The Landau-Pomeranchuk-Migdal effect

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What is the LPM effect?

- Crosssections are usually calculated for interaction on a single isolated atom. This is not limited to the Born approximation (plane waves as wavefunctions at large distances), but also true in higher orders in the nuclear potential, taking into account the slowly decaying Coulomb potential ($\sim 1/r$), which leads to Coulomb corrections. In the case of bremsstrahlung and pair production, these crosssections are often called Bethe-Heitler (BH) crosssections.
- At relativistic energies, the minimum momentum transfer is given by

$$\delta = \frac{m^2\omega}{2EE'} \text{ for bremsstrahlung,} \qquad (1)$$

$$= \frac{m^2\omega}{2E_+E_-} \text{ for pair production.} \qquad (2)$$

At large energies, $\delta \ll m \ll E, \omega$.

What is the LPM effect?

- Semiclassically speaking, Heisenberg's uncertainty relation gives as formation length $\ell_f \sim \hbar/\delta$. At large energies, ℓ_f attains values comparable to the internuclear distance. Therefore, at very high energies, scattering on other atoms has to be taken into account. Additional scattering disturbs the coherence of the wavefunctions and leads to a suppression of the cross section.
- Multiple scattering of the lepton leads to the LPM effect. This was first discussed in Landau and Pomeranchuk 1953, the first quantitative calculation was published in Migdal 1957.
- The effect of Compton scattering on the photon wavefunction is called Ter-Mikaelian effect or dielectric suppression effect Ter-Mikaelian 1969. It is only effective in the region of phase space, where the LPM effect is strong.
- At extremely high energies, ℓ_f can attain values comparable to the radiation length; in this case pair creation can additionally suppress the bremsstrahlung cross section.

Schematic effect on the bremsstrahlung cross section

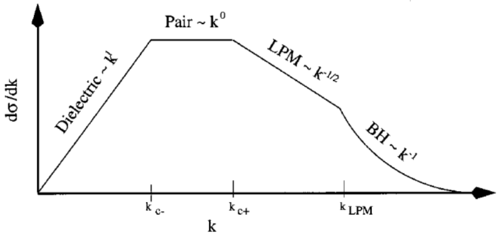

FIG. 3. Schematic view of bremsstrahlung $d\sigma/dk$ when several suppression mechanisms are present. For $E<E_p$, the pair-creation suppression disappears and LPM suppression connects with dielectric suppression.

Figure: Taken from the review paper by Klein 1999.

LPM effect in air showers

- The density in air is small, therefore the internuclear distance is large. Therefore, the LPM effect influences air showers at higher energies than in more dense media such as water or rock.
- The LPM effect suppresses small bremsstrahlung losses and symmetric pair production events.

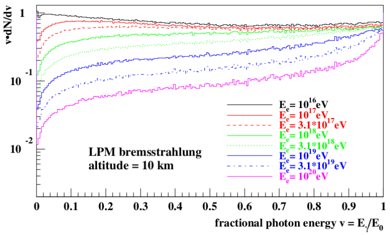

Figure: LPM suppression of bremsstrahlung (from technical report Heck and Knapp 1998).

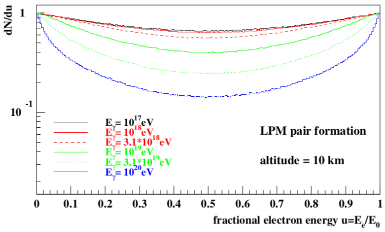

Figure: LPM suppression of pair production (from technical report Heck and Knapp 1998).

LPM effect in air showers

- In the LPM regime, therefore, pair production events are predominantly asymmetric and more rare, and bremsstrahlung losses are predominantly large.
- At LPM energies, we expect photon-induced air showers to develop in two sub-showers
  - The initial photon produces a very asymmetric e^+e^- pair.
  - The low-energy lepton produces a normal (Bethe-Heitler) shower after the first interaction.
  - The high-energy lepton travels farther through the atmosphere because of the suppressed bremsstrahlung loss and produces a second shower deeper in the atmosphere.
- Overall, LPM showers develop more slowly than BH showers and with larger fluctuations (Konishi et al. 1991; Misaki 2019).

Simulation of the LPM effect

- In homogeneous media, PROPOSAL calculates the cross sections taking into account the LPM and TM effect according to Polityko et al. 2002; Stanev et al. 1982. In inhomogeneous media, this approach is not applicable, because the changing density changes the LPM suppression, avoiding its inclusion into the interpolation tables.
- In CORSIKA 7, based on the routines in AIRES (and before that in MOCCA), the LPM effect is taken into account via a variant of Neumann's rejection algorithm:
  - Bremsstrahlung and pair production are sampled according to the BH crosssections.
  - Before writing the secondaries to the stack, the LPM routine checks, whether to discard the interaction: the ratio ξ of the BH and the LPM crosssections is calculated, a uniform random number x between 0 and 1 is determined, and the interaction is discarded if $\xi > x$.

Simulation of the LPM effect

- The rejection procedure correctly takes into account the LPM effect on stochastic interactions. However, when LPM suppression is large, this procedure is inefficient. Also, this approach neglects the LPM effect on the continuous losses. It is therefore most likely not accurate enough for dense media.
- In EmCa (Meighen-Berger et al. 2019), the crosssections are rescaled with a correction factor; as the differential crosssections show, this is not appropriate for strong suppression..

Possible solutions for LPM suppression on continuous losses

1. Ignore LPM suppression on continuous losses.
   - Possible, if energy cuts are chosen such that continuous losses are negligible compared to the stochastic losses.
   - LPM suppression affects small losses, therefore continuous losses are overestimated, possibly by orders of magnitude at sufficiently high energies.
2. Interpolate a correction factor dependent on density and energy.
   - What are appropriate interpolation limits?
   - Interpolation tables would depend on these choices.
3. Choose some effective description to rescale continuous losses.
   - As the effect of continuous losses is not large, the accuracy does not need to be as high as for stochastic losses.
   - Simple choice: LPM-suppression factor ξ at v_{cut}; possibly some overestimation, because softer losses are even more suppressed.

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