Annual Meeting CRC 26th of May 2021

# $t\bar{t}W^{\pm}$ at NLO in QCD: Off-shell effects and precision observables

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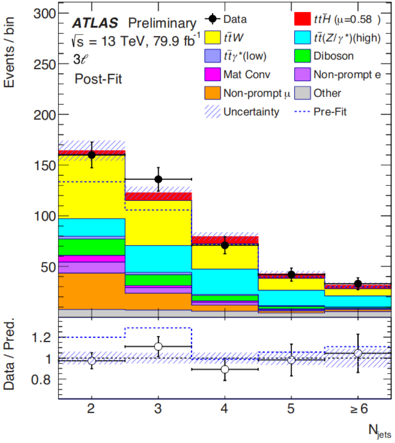
# Outline

- Introduction & Motivation
- $t\bar{t}W$ NLO QCD results
  1. Size of NLO corrections and theoretical uncertainties
  2. Impact of the modelling
- Applications : Ratio $\mathcal{R}$ and Top Quark Charge Asymmetries

# Motivation for $t\bar{t}W^{\pm}$ with multilepton final states

$$pp \to t\bar{t}W^{\pm} \to \ell^{\pm}\nu_{\ell}\ell^{\mp}\nu_{\ell}\ell^{\pm}\nu_{\ell}b\bar{b} \qquad \ell^{\pm} = e^{\pm}, \mu^{\pm}$$




1. Background to $t\bar{t}H$
2. Discrepancy between theory and experiment ATLAS-CONF-2019-045
   -Overall Normalisation (30%-70% for very inclusive cuts)
   -Modelling of top decay
3. Rare same sign lepton signature can be used to constrain BSM physics
4. Phenomenologically relevant for top quark charge asymmetry (more on this later)

# Theoretical predictions for NLO $t\bar{t}W^{\pm}$

- Stable $t\bar{t}W^{\pm}$ $\rightarrow$ General idea about size of NLO corrections

| | |
|---|---|
| NLO QCD | *Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau '11* |
| NLO QCD+EW | *Frixione, Hirschi, Pagani, Shao, Zaro '15* |
| | *Dror, Farina, Salvioni and Serra '16* |
| | *Frederix, Pagani and Zaro '18* |
| | *Frederix, Tsinikos '20* |
| NLO QCD+NNLL | *H.T. Li, C.S. Li and S.A. Li '14* |
| | *Broggio, Ferroglia, Ossola and Pecjak '16* |
| | *Kulesza, Motyka, Schwartländer, Stebel, Theeuwes '19* |

- Include decays for realistic modelling
  1. stable $t\bar{t}W^{\pm}$ +PS $\rightarrow$ No NLO Spin correlations, top and W in PS approx.

| | |
|---|---|
| NLO QCD | *Garzelli, Kardos, Papadopoulos and Trocsanyi '12* |
| | *Maltoni, Pagani and Tsinikos '16* |
| NLO QCD+EW | *Frederix, Tsinikos '20* |

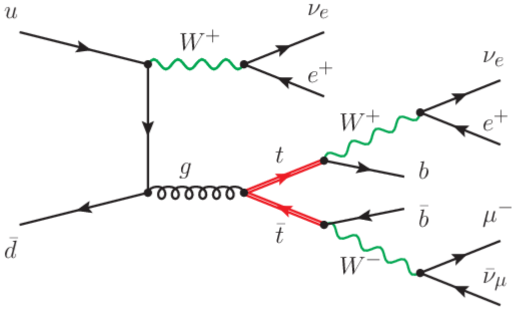
  2. $t\bar{t}W^{\pm}$ in the Narrow-Width Approximation (NWA) + spin corr

| | |
|---|---|
| NLO QCD | *Campbell, Ellis '12* |

  3. Off-shell $t\bar{t}W^{\pm}$

| | |
|---|---|
| NLO QCD | *Bevilacqua, Bi, Hartanto, Kraus, Worek '20* |
| | *Denner, Pelliccioli '20* |
| NLO QCD+EW | *Denner, Pelliccioli '21* |

# Modelling the top decay for $t\bar{t}W^{\pm}$




**Narrow-Width Approximation (NWA):**

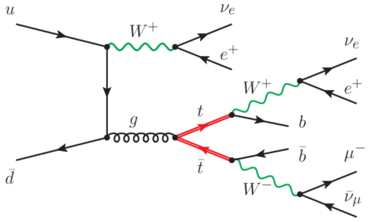
- Includes only diagrams with *double resonant contributions*
- Top and W-Boson are on shell
- Valid for the limit $\Gamma/m \to 0$

$$\frac{1}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \to \frac{\pi}{m_t \Gamma_t} \delta(p_t^2 - m_t^2) + \underbrace{\mathcal{O}\left(\frac{\Gamma_t}{m_t}\right)}_{0.8\%}$$

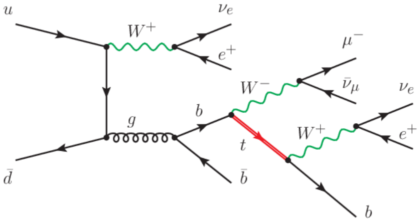
$$\Rightarrow d\sigma^{\text{NWA}} = d\sigma_{t\bar{t}W^+} d\mathcal{B}_{t\to\ell^+\bar{\nu}_\ell b} d\mathcal{B}_{\bar{t}\to\ell^-\nu_\ell b} d\mathcal{B}_{W\to\ell^+\bar{\nu}_\ell}$$

# Modelling the top decay for $t\bar{t}W^{\pm}$

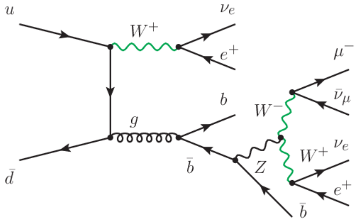
Double resonance




Single resonance



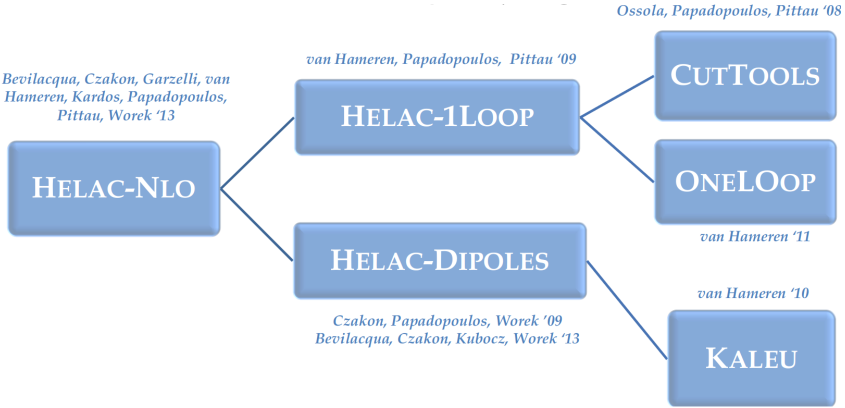

No resonance




**Off-shell effects!**

- Include double, single and non resonant contributions
- Top and W-Boson are described by Breit-Wigner Propagators
- All interference effects consistently incorporated at the matrix element level

# Software




- Output is saved in modified Les Houches & Root files, and Ntuple files.
  *Bern, Dixon, Febres, Cordero, Hoeche, Ita, Kosower, Maitre '14*
- *It can be further modified*: cuts can be added, reweighting to different PDF, renormalization and factorisation scales

# Modelling : Integrated Cross Section

*Bevilacqua, Bi, Hartanto, Kraus, Worek '20*

$pp \to t\bar{t}W^+ \to e^+\nu_e\mu^-\bar{\nu}_\mu e^-\nu_e b\bar{b}$ @$\text{LHC}_{13}$

Modelling (1) Full Off-shell (2) Full NWA (3) $\text{NWA}_{LOdec}$

Central scales $\mu_0 = m_t + m_W/2$

$\mu_0 = H_T/3$ $H_T = \sum_{vis} p_T + p_{T,miss}$

| $t\bar{t}W^+$ | Scale | $\sigma_{\text{LO}}$ [ab] | $\sigma_{\text{NLO}}$ [ab] |
|---|---|---|---|
| full offshell | $H_T/3$ | $115.1^{+26\%}_{-20\%}$ | $124.4^{+3\%}_{-6\%}$ |
| NWA | $H_T/3$ | $115.1^{+26\%}_{-19\%}$ | $124.2^{+3\%}_{-6\%}$ |
| $\text{NWA}_{LOdec}$ | $H_T/3$ | | $130.7^{+10\%}_{-10\%}$ |

- Expected NWA error: $\mathcal{O}\left(\frac{\Gamma_t}{m_t}\right) \approx 0.8\%$

  Here : $\frac{\text{NWA}}{\text{Off-shell}} \approx 0.2\%$ ✓
- NWA and off shell within theoretical uncertainty ✓
- $\text{NWA}_{LOdec}$ has bigger theoretical errors
- Why do off-shell?

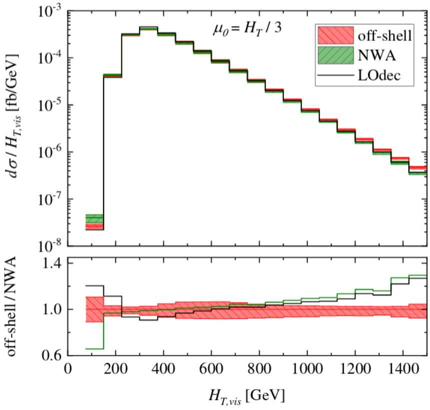
Similar effects for scale $\mu_0 = m_t + m_W/2$

Similar NLO corrections and uncertainty for $t\bar{t}W^-$

# Modelling : Differential Cross Section

*Bevilacqua, Bi, Hartanto, Kraus, Worek '20*

$$pp \to t\bar{t}W^+ \to e^+\nu_e\mu^-\nu_\mu e^+\nu_e b\bar{b} \qquad @ \text{ LHC}_{13}$$



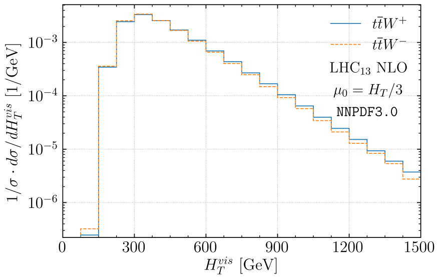

Effects of modelling at differential level:

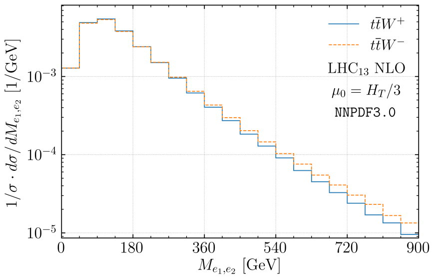
- NWA fails to properly describe fiducial phase space regions, such as tails and kinematic thresholds
  $\frac{\text{Off-shell}}{\text{NWA}}\Big|_{max} \sim 35\%$ vs $\mathcal{O}\left(\frac{\Gamma_t}{m_t}\right) \approx 0.8\%$
- NLO Corrections to decays are sizeable
  NWA vs $\text{NWA}_{LOdec} \sim 57\%$
- For dimensionful observables *off-shell effects can be large for particular phase space regions!* $\to$ Cannot know the latter without full theory predictions

# Cross Section Ratio $\mathcal{R} \equiv \sigma^{NLO}_{t\bar{t}W^+} / \sigma^{NLO}_{t\bar{t}W^-}$

*Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek '20*

**How similar are $t\bar{t}W^+$ and $t\bar{t}W^-$?**




Given that

- the scales of the two processes are *correlated*
- the NLO corrections and theoretical uncertanties are similar

one would expect this observable to exhibit enhanced perturbative stability (NNLO uncertainties!)

# Cross Section Ratio $\mathcal{R}$

*Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek '20*

$$\mathcal{R} \equiv \sigma^{NLO}_{t\bar{t}W^+} / \sigma^{NLO}_{t\bar{t}W^-}$$

| Modelling | Scale | $\mathcal{R} \pm \delta^{theo}$ |
|---|---|---|
| full offshell | $H_T/3$ | $1.81 \pm 0.02$ (1%) |
| NWA | $H_T/3$ | $1.81 \pm 0.03$ (2%) |
| $\text{NWA}_{LOdec}$ | $H_T/3$ | $1.81 \pm 0.02$ (1%) |

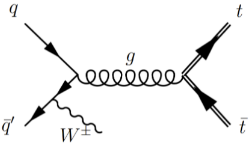
Similar for scale $\mu_0 = m_t + m_W/2$

- Precise observable! $\Rightarrow$ Theoretical error $\sim 2\%$
- Furthermore, it is stable when testing for increasing $p_{T,b}^{min}$ between 25...40 GeV
- Stable with regards to scale choice and modelling
- Error is estimated by scale variation and PDF uncertainty

# $t\bar{t}W^+$: Charge Asymmetry at LHC

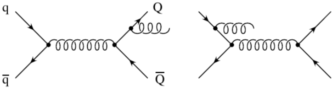
*Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek '20*

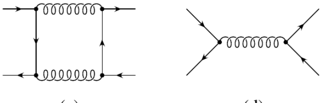
$$pp \to t\bar{t}W^+ \to e^+\nu_e\mu^-\nu_\mu e^+\nu_e b\bar{b} \qquad @ \text{ LHC}_{13}$$




- The process can be thought of as $q\bar{q} \to t\bar{t}$ with the $W^\pm$ emitted from the initial state. $W^\pm$ acts as a polarizer for the quarks, leading to the production of polarized top and anti-top quarks.
  $\to A_c^b, A_c^\ell$ at LO! *Maltoni, Mangano, Tsinikos, and Zaro '14*

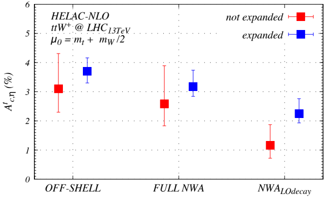
- As for $t\bar{t}$, NLO QCD radiative corrections and interference effects from $q\bar{q}'$ initiated collisions, contribute to the asymmetry at this order. In absence of the symmetric $gg$-channel this leads to a significantly bigger asymmetry than for $t\bar{t}$. *Kühn, Rodrigo '99*



# Charge Asymmetry at the LHC

*Bevilacqua, Bi, Hartanto, Kraus, Nasufi, Worek '20*




$$A_{c,y} = \frac{\sigma\,(\Delta|y| > 0) - \sigma\,(\Delta|y| < 0)}{\sigma\,(\Delta|y| > 0) + \sigma\,(\Delta|y| < 0)} \qquad |y| \equiv |y_t| - |y_{\bar{t}}|$$

| $t\bar{t}W^+$ @ NLO QCD | Off-shell |
|---|---|
| $A^t_{c,\eta}$ [%] | $3.70^{+12\%}_{-11\%}$ |
| $A^t_{c,y}$ [%] | $2.62^{+15\%}_{-13\%}$ |
| $A^\ell_{c,y}$ [%] | $-7.00^{+14\%}_{-11\%}$ |
| $A^b_{c,y}$ [%] | $6.56^{+0.3\%}_{-1.1\%}$ |

- Final results given in terms of *expanded* asymmetries
- The $A^b_{c,y}$ charge asymmetry has quite a small thoretical uncertainty
- After expanding, the uncertainty for $A^t_{c,y}$ and $A^\ell_{c,y}$ comparable to *Maltoni, Mangano, Tsinikos, and Zaro '14*
- Modelling does not impact the size of the errors!

# Summary

- We have studied the modelling of $t\bar{t}W^{\pm}$ @NLO QCD
  1. Full offshell
  2. Full NWA
  3. $\text{NWA}_{LOdec}$
- Off-shell effects *phase space regions* : high $p_T$ regions and kinematical edges
- On the lookout for precision observables : $\mathcal{R} \equiv \sigma^{t\bar{t}W^+}/\sigma^{t\bar{t}W^-}$
  - High precision observable with errors of order $\sim 2\%$
- Charge Asymmetries for $t\bar{t}W^{\pm}$
  - Top and it's decay products : $A_c^t$, $A_c^\ell$, $A_c^b$
  - Size of the theoretical errors : Expanded asymmetries
  - Differential and cumulative asymmetries

# Backup

# Note on $t\bar{t}W^-$ at NLO QCD

*Bevilacqua, Bi, Hartanto, Kraus, Worek '20*

$$pp \to t\bar{t}W^- \to \mu^+\nu_\mu e^-\overline{\nu}_e e^-\nu_e b\bar{b} \qquad @\mathrm{LHC}_{13}$$

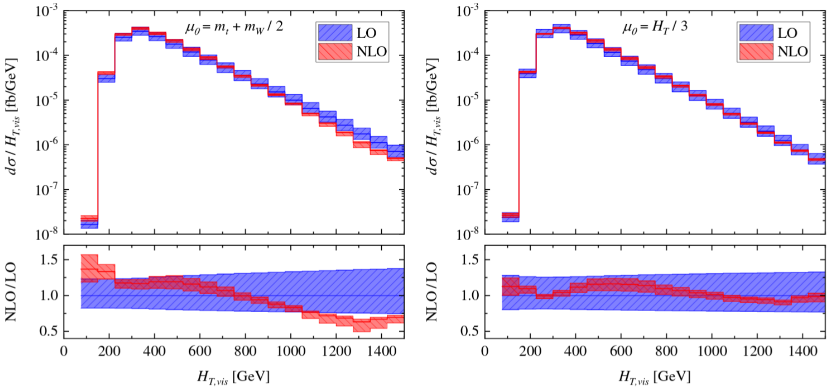
| $t\bar{t}W^-$ | Scale | $\sigma_{\mathrm{LO}}$ [ab] | $\sigma_{\mathrm{NLO}}$ [ab] |
|---|---|---|---|
| full offshell | $H_T/3$ | $62.4^{+27\%}_{-20\%}$ | $68.6^{+5\%}_{-7\%}$ |
| NWA | $H_T/3$ | $62.6^{+27\%}_{-20\%}$ | $68.7^{+5\%}_{-7\%}$ |
| $\mathrm{NWA}_{LOdec}$ | $H_T/3$ | | $72.0^{+11\%}_{-11\%}$ |

- Quite similar to $t\bar{t}W^+$
- Similar NLO corrections, theoretical uncertainties and modelling effects!
- The biggest difference : normalisation! $\to$ almost factor 2 difference with $t\bar{t}W^+$

# Differential Cross Section at NLO QCD

*Bevilacqua, Bi, Hartanto, Kraus, Worek '20*

$t\bar{t}W^+$

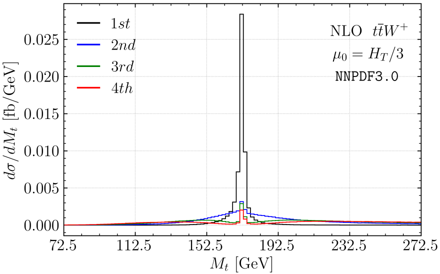



NLO corrections $\pm 35\%$
Theoretical uncertainty $\pm 20\%$

NLO corrections $\pm 10\%$
Theoretical uncertainty $\pm 13\%$
$\rightarrow \mathcal{K}$-factor still not flat!

# Q analysis : Top reconstruction

$$pp \to t\bar{t}W^+ \to e^+\nu_e\mu^-\nu_\mu e^+\nu_e b\bar{b}$$




Consider different histories:

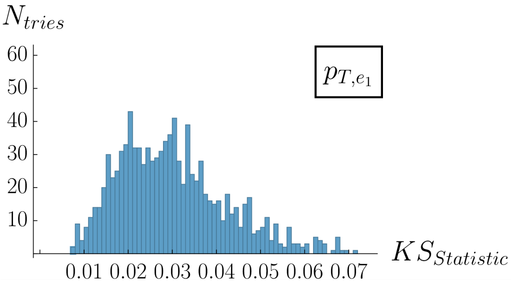
1. $t = e_1^+ + \nu_{e1} + b \;\; ; \;\; \bar{t} = \mu^- + \bar{\nu}_\mu + \bar{b}$
2. $t = e_1^+ + \nu_{e2} + b \;\; ; \;\; \bar{t} = \mu^- + \bar{\nu}_\mu + \bar{b}$
3. $t = e_2^+ + \nu_{e1} + b \;\; ; \;\; \bar{t} = \mu^- + \bar{\nu}_\mu + \bar{b}$
4. $t = e_2^+ + \nu_{e2} + b \;\; ; \;\; \bar{t} = \mu^- + \bar{\nu}_\mu + \bar{b}$

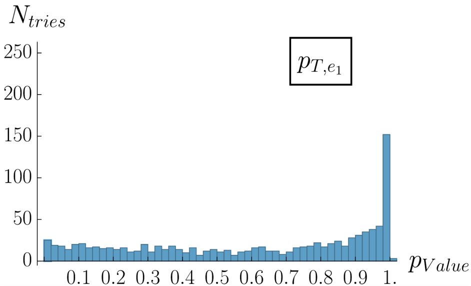
More in case of extra radiation.

Minimize $Q = |M(t) - m_t| + |M(\bar{t}) - m_T|$ with $m_t = 172.5\text{GeV}$

# Kolmogorov Smirnov test

How *similar* are the distributions?



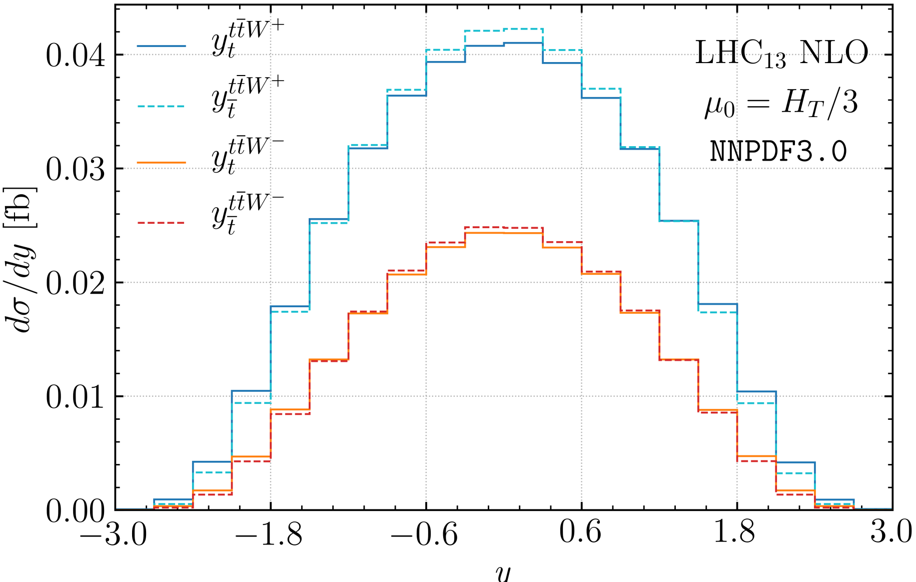




$$KS_{statistic} = \mathrm{sum}_{\mathrm{p_T}} |\mathrm{F}^1_{\mathrm{n_1}}(\mathrm{p_T}) - \mathrm{F}^2_{\mathrm{n_2}}(\mathrm{p_T})|$$

$p \sim 1 \Rightarrow$ Distributions are similar

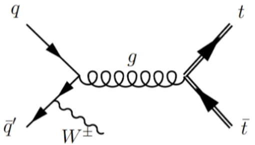
$p \sim 0 \Rightarrow$ Distributions are not similar

# Asymmetries



# How to understand the polarised top production?

**Parke,Shadmi hep-ph 9606419**




(slide from @M.Zaro)

$q\bar{q}' \to t\bar{t}W^{\pm}$ is analogous to $q_L\bar{q}_R \to t\bar{t}$

Possible polarisation states (in beam axis basis):

| | Thres $\beta \to 0$ | High E $\beta \to 1$ |
|---|---|---|
| $\frac{d\sigma_{\uparrow\uparrow}}{d\cos(\theta)} = \frac{d\sigma_{\downarrow\downarrow}}{d\cos(\theta)} = \mathcal{N}(\beta)\frac{\beta^2(1-\beta^2)\sin^2(\theta)}{(1+\beta\cos(\theta))^2}$ | $0$ | $0$ |
| $\frac{d\sigma_{\downarrow\uparrow}}{d\cos(\theta)} = \mathcal{N}(\beta)\frac{\beta^4\sin(\theta)}{(1+\beta\cos(\theta))^2}$ | $0$ | $\mathcal{N}(1)(1-\cos(\theta))^2$ |
| $\frac{d\sigma_{\uparrow\downarrow}}{d\cos(\theta)} = \mathcal{N}(\beta)\frac{\left[(1+\beta\cos(\theta))^2+(1-\beta^2)\right]^2}{(1+\beta\cos(\theta))^2}$ | $4\mathcal{N}(0)$ | $\mathcal{N}(1)(1+\cos(\theta))^2$ |

# Asymmetry at the LHC

## Does the modelling affect the errors?

| $t\bar{t}W^+$ @ NLO QCD | Off-shell | NWA | NWA$_{LOdec}$ | $\delta^{goal}_{scale}$ |
|---|---|---|---|---|
| $A^t_{c,\eta}$ [%] | $3.10(8)^{+39\%}_{-26\%}$ | $2.58(4)^{+51\%}_{-29\%}$ | $1.16(4)^{+61\%}_{-38\%}$ | $^{+19\%}_{-14\%}$ |
| $A^t_{c,y}$ [%] | $2.09(8)^{+51\%}_{-33\%}$ | $1.68(4)^{+60\%}_{-40\%}$ | $0.86(3)^{+77\%}_{-50\%}$ | $^{+19\%}_{-14\%}$ |
| $A^\ell_{c,y}$ [%] | $-7.9(10)^{+27\%}_{-17\%}$ | $-8.43(4)^{+25\%}_{-16\%}$ | $-10.11(3)^{+13\%}_{-9.4\%}$ | $^{+8.5\%}_{-6.0\%}$ |
| $A^b_{c,y}$ [%] | $6.46(8)^{+0.8\%}_{-0.8\%}$ | $6.18(4)^{+2\%}_{-0.8\%}$ | $5.99(3)^{+1.7\%}_{-0.2\%}$ | $^{+2.5\%}_{-2.2\%}$ |

- Modelling shifts the central values, but the size of the theoretical error remains unchanged

# Expanded Asymmetry

$$A^c_{X,y} = \frac{\sigma\left(\Delta_X > 0\right) - \sigma\left(\Delta_X < 0\right)}{\sigma\left(\Delta_X > 0\right) + \sigma\left(\Delta_X < 0\right)} \equiv \frac{\sigma^-}{\sigma^+} \qquad X = t, \ell_t, b \quad \overline{X} = \bar{t}, \ell_{\bar{t}}, \bar{b}$$

$$LO: \qquad A^{c,LO}_y = \frac{\sigma^-_{LO}}{\sigma^+_{LO}}$$

$$NLO: \qquad A^{c,NLO}_y = \frac{\hat{\sigma}^-_{LO} + \delta\sigma^-_{NLO}}{\hat{\sigma}^+_{LO} + \delta\sigma^+_{NLO}}$$

$$\stackrel{\frac{\delta\sigma^{\pm}_{NLO}}{\hat{\sigma}^{\pm}_{LO}}\to 0}{\approx} \frac{\hat{\sigma}^-_{LO}}{\hat{\sigma}^+_{LO}}\left(1 + \frac{\delta\sigma^-_{NLO}}{\hat{\sigma}^-_{LO}} - \frac{\delta\sigma^+_{NLO}}{\hat{\sigma}^+_{LO}}\right)$$

with

$$\hat{\sigma}_{LO} = \left[\sigma_{LO}\left(\Delta_y > 0\right) - \sigma_{LO}\left(\Delta_y < 0\right)\right]\big|_{NLO\ \Gamma_t, PDF}$$

$$\delta\sigma^{\pm}_{NLO} = \left(\sigma_{NLO}\left(\Delta_y > 0\right) - \sigma_{LO}\left(\Delta_y > 0\right)\big|_{NLO\ \Gamma_t, PDF}\right)$$

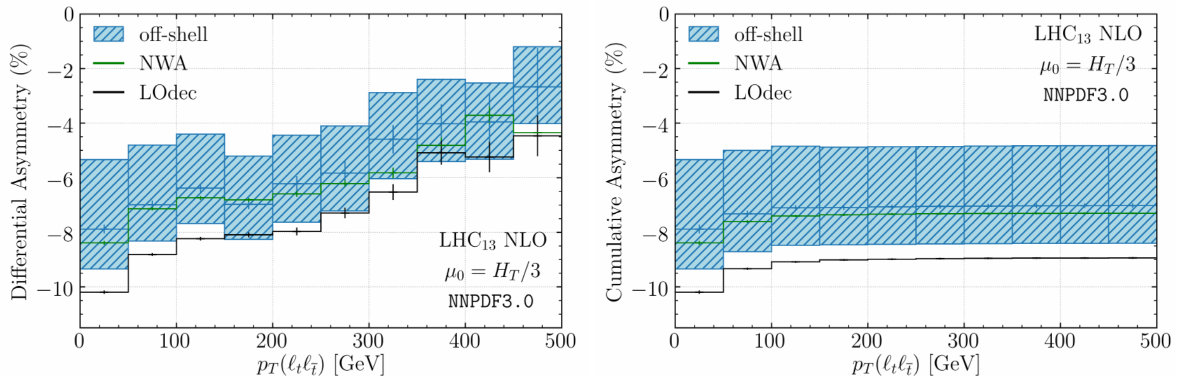
$$\pm \left(\sigma_{NLO}\left(\Delta_y < 0\right) - \sigma_{LO}\left(\Delta_y < 0\right)\big|_{NLO\ \Gamma_t, PDF}\right)$$

# Expanded charge asymmetries

| $\mu_0 = m_t + m_W/2$ | $t\bar{t}W^+$ | $t\bar{t}W^-$ | $t\bar{t}W^\pm$ | $\delta^{old}_{scale}$ |
|---|---|---|---|---|
| $A^t_{c,\eta,exp}$ [%] | $3.70^{+12\%}_{-11\%}$ | $1.31^{+24\%}_{-19\%}$ | $2.87^{+14\%}_{-12\%}$ | $^{+19\%}_{-14\%}$ |
| $A^t_{c,y,exp}$ [%] | $2.62^{+15\%}_{-13\%}$ | $1.97^{+16\%}_{-13\%}$ | $2.40^{+15\%}_{-13\%}$ | $^{+19\%}_{-14\%}$ |
| $A^\ell_{c,y,exp}$ [%] | $-7.00^{+14\%}_{-11\%}$ | $-5.68^{+14\%}_{-11\%}$ | $-6.51^{+14\%}_{-11\%}$ | $^{+8.5\%}_{-6.0\%}$ |
| $A^b_{c,y,exp}$ [%] | $6.56^{+0.3\%}_{-1.1\%}$ | $4.80^{+1\%}_{-1\%}$ | $5.93^{+0.5\%}_{-1.3\%}$ | $^{+2.5\%}_{-2.2\%}$ |

- Expanding the asymmetries reduces the theoretical error
- This is the full theory results, with off-shell modelling and reconstruction of the top from it's most probable decay products.

# Differential and Cumulative asymmetries




- There is good agreement between the full off-shell and NWA.
- Off-shell effects vary between $5 - 30\%$ (left), but are consistently within the MC(!) error bounds.
- $\text{NWA}_{LOdec}$ shows bigger discrepancies (up to $70\%$)