons only arise in higher orders: [Dobrescu, Landsberg, Matchev (2000); Bauer, MN, Thamm (2017)]

$$\mathcal{L}_{\text{eff}}^{D\ge 6} = \frac{C_{ah}}{f^2}\,(\partial_\mu a)(\partial^\mu a)\,\phi^\dagger\phi + \frac{C'_{ah}}{f^2}\,m_{a,0}^2\,a^2\phi^\dagger\phi + \frac{C_{Zh}}{f^3}\,(\partial^\mu a)\left(\phi^\dagger\,iD_\mu\,\phi + \text{h.c.}\right)\phi^\dagger\phi + \dots$$

A redundant operator

- The only possible dimension-5 coupling to the Higgs doublet

$$\mathcal{L}_{\text{eff}}^{D\le 5} \supset c_\phi\, O_\phi = c_\phi\, \frac{\partial^\mu a}{f} \left(\phi^\dagger i D_\mu \phi + \text{h.c.} \right)$$

is a redundant operator, which can be removed by means of the field redefinitions $\phi \to e^{ic_\phi\, a/f}\, \phi$ and $F \to e^{-i\beta_F\, c_\phi\, a/f}\, F$ as long as:

$$\beta_u - \beta_Q = -1\,, \qquad \beta_d - \beta_Q = 1\,, \qquad \beta_e - \beta_L = 1$$

- This adds $\boldsymbol{c}_F \to \boldsymbol{c}_F + \beta_F\, c_\phi\, \mathbb{1}$ to the ALP-fermion couplings, i.e.:

$$O_\phi = \mathcal{O}_\phi + \sum_F \beta_F\, O_F\,, \qquad \text{with} \quad O_F = \frac{\partial^\mu a}{f}\, \bar\psi_F^i \gamma_\mu \psi_F^i$$

vanishes by the EOMs

Alternative operator basis

A useful alternative form of the Lagrangian involves non-derivative couplings:

$$\mathcal{L}_{\text{eff}}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 - \frac{a}{f}\left(\bar{Q}\,\phi\,\tilde{\boldsymbol{Y}}_d\,d_R + \bar{Q}\,\tilde{\phi}\,\tilde{\boldsymbol{Y}}_u\,u_R + \bar{L}\,\phi\,\tilde{\boldsymbol{Y}}_e\,e_R + \text{h.c.}\right)$$

$$+\,\tilde{c}_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + \tilde{c}_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde{W}^{\mu\nu,A} + \tilde{c}_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

where:

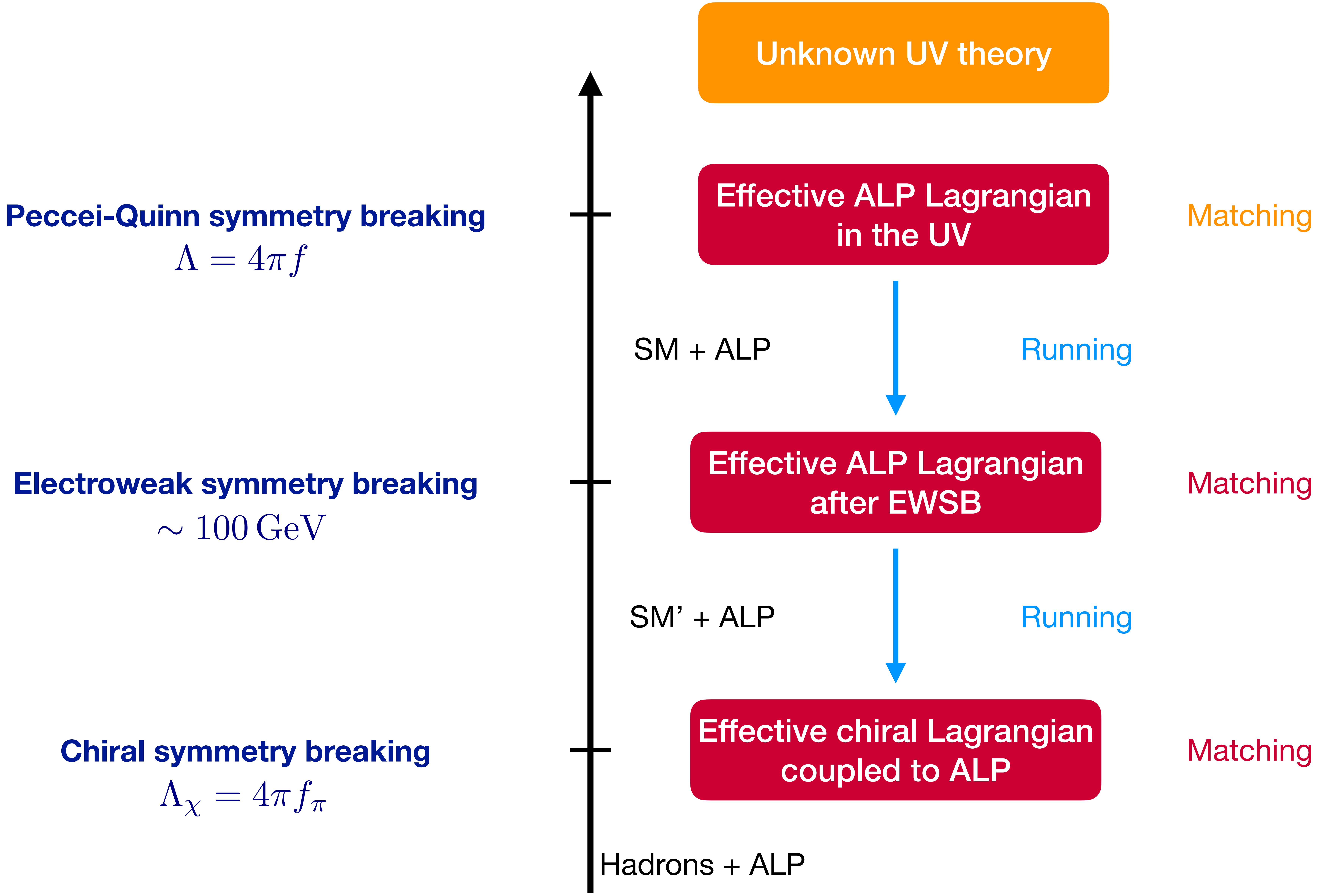
[Bauer, MN, Renner, Schnubel, Thamm (2020)]

$$\tilde{\boldsymbol{Y}}_d = i\left(\boldsymbol{Y}_d\,\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d\right), \qquad \tilde{\boldsymbol{Y}}_u = i\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u\right), \qquad \tilde{\boldsymbol{Y}}_e = i\left(\boldsymbol{Y}_e\,\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e\right)$$

$$\tilde{c}_{GG} = c_{GG} + T_F\,\text{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\,\boldsymbol{c}_Q\right)$$

$$\tilde{c}_{WW} = c_{WW} - T_F\,\text{Tr}\left(N_c\,\boldsymbol{c}_Q + \boldsymbol{c}_L\right)$$

$$\tilde{c}_{BB} = c_{BB} + \text{Tr}\left[N_c\left(\mathcal{Y}_u^2\,\boldsymbol{c}_u + \mathcal{Y}_d^2\,\boldsymbol{c}_d - N_L\,\mathcal{Y}_Q^2\,\boldsymbol{c}_Q\right) + \mathcal{Y}_e^2\,\boldsymbol{c}_e - N_L\,\mathcal{Y}_L^2\,\boldsymbol{c}_L\right]$$



Unknown UV theory

Peccei-Quinn symmetry breaking
$\Lambda = 4\pi f$

Effective ALP Lagrangian in the UV

Matching

SM + ALP

Running

Electroweak symmetry breaking
$\sim 100\,\mathrm{GeV}$

Effective ALP Lagrangian after EWSB

Matching

SM’ + ALP

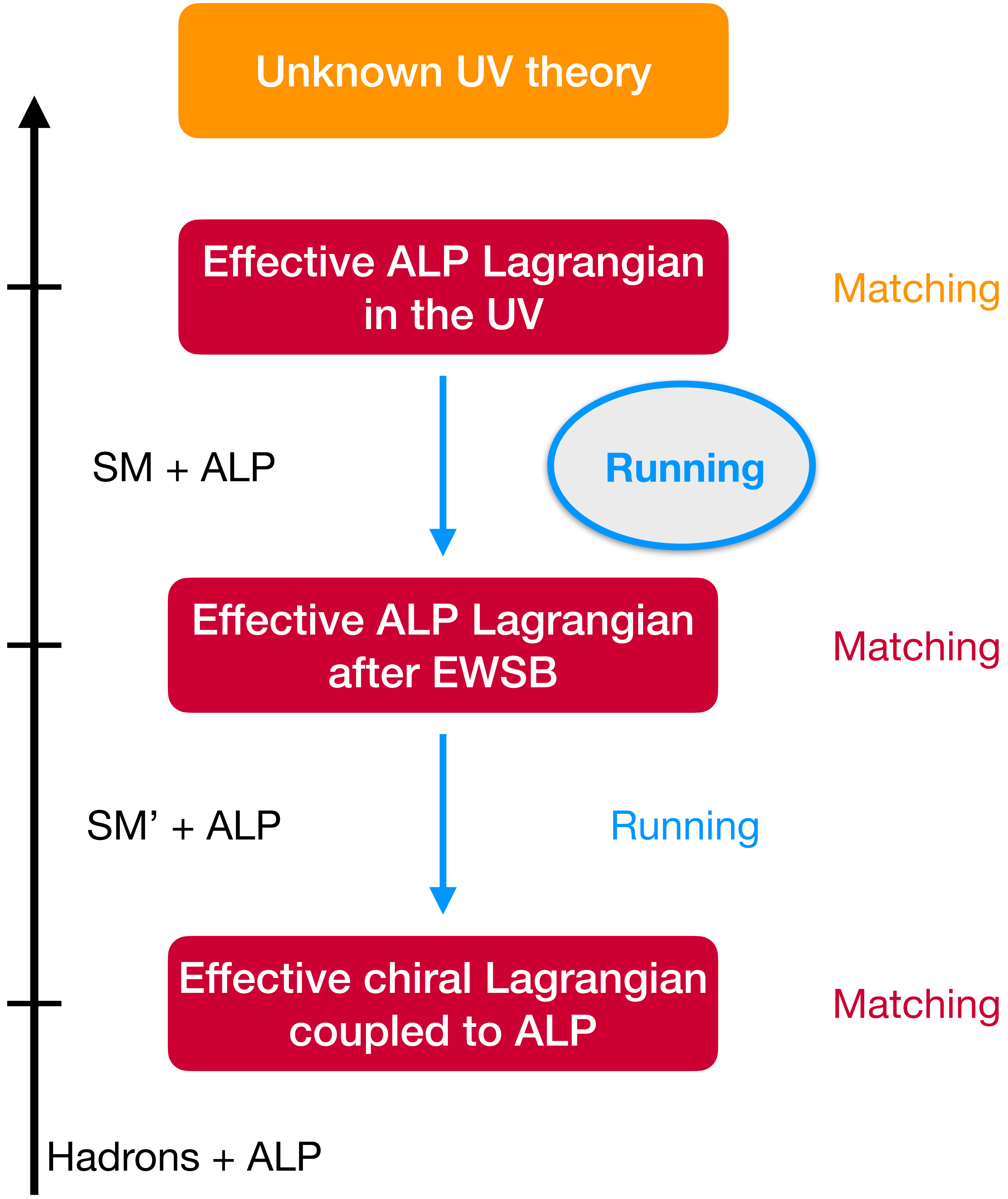
Running

Chiral symmetry breaking
$\Lambda_\chi = 4\pi f_\pi$

Effective chiral Lagrangian coupled to ALP

Matching

Hadrons + ALP



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Evolution to the weak scale

Factoring out the gauge couplings from c_{VV} ensures that (at least to 2 loops):

$$\boxed{\frac{d}{d\ln\mu}\,c_{VV}(\mu)=0\,;\quad V=G,W,B}$$

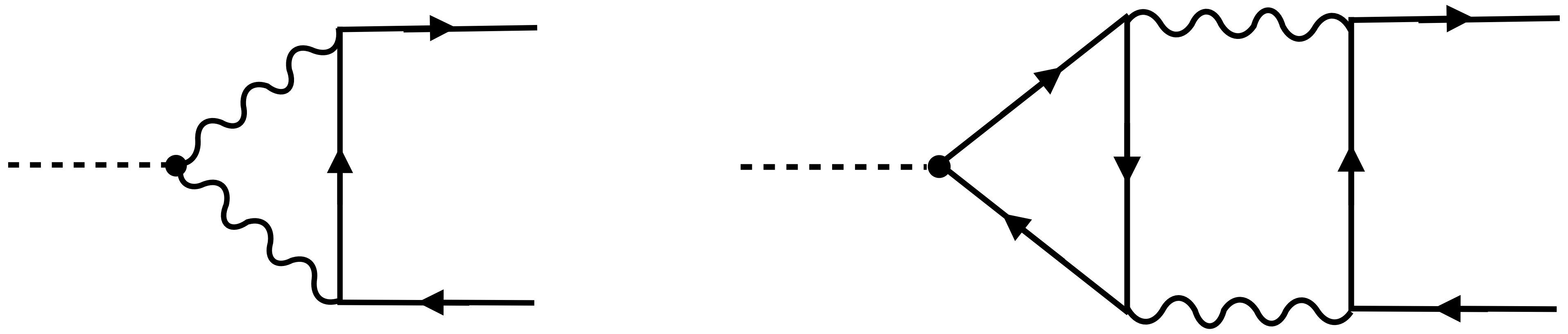
[Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

For the ALP-fermion couplings, we have computed:

1-loop Yukawa int.

[Choi, Im, Park, Yun (2017);
Martin Camalich, Pospelov, Vuong, Ziegler, Zupan (2020);
Heiles, König, MN (2020)]

requires the redundant Higgs operator as counterterm



2-loop gauge int.

[Altarelli, Ross (1988);
Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

[Kodaira (1980); Larin (1993)]

Evolution to the weak scale

We find: [Bauer, MN, Renner, Schnubel, Thamm (2020); see also: Chala, Guedes, Ramos, Santiago (2020)]

$$\begin{aligned}\frac{d}{d\ln\mu}\,\boldsymbol{c}_Q(\mu) &= \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger, \boldsymbol{c}_Q\right\} - \frac{1}{16\pi^2}\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger + \boldsymbol{Y}_d\,\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\right)\\ &\quad + \left[\frac{\beta_Q}{8\pi^2}\,X - \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} - \frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_Q^2\,\tilde{c}_{BB}\right]\mathbb{1}\end{aligned}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_q(\mu) = \frac{1}{16\pi^2}\left\{\boldsymbol{Y}_q^\dagger\boldsymbol{Y}_q, \boldsymbol{c}_q\right\} - \frac{1}{8\pi^2}\,\boldsymbol{Y}_q^\dagger\boldsymbol{c}_Q\boldsymbol{Y}_q + \left[\frac{\beta_q}{8\pi^2}\,X + \frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG} + \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_q^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_L(\mu) = \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger, \boldsymbol{c}_L\right\} - \frac{1}{16\pi^2}\,\boldsymbol{Y}_e\,\boldsymbol{c}_e\boldsymbol{Y}_e^\dagger + \left[\frac{\beta_L}{8\pi^2}\,X - \frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW} - \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_L^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_e(\mu) = \frac{1}{16\pi^2}\left\{\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e, \boldsymbol{c}_e\right\} - \frac{1}{8\pi^2}\,\boldsymbol{Y}_e^\dagger\boldsymbol{c}_L\boldsymbol{Y}_e + \left[\frac{\beta_e}{8\pi^2}\,X + \frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_e^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

with:

$$X = \mathrm{Tr}\left[3\boldsymbol{c}_Q\left(\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger - \boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger\right) - 3\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_u + 3\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\boldsymbol{Y}_d - \boldsymbol{c}_L\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger + \boldsymbol{c}_e\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e\right]$$

Lagrangian at the weak scale

Effective Lagrangian in the broken phase:

$$\mathcal{L}_{\text{eff}}(\mu_w) = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 + \mathcal{L}_{\text{ferm}}(\mu_w) + c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

$$+ c_{\gamma Z}\,\frac{\alpha}{2\pi s_w c_w}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{Z}^{\mu\nu} + c_{ZZ}\,\frac{\alpha}{4\pi s_w^2 c_w^2}\,\frac{a}{f}\,Z_{\mu\nu}\,\tilde{Z}^{\mu\nu} + c_{WW}\,\frac{\alpha}{2\pi s_w^2}\,\frac{a}{f}\,W_{\mu\nu}^+\,\tilde{W}^{-\mu\nu}$$

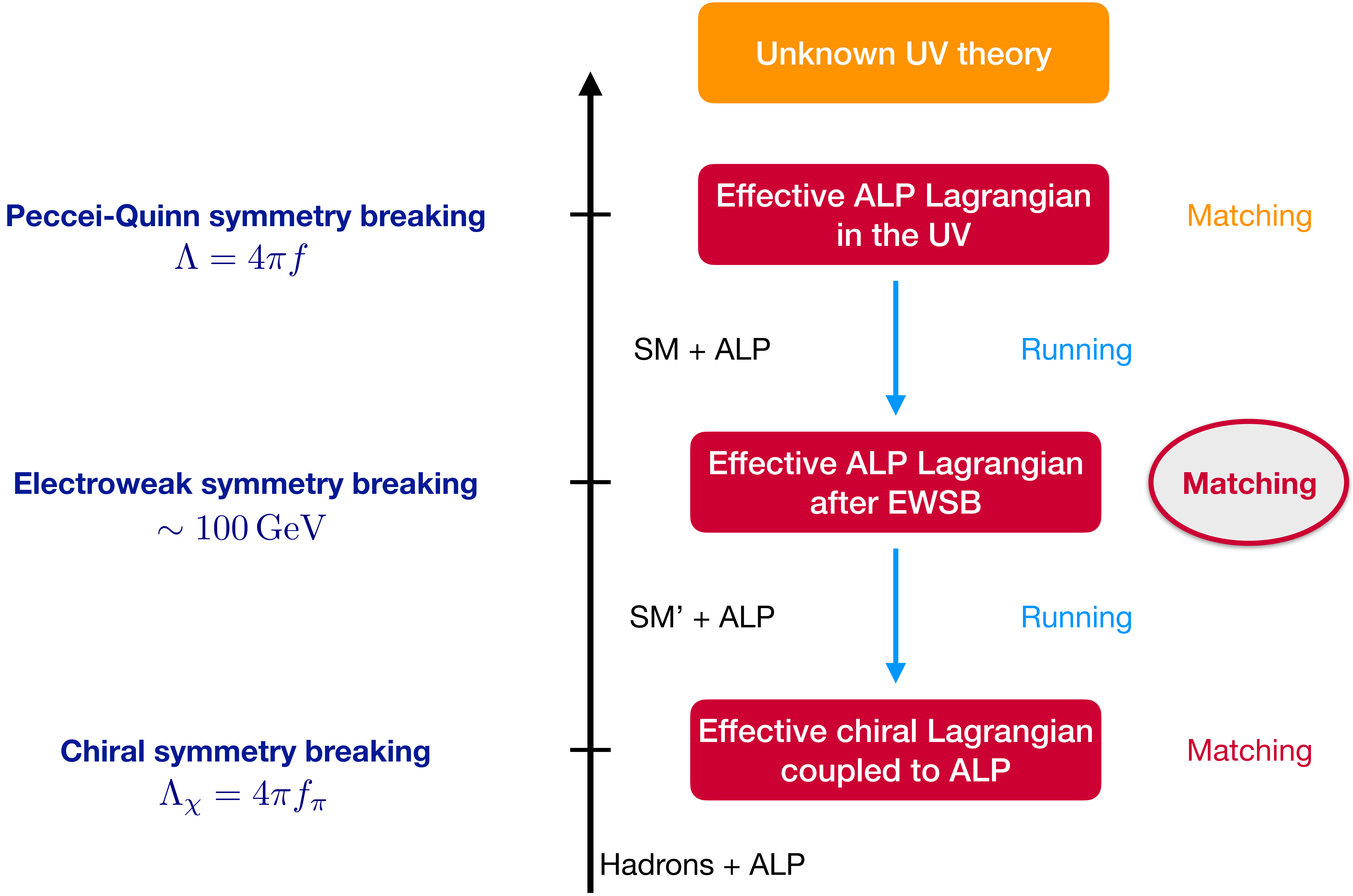
with:

matrices $\boldsymbol{c}_Q$, $\boldsymbol{c}_u$ etc. rotated to the mass basis

$$\mathcal{L}_{\text{ferm}}(\mu_w) = \frac{\partial^\mu a}{f}\,\Big[\bar{u}_L\,\boldsymbol{k}_U\,\gamma_\mu\,u_L + \bar{u}_R\,\boldsymbol{k}_u\,\gamma_\mu\,u_R + \bar{d}_L\,\boldsymbol{k}_D\,\gamma_\mu\,d_L + \bar{d}_R\,\boldsymbol{k}_d\,\gamma_\mu\,d_R$$

$$+ \bar{\nu}_L\,\boldsymbol{k}_\nu\,\gamma_\mu\,\nu_L + \bar{e}_L\,\boldsymbol{k}_E\,\gamma_\mu\,e_L + \bar{e}_R\,\boldsymbol{k}_e\,\gamma_\mu\,e_R\Big]$$

In the next step, we integrate out the heavy particles *t*, *W*, *Z* and *h*.



Unknown UV theory

Peccei-Quinn symmetry breaking

$\Lambda = 4\pi f$

Effective ALP Lagrangian in the UV

Matching

SM + ALP

Running

Electroweak symmetry breaking

$\sim 100\,\text{GeV}$

Effective ALP Lagrangian after EWSB

Matching

SM’ + ALP

Running

Chiral symmetry breaking

$\Lambda_\chi = 4\pi f_\pi$

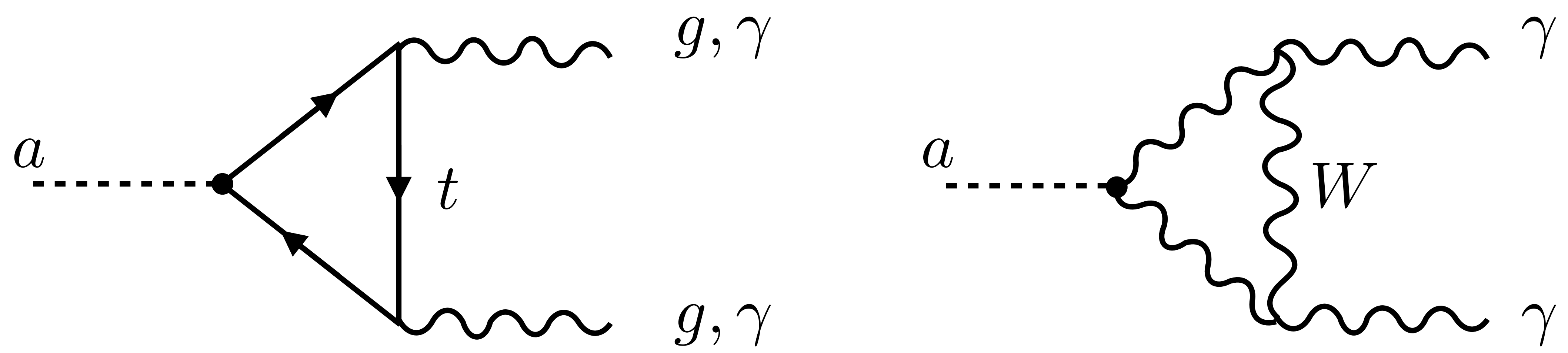
Effective chiral Lagrangian coupled to ALP

Matching

Hadrons + ALP

Weak-scale matching

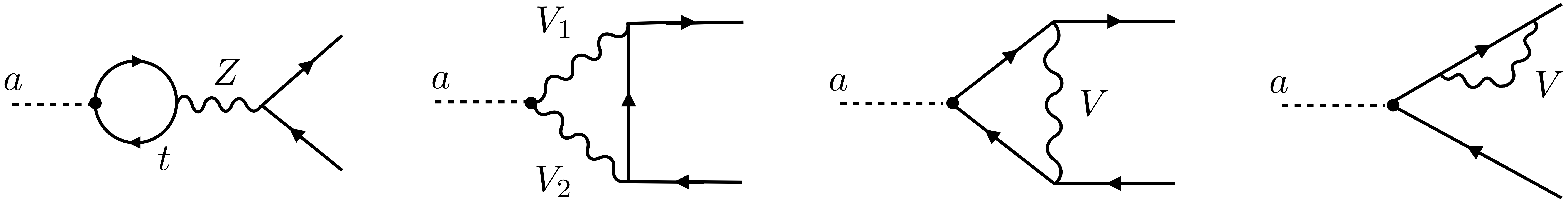
Matching contributions to the ALP-boson couplings are absent in the standard basis (for a light ALP):



$$\sim \frac{m_a^2}{m_t^2}\,, \quad \frac{m_a^2}{m_W^2}$$

[Bauer, MN, Thamm (2017)]

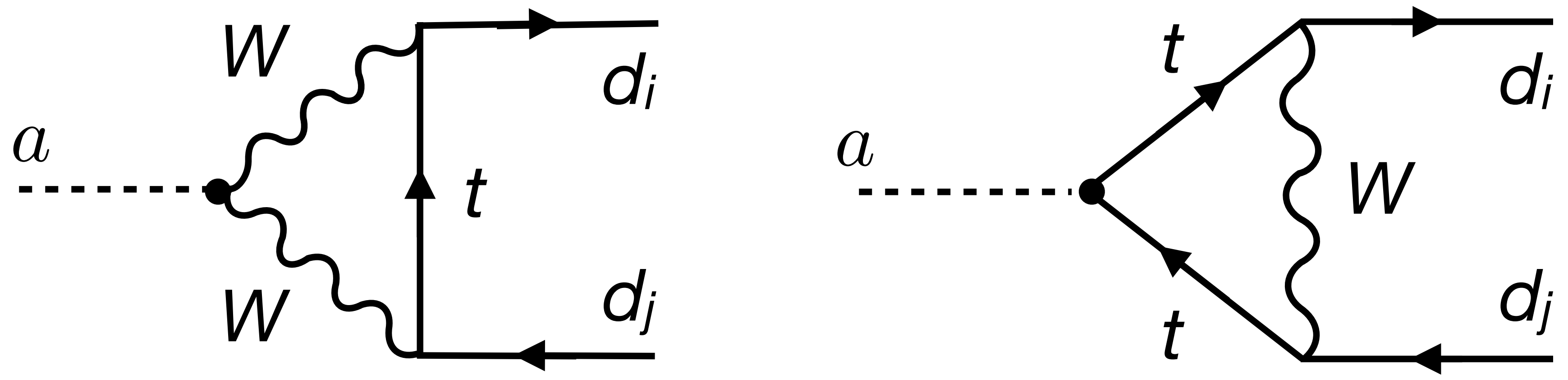
but there are non-trivial matching conditions to the ALP-fermion couplings:



[Bauer, MN, Thamm (2017);
Bauer, MN, Renner, Schnubel, Thamm (2020)]

Weak-scale matching

These include, in particular, flavor-violating contributions to k_D:



$$\left[\hat{\Delta}k_D(\mu_w)\right]_{ij} = \frac{y_t^2}{16\pi^2}\left\{V_{mi}^* V_{nj}\left[k_U(\mu_w)\right]_{mn}(\delta_{m3}+\delta_{n3})\left[-\frac{1}{4}\ln\frac{\mu_w^2}{m_t^2}-\frac{3}{8}+\frac{3}{4}\frac{1-x_t+\ln x_t}{(1-x_t)^2}\right]\right.$$

$$+V_{3i}^* V_{3j}\left[k_U(\mu_w)\right]_{33}+V_{3i}^* V_{3j}\left[k_u(\mu_w)\right]_{33}\left[\frac{1}{2}\ln\frac{\mu_w^2}{m_t^2}-\frac{1}{4}-\frac{3}{2}\frac{1-x_t+\ln x_t}{(1-x_t)^2}\right]$$

$$\left.-\frac{3\alpha}{2\pi s_w^2}c_{WW}\,V_{3i}^* V_{3j}\frac{1-x_t+x_t\ln x_t}{(1-x_t)^2}\right\}$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

ALP couplings at the weak scale

Results for the flavor-diagonal couplings with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{diag}}(\mu) = \sum_{f \neq t} \frac{c_{ff}(\mu)}{2} \frac{\partial^\mu a}{f} \bar{f} \gamma_\mu \gamma_5 f}$$

with $c_{f_i f_i}(\mu) = [k_f(\mu)]_{ii} - [k_F(\mu)]_{ii}$

We find:

$$c_{uu,cc}(\mu_w) \simeq c_{uu,cc}(\Lambda) - 0.116\, c_{tt}(\Lambda) - \Big[6.35\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.02\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{dd,ss}(\mu_w) \simeq c_{dd,ss}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \Big[7.08\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{bb}(\mu_w) \simeq c_{bb}(\Lambda) + 0.097\, c_{tt}(\Lambda) - \Big[7.02\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

$$c_{e_i e_i}(\mu_w) \simeq c_{e_i e_i}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \Big[0.37\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.05\, \tilde{c}_{BB}(\Lambda) \Big] \cdot 10^{-3}$$

ALP couplings at the weak scale

Flavor off-diagonal coefficients with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{FCNC}}(\mu) = -\frac{ia}{2f}\sum_f \Big[(m_{f_i} - m_{f_j})\,(k_f + k_F)_{ij}\,\bar{f}_i f_j + (m_{f_i} + m_{f_j})\,(k_f - k_F)_{ij}\,\bar{f}_i\gamma_5 f_j\Big]}$$

with:

$$[k_u(\mu_w)]_{ij} = [k_u(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

$$[k_U(\mu_w)]_{ij} = [k_U(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

(top quark has been integrated out)

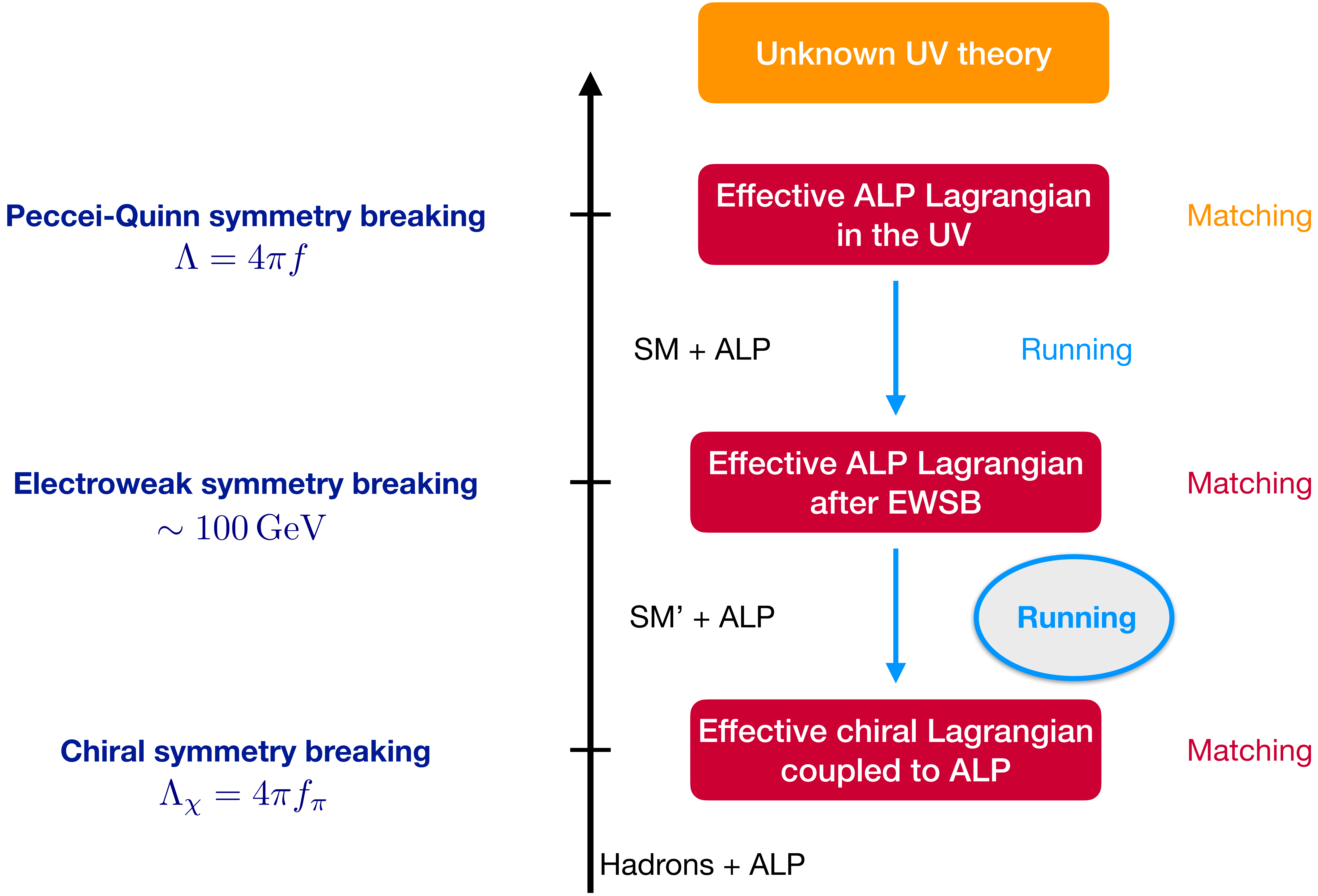
$$[k_d(\mu_w)]_{ij} = [k_d(\Lambda)]_{ij}\,,$$

$$[k_e(\mu_w)]_{ij} = [k_e(\Lambda)]_{ij}\,,$$

$$[k_L(\mu_w)]_{ij} = [k_L(\Lambda)]_{ij}\,.$$

RG running generates MFV-type flavor violation in the left-handed down-quark sector

$$[k_D(m_t)]_{ij} \simeq [k_D(\Lambda)]_{ij} + \boxed{0.019\,V_{ti}^* V_{tj}\Big[c_{tt}(\Lambda) - 0.0032\,\tilde{c}_{GG}(\Lambda) - 0.0057\,\tilde{c}_{WW}(\Lambda)\Big]}$$



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Running

Chiral symmetry breaking
$\Lambda_\chi = 4\pi f_\pi$

Effective chiral Lagrangian coupled to ALP

Matching

Hadrons + ALP

Evolution below the weak scale

In this case only gluon and photon loops contribute:

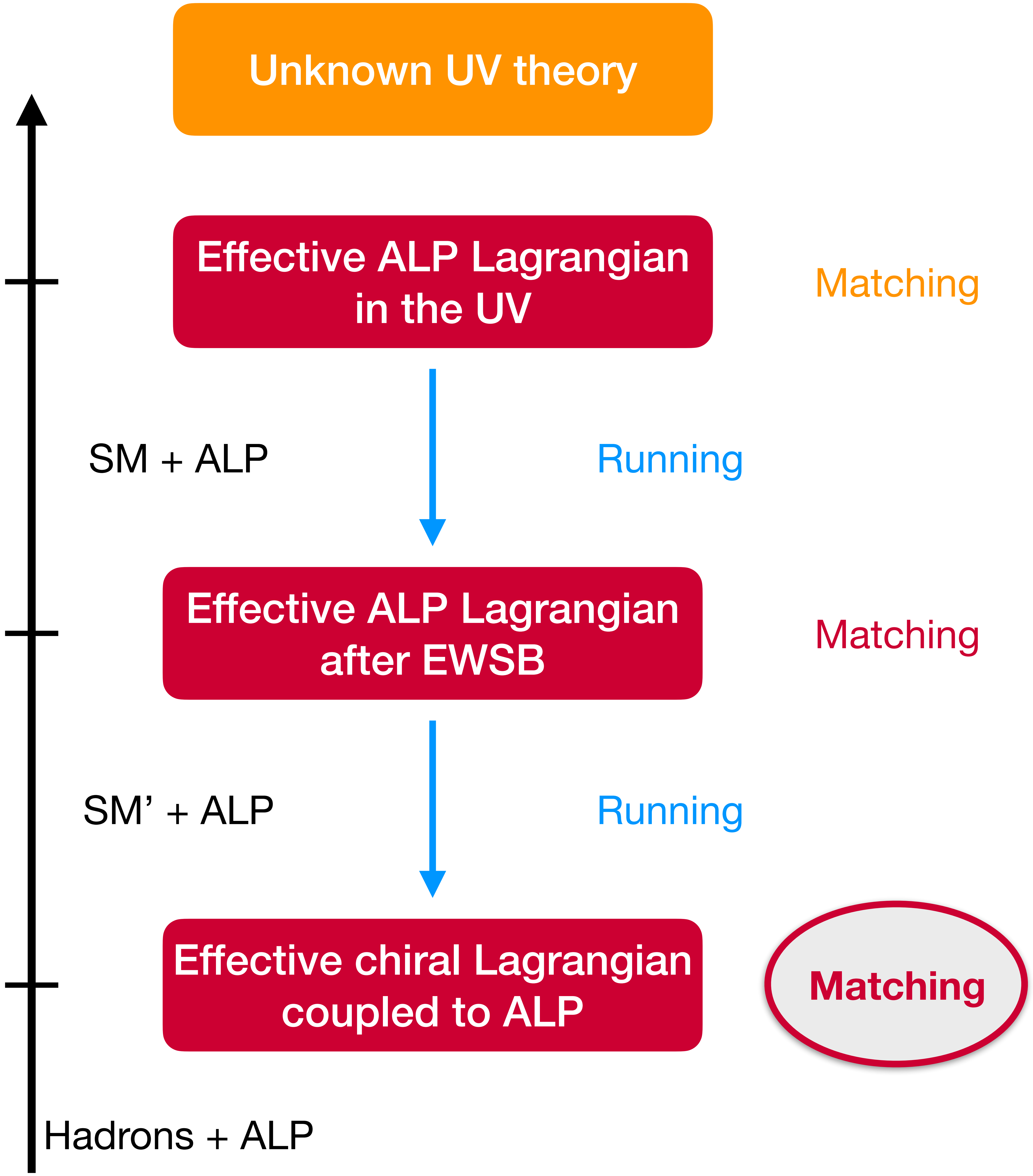
We find numerically with $\mu_0 = 2$ GeV:

$$c_{qq}(\mu_0) = c_{qq}(m_t) + \Big[3.0\, \tilde{c}_{GG}(\Lambda) - 1.4\, c_{tt}(\Lambda) - 0.6\, c_{bb}(\Lambda) \Big] \cdot 10^{-2}$$

$$+ Q_q^2 \Big[3.9\, \tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\, c_{tt}(\Lambda) - 0.2\, c_{bb}(\Lambda) \Big] \cdot 10^{-5} \,,$$

$$c_{\ell\ell}(\mu_0) = c_{\ell\ell}(m_t) + \Big[3.9\, \tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\, c_{tt}(\Lambda) - 0.2\, c_{bb}(\Lambda) \Big] \cdot 10^{-5} \,.$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]



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Fun facts about $K \to \pi a$

Matching to the chiral Lagrangian

Georgi, Kaplan, Randall (1986) have developed a model-independent chiral Lagrangian approach valid for any ALP model

In the quark mass basis, the starting point is (at $\mu_\chi \approx 4\pi f_\pi$):

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \mathcal{L}_{\text{QCD}} + \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 \\ &\quad + c_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu} \\ &\quad + \frac{\partial^\mu a}{f}\left(\bar{q}_L\,\boldsymbol{k}_Q\,\gamma_\mu\,q_L + \bar{q}_R\,\boldsymbol{k}_q\,\gamma_\mu\,q_R + \dots\right)\end{aligned}$$

three light quarks *u*, *d*, *s*

Matching to the chiral Lagrangian

To bosonize this theory, one first eliminates the ALP-gluon coupling using the chiral rotation: [Srednicki (1985); Georgi, Kaplan, Randall (1986); Krauss, Wise (1986); Bardeen, Peccei, Yanagida (1987)]

$$q(x) \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q \gamma_5\right) c_{GG}\, \frac{a(x)}{f}\right] q(x) \qquad \text{with} \qquad \mathrm{Tr}\,\boldsymbol{\kappa}_q = \kappa_u + \kappa_d + \kappa_s = 1$$

diagonal in the quark mass basis

Modified quark mass matrix and ALP couplings:

$$\hat{\boldsymbol{m}}_q(a) = \exp\left(-2i\boldsymbol{\kappa}_q\, c_{GG}\, \frac{a}{f}\right) \boldsymbol{m}_q$$

$$\hat{c}_{\gamma\gamma} = c_{\gamma\gamma} - 2N_c\, c_{GG}\, \mathrm{Tr}\, \boldsymbol{Q}^2 \boldsymbol{\kappa}_q$$

$$\left.\begin{aligned} \hat{\boldsymbol{k}}_Q(a) &= e^{i\boldsymbol{\phi}_q^- a/f} \left(\boldsymbol{k}_Q + \boldsymbol{\phi}_q^-\right) e^{-i\boldsymbol{\phi}_q^- a/f} \\ \hat{\boldsymbol{k}}_q(a) &= e^{i\boldsymbol{\phi}_q^+ a/f} \left(\boldsymbol{k}_q + \boldsymbol{\phi}_q^+\right) e^{-i\boldsymbol{\phi}_q^+ a/f} \end{aligned}\right\} \quad \text{with} \quad \boldsymbol{\phi}_q^\pm = c_{GG}\left(\boldsymbol{\delta}_q \pm \boldsymbol{\kappa}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

Matching to the chiral Lagrangian

- The light pseudoscalar mesons are described by $\boldsymbol{\Sigma}(x) = \exp\left[\frac{i\sqrt{2}}{f_\pi}\,\lambda^a\,\pi^a(x)\right]$
- The derivative ALP couplings to fermions are included in the covariant derivative:

$$iD_\mu\boldsymbol{\Sigma} = i\partial_\mu\boldsymbol{\Sigma} + e\,A_\mu\,[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(\hat{\boldsymbol{k}}_Q\,\boldsymbol{\Sigma} - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

- Leading-order effective chiral Lagrangian:

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\mathrm{Tr}\left[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4}\,B_0\,\mathrm{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\right]$$
$$+\,\frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

[Gasser, Leutwyler (1985)]

- Periodic potential breaks the shift symmetry and provides a mass for the axion (QCD instantons) [Weinberg (1978); Wilczek (1978)]

Weak decay $K \to \pi a$

- Strongest particle-physics constraint on ALP couplings for mass range $m_a < m_K - m_\pi \approx 354$ MeV

- Despite a 35-year history, we find that even nowadays most papers on this process are based on inconsistent equations

- The chiral implementation of the leading SU(3)-octet weak-interaction operator is: [Bernard, Draper, Soni, Politzer, Wise (1985); Crewther (1986); Kambor, Missimer, Wyler (1990)]

$$\mathcal{L}_{\text{weak}} = -\frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8 \left[L_\mu L^\mu\right]^{32}$$

where L_μ^{ji} is the chiral representation of the left-handed current $\bar{q}_L^i \gamma_\mu q_L^j$

Weak decay $K \to \pi a$

Georgi, Kaplan, Randall used:

$$L_\mu^{ij} = -\frac{if_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\,a/f} \left[\boldsymbol{\Sigma}\,\partial_\mu \boldsymbol{\Sigma}^\dagger\right]^{ij}$$

where the phase factor results from the chiral rotation, but the Noether theorem gives instead: [Bauer, MN, Renner, Schnubel, Thamm (2021)]

$$\boxed{\begin{aligned} L_\mu^{ji} &= -\frac{if_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\,a/f} \left[\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu \boldsymbol{\Sigma})^\dagger\right]^{ji} \\ &\ni -\frac{if_\pi^2}{4} \left[1 + i(\delta_{q_i} - \delta_{q_j} - \kappa_{q_i} + \kappa_{q_j})\, c_{GG}\,\frac{a}{f}\right] \left[\boldsymbol{\Sigma}\,\partial_\mu \boldsymbol{\Sigma}^\dagger\right]^{ji} \\ &\quad + \frac{f_\pi^2}{4}\,\frac{\partial^\mu a}{f} \left[\hat{\boldsymbol{k}}_Q - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\,\boldsymbol{\Sigma}^\dagger\right]^{ji} \qquad \leftarrow \text{crucial extra terms!} \end{aligned}}$$

Weak decay $K \to \pi a$

Cancellation of auxiliary parameters:

$$D_1 \ni \frac{N_8}{2f}\, c_{GG}\, (\kappa_u - \kappa_d)\, (m_\pi^2 - m_a^2)$$

$$D_2 \ni -\frac{N_8}{6f}\, c_{GG}\, (2m_K^2 + m_\pi^2 - 3m_a^2)\, (\kappa_u + \kappa_d - 2\kappa_s)$$

$$\begin{aligned} D_3 \ni \frac{N_8}{2f}\, c_{GG} \Big[&- (\delta_d - \delta_s - \kappa_d + \kappa_s)\, (m_K^2 + m_\pi^2 - m_a^2) \\ &+ (\delta_u - \delta_d + \kappa_u + \kappa_s)\, (m_K^2 - m_\pi^2 + m_a^2) \\ &+ (\delta_u - \delta_s + \kappa_u + \kappa_d)\, (m_K^2 - m_\pi^2 - m_a^2) \Big] \end{aligned}$$

previously omitted contributions

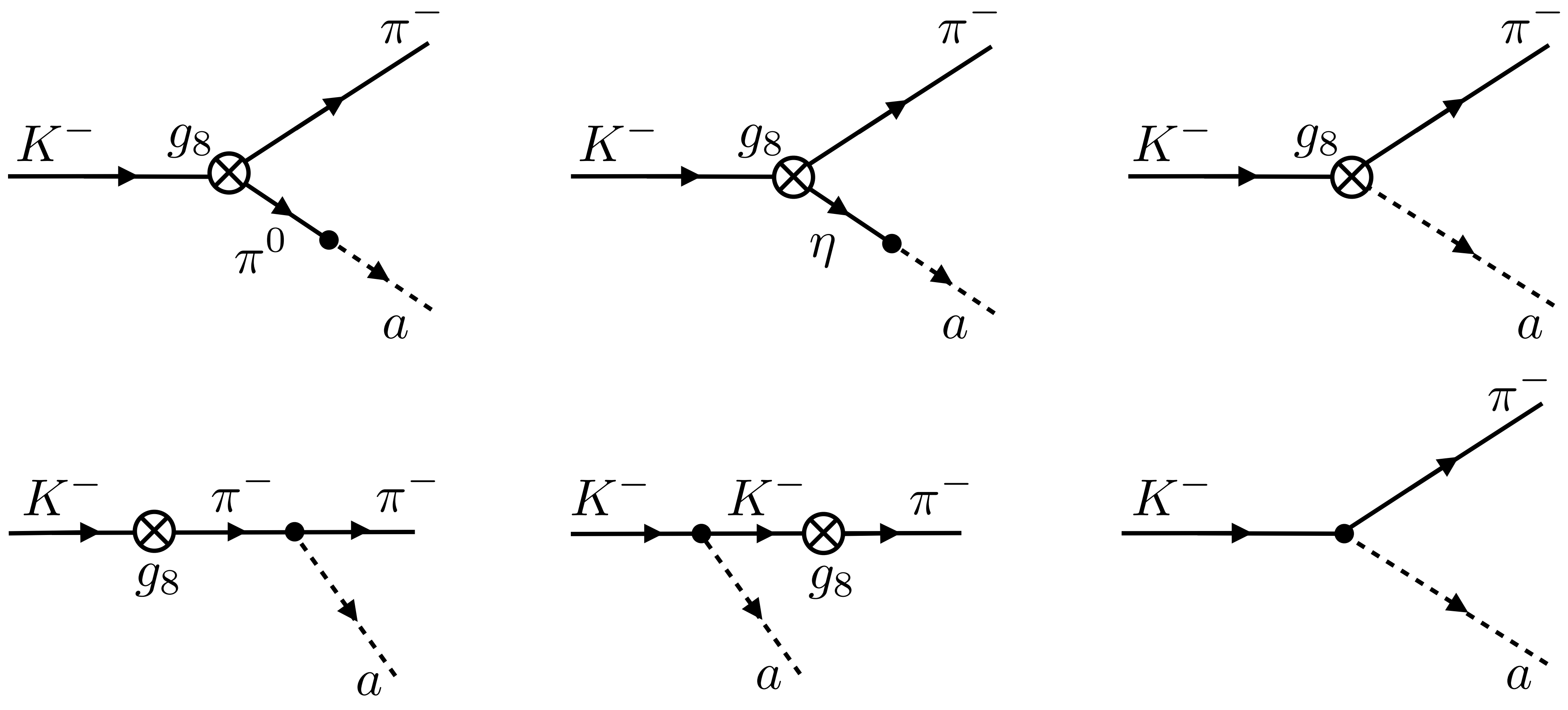
$$D_4 \ni -\frac{N_8}{f}\, c_{GG}\, m_K^2\, (\delta_u - \delta_d)$$

$$D_5 \ni \frac{N_8}{f}\, c_{GG}\, m_\pi^2\, (\delta_u - \delta_s)$$

with:

$$N_8 = -\frac{G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8\, f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]



ALP-pion mixing

ALP-η mixing

"direct" contribution

Final-state radiation

Initial-state radiation

"direct" flavor-changing ALP contribution

- Find that omitted contributions have a large effect (parametrically leading terms)
- Including only the first two diagrams (ALP-meson mixing) gives an uncontrolled approximation (except in very special cases)

Weak decay $K \to \pi a$

Decay amplitude:

$$
\begin{aligned}
i\mathcal{A}_{K^-\to\pi^- a} = \frac{N_8}{4f}\Big[& 16\,c_{GG}\,\frac{(m_K^2-m_\pi^2)(m_K^2-m_a^2)}{4m_K^2-m_\pi^2-3m_a^2} \\
& + 6\,(c_{uu}+c_{dd}-2c_{ss})\,m_a^2\,\frac{m_K^2-m_a^2}{4m_K^2-m_\pi^2-3m_a^2} \\
& + (2c_{uu}+c_{dd}+c_{ss})\,(m_K^2-m_\pi^2-m_a^2) + 4c_{ss}m_a^2 \\
& + (k_d+k_D-k_s-k_S)\,(m_K^2+m_\pi^2-m_a^2)\Big] \\
& - \frac{m_K^2-m_\pi^2}{2f}\,\big[k_q+k_Q\big]^{23}
\end{aligned}
$$

with:

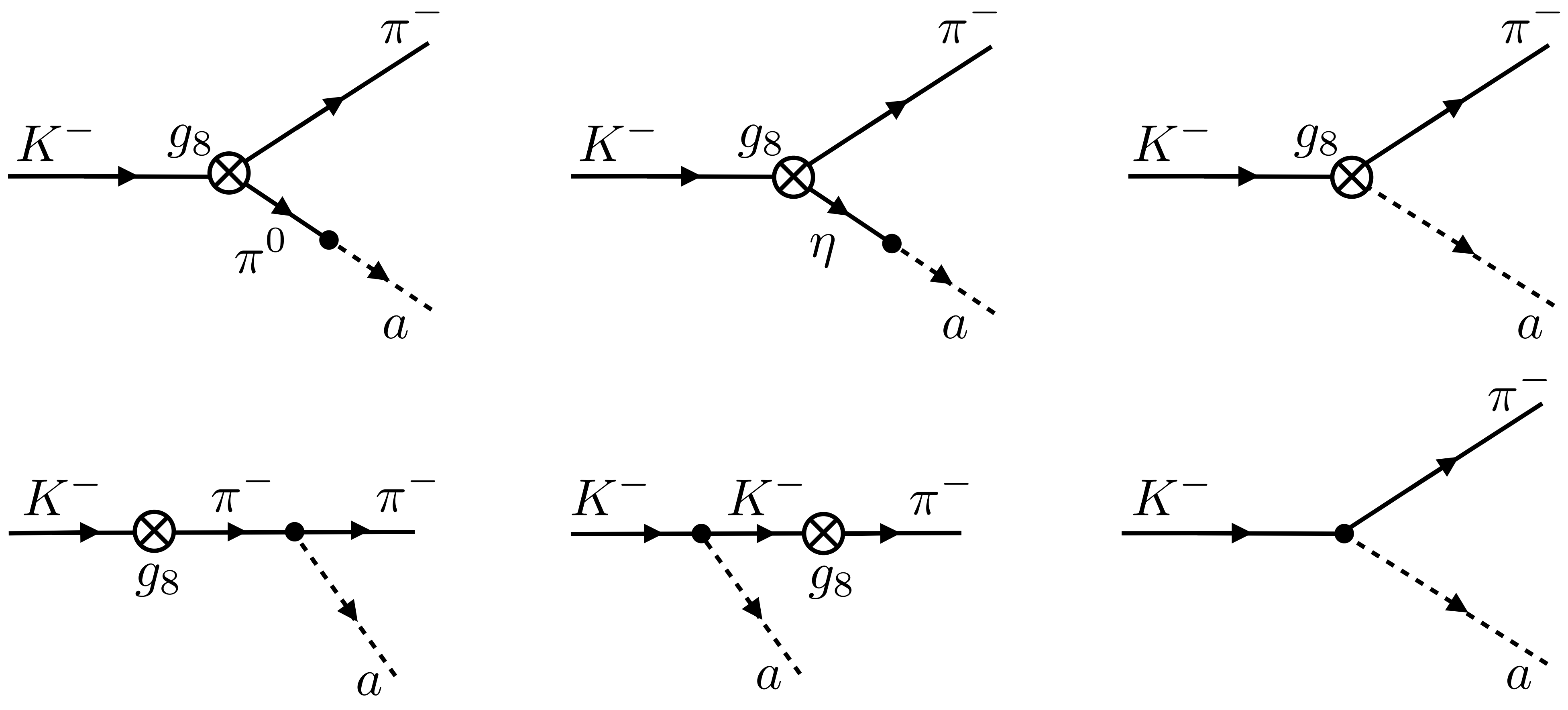
$$N_8 = -\frac{G_F}{\sqrt{2}}\,V_{ud}^*V_{us}\,g_8\,f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

ALP-pion mixing

ALP-η mixing

“direct” contribution



Final-state radiation

Initial-state radiation

Flavor-changing ALP coupling

Georgi, Kaplan and Randall have only considered the axion-gluon coupling c_{GG} and find a result smaller by a factor

$$\frac{m_u}{2(m_u+m_d)} \approx 0.16$$

$K \to \pi a$ phenomenology

Expressing the ALP couplings in terms of the couplings at the scale $\Lambda = 4\pi f$ with $f = 1$ TeV, and assuming MFV, we find:

strong-interaction phase of g_8

$$|\mathcal{A}_{K^- \to \pi^- a}| \simeq 10^{-11}\,\text{GeV}\left[\frac{1\,\text{TeV}}{f}\right] \times \Big[e^{i\delta_8}\left(3.58\,c_{GG} + 1.79\,c_{uu}(\Lambda) + 1.81\,c_{dd}(\Lambda)\right)$$

$$+\, e^{i\alpha}\left(-\,65.8\,c_{uu}(\Lambda) + 0.32\,c_{dd}(\Lambda) + 0.21\,c_{GG} + 0.38\,c_{WW}\right)$$

$$-\,1.12 \cdot 10^7\,k_D^{12}(\Lambda)\Big]$$

weak phase of V_{td}^*

← proportional to $V_{td}^* V_{ts}$ in MFV

The coefficients refer to $m_a = 0$, but they vary by less than 10% over the entire allowed mass range. Two “benchmarks”: [see e.g.: Gori, Perez, Tobioka (2020)]

- only $c_{GG} \neq 0$: “indirect” contribution (g_8) dominates
- only $c_{WW} \neq 0$: “direct” contribution (from RG running) dominates

Flavor benchmarks

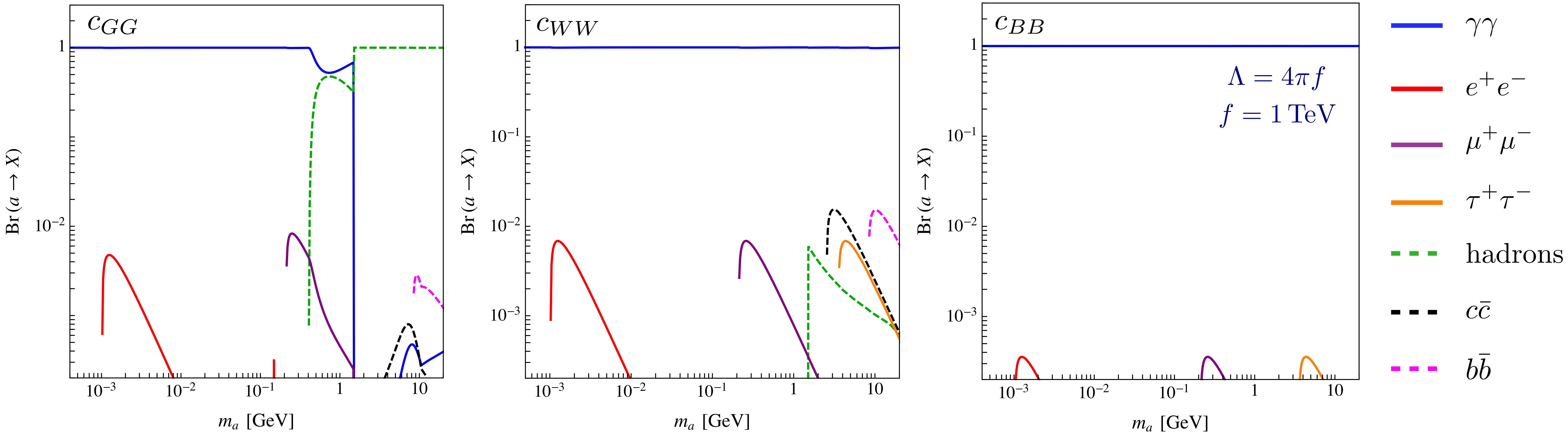
Flavor physics benchmarks

- RG evolution effects have a profound impact on phenomenology, for instance in flavor physics
- General lesson: no ALP couplings can be avoided !
- Below we consider **benchmark scenarios**, starting with a **single ALP coupling in the UV** (at $\Lambda = 4\pi f$) and assuming **flavor universality**
- We then calculate the contributions to various flavor observables and derived bounds on the UV couplings as a function of the ALP mass
- In this process, we carefully account for the effects of the ALP lifetime and its various decay modes

based on ongoing work with M. Bauer, S. Renner, M. Schnubel & A. Thamm

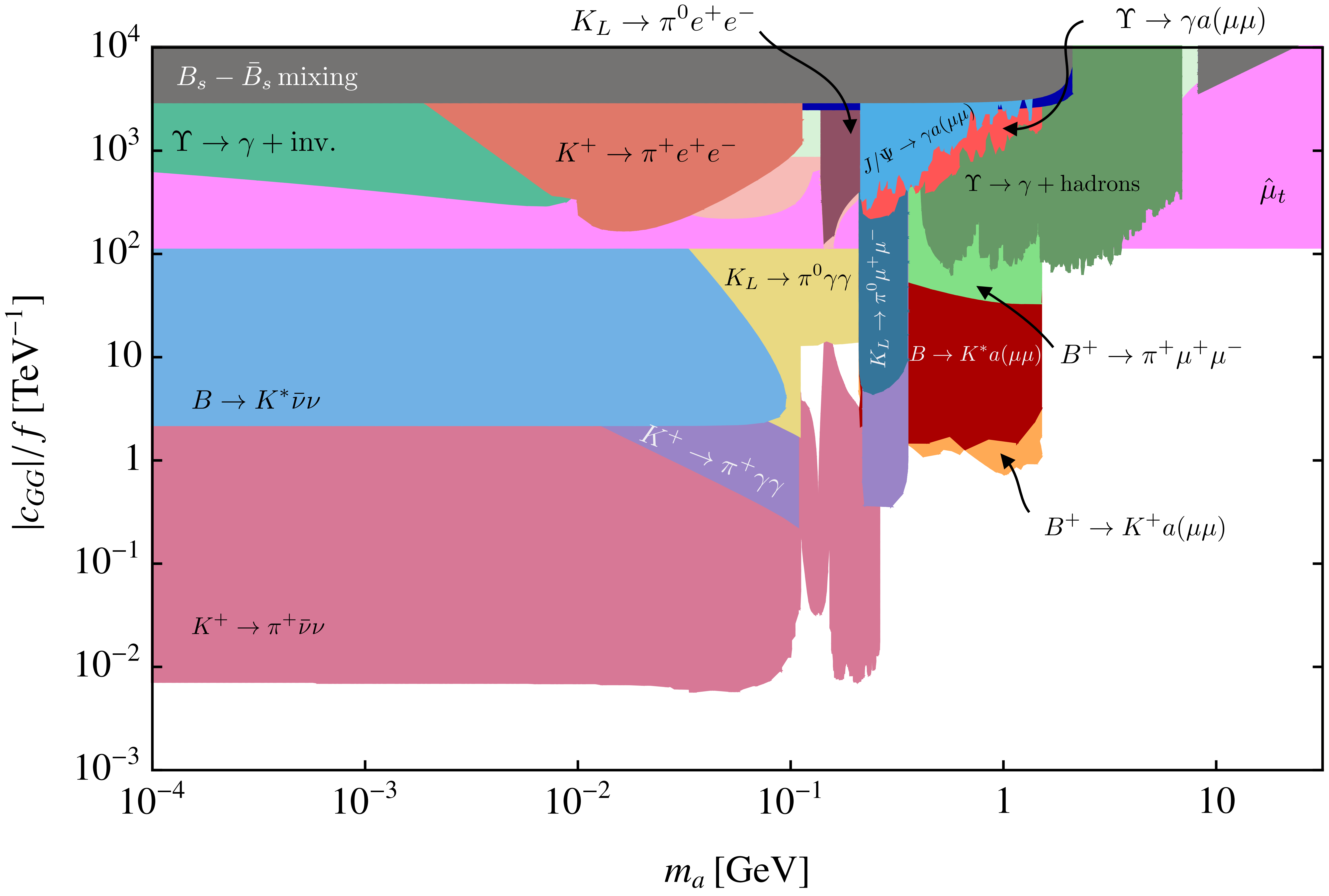
Flavor physics benchmarks

ALP branching fractions in the benchmarks with a single non-vanishing ALP-gauge boson coupling at the UV scale: [Bauer, MN, Thamm (2017)]



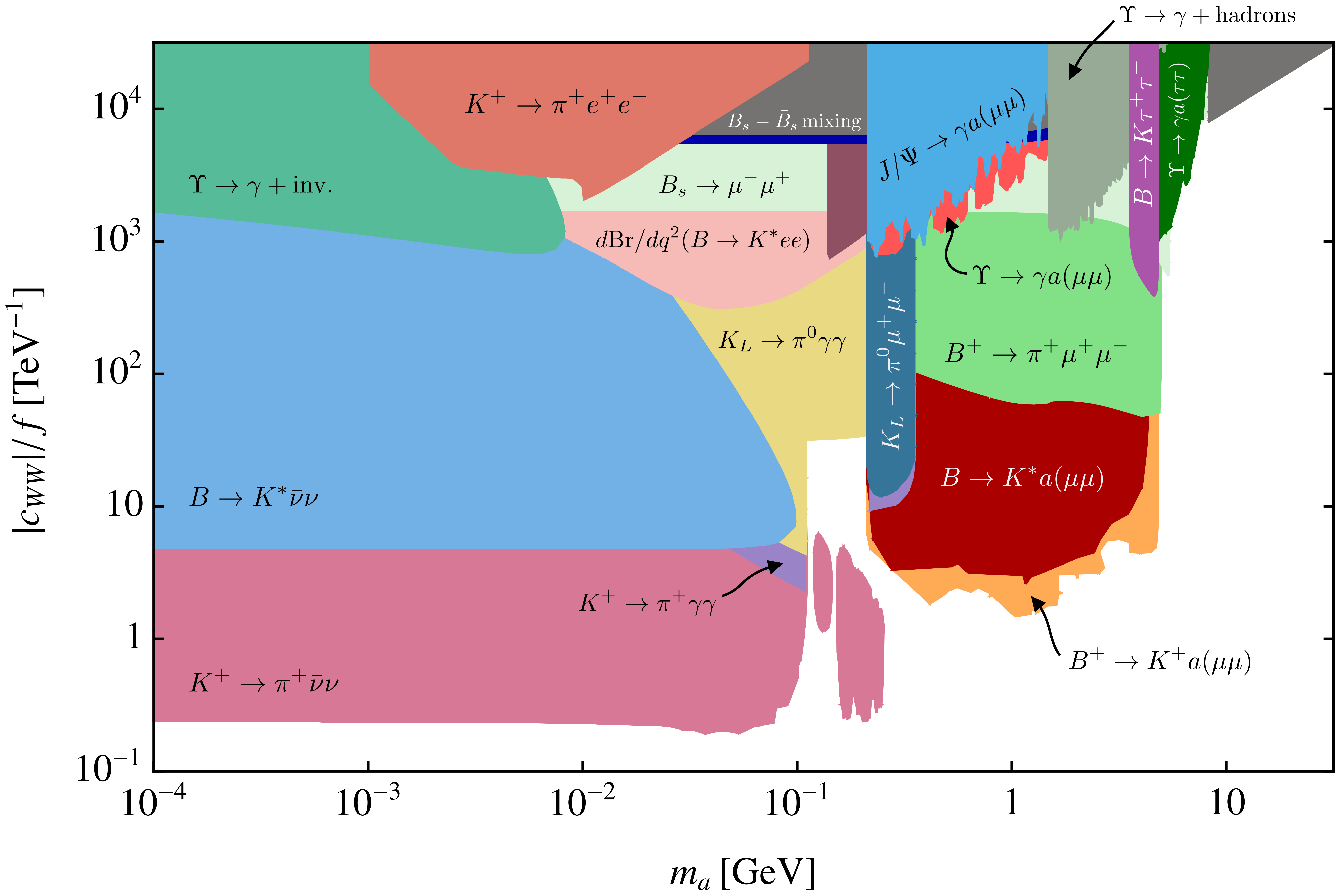
Flavor physics benchmarks

ALP-gluon coupling in the UV:



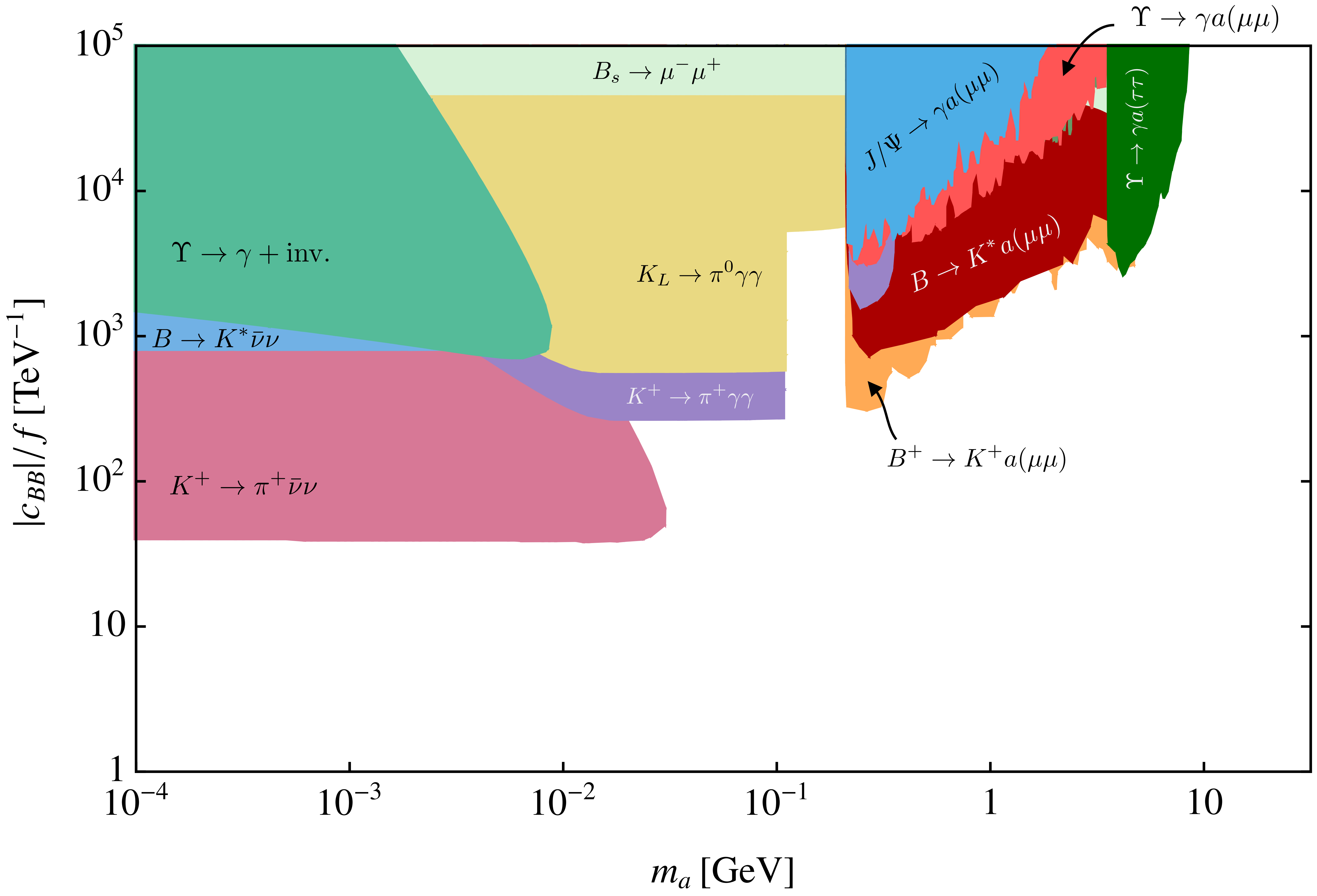
Flavor physics benchmarks

ALP-*W* coupling in the UV (note change in scale):



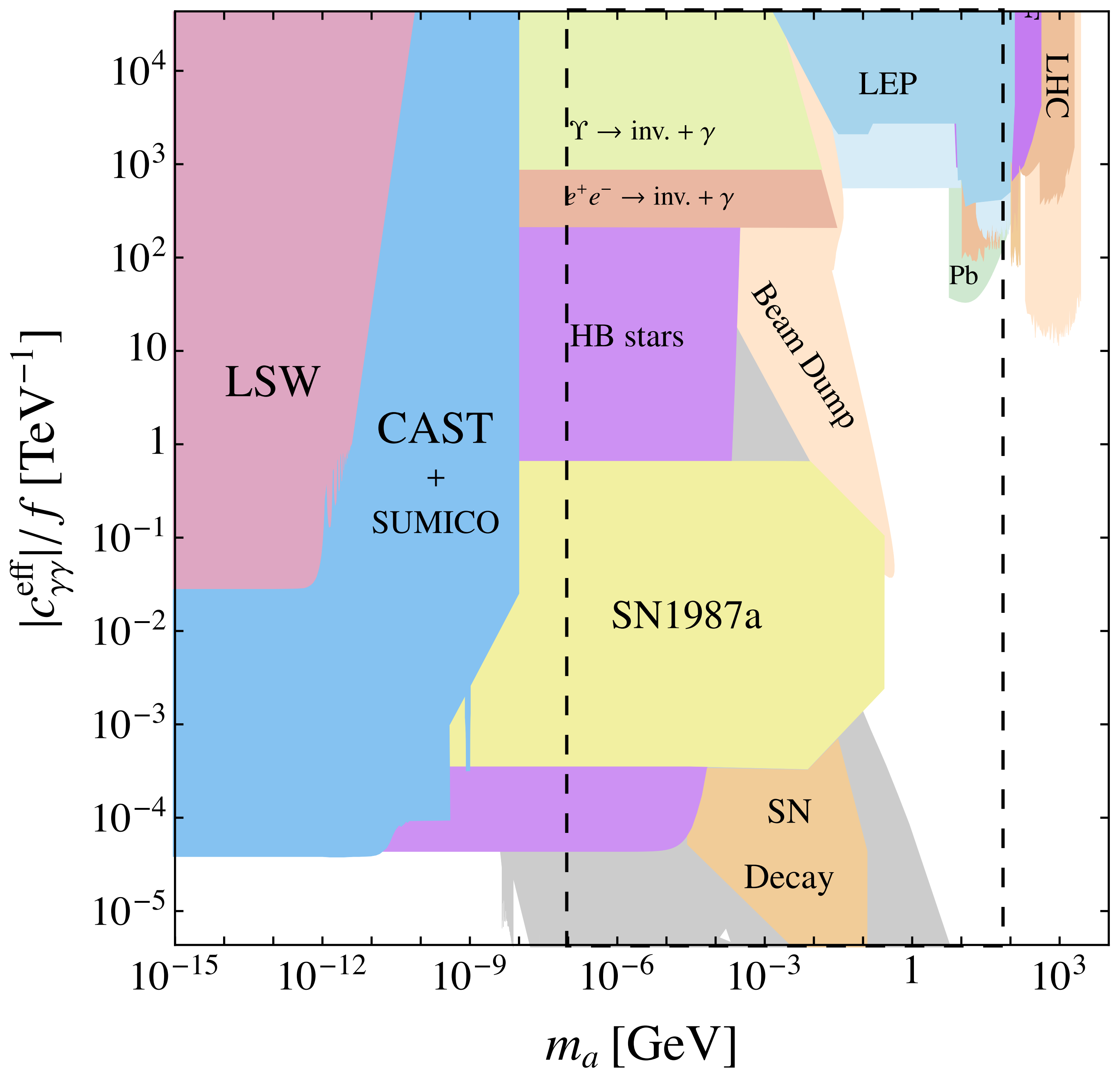
Flavor physics benchmarks

ALP-*B* coupling in the UV (note change in scale):

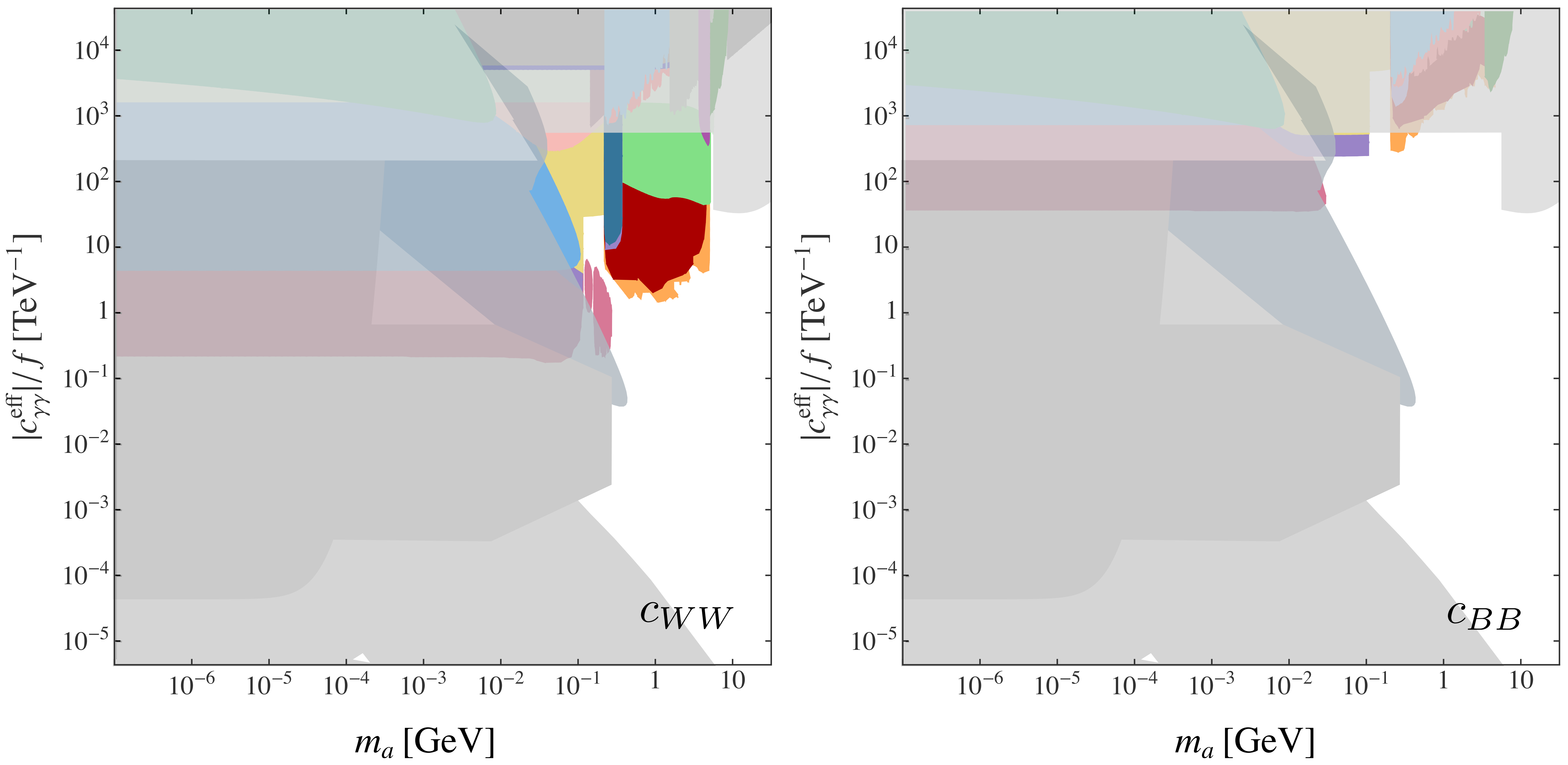


Flavor physics benchmarks

Impact on the chart for the ALP-photon coupling:



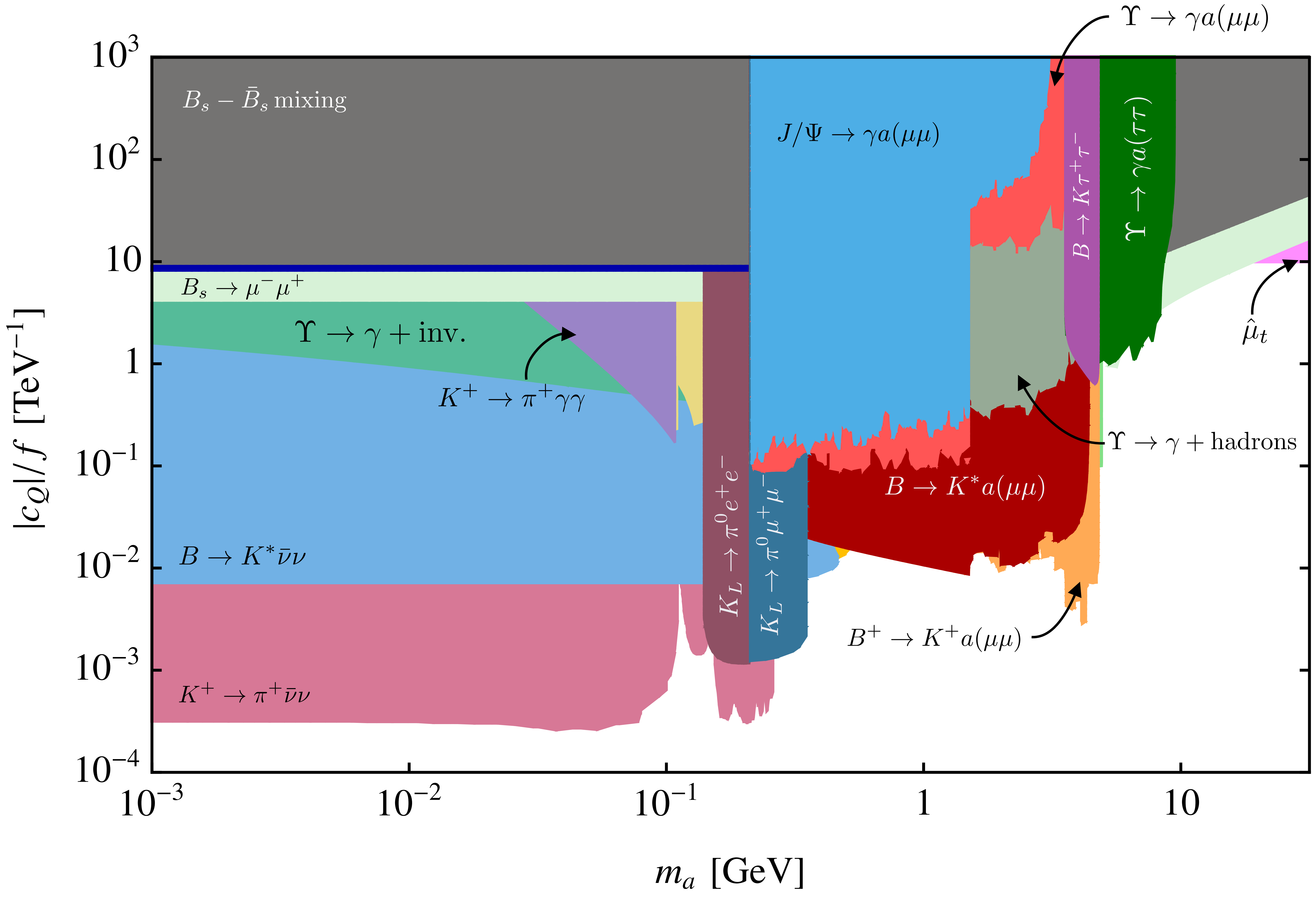
bounds from cosmology, astrophysics and collider physics



$c_{\gamma\gamma} = c_{WW} + c_{BB}$

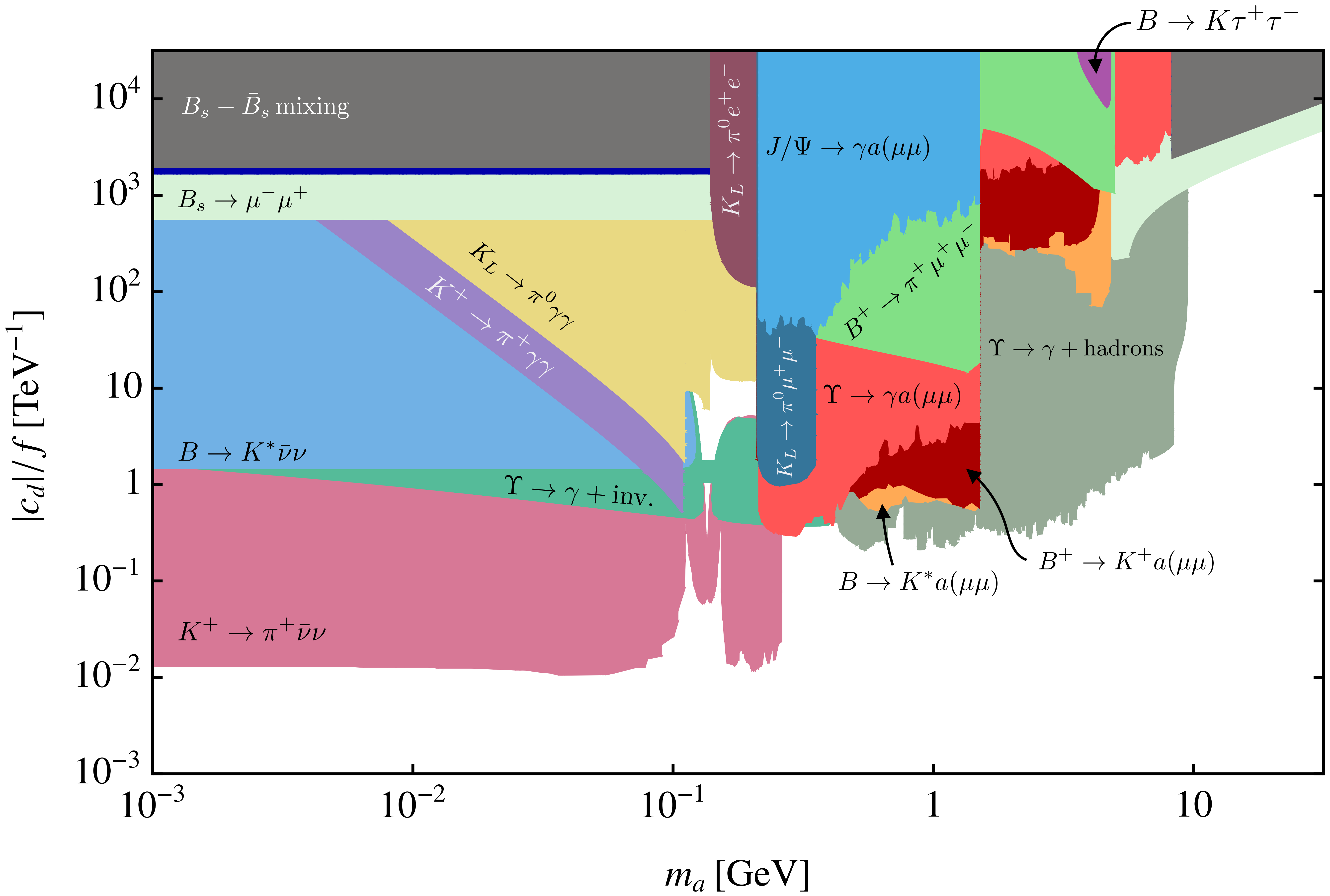
Flavor physics benchmarks

Flavor-universal ALP-Q_L coupling in the UV:



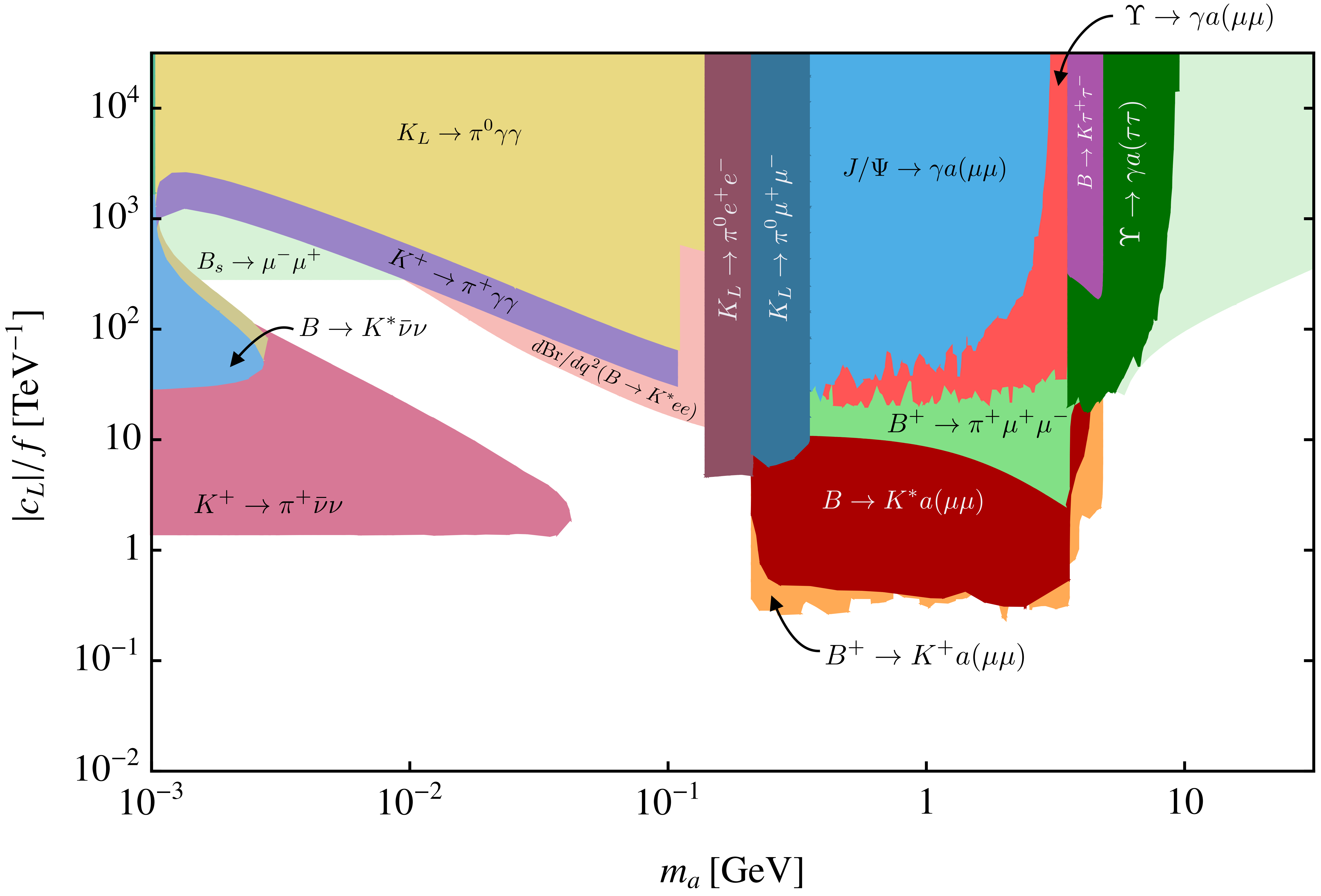
Flavor physics benchmarks

Flavor-universal ALP-d_R coupling in the UV:



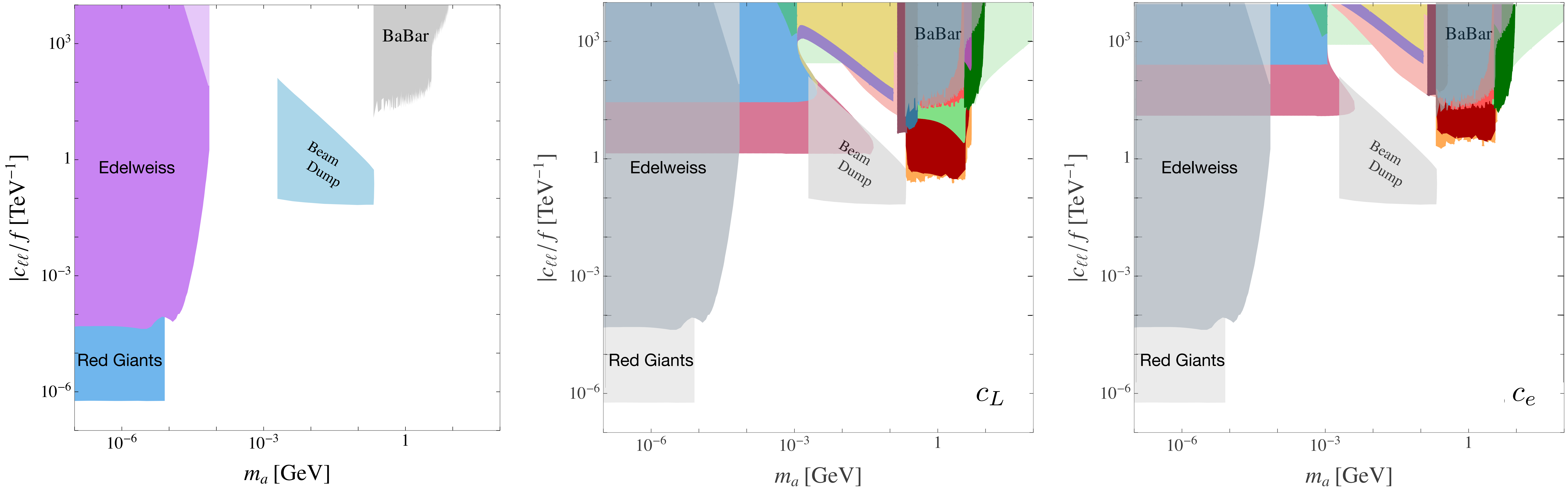
Flavor physics benchmarks

Flavor-universal ALP-L_L coupling in the UV:



Flavor physics benchmarks

Impact on the chart for the ALP-electron coupling:

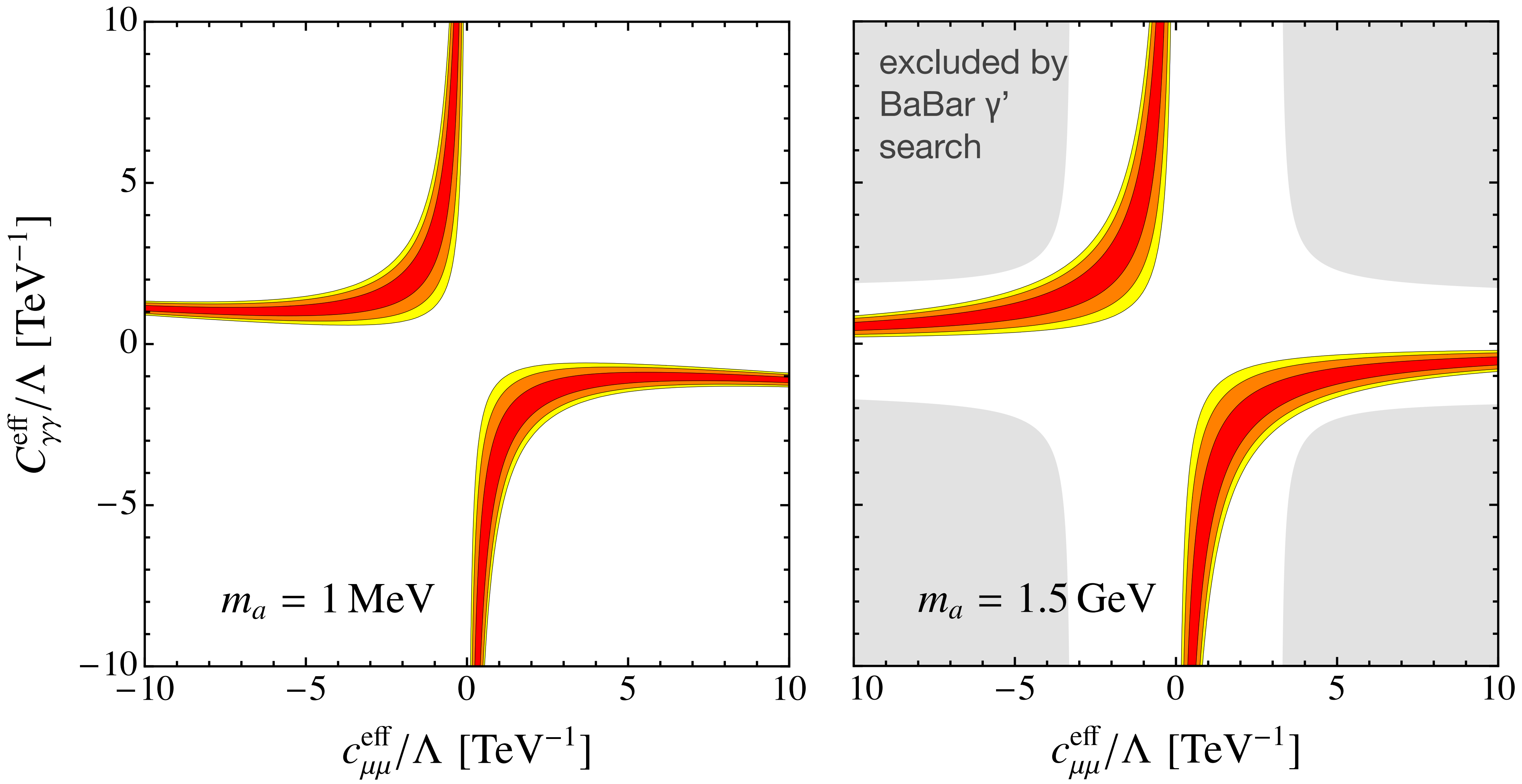


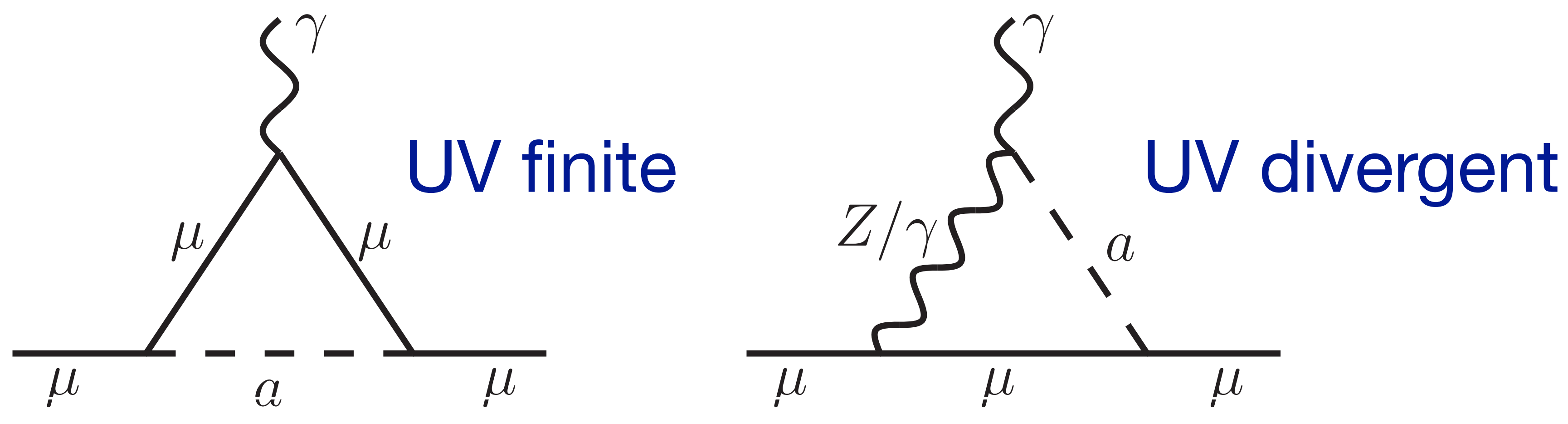
ALP—SMEFT interference

ALP—SMEFT interference

It is well-known that one-loop diagrams with virtual ALP exchange can be UV divergent. This was first studied in the context of $(g\text{-}2)_\mu$:

[Marciano, Masiero, Paradisi, Passera (2016); Bauer, MN, Thamm (2017)]





$$\delta a_\mu = \frac{m_\mu^2}{\Lambda^2}\left\{ K_{a_\mu}(\mu) - \frac{(c_{\mu\mu})^2}{16\pi^2}\,h_1\!\left(\frac{m_a^2}{m_\mu^2}\right) - \frac{2\alpha}{\pi}\,c_{\mu\mu}\,C_{\gamma\gamma}\left[\ln\frac{\mu^2}{m_\mu^2} + \delta_2 + 3 - h_2\!\left(\frac{m_a^2}{m_\mu^2}\right)\right] \right.$$
$$\left. - \frac{\alpha}{2\pi}\,\frac{1-4s_w^2}{s_w c_w}\,c_{\mu\mu}\,C_{\gamma Z}\left(\ln\frac{\mu^2}{m_Z^2} + \delta_2 + \frac{3}{2}\right)\right\}$$

needs a D=6 counterterm not contained in the ALP effective Lagrangian

ALP—SMEFT interference

A systematic treatment of these UV divergences requires an embedding of the ALP model in the SMEFT: [Buchmüller, Wyler (1986)]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}\,a^2 + \mathcal{L}_{\text{SM+ALP}} + \mathcal{L}_{\text{SMEFT}}$$

where:

$$\mathcal{L}_{\text{SM+ALP}}^{D=5} = C_{GG}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + C_{WW}\,\frac{a}{f}\,W_{\mu\nu}^I\,\tilde{W}^{\mu\nu,I} + C_{BB}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

$$- \frac{a}{f}\Big(\bar{Q}\,\tilde{H}\,\widetilde{\boldsymbol{Y}}_u\,u_R + \bar{Q}\,H\,\widetilde{\boldsymbol{Y}}_d\,d_R + \bar{L}\,H\,\widetilde{\boldsymbol{Y}}_e\,e_R + \text{h.c.}\Big)$$

Irrespective of the existence of other new physics, the presence of a light ALP provides source terms S_i for the D=6 SMEFT Wilson coefficients:

$$\boxed{\frac{d}{d\ln\mu}\,C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}}\,C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2} \qquad (\text{for } \mu < 4\pi f)}$$

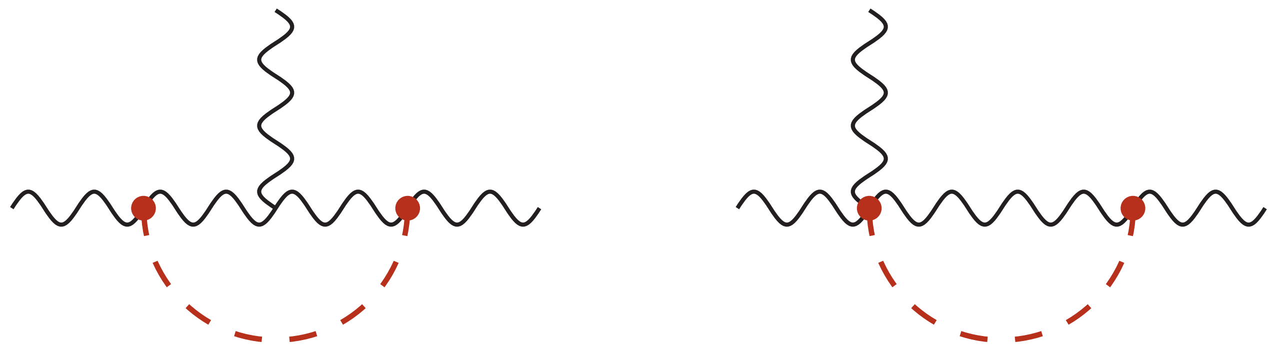
[Galda, MN, Renner: 2105.01078]

ALP—SMEFT interference

Systematic study of divergent Green's functions with ALP exchange

[Galda, MN, Renner: 2105.01078]

Sample calculation: UV divergences of the three-gluon amplitude



$$\mathcal{A}(gg(g)) = -\frac{C_{GG}^2}{\epsilon}\left[4g_s\,\langle Q_G\rangle + \frac{4}{3}\,\langle\widehat{Q}_{G,2}\rangle - 2m_a^2\,\langle G_{\mu\nu}^a\,G^{\mu\nu,a}\rangle\right] + \text{finite}$$

Source term for Weinberg operator:

$$S_G = 8\,g_s\,C_{GG}^2$$

Eliminate redundant operator $\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a\,(D_\omega G^{\omega\mu})^a$ using the EOMs:

$$\begin{aligned}\widehat{Q}_{G,2} &\cong g_s^2\,\big(\bar{Q}\gamma_\mu T^a Q + \bar{u}\gamma_\mu T^a u + \bar{d}\gamma_\mu T^a d\big)^2 \\ &= g_s^2\left[\frac{1}{4}\Big([Q_{qq}^{(1)}]_{prrp} + [Q_{qq}^{(3)}]_{prrp}\Big) - \frac{1}{2N_c}\,[Q_{qq}^{(1)}]_{pprr} + \frac{1}{2}\,[Q_{uu}]_{prrp} - \frac{1}{2N_c}\,[Q_{uu}]_{pprr}\right. \\ &\qquad \left. + \frac{1}{2}\,[Q_{dd}]_{prrp} - \frac{1}{2N_c}\,[Q_{dd}]_{pprr} + 2\,[Q_{qu}^{(8)}]_{pprr} + 2\,[Q_{qd}^{(8)}]_{pprr} + 2\,[Q_{ud}^{(8)}]_{pprr}\right]\end{aligned}$$

[Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)]

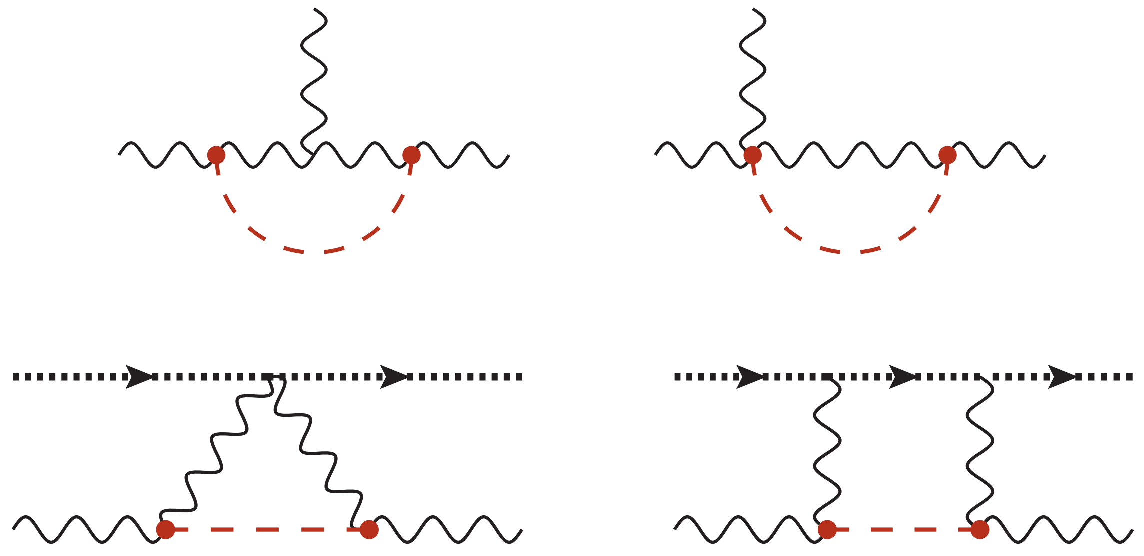
→ generates further source terms

ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

Operator class	Warsaw basis	Way of generation	
Purely bosonic			
X^3	yes	direct	—
X^2D^2	no	direct	
X^2H^2	yes	direct	—
XH^2D^2	no	—	
H^6	yes	—	EOM
H^4D^2	yes	—	EOM
H^2D^4	no	—	

[Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)]



$$S_G = 8g_s C_{GG}^2\,, \qquad S_{\widetilde{G}} = 0$$
$$S_W = 8g_2 C_{WW}^2\,, \qquad S_{\widetilde{W}} = 0$$

$$S_{HG} = 0\,, \qquad S_{H\widetilde{G}} = 0$$
$$S_{HW} = -2g_2^2\,C_{WW}^2\,, \qquad S_{H\widetilde{W}} = 0$$
$$S_{HB} = -2g_1^2\,C_{BB}^2\,, \qquad S_{H\widetilde{B}} = 0$$
$$S_{HWB} = -4g_1g_2\,C_{WW}\,C_{BB}\,, \qquad S_{H\widetilde{W}B} = 0$$

$$S_H = \frac{8}{3}\,\lambda g_2^2\,C_{WW}^2\,,$$
$$S_{H\Box} = 2g_2^2\,C_{WW}^2 + \frac{8}{3}\,g_1^2\,\mathcal{Y}_H^2\,C_{BB}^2$$
$$S_{HD} = \frac{32}{3}\,g_1^2\,\mathcal{Y}_H^2\,C_{BB}^2\,.$$

ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 X D$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 X H$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 H D^2$	no	—	

$$\boldsymbol{S}_{eW} = -i g_2\, \widetilde{\boldsymbol{Y}}_e\, C_{WW}$$

$$\boldsymbol{S}_{eB} = -2 i g_1\, (\mathcal{Y}_L + \mathcal{Y}_e)\, \widetilde{\boldsymbol{Y}}_e\, C_{BB}$$

$$\boldsymbol{S}_{uG} = -4 i g_s\, \widetilde{\boldsymbol{Y}}_u\, C_{GG}$$

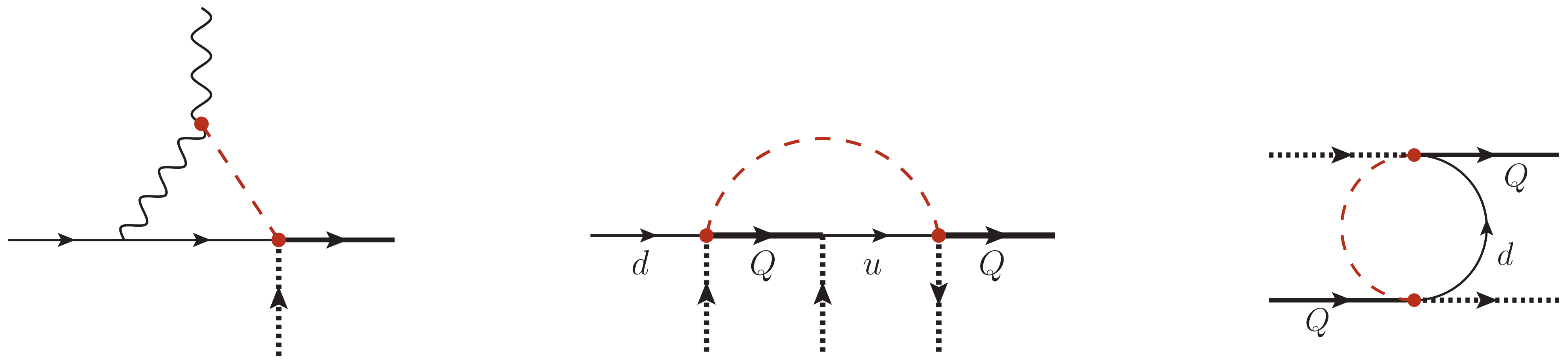
$$\boldsymbol{S}_{uW} = -i g_2\, \widetilde{\boldsymbol{Y}}_u\, C_{WW}$$

$$\boldsymbol{S}_{uB} = -2 i g_1\, (\mathcal{Y}_Q + \mathcal{Y}_u)\, \widetilde{\boldsymbol{Y}}_u\, C_{BB}$$

$$\boldsymbol{S}_{dG} = -4 i g_s\, \widetilde{\boldsymbol{Y}}_d\, C_{GG}$$

$$\boldsymbol{S}_{dW} = -i g_2\, \widetilde{\boldsymbol{Y}}_d\, C_{WW}$$

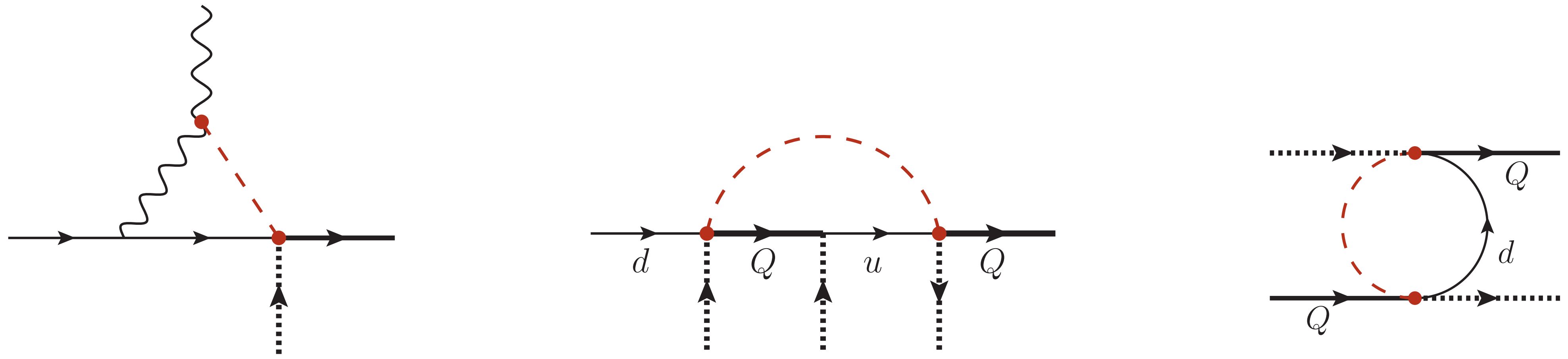
$$\boldsymbol{S}_{dB} = -2 i g_1\, (\mathcal{Y}_Q + \mathcal{Y}_d)\, \widetilde{\boldsymbol{Y}}_d\, C_{BB}$$



ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 XD$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 XH$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 HD^2$	no	—	



$$\boldsymbol{S}_{Hl}^{(1)} = \frac{1}{4}\,\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_L\,C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hl}^{(3)} = \frac{1}{4}\,\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger + \frac{4}{3}\,g_2^2\,C_{WW}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{He} = -\frac{1}{2}\,\widetilde{\boldsymbol{Y}}_e^\dagger \widetilde{\boldsymbol{Y}}_e + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_e\,C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hq}^{(1)} = \frac{1}{4}\left(\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger - \widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger\right) + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_Q\,C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hq}^{(3)} = \frac{1}{4}\left(\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger + \widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger\right) + \frac{4}{3}\,g_2^2\,C_{WW}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hu} = \frac{1}{2}\,\widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_u + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_u\,C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hd} = -\frac{1}{2}\,\widetilde{\boldsymbol{Y}}_d^\dagger \widetilde{\boldsymbol{Y}}_d + \frac{16}{3}\,g_1^2\,\mathcal{Y}_H \mathcal{Y}_d\,C_{BB}^2\,\mathbf{1}$$

$$\boldsymbol{S}_{Hud} = -\widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_d$$

ALP—SMEFT interference

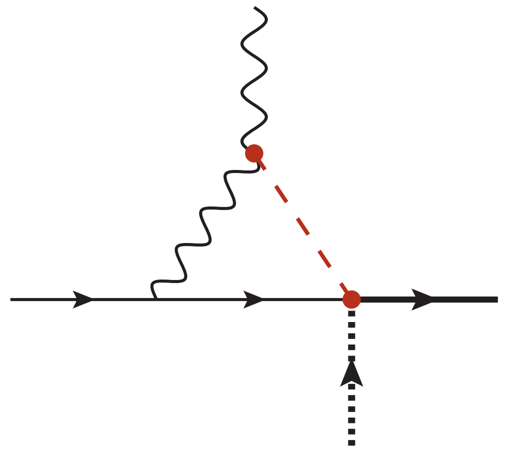
One-loop results for the ALP source terms in the Warsaw basis:

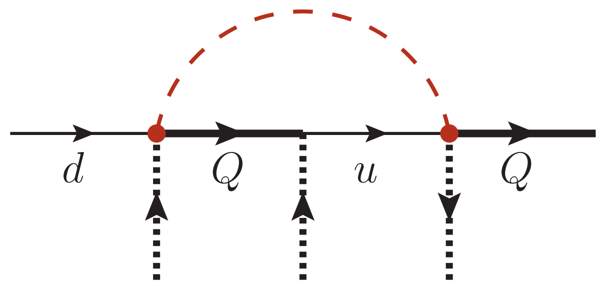
Operator class	Warsaw basis	Way of generation	
Single fermion current			
$\psi^2 XD$	no	—	
$\psi^2 D^3$	no	—	
$\psi^2 XH$	yes	direct	—
$\psi^2 H^3$	yes	direct	EOM
$\psi^2 H^2 D$	yes	direct	EOM
$\psi^2 HD^2$	no	—	

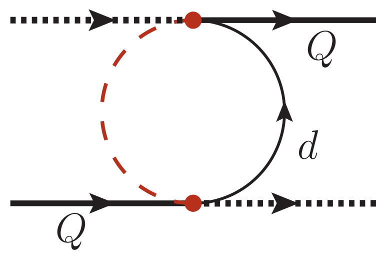
$$\boldsymbol{S}_{eH} = -2\widetilde{\boldsymbol{Y}}_e \boldsymbol{Y}_e^\dagger \widetilde{\boldsymbol{Y}}_e - \frac{1}{2}\widetilde{\boldsymbol{Y}}_e \widetilde{\boldsymbol{Y}}_e^\dagger \boldsymbol{Y}_e - \frac{1}{2}\boldsymbol{Y}_e \widetilde{\boldsymbol{Y}}_e^\dagger \widetilde{\boldsymbol{Y}}_e + \frac{4}{3}\, g_2^2\, C_{WW}^2\, \boldsymbol{Y}_e$$

$$\boldsymbol{S}_{uH} = -2\widetilde{\boldsymbol{Y}}_u \boldsymbol{Y}_u^\dagger \widetilde{\boldsymbol{Y}}_u - \frac{1}{2}\widetilde{\boldsymbol{Y}}_u \widetilde{\boldsymbol{Y}}_u^\dagger \boldsymbol{Y}_u - \frac{1}{2}\boldsymbol{Y}_u \widetilde{\boldsymbol{Y}}_u^\dagger \widetilde{\boldsymbol{Y}}_u + \frac{4}{3}\, g_2^2\, C_{WW}^2\, \boldsymbol{Y}_u$$

$$\boldsymbol{S}_{dH} = -2\widetilde{\boldsymbol{Y}}_d \boldsymbol{Y}_d^\dagger \widetilde{\boldsymbol{Y}}_d - \frac{1}{2}\widetilde{\boldsymbol{Y}}_d \widetilde{\boldsymbol{Y}}_d^\dagger \boldsymbol{Y}_d - \frac{1}{2}\boldsymbol{Y}_d \widetilde{\boldsymbol{Y}}_d^\dagger \widetilde{\boldsymbol{Y}}_d + \frac{4}{3}\, g_2^2\, C_{WW}^2\, \boldsymbol{Y}_d$$







ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$[S_{ll}]_{prst} = \frac{2}{3}\, g_2^2\, C_{WW}^2\, (2\delta_{pt}\,\delta_{sr} - \delta_{pr}\,\delta_{st}) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_L^2\, C_{BB}^2\, \delta_{pr}\,\delta_{s}$$

$$[S_{qq}^{(1)}]_{prst} = \frac{2}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\,\delta_{sr} - \frac{2}{N_c}\,\delta_{pr}\,\delta_{st}\right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_Q^2\, C_{BB}^2\, \delta_{pr}\,\delta_{st}$$

$$[S_{qq}^{(3)}]_{prst} = \frac{2}{3}\, g_s^2\, C_{GG}^2\, \delta_{pt}\,\delta_{sr} + \frac{2}{3}\, g_2^2\, C_{WW}^2\, \delta_{pr}\,\delta_{st}$$

$$[S_{lq}^{(1)}]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_Q\, C_{BB}^2\, \delta_{pr}\,\delta_{st}$$

$$[S_{lq}^{(3)}]_{prst} = \frac{4}{3}\, g_2^2\, C_{WW}^2\, \delta_{pr}\,\delta_{st}$$

ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

Operator class	Warsaw basis	Way of generation	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$\big[S_{ee}\big]_{prst} = \frac{8}{3}\, g_1^2\, \mathcal{Y}_e^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{uu}\big]_{prst} = \frac{4}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\, \delta_{sr} - \frac{1}{N_c}\, \delta_{pr}\, \delta_{st} \right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_u^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{dd}\big]_{prst} = \frac{4}{3}\, g_s^2\, C_{GG}^2 \left(\delta_{pt}\, \delta_{sr} - \frac{1}{N_c}\, \delta_{pr}\, \delta_{st} \right) + \frac{8}{3}\, g_1^2\, \mathcal{Y}_d^2\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{eu}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_e \mathcal{Y}_u\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ed}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_e \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ud}^{(1)}\big]_{prst} = \frac{16}{3}\, g_1^2\, \mathcal{Y}_u \mathcal{Y}_d\, C_{BB}^2\, \delta_{pr}\, \delta_{st}$$

$$\big[S_{ud}^{(8)}\big]_{prst} = \frac{16}{3}\, g_s^2\, C_{GG}^2\, \delta_{pr}\, \delta_{st}$$

ALP—SMEFT interference

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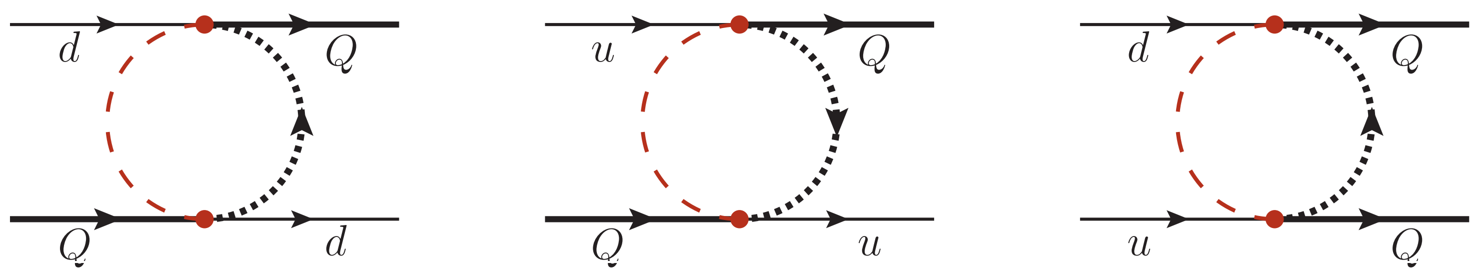
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—



$$[S_{le}]_{prst} = \big(\widetilde{\boldsymbol{Y}}_e\big)_{pt}\big(\widetilde{\boldsymbol{Y}}_e^\dagger\big)_{sr} + \frac{16}{3}\,g_1^2\,\mathcal{Y}_L\,\mathcal{Y}_e\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$[S_{lu}]_{prst} = \frac{16}{3}\,g_1^2\,\mathcal{Y}_L\,\mathcal{Y}_u\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$[S_{ld}]_{prst} = \frac{16}{3}\,g_1^2\,\mathcal{Y}_L\,\mathcal{Y}_d\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$[S_{qe}]_{prst} = \frac{16}{3}\,g_1^2\,\mathcal{Y}_Q\,\mathcal{Y}_e\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$\big[S_{qu}^{(1)}\big]_{prst} = \frac{1}{N_c}\,\big(\widetilde{\boldsymbol{Y}}_u\big)_{pt}\big(\widetilde{\boldsymbol{Y}}_u^\dagger\big)_{sr} + \frac{16}{3}\,g_1^2\,\mathcal{Y}_Q\,\mathcal{Y}_u\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$\big[S_{qu}^{(8)}\big]_{prst} = 2\,\big(\widetilde{\boldsymbol{Y}}_u\big)_{pt}\big(\widetilde{\boldsymbol{Y}}_u^\dagger\big)_{sr} + \frac{16}{3}\,g_s^2\,C_{GG}^2\,\delta_{pr}\,\delta_{st}$$

$$\big[S_{qd}^{(1)}\big]_{prst} = \frac{1}{N_c}\,\big(\widetilde{\boldsymbol{Y}}_d\big)_{pt}\big(\widetilde{\boldsymbol{Y}}_d^\dagger\big)_{sr} + \frac{16}{3}\,g_1^2\,\mathcal{Y}_Q\,\mathcal{Y}_d\,C_{BB}^2\,\delta_{pr}\,\delta_{st}$$

$$\big[S_{qd}^{(8)}\big]_{prst} = 2\,\big(\widetilde{\boldsymbol{Y}}_d\big)_{pt}\big(\widetilde{\boldsymbol{Y}}_d^\dagger\big)_{sr} + \frac{16}{3}\,g_s^2\,C_{GG}^2\,\delta_{pr}\,\delta_{st}$$

ALP—SMEFT interference

One-loop results for the ALP source terms in the Warsaw basis:

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$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

$$\big[S_{ledq}\big]_{prst} = -2\big(\widetilde{\boldsymbol{Y}}_e\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_d^\dagger\big)_{st}$$

$$\big[S_{quqd}^{(1)}\big]_{prst} = -2\big(\widetilde{\boldsymbol{Y}}_u\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_d\big)_{st}$$

$$\big[S_{quqd}^{(8)}\big]_{prst} = 0$$ (starts at 2 loops)

$$\big[S_{lequ}^{(1)}\big]_{prst} = 2\big(\widetilde{\boldsymbol{Y}}_e\big)_{pr}\big(\widetilde{\boldsymbol{Y}}_u\big)_{st}$$

$$\big[S_{lequ}^{(3)}\big]_{prst} = 0$$ (starts at 2 loops)

With very few exceptions, all operators in the Warsaw basis are generated at one-loop order in the ALP model !

Top chromo-magnetic moment

Sample application: chromo-magnetic dipole moment of the top quark

$$\mathcal{L}_{t\bar{t}g} = g_s \left(\bar{t}\gamma^\mu T^a t\, G_\mu^a + \frac{\hat{\mu}_t}{2m_t}\, \bar{t}\,\sigma^{\mu\nu} T^a t\, G_{\mu\nu}^a + \frac{i\,\hat{d}_t}{2m_t}\, \bar{t}\,\sigma^{\mu\nu}\gamma_5\, T^a\, t\, G_{\mu\nu}^a \right)$$

with:

$$\hat{\mu}_t = \frac{y_t v^2}{g_s}\, \Re e\, C_{uG}^{33}\,, \qquad \hat{d}_t = \frac{y_t v^2}{g_s}\, \Im m\, C_{uG}^{33}$$

ALP-induced contribution follows from the solution of:

$$\frac{d}{d\ln\mu}\, \Re e\, C_{uG}^{33} = \boxed{\frac{S_{uG}^{33}}{(4\pi f)^2}} + \left(\frac{15\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi} \right) \Re e\, C_{uG}^{33} + \frac{9\alpha_s}{4\pi}\, y_t\, C_G + \frac{g_s y_t}{4\pi^2}\, C_{HG}$$

$$\frac{d}{d\ln\mu}\, C_G = \boxed{\frac{S_G}{(4\pi f)^2}} + \frac{15\alpha_s}{4\pi}\, C_G$$

$$\frac{d}{d\ln\mu}\, C_{HG} = \left(\frac{3\alpha_t}{2\pi} - \frac{7\alpha_s}{2\pi} \right) C_{HG} + \frac{g_s y_t}{4\pi^2}\, \Re e\, C_{uG}^{33}$$

Top chromo-magnetic moment

At lowest logarithmic order, one finds: [Galda, MN, Renner: 2105.01078]

$$\hat{\mu}_t \approx -\frac{8m_t^2}{(4\pi f)^2}\left[c_{tt}\,C_{GG}\,\ln\frac{4\pi f}{m_t} - \frac{9\alpha_s}{4\pi}\,C_{GG}^2\,\ln^2\frac{4\pi f}{m_t}\right]$$

$$\approx -\left(5.87\,c_{tt}\,C_{GG} - 1.98\,C_{GG}^2\right)\cdot 10^{-3} \times \left[\frac{1\,\text{TeV}}{f}\right]^2$$

Combined with experimental bounds from CMS (2019), we obtain:

$$-0.68 < \left(c_{tt}\,C_{GG} - 0.34\,C_{GG}^2\right) \times \left[\frac{1\,\text{TeV}}{f}\right]^2 < 2.38 \qquad (95\%\ \text{CL})$$

color dipole operator

Weinberg 3-gluon operator

Comparable to strongest bounds following from collider and flavor physics !

Summary

- Axions and axion-like particles appear in many well-motivated extensions of the SM, including those addressing the strong CP problem
- They are an interesting target for searches in high-energy physics, using flavor, collider and precision probes
- If the scale of global symmetry breaking is far above the weak scale, it is important to connect the low-energy ALP couplings in a systematic way with the couplings in the UV theory
- A correct implementation of the left-handed quark currents in the chiral Lagrangian is required to correctly obtain the $K \to \pi a$ decay amplitude
- ALP unavoidably provide source terms for D=6 SMEFT operators

Backup slides

$K \to \pi a$ phenomenology

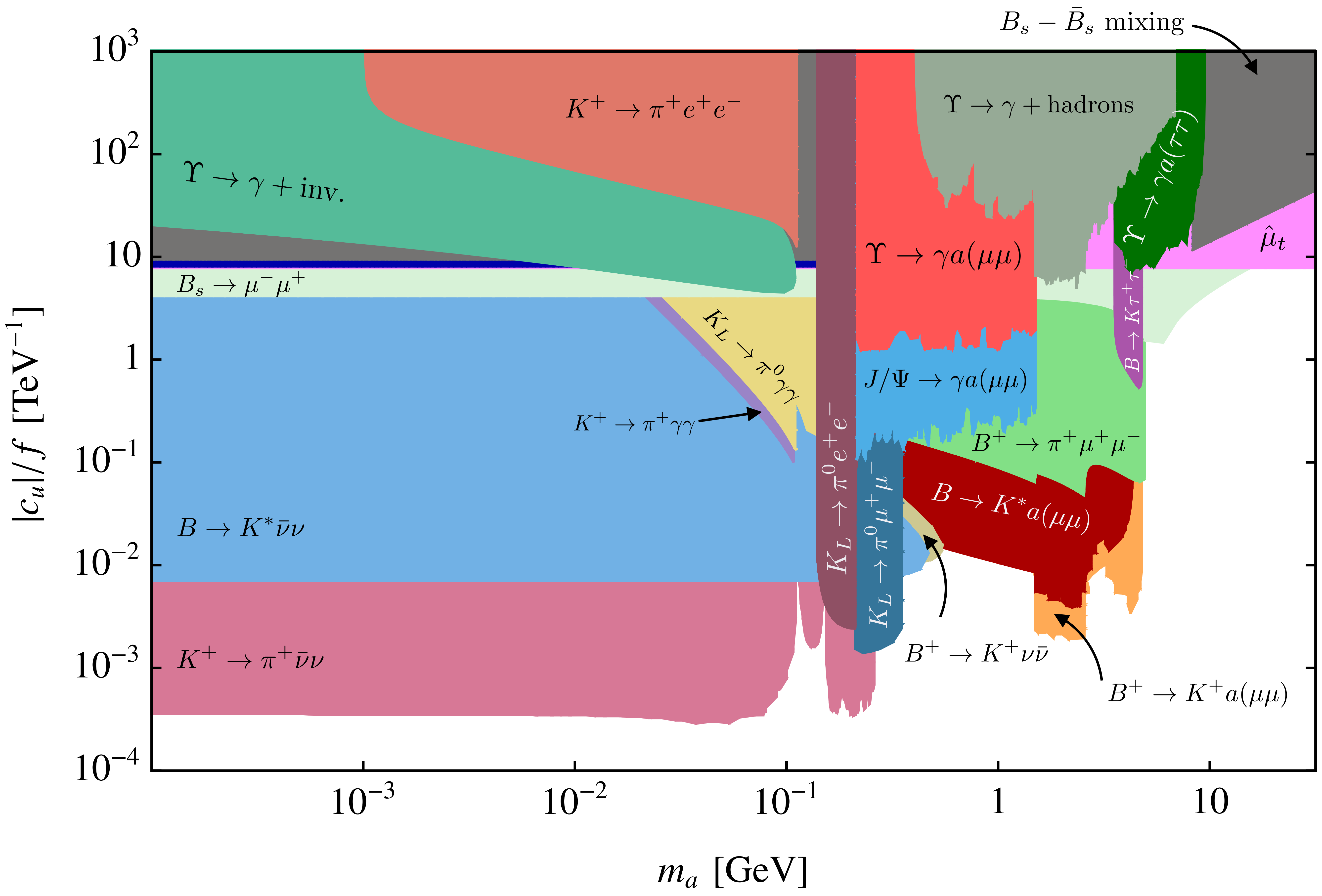
More generally, one can derive bounds $|c_{ii}|/f < \left[\Lambda_{ii}^{\text{eff}}\right]^{-1}$ for all relevant ALP couplings using the NA62 upper limit $\text{Br}(K^- \to \pi^- X) < 2.0 \cdot 10^{-10}$ (90% CL), which implies:

c_{ii}	c_{GG}	c_{WW}	c_{uu}	c_{dd}	$k_D{}^{12}$	$k_D{}^{12}/\lvert V_{td}V_{ts}\rvert$
$\Lambda_{ii}^{\text{eff}}$ [TeV]	61.3	6.5	1126	31.0	$1.9 \cdot 10^8$	60 000

- very strong bounds on flavor-changing ALP couplings in the UV
- strong bounds on ALP couplings to fermions (c_u or c_Q)
- relatively strong bounds on ALP-boson couplings

Flavor physics benchmarks

Flavor-universal ALP-u_R coupling in the UV:



Flavor physics benchmarks

Flavor-universal ALP-e_R coupling in the UV: