Top-quark fragmentation into a Higgs boson with next-to-leading order accuracy

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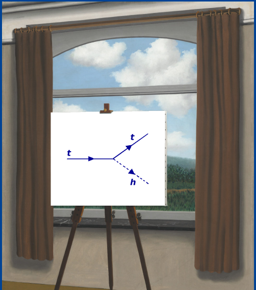


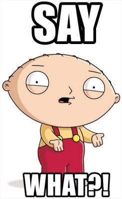
Outline

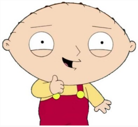
Goals & Motivations

PHENOMENOLOGICAL APPLICATIONS

WHAT?

Computation of the top-quark fragmentation into a Higgs boson with NLO accuracy.





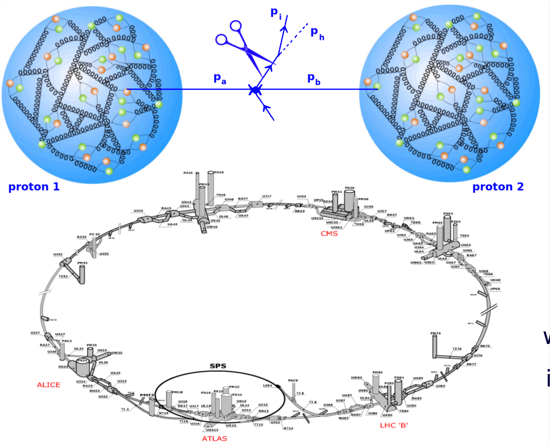
WHY?

- Testing NLO $t\bar{t}h$ production in collinear approximation with the aim of extending the procedure to NNLO $t\bar{t}h$ cross section[1,2],
- Investigating BSM physics[3,4].

[1] E. Braaten, H. Zhang, Phys. Rev. D, 93.5 (2016): 053014,
[2] S. Dawson, L. Reina, Phys. Rev. D, 57.9 (1998): 5851,
[3] F. Boudjema et al., Phys. Rev. D, 92.1 (2015): 015019,
[4] U. Haisch, G. Polesello, Journal of HEP, 2019(2), 29.

The fragmentation function

The final state factorization



In the collinear limit:

$$\hat{s} \gg q^2 \sim m_t^2$$

hard scattering and collinear emission factorise.

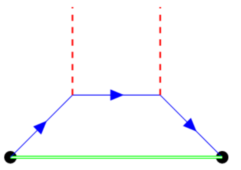
Momenta definitions:

$$z = \frac{n \cdot p_h}{n \cdot p_i},$$

where $n^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$ is the light-cone vector in the Higgs direction.

$$d\hat{\sigma}_{ab\to h+X}(p_a,p_b,p_h) = \sum_i \int_0^1 dz\, d\tilde{\sigma}_{ab\to i+X}(p_a,p_b,p_i;\mu) D_{i\to h}(z;\mu)$$

DEFINITION OF THE FRAGMENTATION FUNCTION



[LO example of the following general formula]

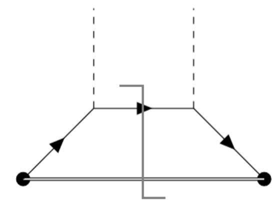
$$D_{q\to h}(z) = \frac{z^{d-3}}{4\pi}\int dx^- e^{-ip_h^+ x^-/z}\frac{1}{2N_c} Tr_{colour} Tr_{Dirac}\Big[\not{n}\,\langle 0|\,\psi_q(0)$$
$$\bar{\mathrm{P}}\exp\left(ig\int_0^\infty dy^- n\cdot A_a(y^- n)T_a^T\right) a_h^\dagger(p_h)a_h(p_h)$$
$$\mathrm{P}\exp\left(-ig\int_{x^-}^\infty dy^- n\cdot A_b(y^- n)T_b^T\right)\bar{\psi}_q(x^- n)\,|0\rangle\Big]$$

Wilson Lines

This definition is Gauge Invariant!

[5] J. Collins, D.E. Soper, Nucl. Phys. B 194.3 (1982): 445-492.

EXAMPLE: THE LO FRAGMENTATION FUNCTION

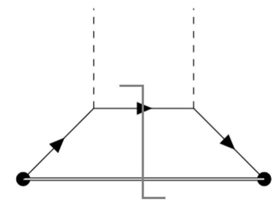


Applying the formula of the previous slide at LO, the fragmentation $D_{t\to h}$ reads:

$$D_{t\to h} = \frac{z^{d-3}}{4\pi}\int \frac{d^d p_t}{(2\pi)^d}(2\pi)\delta^+(p_t^2-m_t^2)(2\pi)\delta^+(p_h^+/z-(p_t+p_h)^+)\ \frac{y_t^2\tilde{\mu}^{2\epsilon}}{2N_c}$$

$$\times \sum_{spins,colours} Tr[\not{n}\frac{\not{p}_t+\not{p}_h+m_t}{(p_t+p_h)^2-m_t^2}(\not{p}_t+m_t)\frac{\not{p}_t+\not{p}_h+m_t}{(p_t+p_h)^2-m_t^2}].$$

EXAMPLE: THE LO FRAGMENTATION FUNCTION



Using Cutkosky rules[6], the phase-space becomes a loop integral[7]:

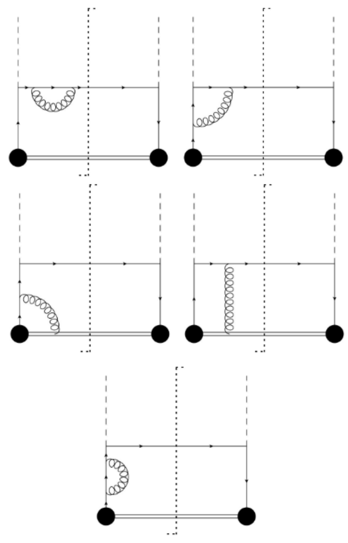
$$\delta(x) \to \frac{1}{2\pi i}\left(\frac{1}{x-i\varepsilon} - \frac{1}{x+i\varepsilon}\right)$$

$$D_{t\to h} = \frac{z^{d-3}}{4\pi}\int \frac{d^d p_t}{(2\pi)^d}(2\pi)\,\boxed{\delta^+(p_t^2 - m_t^2)}\,(2\pi)\,\boxed{\delta^+(p_h^+/z - (p_t+p_h)^+)}\,\frac{y_t^2\tilde{\mu}^{2\epsilon}}{2N_c}$$
$$\times \sum_{spins,colours} Tr[\not{n}\frac{\not{p}_t+\not{p}_h+m_t}{(p_t+p_h)^2-m_t^2}(\not{p}_t+m_t)\frac{\not{p}_t+\not{p}_h+m_t}{(p_t+p_h)^2-m_t^2}].$$

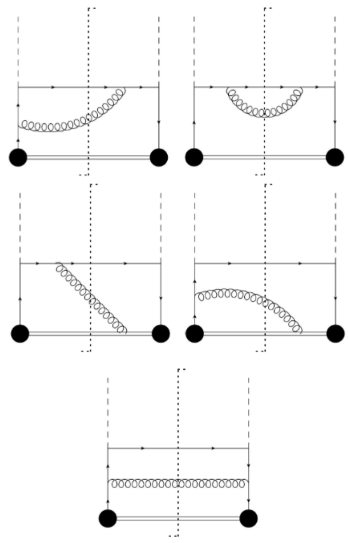
[6]R. Cutkosky, J.Math.Phys. 1.5 (1960): 429-433,
[7]K. Melnikov, A. Mitov, Phys. Rev. D, 70.3 (2004): 034027.

THE NLO FRAGMENTATION FUNCTION CONTRIBUTIONS

Virtual corrections



Real corrections



Methods for loop computation

Differential equations method

- Reduction to MIs can be performed with the usual techniques[8,9]:
  - ✓ 8 master found for virtual corrections,
  - ✓ 8 master found for real corrections.
- MI derivatives with respect to each kinematic invariant x_i can be computed by introducing the differential operators:

$$O_{jk} = p_j^\mu \sum_{i=1}^{n} \frac{\partial x_i}{\partial p_k^\mu} \frac{\partial f(\vec{x},\epsilon)}{\partial x_i} = \sum_{i=1}^{n} a_{i,jk}(x_i) \frac{\partial f(\vec{x},\epsilon)}{\partial x_i}.$$

- A system of first order linear differential equations for the MIs can be derived:

$$\partial_{x_i} \vec{f}(\vec{x},\epsilon) = A_{x_i}(\vec{x},\epsilon)\, \vec{f}(\vec{x},\epsilon).$$

[8] R.N. Lee, arXiv:1310.1145 (2013),
[9] A.V. Smirnov, Comp. Phys. Com. 189 (2015): 182-191.

Canonical basis approach

- It is possible to choose a basis of MIs, the Canonical basis[10], such that:

$$d\vec{f}(\vec{x},\epsilon) = \epsilon\, dA(\vec{x})\, \vec{f}(\vec{x},\epsilon),$$

with

$$dA(\vec{x})_{ij} = \sum_k c_{ijk}\, \mathrm{dlog}(\alpha_k(\vec{x})).$$

- The solution of the differential equations system is:

$$\vec{f}(\vec{x},\epsilon) = \mathrm{P}\exp\left[\epsilon \int_\gamma d\tilde{A}(\vec{x}')\right] \vec{f}(\vec{x}_0,\epsilon).$$

Canonical MIs can be expanded in Taylor series around $\epsilon = 0$:

$$\vec{f}(\vec{x},\epsilon) = \sum_{k=0}^{\infty} \epsilon^k\, \vec{f}^{(k)}(\vec{x}) \longmapsto \vec{f}^{(k)}(\vec{x}) = \int_\gamma d\tilde{A}(\vec{x}')\, \vec{f}^{(k-1)}(\vec{x}') + \vec{f}(\vec{x}_0,\epsilon)$$

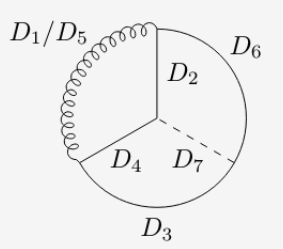
[10] J.M. Henn, Phys. Rev. Lett. 110.25 (2013): 251601.

Results

A semi-algorithmic approach to obtain the canonical basis

- Generic canonical master: $f_i^{virt} = \epsilon^{n_i}\, M(m_t, m_h, z)\; T_i^{virt}$.
- Topology definition (here for the virtual corrections):

D_1	$n \cdot p_g$
D_2	$(p_t - p_g)^2 - m_t^2$
D_3	$(p_h + p_t)^2 - m_t^2$
D_4	$(p_t + p_h - p_g)^2 - m_t^2$
D_5	p_g^2
D_6	$p_t^2 - m_t^2$
D_7	$n \cdot p_t - \frac{1-z}{z}\, n \cdot p_h$



- Semi-algorithmic approach:
  - ✓ T_i^{virt} found by maximizing the symmetries,
  - ✓ $M(m_t, m_h, z)$ found by applying Magnus transformations [11].

[11] M. Argeri, et al. Journal of HEP 2014.3 (2014): 82.

CANONICAL BASIS FOR VIRTUAL CORRECTIONS

Canonical form for MIs of the virtual topology:

$$f_1^{virt} = \epsilon^2\, n\cdot p_h\, T_1^{virt},$$

$$f_2^{virt} = \epsilon^2 m_h \sqrt{4m_t^2 - m_h^2}\, n\cdot p_h\, T_2^{virt},$$

$$f_3^{virt} = \epsilon^3\, (n\cdot p_h)^2\, T_3^{virt},$$

$$f_4^{virt} = \epsilon^2\, n\cdot p_h\, T_4^{virt},$$

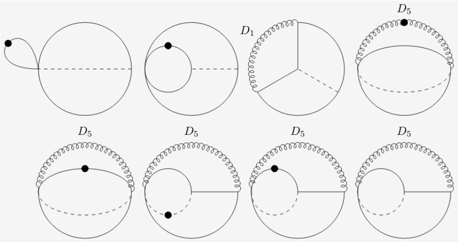
$$f_5^{virt} = \epsilon^2\, n\cdot p_h\, T_5^{virt},$$

$$f_6^{virt} = \epsilon^2\, \frac{1-z}{z}\, (n\cdot p_h)^2\, T_6^{virt},$$

$$f_7^{virt} = \epsilon^2 m_h \sqrt{4m_t^2 - m_h^2}\, n\cdot p_h\, T_7^{virt},$$

$$f_8^{virt} = \epsilon^3\, n\cdot p_h\, T_8^{virt}.$$

Pre-canonical T_i^{virt}:



CANONICAL MATRIX FOR VIRTUAL CORRECTIONS

- For a dlog shape of the canonical matrix, the change of variable $m_t^2 \to \frac{m_h^2}{4}(-\tau^2+1)$ is performed.
- The canonical matrix reads:

$$\begin{aligned} \mathrm{d}A_\tau = & M_1 \,\mathrm{dlog}\,(\tau) + M_2 \,\mathrm{dlog}\,(1-\tau) + M_3 \,\mathrm{dlog}\,(1+\tau) \\ & + M_4 \,\mathrm{dlog}\,(2 - z\,(1-\tau)) + M_5 \,\mathrm{dlog}\,(-2 + z\,(1+\tau)) \\ & + M_6 \,\mathrm{dlog}\,\left(-4 + z\left(3+\tau^2\right)\right), \end{aligned}$$

  where the M_i are 8×8 matrices with purely rational entries.
- Solution given in terms of GPLs.
- Integration constants are m_h and z dependent!

Collinear Renormalization

The bare $t \to h$ fragmentation function is:

$$D_{h\to h} = \delta(1-z) + \mathcal{O}(y_t^2)$$

$$D^B_{t\to h}(z) = (Z_{th} \otimes D_{h\to h})(z) + (Z_{tt} \otimes D_{t\to h})(z) + \sum_{i\neq t,h} (Z_{ti} \otimes D_{i\to h})(z)$$

$$= Z_{th}(z) + (Z_{tt} \otimes D_{t\to h})(z) + \mathcal{O}(y_t^2 \alpha_s^2, y_t^4),$$

where the renormalization constants are:

$$\begin{aligned} Z_{th}(z) =& \frac{y_t^2}{16\pi^2} \frac{1}{\epsilon} P^{(0)T}_{th}(z) \\ &+ \frac{y_t^2}{16\pi^2} \frac{\alpha_s}{2\pi} \left(\frac{1}{2\epsilon} P^{(1)T}_{th}(z) + \frac{1}{2\epsilon^2} (P^{(0)T}_{qq} \otimes P^{(0)T}_{th})(z) - \frac{\beta^{(0)}_{th}}{4\epsilon^2} P^{(0)T}_{th}(z) \right) \\ &+ \mathcal{O}(y_t^2 \alpha_s^2, y_t^4), \\ Z_{tt}(z) =& \delta(1-z) + \frac{\alpha_s}{2\pi} \frac{1}{\epsilon} P^{(0)T}_{qq} + \mathcal{O}(\alpha_s^2, y_t^2), \end{aligned}$$

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where the renormalization constants are:

$$Z_{th}(z) = \frac{y_t^2}{16\pi^2}\frac{1}{\epsilon}P^{(0)T}_{th}(z) + \frac{y_t^2}{16\pi^2}\frac{\alpha_s}{2\pi}\left(\underbrace{\boxed{\frac{1}{2\epsilon}P^{(1)T}_{th}(z)}}_{?} + \boxed{\frac{1}{2\epsilon^2}(P^{(0)T}_{qq} \otimes P^{(0)T}_{th})(z)} - \boxed{\frac{\beta^{(0)}_{th}}{4\epsilon^2}P^{(0)T}_{th}(z)}\right) + \mathcal{O}(y_t^2\alpha_s^2, y_t^4),$$

$$Z_{tt}(z) = \delta(1-z) + \boxed{\frac{\alpha_s}{2\pi}\frac{1}{\epsilon}P^{(0)T}_{qq}} + \mathcal{O}(\alpha_s^2, y_t^2).$$

SPLITTING FUNCTION RESULTS

$$P_{th}^{(0)T}(z) = z$$

$$P_{th}^{(1)T}(z) = C_F \Bigg[-8z\, Li_2(z) + z\, ln^2(1-z) - \frac{1}{2} z\, ln^2(z) + 3z\, ln(1-z) \\ - 4z\, ln(z)\, ln(1-z) + \left(-1 + \frac{1}{2}z\right) ln(z) + \left(-\frac{13}{2} + 15z\right) \Bigg]$$

$$P_{gh}^{(1)T}(z) = 2T_F \Big[2(-3 + 2z + z^2) - (1 + 5z)\, ln(z) + z\, ln^2(z) \Big]$$

Fragmentation Function Results

$$
\begin{aligned}
D^{(1)}_{t\to H}(z) = {} & \frac{\alpha_s C_F}{2\pi}\frac{y_t^2}{16\pi^2}\Bigg\{\left[\frac{1}{4}(2-5z)+z\ln(1-z)-\frac{z}{2}\ln(z)\right]L^2+\Bigg[6z\,\mathrm{Re}\left[\mathrm{Li}_2\left(\frac{z}{x+}\right)\right] \\
& +2z\mathrm{Li}_2(r)+4z\mathrm{Li}_2(z)+4z\arg\left(\frac{x^+}{x^-}\right)\mathrm{Im}\left[\ln\left(1-\frac{z-x^+}{z-x^-}\right)\right]-\frac{5z}{4}\arg^2\left(\frac{x^+}{x^-}\right) \\
& -2z\,\mathrm{Im}^2\left[\ln\left(1-\frac{z-x^+}{z-x^-}\right)\right]+\frac{3z}{4}\ln^2(1-r)+\frac{z}{4}\ln^2(r)-\frac{3z}{2}\ln(1-r)\ln(1-z) \\
& -\frac{z}{2}\ln(r)\ln(1-z)+\frac{z}{2}\ln(1-r)\ln(r)-\frac{z}{4}\ln^2(1-z)-\frac{z}{2}\ln^2(z)+4z\ln(1-z)\ln(z) \\
& -\frac{r}{z}\left(4\xi^2(1-z)+8-8z+3z^2\right)\ln(1-z)-\frac{r}{\xi z^2}\Bigg(8\pi\xi^3(1-z)z+2\pi\xi z^3 \\
& +\beta\left(32\xi^4(1-z)-4\xi^2(2-3z-z^2)+(2-z)z^2\right)\Bigg)\left(\mathrm{Im}\left[\ln\left(1-\frac{z-x^+}{z-x^-}\right)\right]\right. \\
& \left.-\arg\left(\frac{x^+}{x^-}\right)\right)-\frac{r}{2\xi^2z^2}\left(32\xi^6(1-z)-8\xi^4(2-3z)-4\xi^2z+z^3\right)\ln(r)+\Bigg(8\xi^2-3 \\
& +\frac{z}{2}\Bigg)\ln(z)+\frac{r}{6\xi z^2}\Bigg(4\xi^3(1-z)(27-(45+4\pi^2)z)-\xi z(36-63z+(54+4\pi^2)z^2) \\
& -3\beta\pi\left(32\xi^4(1-z)-4\xi^2(2-3z-z^2)+(2-z)z^2\right)\Bigg)\Bigg]L \\
& -\frac{2\xi^2z-(2+z)}{\beta\xi}\Bigg(2\,\mathrm{Im}[\mathrm{Li}_3(x^+)]-\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{z}{x+}\right)\right]-2\,\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{z-1}{z-x+}\right)\right] \\
& -\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{z}{z-x+}\right)\right]-\mathrm{Im}\left[\mathrm{Li}_3\left(-\frac{(1-z)x^+}{z-x+}\right)\right]-\mathrm{Im}\left[\mathrm{Li}_3\left(1-\frac{1-x^+}{1-t}\right)\right] \\
& -\mathrm{Im}\left[\mathrm{Li}_3\left(1-\frac{x^+}{t}\right)\right]-\mathrm{Im}\left[\mathrm{Li}_3\left(1-\frac{z}{1-w+}\right)\right]+\frac{1}{2}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{r+z(w^+-1)}{zw+}\right)\right] \\
& +\frac{1}{2}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{zw^+}{r+z(w^+-1)}\right)\right]-\frac{1}{2}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{rw^+}{r+z(w^+-1)}\right)\right] \\
& -\frac{1}{2}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{r+z(w^+-1)}{rw^+}\right)\right]\Bigg)-9z\mathrm{Li}_3(1-z)-2z\mathrm{Li}_3\left(1-\frac{z}{r}\right)-4z\mathrm{Li}_3(1-r)
\end{aligned}
$$

Conclusions

SUMMARY & OUTLOOK

ACHIEVEMENTS

- ✓ Analytic computation of the MIs,
- ✓ $D_{t\to h}(z)$ fragmentation function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $D_{g\to h}(z)$ fragmentation function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $P_{th}^{T}(z)$ splitting function at $\mathcal{O}(y_t^2\alpha_s)$,
- ✓ $P_{gh}^{T}(z)$ splitting function at $\mathcal{O}(y_t^2\alpha_s)$.

OUTLOOK

- × Convolution of the fragmentation functions with $t\bar{t}$,
- × NNLO computation of the fragmentation functions in $m_h \to 0$.


That's all folks!