

# Top-Pair Events with B-hadrons at the LHC

Michał Czakon, **Terry Generet**, Alexander Mitov, René Poncelet  
arXiv:2102.08267

Annual CRC Meeting 2021

May 27, 2021  
RWTH Aachen University

# Table of Contents

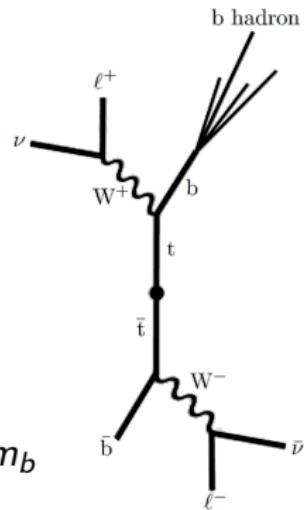
- 1 Introduction
- 2 NNLO Subtraction Schemes and Fragmentation
- 3 Results

# Top-pairs with B-Hadrons

- Process considered:

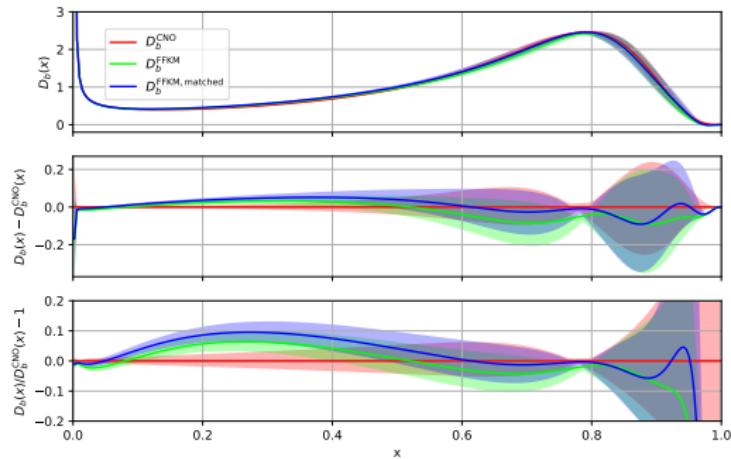
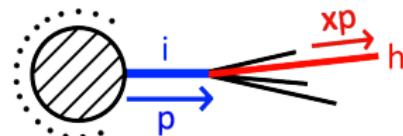
$$\begin{aligned} p \, p \rightarrow t(\rightarrow B \, W^+ + X) \, \bar{t}(\rightarrow \bar{b} \, W^-) \\ \downarrow \ell^+ \nu_\ell \qquad \qquad \qquad \downarrow \ell^- \bar{\nu}_\ell \end{aligned}$$

- Measurements of B-hadrons very precise  
 $\Rightarrow$  high-precision top-mass determination
- High top mass  $\Rightarrow$  small power corrections in  $m_b$
- Production of hadrons is a non-perturbative effect



# Fragmentation Functions

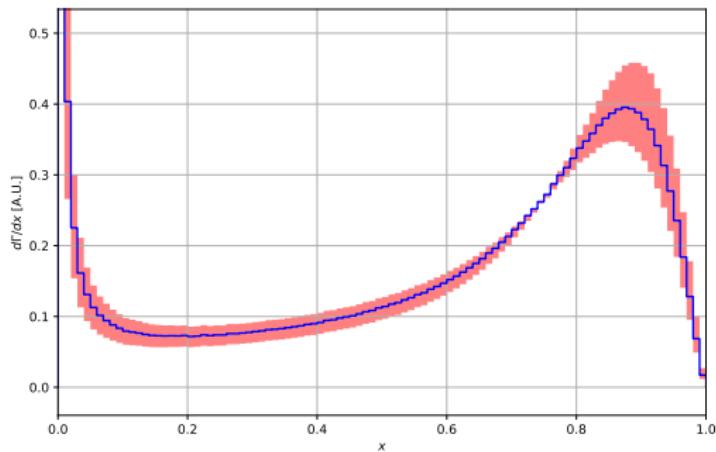
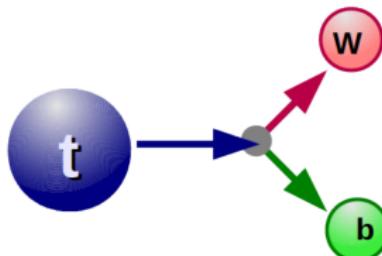
- Transition parton  $\rightarrow$  hadron in the final state
- "Probability distribution" to find a hadron  $h$  with a fraction  $x$  of the parton  $i$ 's momentum:  $D_{i \rightarrow h}(x)$
- Only considers longitudinal kinematics;  $i, h$  massless
- Non-perturbative: fitted to data
- Scale dependent
- Analogous to PDFs



# Example: Top-decay to a B-meson at LO

- $t \rightarrow B W^+ + X$

- Partonically at LO:  
 $t \rightarrow b W^+$

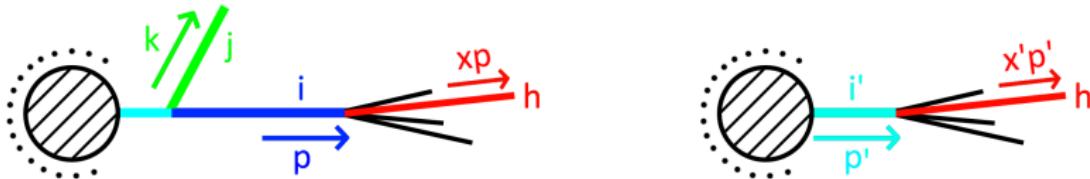


# Introduction to Subtraction Schemes

- Strategy for numerical integration of cross sections
- Cross sections contain singularities in  $d=4$  (soft, collinear)
- Idea: subtract divergences differentially (subtraction terms), add them in integrated form (integrated subtraction terms)
- Both value and kinematics of subtraction term must match cross section in singular limit
- Here: sector-improved residue subtraction scheme (previously implemented in C++ library STRIPPER)

# Subtraction Schemes and Fragmentation

- Without fragmentation: cannot distinguish collinear quark-pair  $q(p_1) + \bar{q}(p_2)$  from  $g(p_1 + p_2)$
- With fragmentation: both momentum of fragmenting particle and flavour matter  
 $\Rightarrow$  must store flavour and e.g.  $p_1^0/(p_1^0 + p_2^0)$
- Introduce concept of reference observables: match reference observable for cross section and subtraction term by rescaling the momentum fraction



# Subtraction Schemes and Fragmentation (Continued)

- Without fragmentation: cannot distinguish  $q(p) + g(0)$  from  $q(p)$
- With fragmentation: cannot remove gluon if it is the fragmenting particle
- Usually: have to recalculate integrated subtraction terms
- Important observation: not necessary if each subtraction term cancels only one singularity
- This is the case for STRIPPER  $\Rightarrow$  major simplification

# Isolated Top Decay: Setup

- Previously considered through NLO

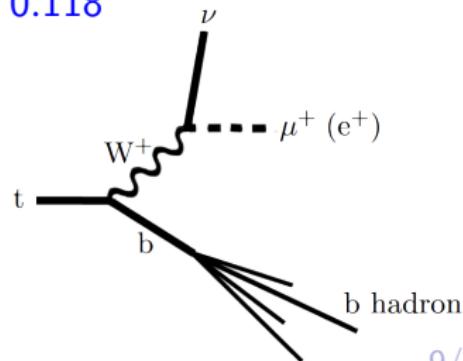
*S. Biswas, K. Melnikov and M. Schulze (2010)*

- On-shell  $W^+$  (narrow width approximation)

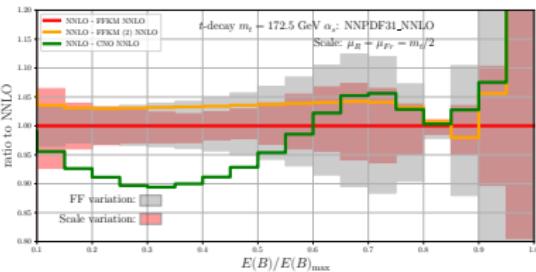
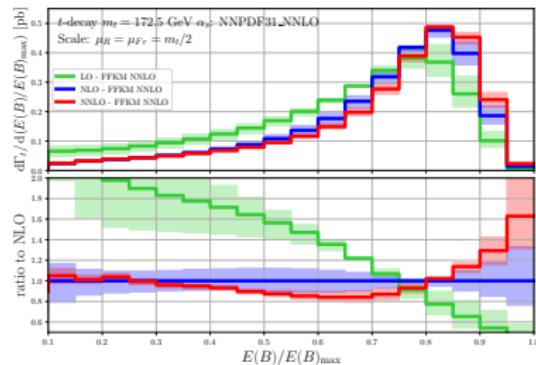
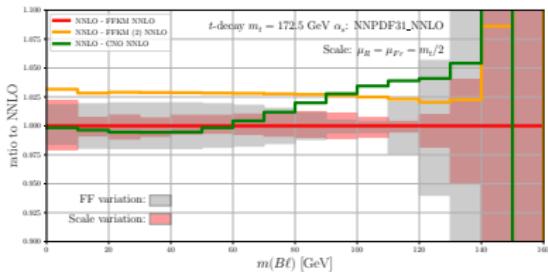
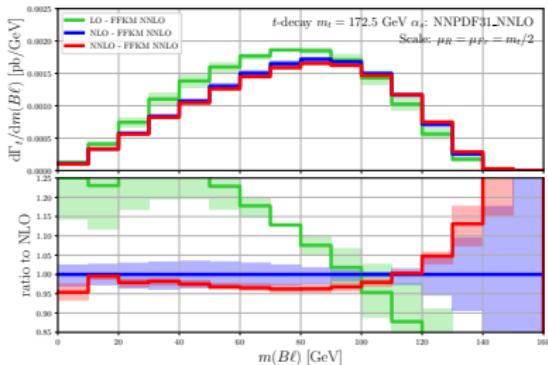
- Parameters:

$$m_t = 172.5 \text{ GeV}, m_W = 80.385 \text{ GeV}, \Gamma_W = 2.0928 \text{ GeV}, \\ m_b = (4.66 \text{ GeV}, 4.75 \text{ GeV}), \alpha_s(M_Z) = 0.118$$

- 7-point scale variation with central scales  $\mu_R = \mu_{Fr} = m_t/2$
- Single cut:  $E(B) > 5 \text{ GeV}$



# Isolated Top Decay: Plots



# Top-Pair Events with B-hadrons at the LHC: Setup

- Previously studied at NLO

*A. Kharchilava (2000), S. Biswas, K. Melnikov and M. Schulze (2010)*

*K. Agashe, R. Franceschini and D. Kim (2013), K. Agashe, R. Franceschini, D. Kim and M. Schulze (2016)*

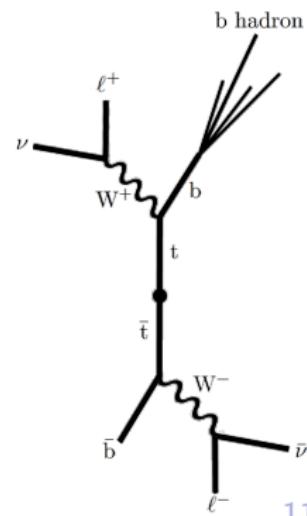
- On-shell  $W^+$  (narrow width approximation)

- Parameters as before

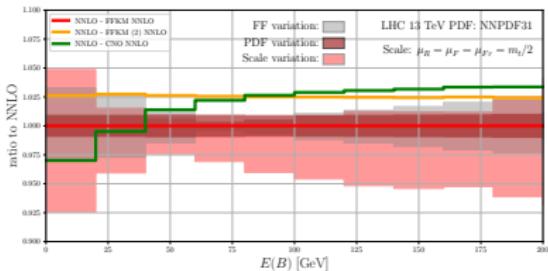
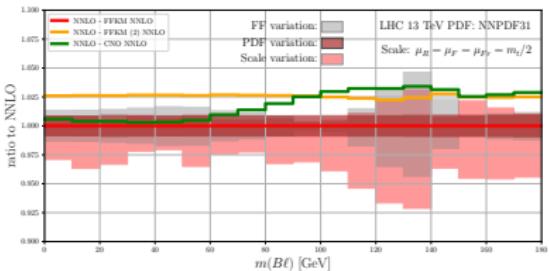
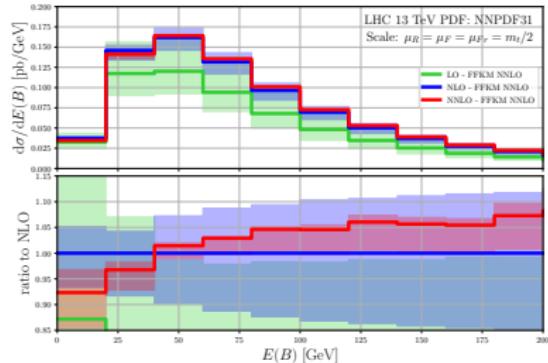
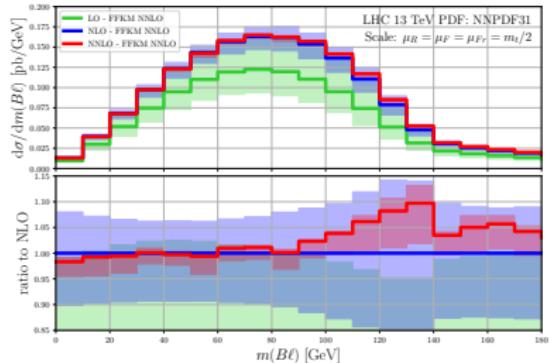
- 15-point scale variation with central scales  $\mu_R = \mu_F = \mu_{Fr} = m_t/2$  and  $1/2 \leq \mu_i/\mu_j \leq 2$

- PDF set: NNPDF3.1

- $p_T(B) > 10 \text{ GeV}$  and  $|\eta(B)| < 2.4$

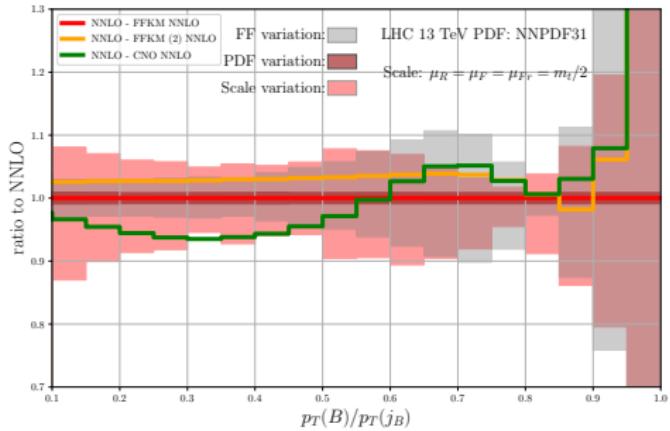
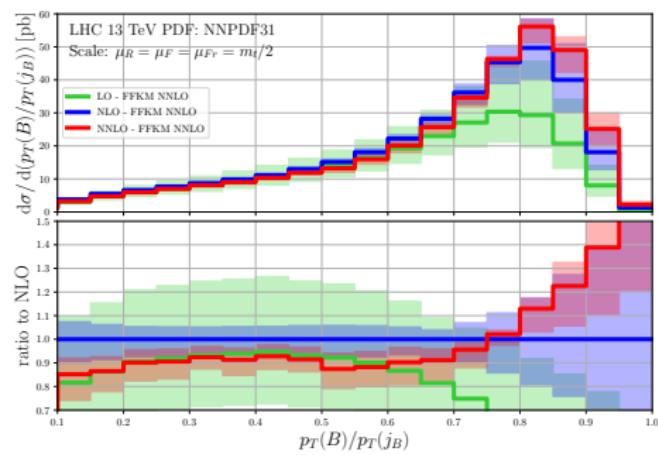


# Top-Pair Events with B-hadrons at the LHC: Plots



# Top-Pair Events with B-hadrons at the LHC: Jet Ratio

- Jet algorithm: anti- $k_T$  with  $R = 0.8$



## Conclusion and Outlook

- Fragmentation has been implemented in STRIPPER
- First application: top-quark pairs at the LHC
- Big reduction in scale uncertainties from NLO to NNLO
- $\Rightarrow$  potential for more accurate top-mass determination
- PDF-insensitive extraction of FFs at LHC plausible
- Framework completely general: can describe the production of any hadron in any process at NNLO

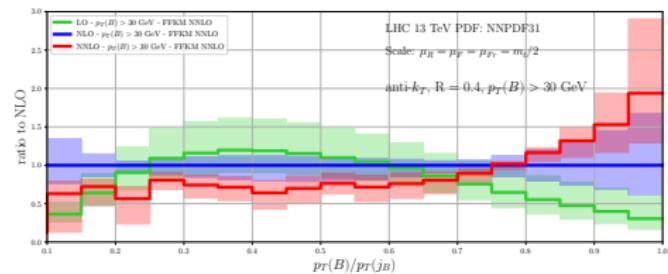
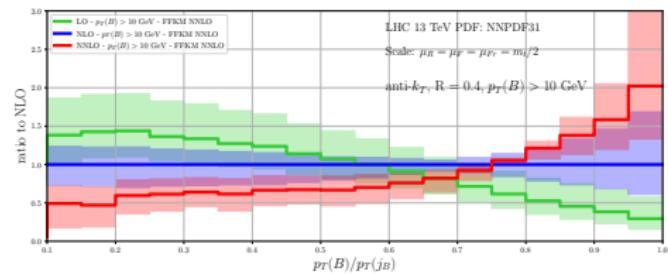
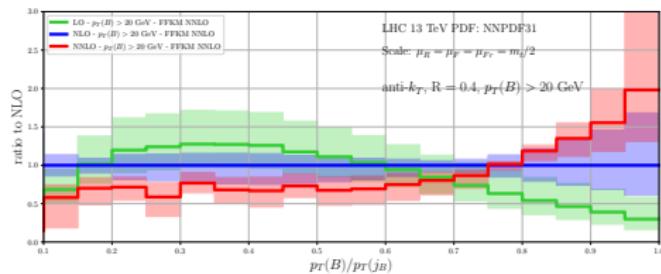
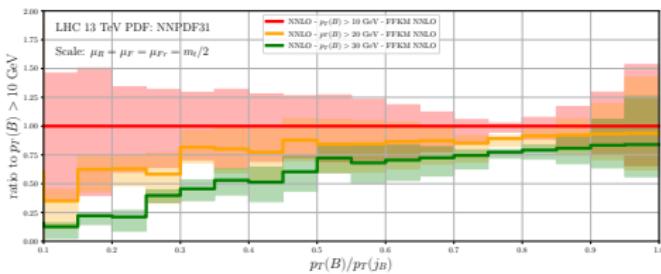
# Perturbative Vs. Non-perturbative

- Production of b-quarks is perturbative  
⇒ Can separate perturbative and non-perturbative parts
- Perturbative part: previously calculated through NNLO  
*K. Melnikov and A. Mitov (2004), A. Mitov (2005)*
- Non-perturbative part: a single function fitted to data

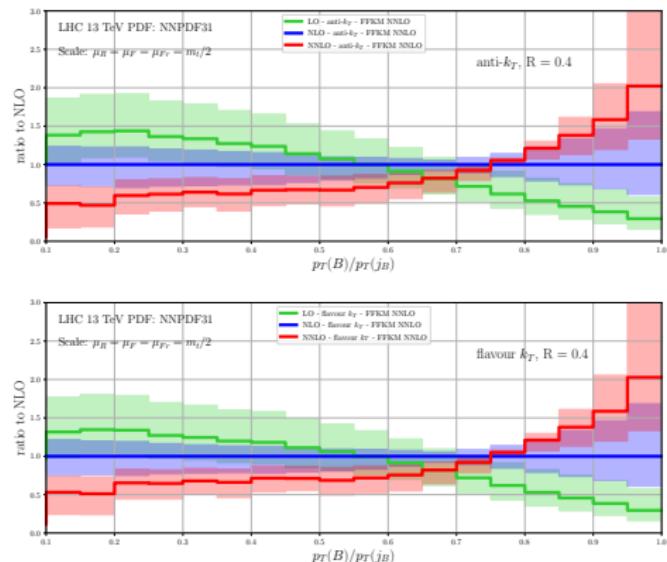
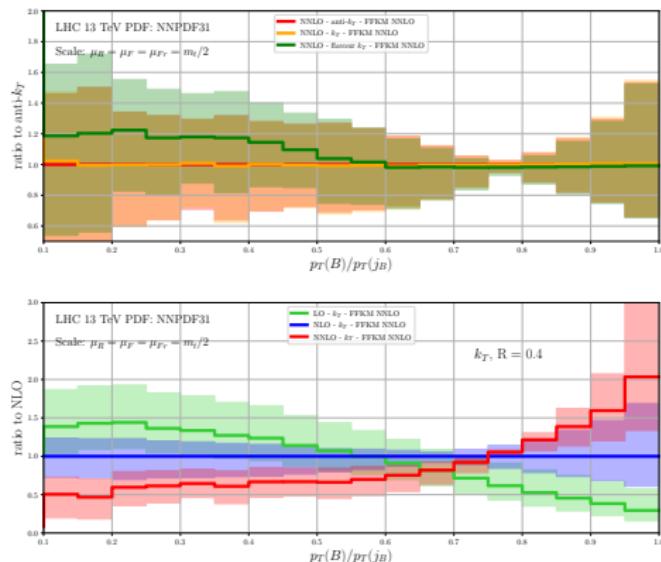
*M. Cacciari, P. Nason and C. Oleari (2006)*

*M. Fickinger, S. Fleming, C. Kim and E. Mereghetti (2016)*

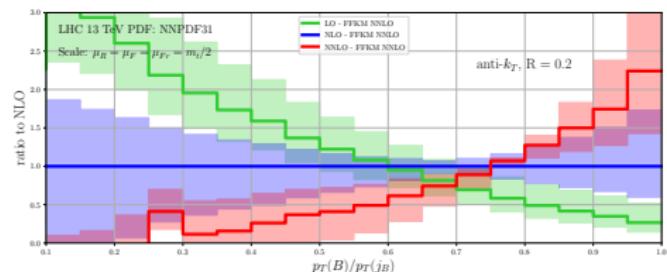
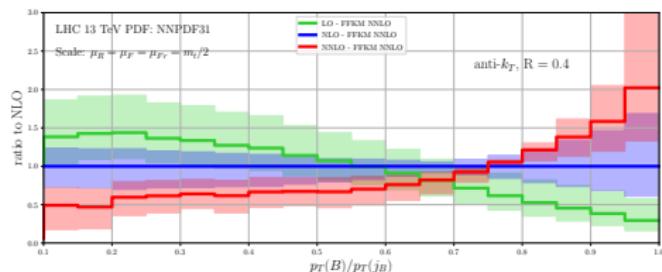
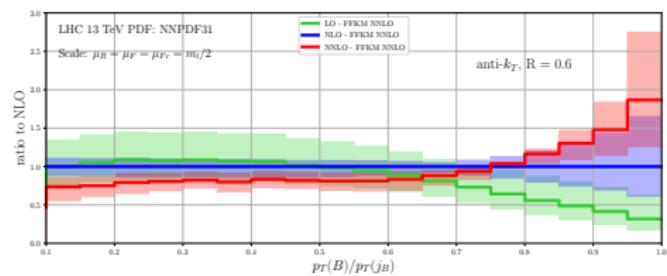
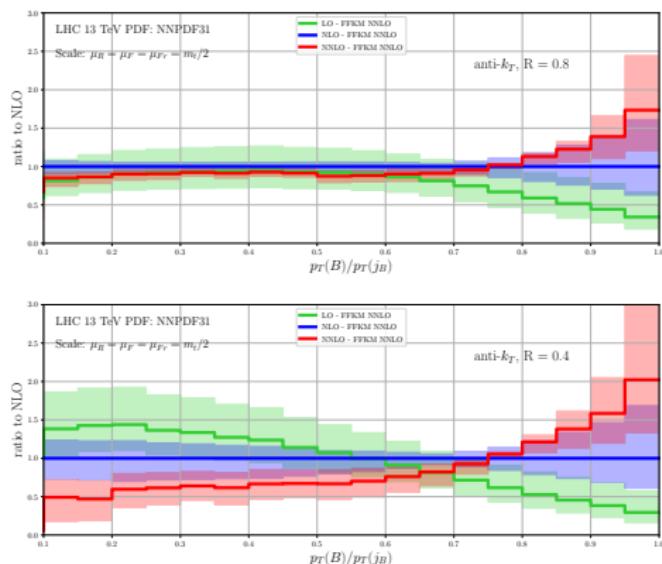
# Jet Ratio: $p_T$ -Cut-Dependence



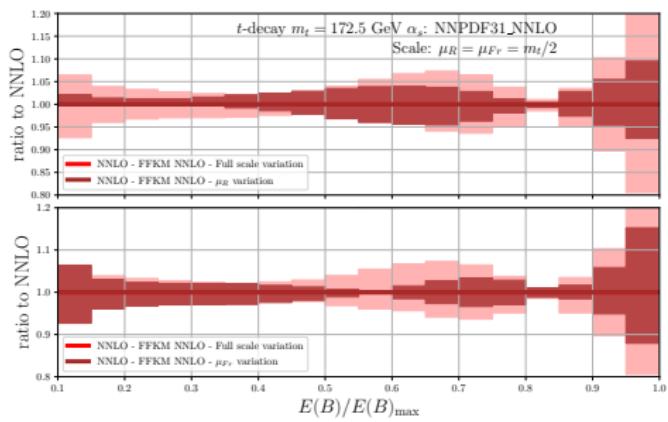
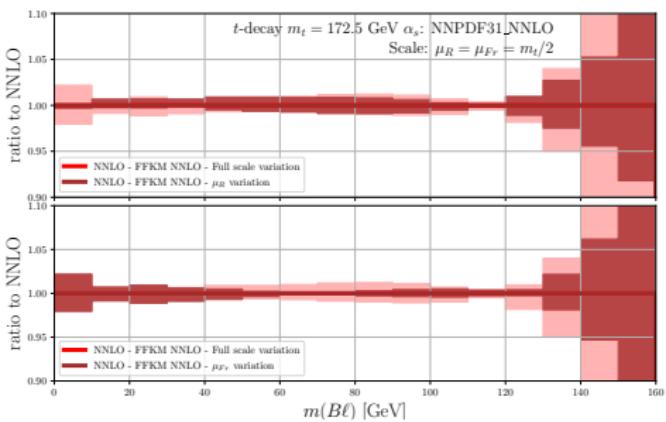
# Jet Ratio: Jet-Algorithm-Dependence



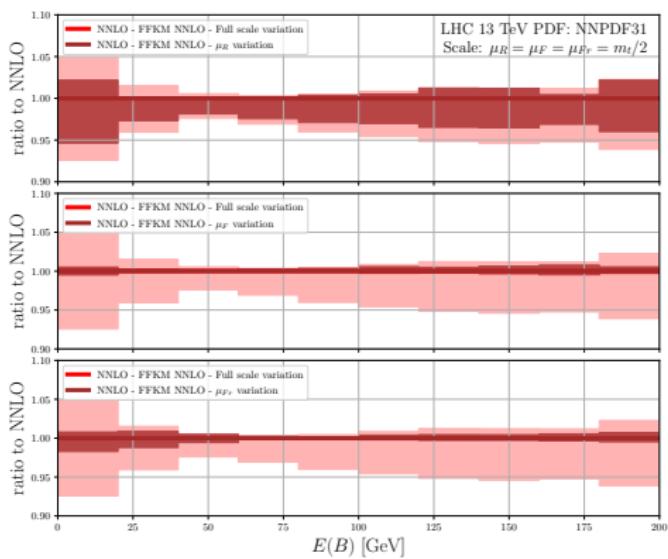
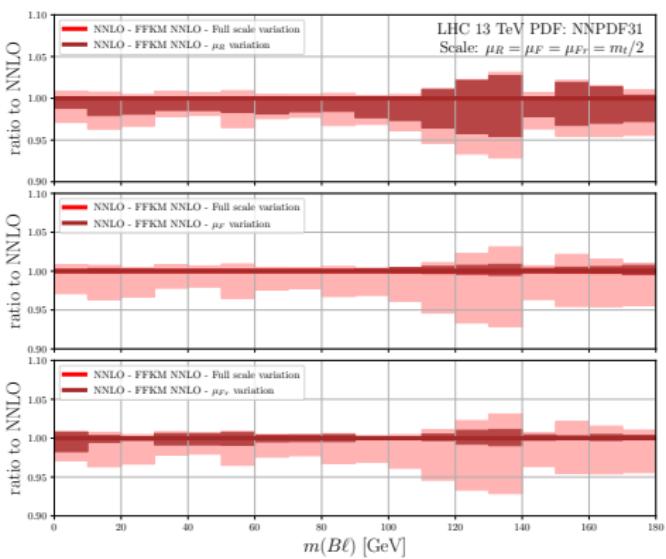
# Jet Ratio: $R$ -Dependence



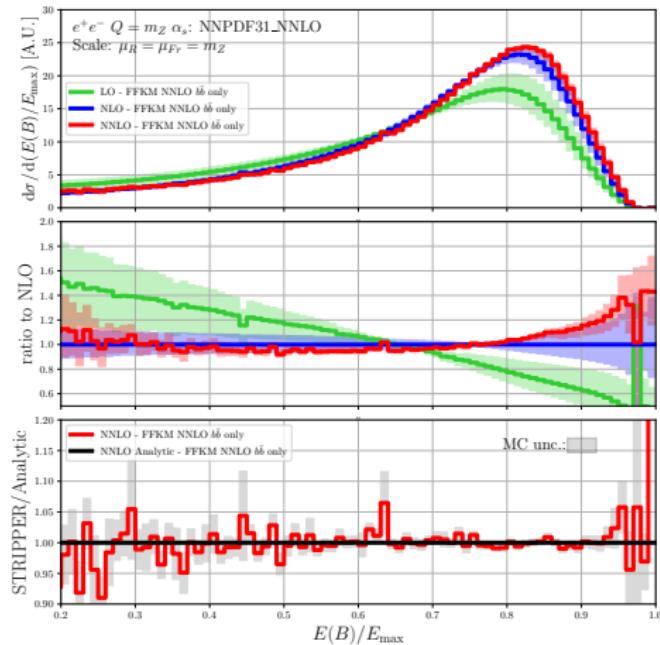
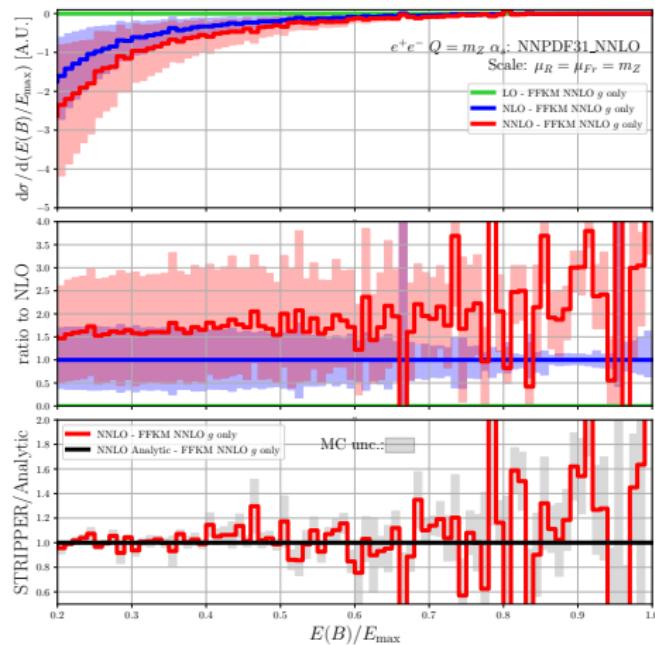
# Isolated Top Decay: Separated Scale Dependence



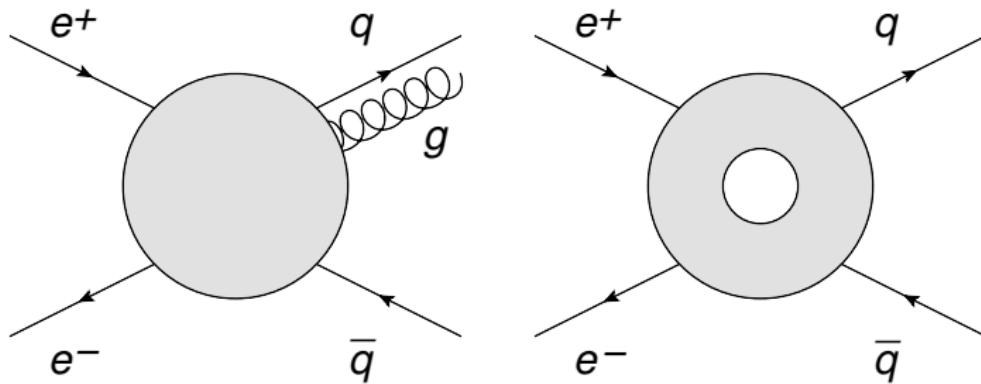
# Top-Pair Events with B-hadrons at the LHC: Separated Scale Dependence



# Cross-Check: B-Hadrons at LEP



# Collinear Renormalisation



# Collinear Renormalisation

$$D_i^B(x) = \sum_j (Z_{ij} \otimes D_j)(x), \quad (f \otimes g)(x) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$

$$\begin{aligned}
 Z_{ij}(x) = & \delta_{ij} \delta(1-x) + \frac{1}{\epsilon} \left( \frac{\mu_R^2}{\mu_{Fr}^2} \right)^\epsilon \frac{\alpha_s}{2\pi} P_{ij}^{(0)\text{T}}(x) \\
 & + \left( \frac{\alpha_s}{2\pi} \right)^2 \left[ \frac{1}{2\epsilon} \left( \frac{\mu_R^2}{\mu_{Fr}^2} \right)^{2\epsilon} P_{ij}^{(1)\text{T}}(x) \right. \\
 & + \frac{1}{2\epsilon^2} \left( \frac{\mu_R^2}{\mu_{Fr}^2} \right)^{2\epsilon} \sum_k (P_{ik}^{(0)\text{T}} \otimes P_{kj}^{(0)\text{T}})(x) \\
 & \left. + \frac{\beta_0}{4\epsilon^2} \left\{ \left( \frac{\mu_R^2}{\mu_{Fr}^2} \right)^{2\epsilon} - 2 \left( \frac{\mu_R^2}{\mu_{Fr}^2} \right)^\epsilon \right\} P_{ij}^{(0)\text{T}}(x) \right]
 \end{aligned}$$

# DGLAP Scale Evolution

- Collinear renormalisation of fragmentation functions
- $\Rightarrow$  'RGEs' for fragmentation functions
- $\Rightarrow$  DGLAP evolution equations:

$$\mu_{Fr}^2 \frac{d}{d\mu_{Fr}^2} D_i = \sum_j P_{ij}^T \otimes D_j \equiv \sum_j \sum_n \left( \frac{\alpha_s}{2\pi} \right)^{n+1} P_{ij}^{(n)T} \otimes D_j$$

- Scale dependence predicted by theory, need only  $x$ -dependence

# Fragmentation and Subtraction Schemes

- IR singularities at higher orders (soft/collinear)
- Subtraction schemes:  
$$d\sigma = d\sigma_0 + \sum_{j=1}^n \sum_{m=0}^n \int_m \left\{ \left( d\sigma_j^m - \sum_i d\sigma_{j,i}^m \right) + \sum_i d\sigma_{j,i}^m \right\}$$
- Lower-order matrix elements with factors to match singular behaviour
- Without fragmentation: kinematics match at jet-level in singular limits
- With fragmentation: kinematics mismatch
- $\Rightarrow$  Use full kinematics in singular limit in subtraction terms

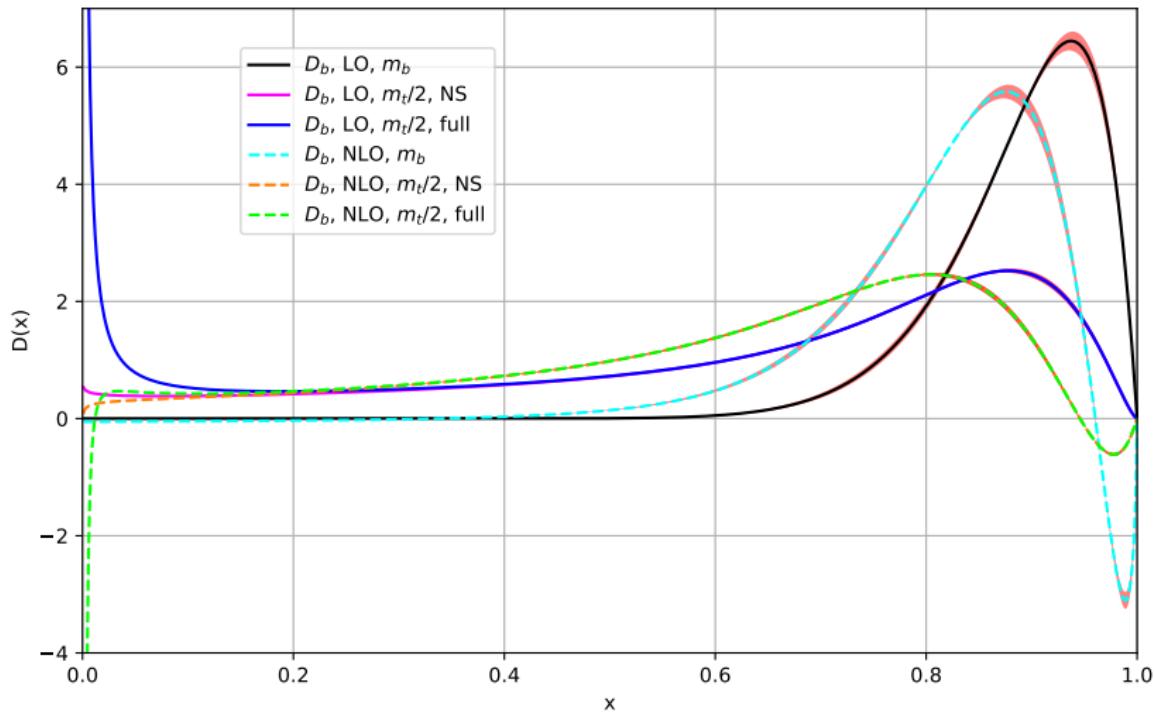
# Subtraction Terms

- Singular factors can be reused from case without fragmentation
- Integrated subtraction terms usually cannot be reused
- Calculation often assumes independence of observables w.r.t. collinear kinematics
- Especially when one subtraction term regulates multiple singularity types (e.g. CS dipoles)
- $\Rightarrow$  Need to redo integration
- Leads to left-over convolution with fragmentation function
- Convenient: in STRIPPER only one singularity is regulated per subtraction term

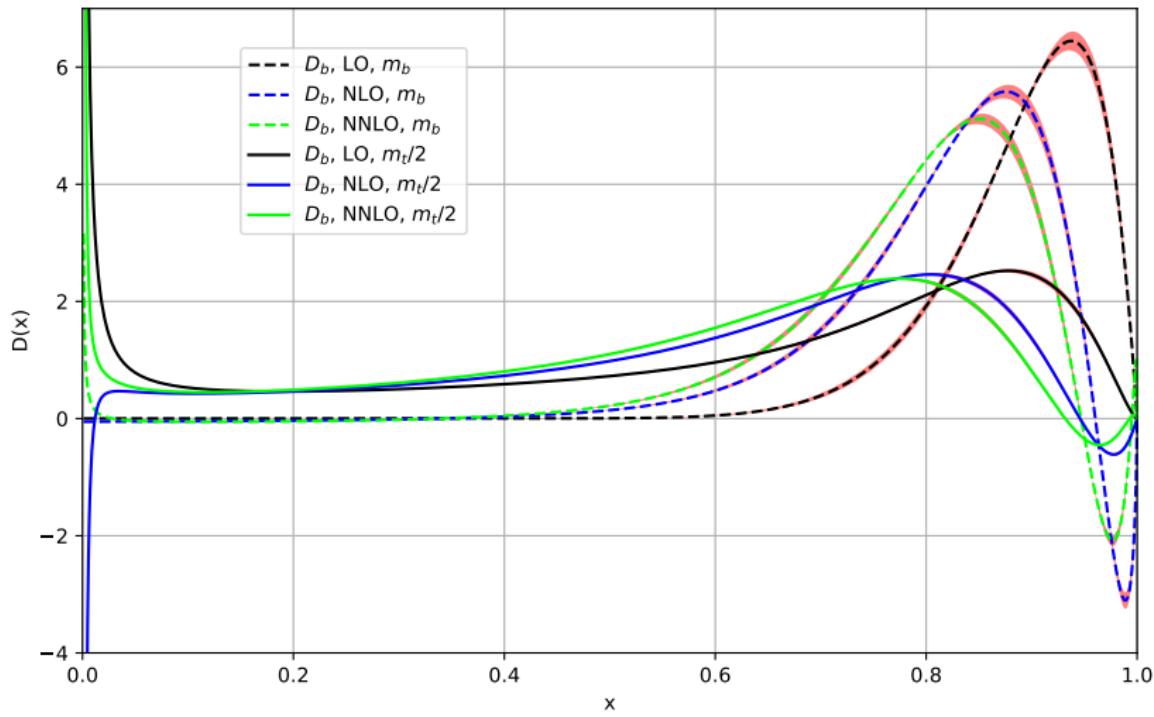
# Reference Observables

- Momentum fraction of subtraction terms not fully constrained
- Must be the same distribution for full/integrated subtraction terms
- Must match fraction of real contribution in relevant singular limit
- $\Rightarrow$  Can use freedom to improve numerical convergence
- Idea: rescale fractions per event to make all terms land in the same histogram bin
- Significantly reduce poor convergence due to "missed binning"
- Process requires "reference observable"

# Full Vs. Non-Singlet Scale Evolution



# A Fragmentation Function Through NNLO



# Fragmentation Functions of Different Flavours

- Three different fragmentation functions with large- $x$  resummation
- $D^{\text{CNO}}$  resums large logarithms through NLL.

*M. Cacciari, P. Nason and C. Oleari (2006)*

- $D^{\text{FFKM}}$  and  $D^{\text{FFKM,matched}}$  resum through  $\text{N}^3\text{LL}$ .

*M. Fickinger, S. Fleming, C. Kim and E. Mereghetti (2016)*

- Definition of FFs of type FFKM does not match the one used here.
- Ambiguity resolved in two different ways.