Automated calculation of beam functions in SCET

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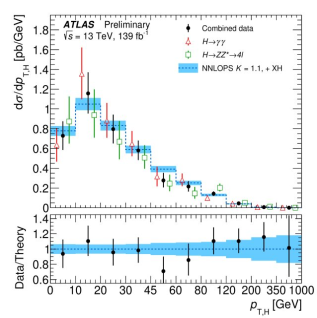
Motivation

- strength of SCET is to separate hard $\leftrightarrow$ soft $\leftrightarrow$ collinear modes and describe IR nature of QCD involving different scales
- different modes scale diffently with respect to the small parameter $\lambda = \frac{p_T}{m_H}$ in the power expansion

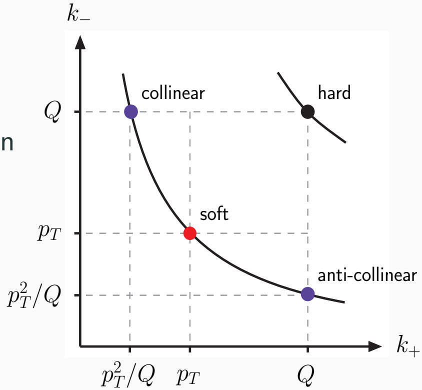


- possible to derive factorization theorems for different observables with general form

$$\mathrm{d}\sigma \approxeq H(\mu_F) \cdot B_n(\omega, \mu_F) \circledast B_{\bar{n}}(\omega, \mu_F) \circledast S(\omega, \mu_F) \qquad (1)$$

Motivation

- other factorization theorems include also jet functions accounting for the final state radiation
  $\rightarrow$ we are working in parallel on an automated approach for jet functions as well
- for SCET-II observables an additional regulator is needed:
  $\int \frac{\mathrm{d}^d p}{(2\pi)^d} \left(\frac{\nu}{p_+ + p_-}\right)^\alpha \cdot (2\pi)\delta(p^2)\Theta(p^0)$
  [Becher, Bell; 2011]



- soft function is provided by the program `SoftSERVE` for a general class of observables [Bell, Rahn, Talbert; 2018, 2020]

Motivation

- goal: design a general setup such that beam functions can be derived for this general class of observables in an automated way
- current status:
  - approach works at NLO [Kevin Brune´s master thesis; 2018]
  - approach extended to NNLO Real-Virtual (RV)-contributions
  - currently investigate NNLO Real-Real (RR)-contributions

Automated framework at NLO

- Definition collinear quark beam function:

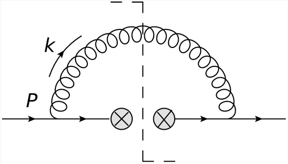
$$\mathcal{B}_{qq}(\tau, x) = \sum_{X_c} \delta\big((\bar{n}\cdot P)(1-x) - \bar{n}\cdot k_{X_c}\big)\mathcal{M}_x(\tau, k_{X_c}) \times \quad (2)$$

$$\langle P|\,\bar{\Psi}(0)\frac{\not{\bar{n}}\not{n}}{4}\underbrace{W_c(0)}_{=1}|X_c\rangle\,\frac{\not{\bar{n}}}{2}\,\langle X_c|\underbrace{W_c^\dagger(0)}_{=1}\frac{\not{n}\not{\bar{n}}}{4}\Psi(0)\,|P\rangle$$

$$\mathcal{M}_1(\tau, k) = \exp\left[-\tau k_T\Big(\frac{k_T}{P_-(1-x)}\Big)^n f\,(\Theta_k)\right] \quad (3)$$

- $f\,(\Theta_k)$ measurement function, varies for different observables
- example:
  - $\mathrm{p_T}$-resummation: $f\,(\Theta_k) = -2i\cos(\Theta_k)$, $n = 0$
  - $\mathrm{p_T}$-veto: $f\,(\Theta_k) = 1$, $n = 0$

Automated framework at NLO



- $k_- = (1-x)P_-\ , \qquad x \in [0,1]$

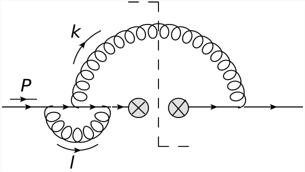
- master formula:

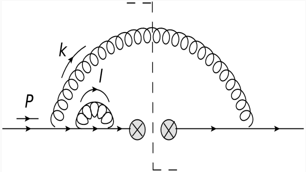
$$\mathcal{B}^1_{qq}(\tau, x) = \frac{4}{\sqrt{\pi}(n+1)} \frac{Z_\alpha \alpha_s}{4\pi} C_F \left(\frac{\mu^2 \tau^{\frac{2}{n+1}}}{(xq_-)^{\frac{2n}{n+1}}} \right)^{\epsilon} \left(\frac{\nu}{xq_-} \right)^{\alpha} \frac{\Gamma\left(\frac{-2\epsilon}{n+1}\right)}{\Gamma\left(\frac{1}{2}-\epsilon\right)} \times$$
$$\exp\left(\gamma_E \epsilon \left(\frac{-2}{n+1}+1\right)\right) \left(1-x\right)^{-1-\frac{2n\epsilon}{n+1}-\alpha} \left[(1-\epsilon)(1-x)^2 + 2x\right] \times$$
$$\int_{-1}^{1} \mathrm{d}\cos(\theta) \sin^{-1-2\epsilon}(\theta) \cdot f^{\frac{2\epsilon}{n+1}}(\theta) \qquad (4)$$

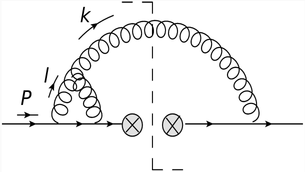
- singularities are completely factorized
- beam functions distribution-valued in x
  $\rightarrow$ Mellin transformation, introduce Mellin parameter N

Automated framework at NNLO - RV contribution

- contributions are related to the NLO splitting function $P_{qg}^{(1)}$ by crossing $P \to -P$
  [Furmanski, Petronzio; 1980]
- as in the case of NLO the same phase space parameterization applies
  $\to$ results can be still given analytically







Automated framework at NNLO - RV contribution

$$\mathcal{B}_{qq}^{2,RV}(\tau,x) = \left(\frac{Z_\alpha \alpha_s}{4\pi}\right)^2 \left(\frac{\mu^2 \bar{\tau}^{\frac{2}{n+1}}}{(xq_-)^{\frac{2n}{n+1}}}\right)^{2\epsilon} \left(\frac{\nu}{xq_-}\right)^{\alpha} g(\epsilon) \cdot \frac{\Gamma\left(\frac{-4\epsilon}{n+1}\right)}{\left(n+1\right)} \exp\left(\gamma_E \epsilon\left(\frac{-4}{n+1}+2\right)\right)\times$$

$$\left(1-x\right)^{-1-\frac{4n\epsilon}{n+1}-\alpha} |P_{qg}^{(1)}(x)|^2 \int_{-1}^{1} \mathrm{d}\cos(\theta) \sin^{-1-2\epsilon}(\theta) \cdot f^{\frac{4\epsilon}{n+1}}(\theta) \qquad (5)$$

- measurement function $f(\Theta_k)$ still sufficient to characterize observables

Automated framework at NNLO - RR contribution

- relevant color structures: $C_F^2, C_F C_A, C_F T_F n_f$
- measurement function gets modified:

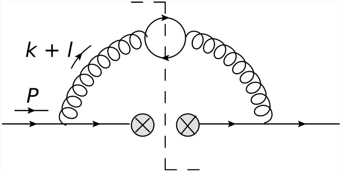
$$\mathcal{M}_2(\tau, k, l) = \exp\left[-\tau q_T \left(\frac{q_T}{P_-(1-x_3)}\right)^n F\,(...)\right] \tag{6}$$

- argument of measurement function F depends on parameterization and generally contains i.e. ratios of energies, angles, ...
- physical limits are protected by infrared safety:

$$\begin{aligned} \mathcal{M}_2(\tau, k, l) &\xrightarrow{k||l} \mathcal{M}_1(\tau, k+l) \\ F\,(...) &\longrightarrow f\,(\Theta_k) \end{aligned} \tag{7}$$

Automated framework at NNLO - $C_F T_F n_f$ contribution

- diagram can be related to the $P_{\bar{q}'q'q}$ splitting function [Catani, Grazzini; 1999]



- parameterization:

$$a = \frac{l_T}{k_T}\frac{k_-}{l_-} \qquad b = \frac{k_T}{l_T} \qquad x_{12} = \frac{k_- + l_-}{P_-}$$

$$q_T = \sqrt{(k_+ + l_+)(k_- + l_-)} \tag{8}$$

- angles (in the transverse plane):

$$t_k = \frac{1-\cos(\Theta_k)}{2} \qquad t_l = \frac{1-\cos(\Theta_l)}{2} \qquad t_{kl} = \frac{1-\cos(\Theta_{kl})}{2}$$

Automated framework at NNLO - $C_F T_F n_f$ contribution

- we use `pySecDec` for all color structures [Borowka et al.; 2017]
- overlapping divergence for $a \to 1$ and $t_{kl} \to 0$
- disentangle with non-linear transformation

$$a \to 1 - u(1-v) \qquad t_{kl} \to \frac{u^2 v}{1 - u(1-v)} \tag{9}$$

- singularities completely factorized: $x_{12}^{-1-2\alpha}$, $u^{-1-2\epsilon}$, $q_T^{-1-4\epsilon}$

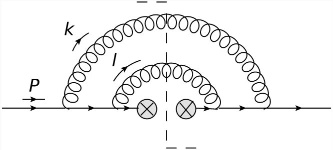
Automated framework at NNLO - C_F^2 contribution

- diagrams are related to the $P_{\bar{q}'q'q}$, $P_{\bar{q}qq}$, $P^{(id)}_{\bar{q}qq}$ splitting functions [Catani, Grazzini; 1999]
- diagrammatic expression can be divided into 3 structures:
  - $\frac{\mathcal{N}_1}{s_{12}s_{13}x_1x_2}$: $x_1 = \frac{k_-}{P_-}$; $\quad x_2 = \frac{l_-}{P_-}$; $\quad b = \frac{k_T}{l_T}$; $\quad q_T = k_T + l_T$

    $\rightarrow$ factorized singularities: $x_1^{-1-\alpha}$, $x_2^{-1-\alpha}$, $b^{-1-2\epsilon}$, $q_T^{-1-4\epsilon}$
  - $\frac{\mathcal{N}_2}{s_{123}^2}$: $\tilde{a} = \frac{k_T}{l_T}\frac{l_-}{k_-}$; $\quad r = \frac{k_-}{l_-}$; $\quad x_{12} = \frac{k_- + l_-}{P_-}$;

    $$q_T = \sqrt{(k_+ + l_+)(k_- + l_-)}$$

    $\rightarrow$ factorized singularities: $\tilde{a}^{-1-2\epsilon}$, $q_T^{-1-4\epsilon}$



Automated framework at NNLO - C_F^2 contribution

- $\frac{1}{s_{123}}$: $x_1 = \frac{k_-}{P_-}; \qquad x_2 = \frac{l_-}{P_-}; \qquad b = \frac{k_T}{l_T}; \qquad q_T = k_T + l_T$

  $\rightarrow$ still overlapping divergences, decomposed with sector decomposition in `pySecDec` [Borowka et al.; 2017]
- structures from $P_{\bar{q}qq}^{(id)}$ can be integrated using the second parameterization

Automated framework at NNLO - Renormalization

- relate beam function to matching kernel which can be calculated perturbatively:

$$\mathcal{B}_{qq}(\bar{\tau}, N) = \sum_{k=q,\bar{q},g} \mathcal{I}_{qk}(\bar{\tau}, N) \cdot \phi_{kq}(N) \qquad (10)$$

- $\phi_{kq}(N) = \delta_{kq} \Rightarrow \mathcal{B}_{qq}(\bar{\tau}, N) = \mathcal{I}_{qq}(\bar{\tau}, N)$

- SCET-II renormalization: collinear anomaly approach

[Becher, Neubert ; 2010]

$$\left[\mathcal{S}(\bar{\tau},\nu)\mathcal{I}_{qq}(N_1,\bar{\tau},\nu)\bar{\mathcal{I}}_{\bar{q}\bar{q}}(N_2,\bar{\tau},\nu)\right]_{q^2} \overset{\alpha=0}{=} \left(\bar{\tau}^2 q^2\right)^{2\mathcal{F}^0_{q\bar{q}}(\bar{\tau})} I^0_{qq}(N_1,\bar{\tau})\, \bar{I}^0_{\bar{q}\bar{q}}(N_2,\bar{\tau})$$

- RGE for anomaly coefficient $\mathcal{F}^0_{q\bar{q}} = \mathcal{F}_{q\bar{q}} + \mathcal{Z}_F$:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} F_{q\bar{q}}(\bar{\tau},\mu) = -\Gamma_{cusp}(\alpha_s) \qquad (11)$$

Automated framework at NNLO - Renormalization

- solution to this RGE: $L = \ln(\mu\bar{\tau})$

$$F^B_{q\bar{q}}(\bar{\tau},\mu) = -\frac{\alpha_s}{4\pi}\left[\Gamma_0 L - d_1^B\right] - \left(\frac{\alpha_s}{4\pi}\right)^2\left[\beta_0\Gamma_0 L^2 + \Gamma_1 L + \beta_0 d_1^B L - d_2^B\right] \tag{12}$$

- d_1^B and d_2^B will be plotted and investigated
- RGE for the remainder function:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} I_{qq}(N_1,\bar{\tau},\mu) = \left[2\Gamma_{cusp} L + 2\gamma^I\right] I_{qq}(N_1,\bar{\tau},\mu) - 2\sum_k I_{qk}(N_1,\bar{\tau},\mu)\cdot P_{kq}(N_1,\mu) \tag{13}$$

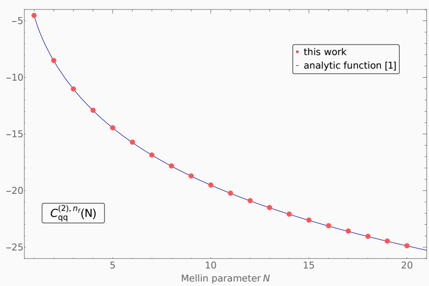
- solution contains non-logarithmic piece $C^{(2)}_{qq}(N_1)$ which will be plotted

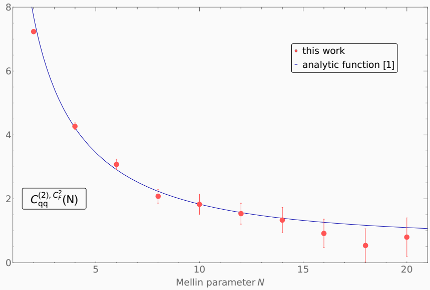
Automated framework at NNLO - Preliminary results for p_T - resummation

- definition p_T-resummation: $\omega_{p_T} = -2i \sum_j |\vec{k}_{j,T}| \cos(\theta_j)$

d_2^B	C_F^2	$C_F T_F n_f$
Literature [1]	0	≈ 4.148
Numerical result	0.022(64)	4.147(4)

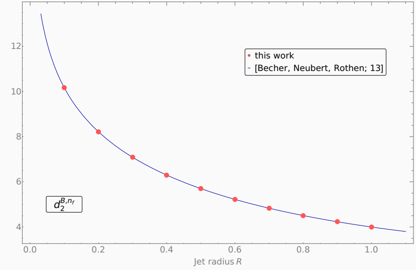
- renormalized matching kernel: [1]: [Gehrmann, Lübbert, Yang; 2014]

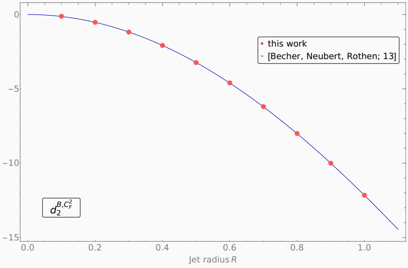




Automated framework at NNLO - Preliminary results for p_T - veto

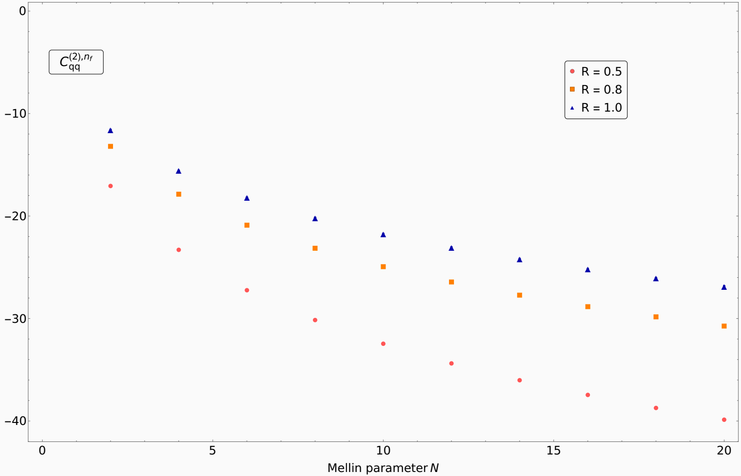
- introduce a distance measure: $\Delta^2 = \frac{1}{4}\ln^2\left(\frac{k_- l_+}{l_- k_+}\right) + \Theta_{kl}^2$
- definition p_T-veto:
  $\omega_{p_T - veto} = \Theta(\Delta - R)\max(\{k_{i,T}\}) + \Theta(R - \Delta)|\sum_i \vec{k}_{i,T}|$
- anomaly coefficient d_2^B depends now on the jet radius R:





Automated framework at NNLO - Preliminary results for p_T - veto

- renormalized matching kernel at $\mathcal{O}(\alpha_s^2)$:



Conclusions and Outlook

We are aiming at developing an automated approach for NNLO beam functions

- NLO and NNLO-RV contributions are done for general observables
- two out of three color structures of the NNLO-RR contribution implemented

- numerical improvement of C_F^2 color structure
- last NNLO color structure in progress
- include SCET-I observables
- compute the remaining matching kernels at NNLO