

Harmonic cavity studies for the SOLEIL Upgrade

Alexis Gamelin on behalf of the SOLEIL upgrade team

In (most) 4th generation low emittance storage rings, harmonic cavities (also called Landau cavities) are critical components needed to reach design performances.

They are mainly used to lengthen the bunches which provide:

- Reduced intra-beam scattering (IBS)
- Increased Touschek lifetime
- Reduced bunch spectral width



- Reduced heating
- Reduced overlap with the high frequency region of the impedance spectrum

- Synchrotron tune spread



- Help to damp longitudinal (SB & MB) instabilities.
- Can help to damp some transverse instabilities (if the head-tail mode number is > 0).

But harmonic cavities (HC) have also some drawbacks:

- They can induce several instabilities or reduce instability thresholds.
- Transient beam loading (TBL) if the beam filling scheme is not symmetric.

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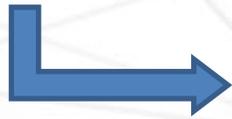
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Subject of this talk

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1. Introduction
 1. Flat potential (FP) conditions
 2. Near flat potential (NFP) conditions
 3. SC passive vs NC passive
 4. Application to SOLEIL-U

2. Super conducting passive harmonic cavity: 3rd vs 4th harmonic
 1. Slow-moving transient instabilities
 2. Low current limitations
 3. Impact of transient beam loading

3. Other options considered:
 1. Normal conducting passive harmonic cavity
 2. Normal conducting active harmonic cavity

The total voltage given by an RF system with a m^{th} harmonic cavity (HC) can be expressed as:

$$V_{tot}(t) = V_1 \cos(\omega_{RF}t + \phi_1) + V_2 \cos(m\omega_{RF}t + \phi_2)$$

Voltage and phase of the main cavity
Voltage and phase of the harmonic cavity

Where the following condition is imposed to insure energy balance:

$$V_{tot}(0) = \frac{U_{loss}}{e}$$

Losses per turn

The RF system is ideally operated at the “flat potential” (FP) conditions:

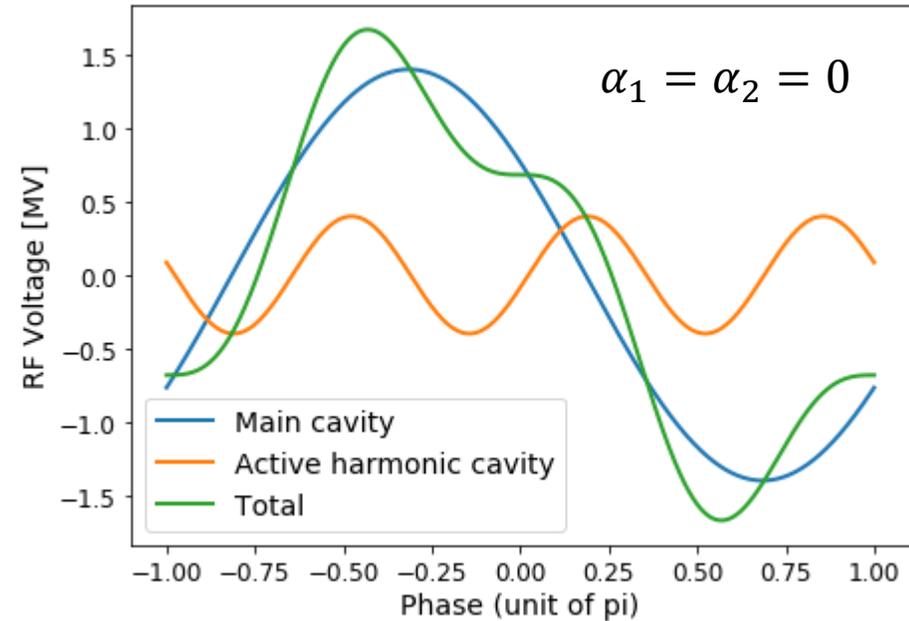
$$\frac{dV_{tot}}{dt}(0) = \alpha_1 = 0 \quad \frac{d^2V_{tot}}{dt^2}(0) = \alpha_2 = 0$$

Which gives the following conditions:

$$\cos(\phi_1) = \frac{m^2}{m^2 - 1} \frac{U_{loss}}{eV_1}$$

$$\tan(\phi_2) = m \tan(\phi_1)$$

$$V_2 = -\frac{V_1 \cos(\phi_1)}{m^2 \cos(\phi_2)}$$



Near flat potential conditions (NFP)

The system of equation is then limited to the “near flat potential” (NFP) conditions:

$$V_{tot}(0) = \frac{U_{loss}}{e} \qquad \frac{dV_{tot}}{dt}(0) = \alpha_1 = 0$$

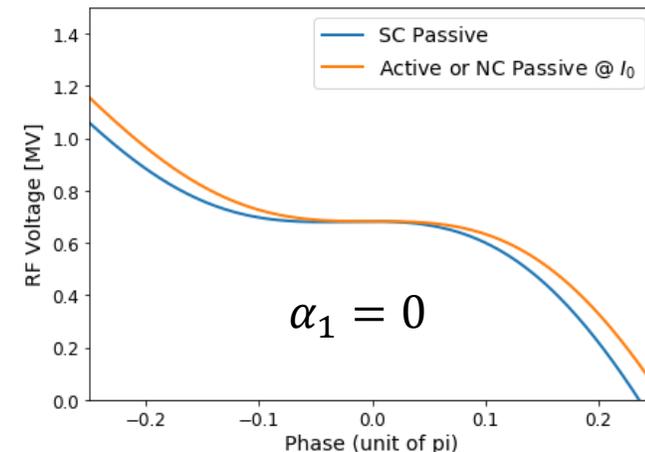
Introducing $\xi = -\frac{mV_2 \sin(\phi_2)}{V_1 \sin(\phi_1)}$, the ratio of the harmonic “force” to the main cavity (MC) one, gives:

$$\cos(\phi_1) = \frac{U_{loss}}{eV_1} + \xi \frac{\sin(\phi_1)}{m \tan(\phi_2)} \qquad V_2 = -\xi \frac{V_1 \sin(\phi_1)}{m \sin(\phi_2)}$$

Now expressing the α_1 and α_2 using ξ gives:

$$\alpha_1 = \xi \omega_{RF} \sin(\phi_1) \left(1 - \frac{1}{\xi}\right) \xrightarrow{\text{NFP } (\phi_1, \xi = 1)} \alpha_1 = 0$$

$$\alpha_2 = -V_1 \omega_{RF}^2 \left[\cos(\phi_1) - \xi m \frac{\sin(\phi_1)}{\tan(\phi_2)} \right] \xrightarrow{\text{NFP } (\phi_1, \xi = 1)} \alpha_2 = -V_1 \omega_{RF}^2 \left[\frac{U_{loss}}{eV_1} - \frac{m^2 - 1}{m^2} \frac{\sin(\phi_1)}{\tan(\phi_2)} \right] \neq 0$$



For a super conducting (SC) passive HC, the shunt impedance R_s is very high, typically $R_s \approx G\Omega$, it implies that :

- The current needed to be at FP condition is very low.
- The tuning angle is fixed to $\psi \approx \pi/2$ for most of the useful tuning range.

In that case, assuming $\psi \approx \pi/2$ and $\Phi \approx 0$ gives a fixed value for ϕ_2 : $\phi_2 = \pi + (\psi - \Phi) \approx -\pi/2$

The previous equations are much simplified as $\tan(\phi_2) \rightarrow -\infty$ which gives:

$$\cos(\phi_1) = \frac{U_{loss}}{eV_1} \quad V_2 = \xi \frac{V_1 \sin(\phi_1)}{m} \quad \alpha_1 = \xi \omega_{RF} \sin(\phi_1) \left(1 - \frac{1}{\xi}\right) \quad \alpha_2 = -\frac{\omega_{RF} U_{loss}}{e}$$

So, for a SC passive HC:

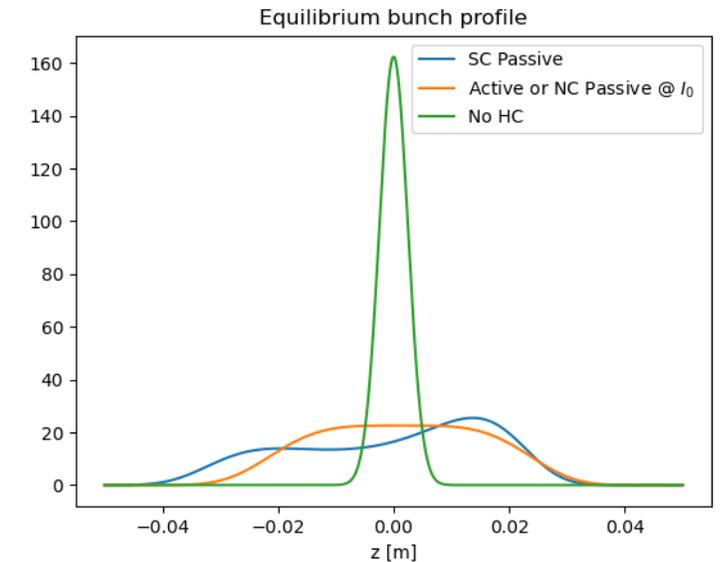
- The MC phase ϕ_1 is the same as the one used without HC.
- The second derivative of the RF voltage α_2 can never be cancelled and is fixed by the losses.
- The MC and HC are totally independent systems.

For a normal conducting (NC) passive HC, the shunt impedance R_s is in the $M\Omega$ region, it implies that:

- You can design the system in such a way to achieve FP conditions for given current value I_0 .
- For all other currents, the cavity tuning ψ needs to change to get the correct voltage in the harmonic cavity which also change the HC phase ϕ_2 .

Assuming $F \approx 1$, $\Phi \approx 0$ and $V_2 \approx \xi \frac{V_1}{m}$ gives a value for ϕ_2 which allows to solve self-consistently the NFP equation:

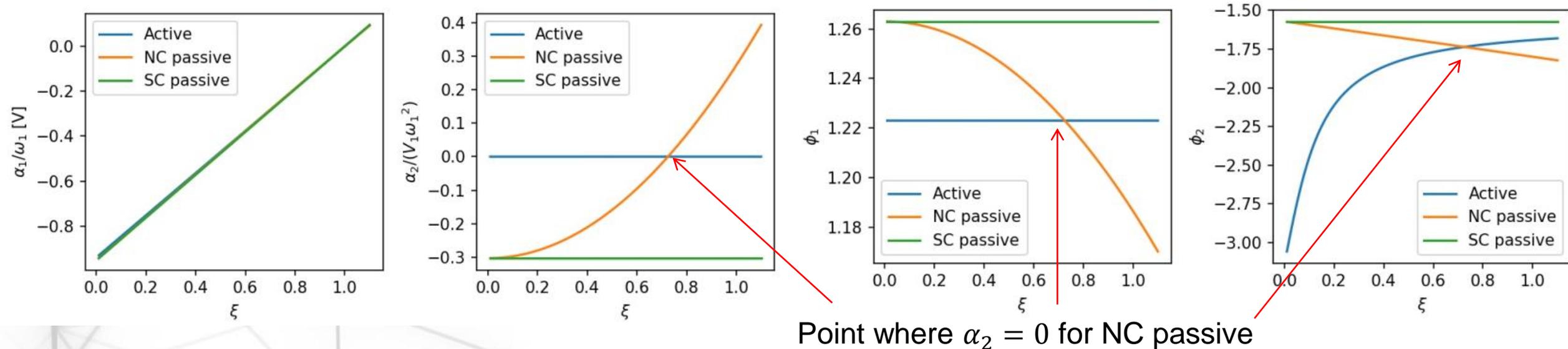
$$\phi_2 \approx \arccos(\xi V_1 / (2mIR_s))$$



So, for a NC passive HC:

- The MC phase ϕ_1 needs to change when the HC is tuned in or out.
- The second derivative of the RF voltage α_2 can be cancelled for a single current I_0 .
- The MC and HC are interdependent systems.

For the 3rd harmonic using $V_1 = 1.7$ MV, $R_s = 2.5$ M Ω and $I = 500$ mA for the NC case gives:



The result presented here is an approximation as the “ideal” cavity equations are used.

The most general case, where both the MC and HC beam loading is taken into account, can be treated with the same equations using the total cavity voltage V and phase ϕ from the phasor addition of the generator phasor \tilde{V}_g and the beam phasor \tilde{V}_b :

$$\tilde{V}_c = V e^{j\phi} = \tilde{V}_g + \tilde{V}_b = V_g e^{j\phi_g} + \frac{2I_0 R_s F}{1 + \beta} \cos \psi e^{j(\psi + \pi - \phi)} \quad V = |\tilde{V}_c| \quad \phi = \arg(\tilde{V}_c)$$

Fixed parameters:

- Main RF : ESRF - EBS fundamental cavity
 - R_s (per cavity) = 5 $M\Omega$
 - $Q_0 = 35\,000$
 - $N_{cav} = 3$ or 4
 - $\beta = 5$ ($Q_L = 5\,833$)
 - $V_c = 1,7$ MV
 - $f_{RF} = 352$ MHz

- SOLEIL Upgrade w/ Operation modes

<ul style="list-style-type: none"> – $h = 416$ – $L = 354,7$ m – $E_0 = 2,75$ GeV – $\alpha_c = 9,12 \times 10^{-5}$ – $\sigma_{\tau_0} = 9$ ps – $\tau = 11,7$ ms – $\sigma_\delta = 9 \times 10^{-4}$ – $U_0 = 515$ keV (w/o IDs) 	<ul style="list-style-type: none"> – Uniform @ 500 mA – 8 bunches @ 100 mA – 1 bunch @ 20 mA – 16/32 bunches @ TBD
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Options considered for the HC:

1. Passive SC : Super3HC (scaled at $m = 3$ or $m = 4$)
 - R_s (per cavity) = 4,5 $G\Omega$
 - $Q_0 = 10^8$
 - $R/Q = 45$ Ω
 - $N_{cav} = 2$

2. Passive NC : ESRF harmonic 2-cell cavity ($m = 4$)
 - R_s (per cavity) = 2,4 $M\Omega$
 - $Q_0 = 27\,000$
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3. Active NC : ESRF harmonic 2-cell cavity ($m = 4$)
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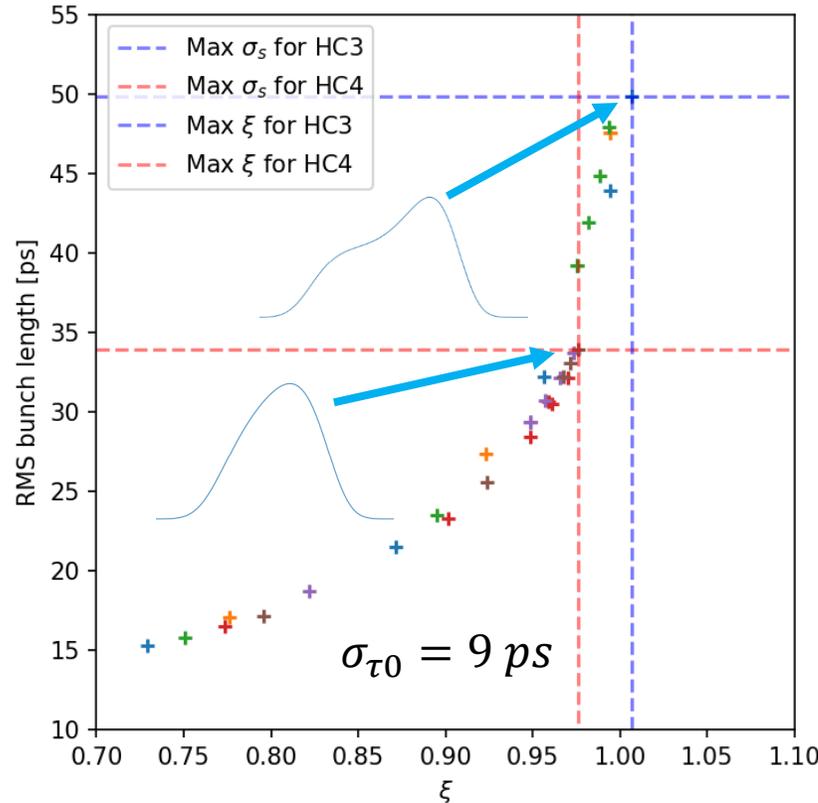
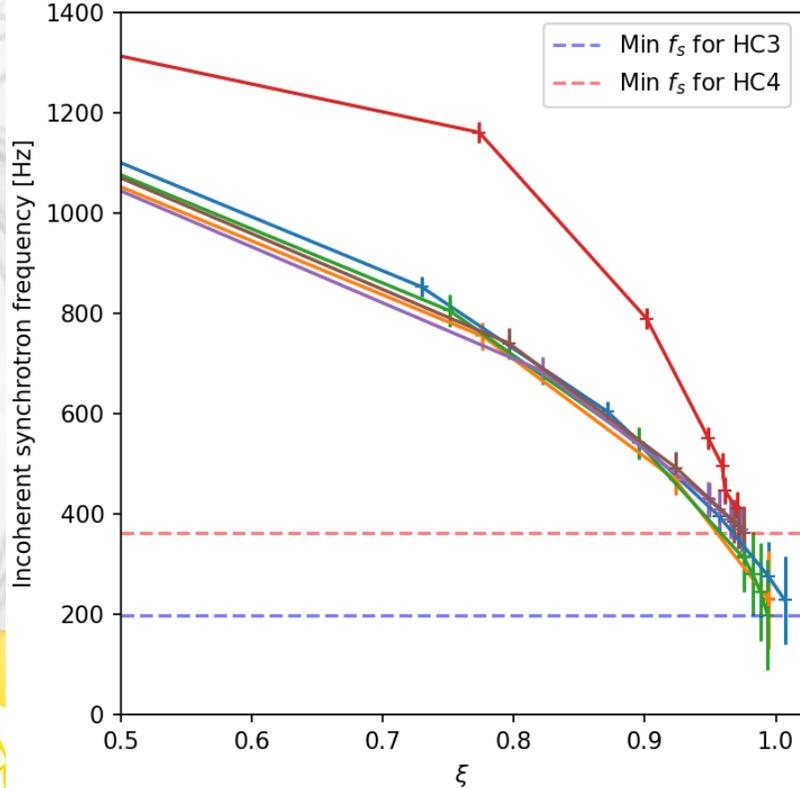
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Bunch lengthening at high current (500 mA)

Here are the stable settings found for a 3rd HC and a 4th HC for the SOLEIL Upgrade using multi-bunch tracking taking into account the beam loading in the main and harmonic cavity (mbtrack2^[1]):

$$\omega_s^2 \approx \frac{e\eta\omega_{RF}}{E_0T_0} [1 - \xi]V_1 \sin(\phi_1)$$

$$\sigma_\tau \approx \frac{\alpha_c \sigma_\delta}{\omega_s}$$



At high current, 500 mA in uniform filling, the 3rd HC allows to get a bit past NFP while the 4th HC is limited before $\xi = 1$.

The limitation, in both cases, is the “periodic transient beam loading instability^[2]” but it is happening at different distance from the NFP conditions.

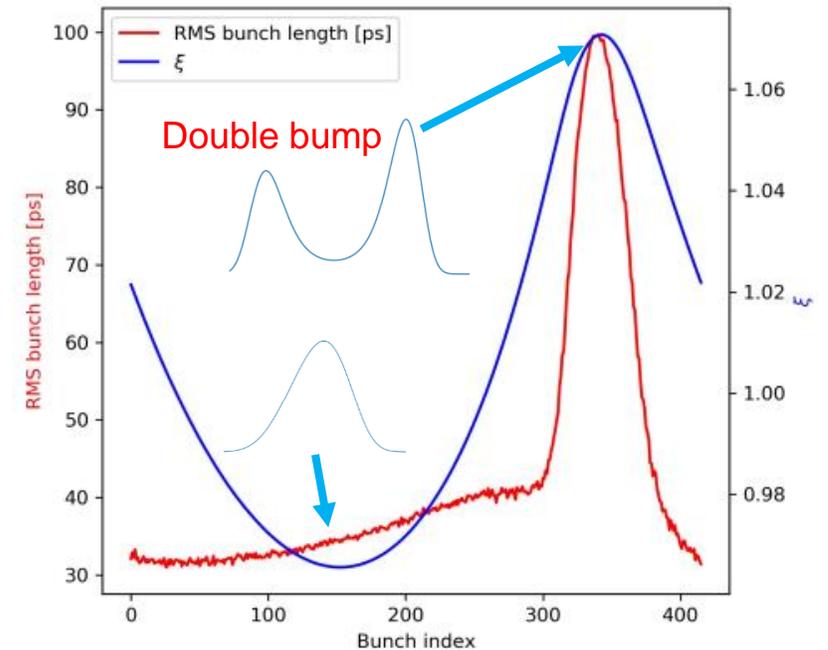
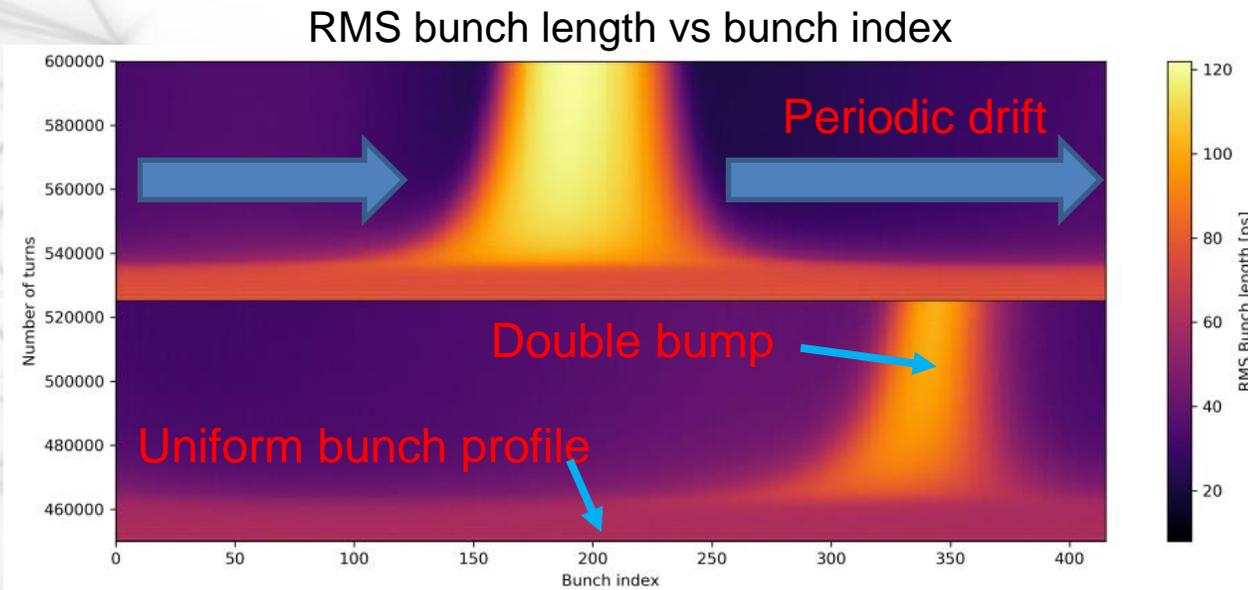
[1] Gamelin, A., Foosang, W., & Nagaoka, R. mbtrack2, a Collective Effect Library in Python. IPAC'21 - <https://gitlab.synchrotron-soleil.fr/PA/collective-effects/mbtrack2>

[2] He, T., Li, W., Bai, Z., & Wang, L. (2022). Periodic transient beam loading effect with passive harmonic cavities in electron storage rings. *PRAB*, 25(2), 024401.

Periodic transient beam loading instability

This “periodic transient beam loading instability^{[1]” (PTBL), also called “slow moving transient instability^{[2]”, seems to happen when the usual stable solution found in the case of a uniform filling beam where all the bunches have the same bunch profile break down:}}

- When getting past the threshold, the different bunches have different bunch profiles.
- The system fall back to a quasi-stable state which drifts slowly in the bunch index space with time period which depends on the cavity parameters, but which can be rather long ($f \approx \text{Hz}$).



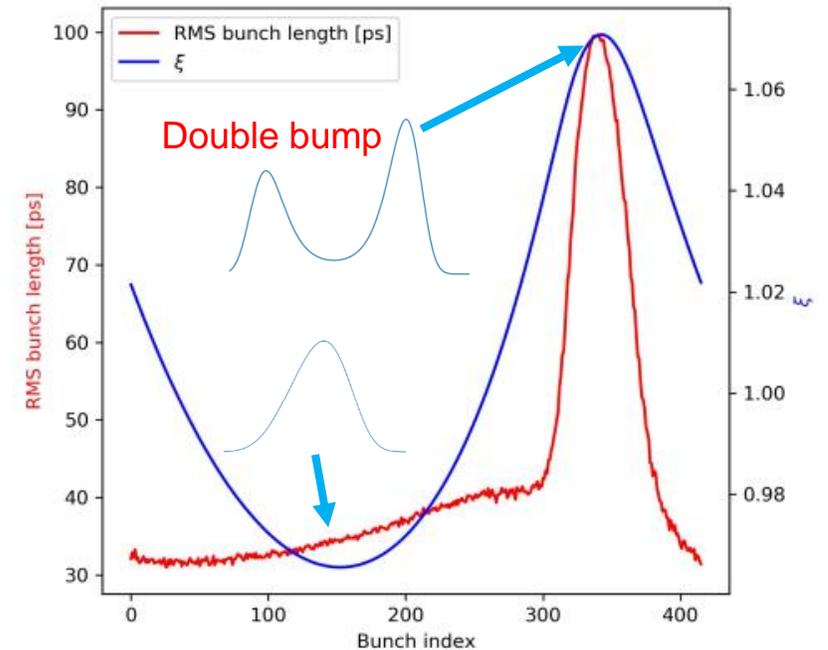
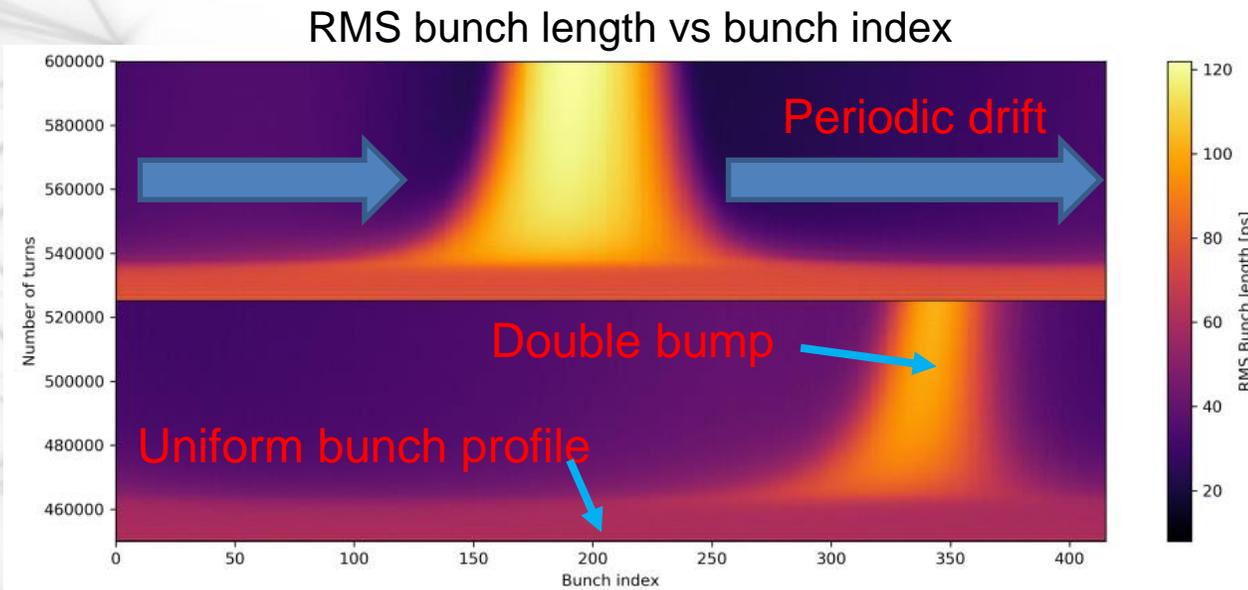
[1] He, T., Li, W., Bai, Z., & Wang, L. (2022). Periodic transient beam loading effect with passive harmonic cavities in electron storage rings. *PRAB*, 25(2), 024401.

[2] Private communication with T. Olson (Diamond) and F. Cullinan (MAXIV), Nov 2020. [3] A. Gamelin, Collective effects studies for the SOLEIL Upgrade, ESRW'22

Periodic transient beam loading instability

It has been observed in simulations (SOLEIL-U^[2,3], Diamond-II^[2], HALF^[1]) and possibly experimentally at MAX IV^[2] and has yet to be fully understood.

From [1], it seems that the PTBL threshold increases with the bunch length. It then explains why the 3rd HC threshold is higher than the 4th HC as it naturally produce longer bunches.

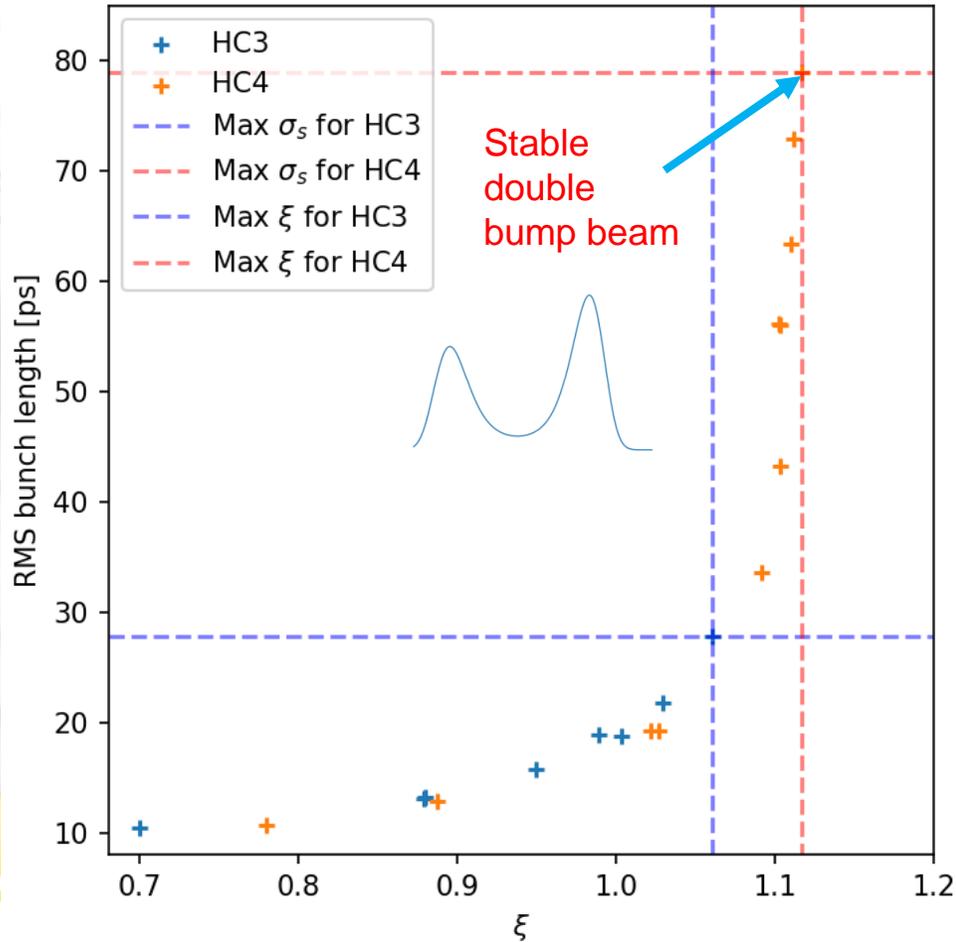


[1] He, T., Li, W., Bai, Z., & Wang, L. (2022). Periodic transient beam loading effect with passive harmonic cavities in electron storage rings. *PRAB*, 25(2), 024401.

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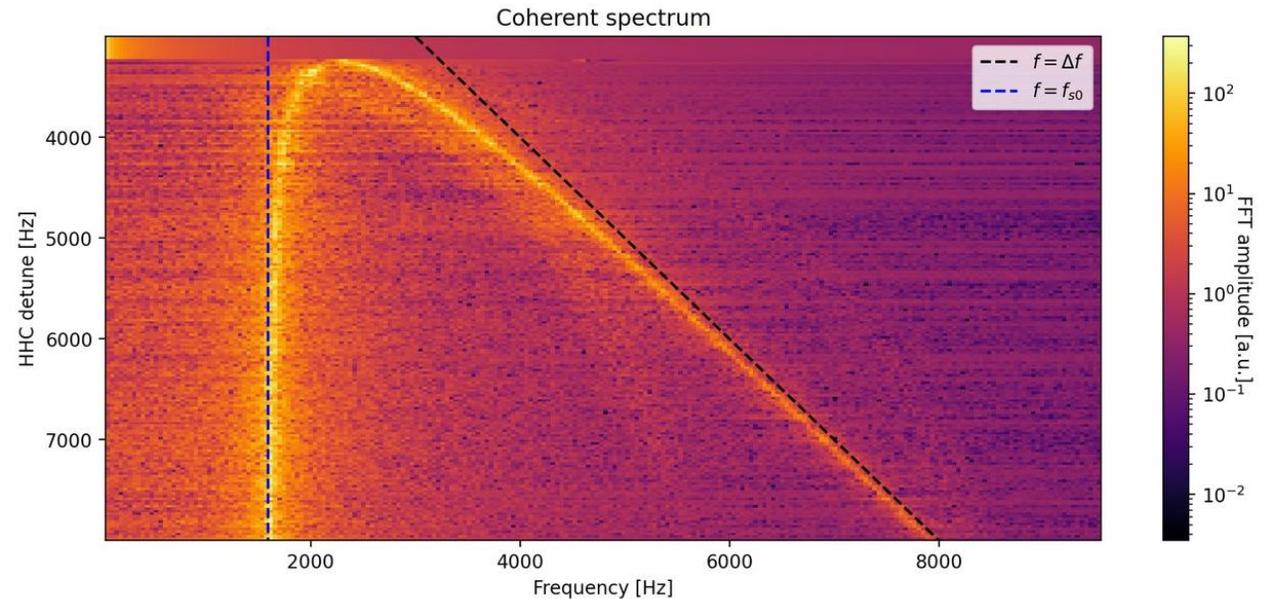
Bunch lengthening at low current (20 mA)

We also want to have the possibility to lengthen the bunches at low current, for example for the single bunch mode (at 20 mA) where the Touschek lifetime and IBS are very critical.



In that case, the picture is very different as the 4th HC allow to lengthen the bunches all the way to double bump bunches in a stable way.

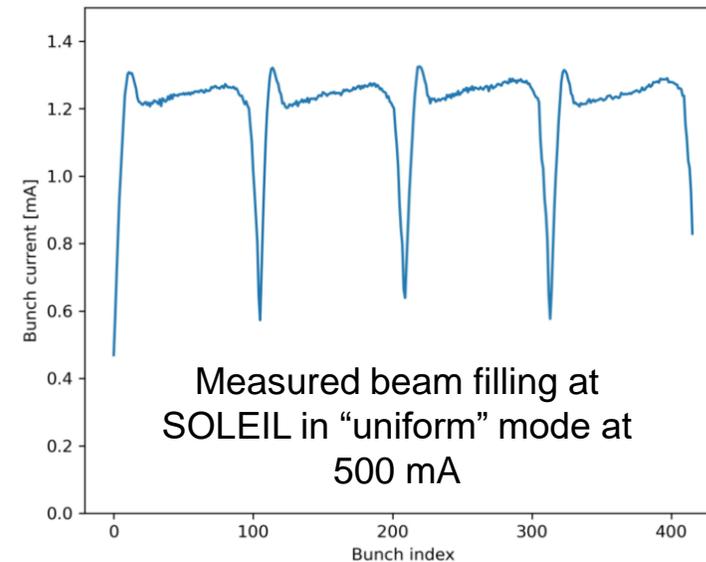
The 3rd HC is very rapidly limited by another instability which could be the “fast mode coupling instability” [1] or some type of Robinson instability (?).



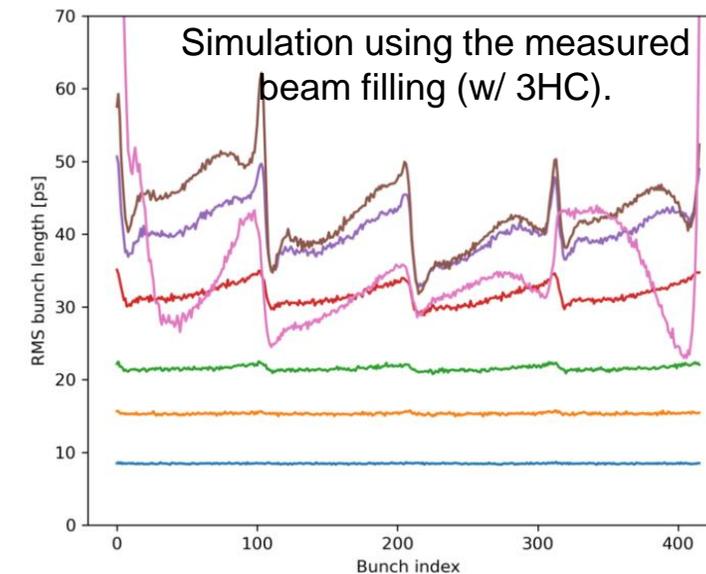
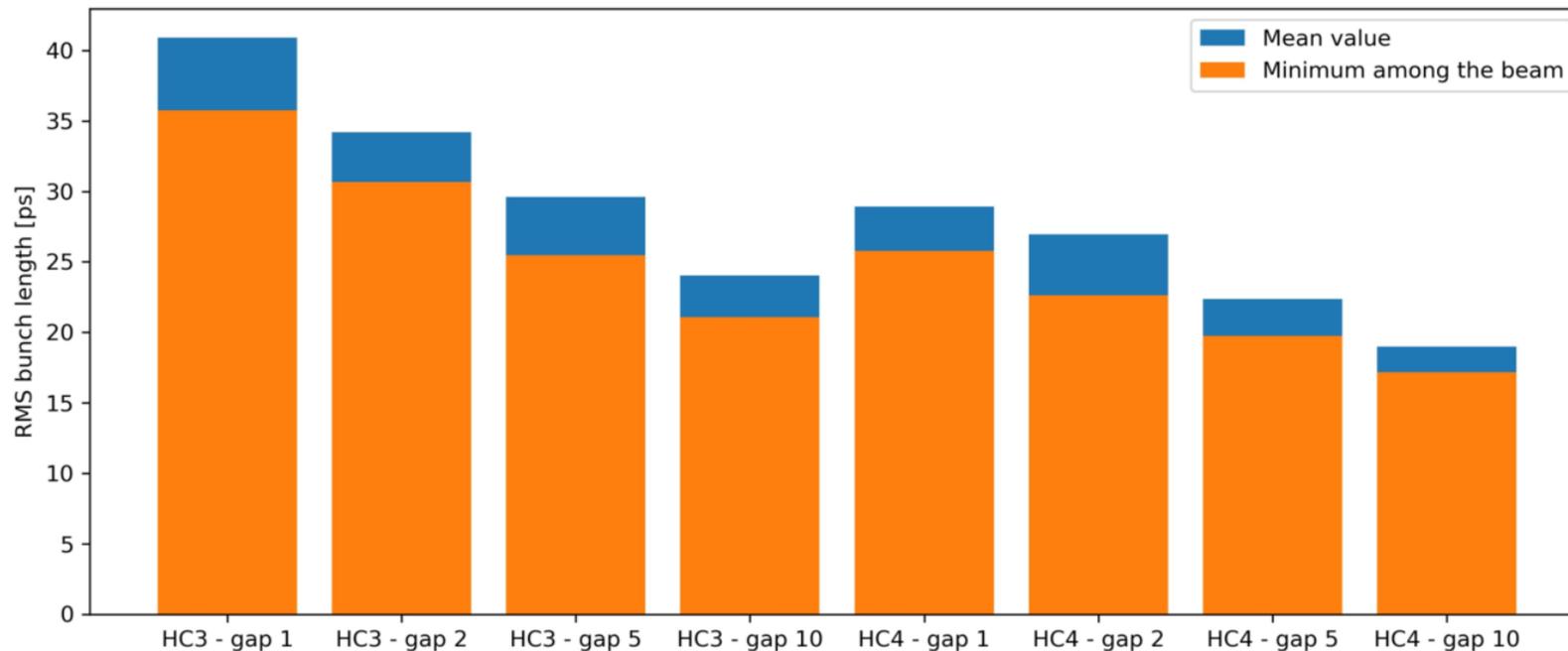
[1] R. A. Bosch, K. J. Kleman, and J. J. Bisognano, “Robinson instabilities with a higher-harmonic cavity”, Physical Review Special Topics-Accelerators and Beams 4, Publisher: APS, 074401 (2001).

Impact of gaps in the beam filling pattern

In today SOLEIL, the “uniform” filling pattern is injected by steps of 104 bunches ($\frac{1}{4}$ of the full filling). Due to the transmission from the booster, there is some variation of the current per bunch depending on the bunch index as shown in the **measured filling pattern taken during an operation run**:



Impact of gaps in the beam filling pattern



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Threshold shunt impedance for longitudinal coupled bunch instabilities driven by the main mode of the HC, sometime called Robinson or AC Robinson instabilities, can be computed using^[1]:

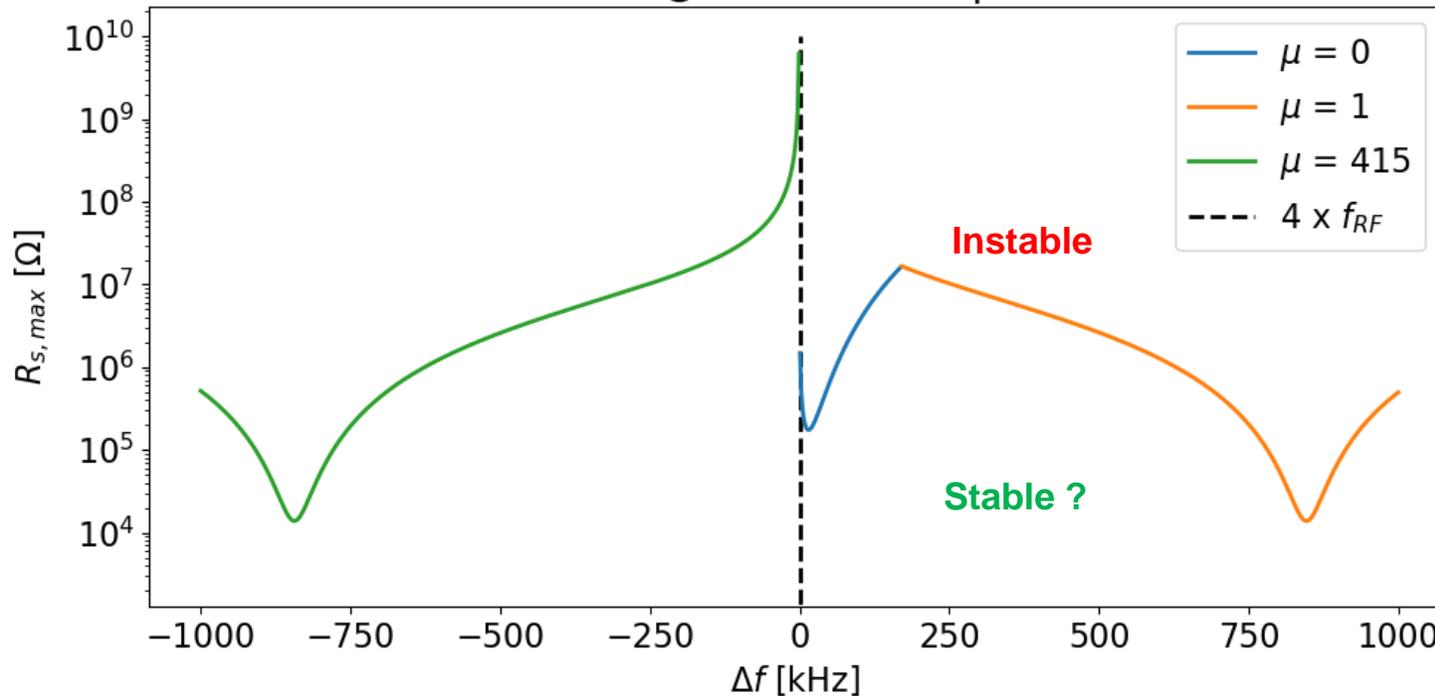
$$\tau^{-1} = \frac{e\alpha I}{4\pi E_0 v_s} \left\{ \sum_{n=0}^{\infty} \omega_{\mu,n}^+ \operatorname{Re}[Z(\omega_{\mu,n}^+)] - \sum_{n=0}^{\infty} \omega_{\mu,n}^- \operatorname{Re}[Z(\omega_{\mu,n}^-)] \right\}$$

$$\omega_{\mu,n}^{\pm} = \{nM \pm (\mu + \nu_s)\}\omega_0$$

↖ Coupled bunch mode number
↖ Number of bunches

$$Z(\omega) = \frac{R_s}{1 + iQ_L \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right)}$$

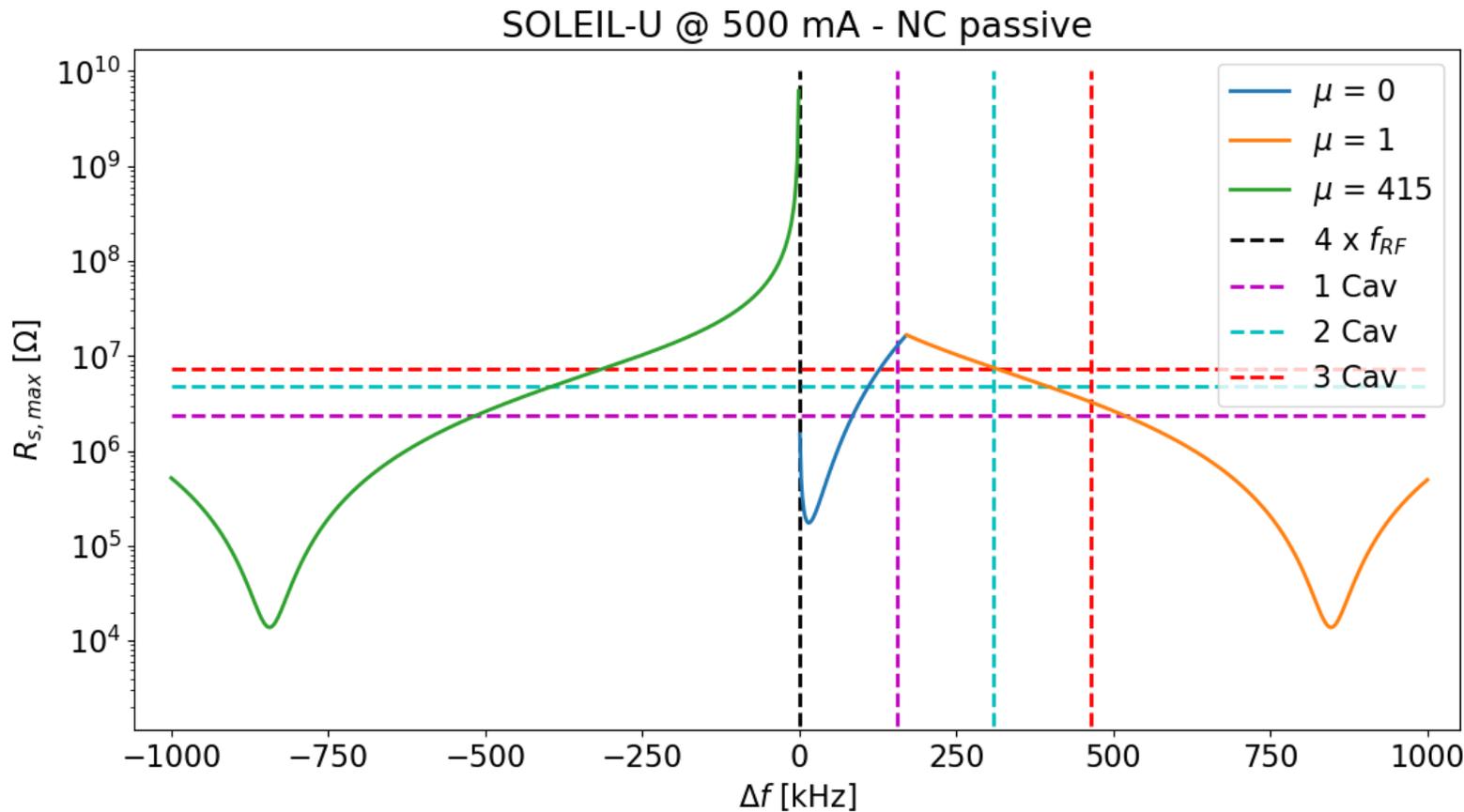
SOLEIL-U @ 500 mA - NC passive



[1] Akai, K. (2002). RF System for Electron Storage Rings. In *Physics and Engineering of High Performance Electron Storage Rings and Application of Superconducting Technology* (pp. 118-149).

The working point of each system ($R_s, \Delta f$) close to the NFP is given by the number of cavities N_{cav} and the beam current I :

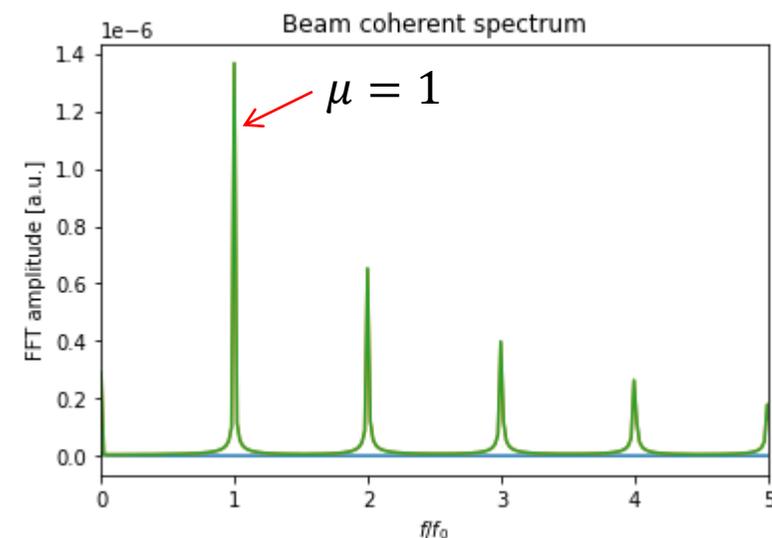
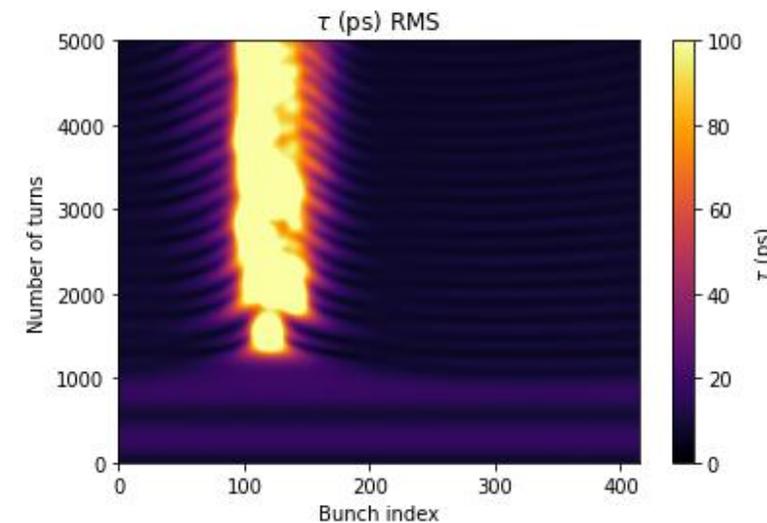
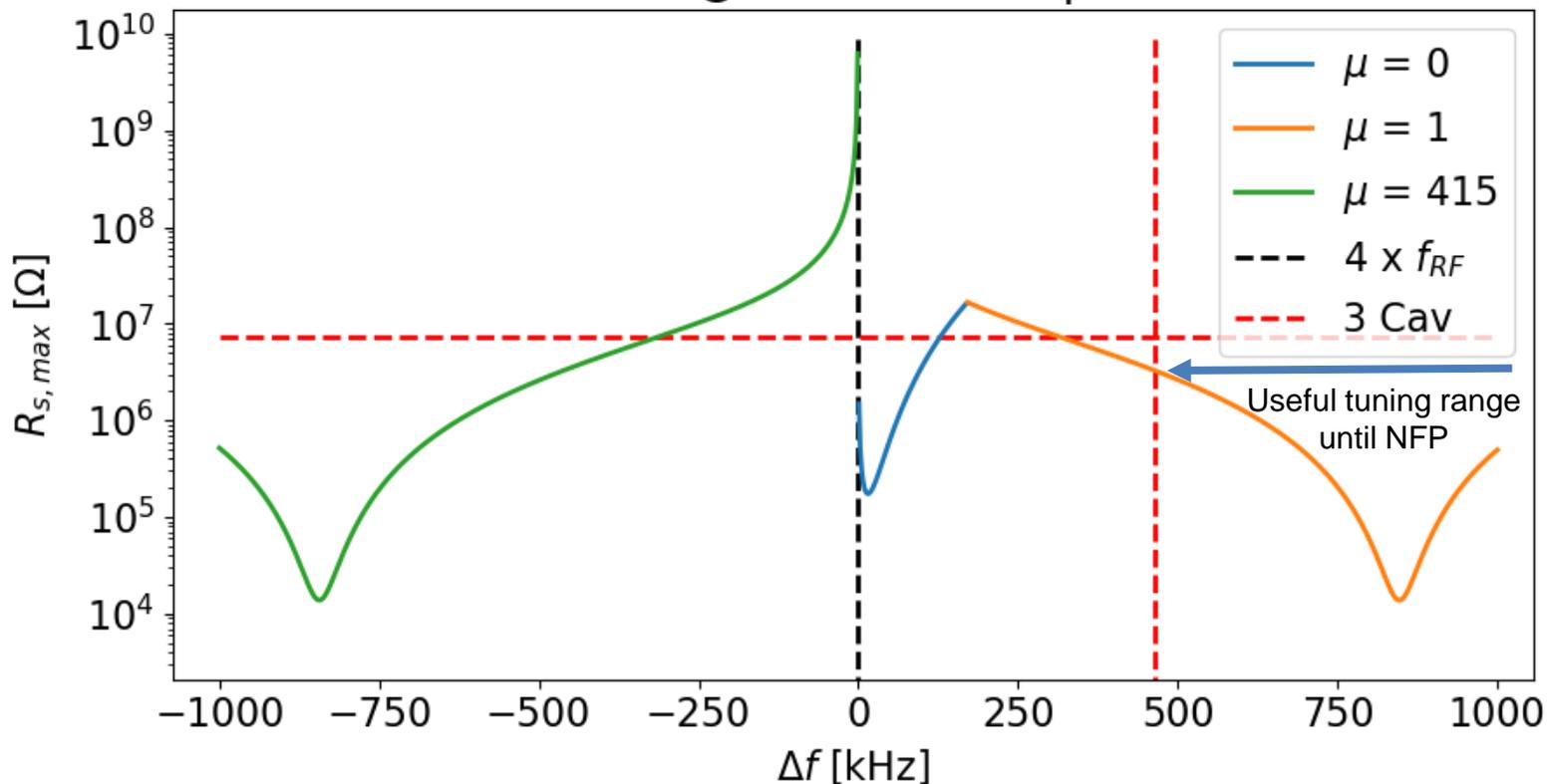
$$\Delta f = I \frac{R_s m f_{RF}}{Q V_2}$$



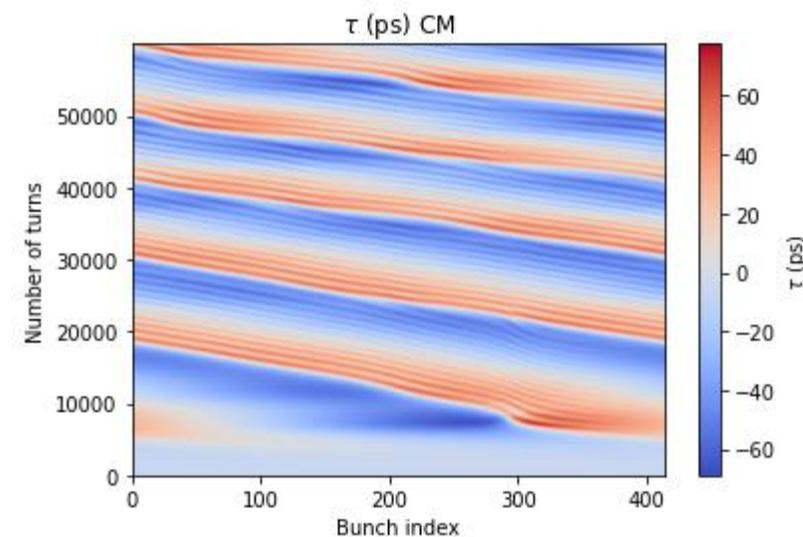
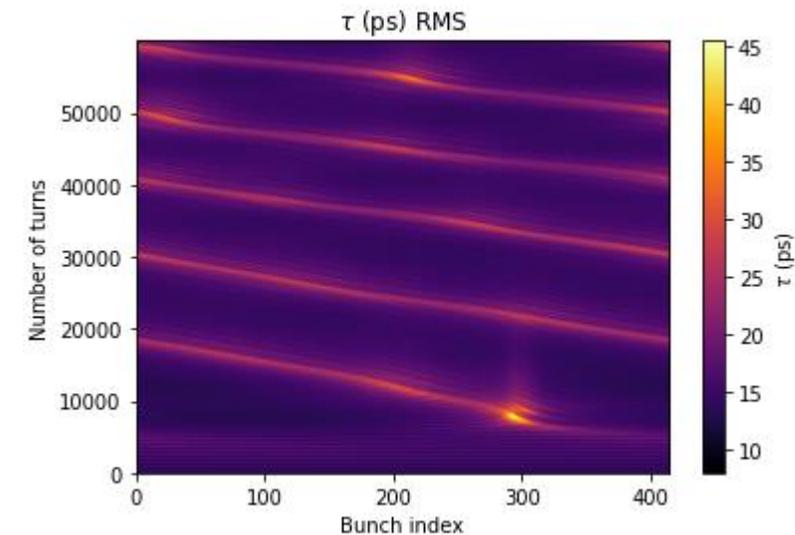
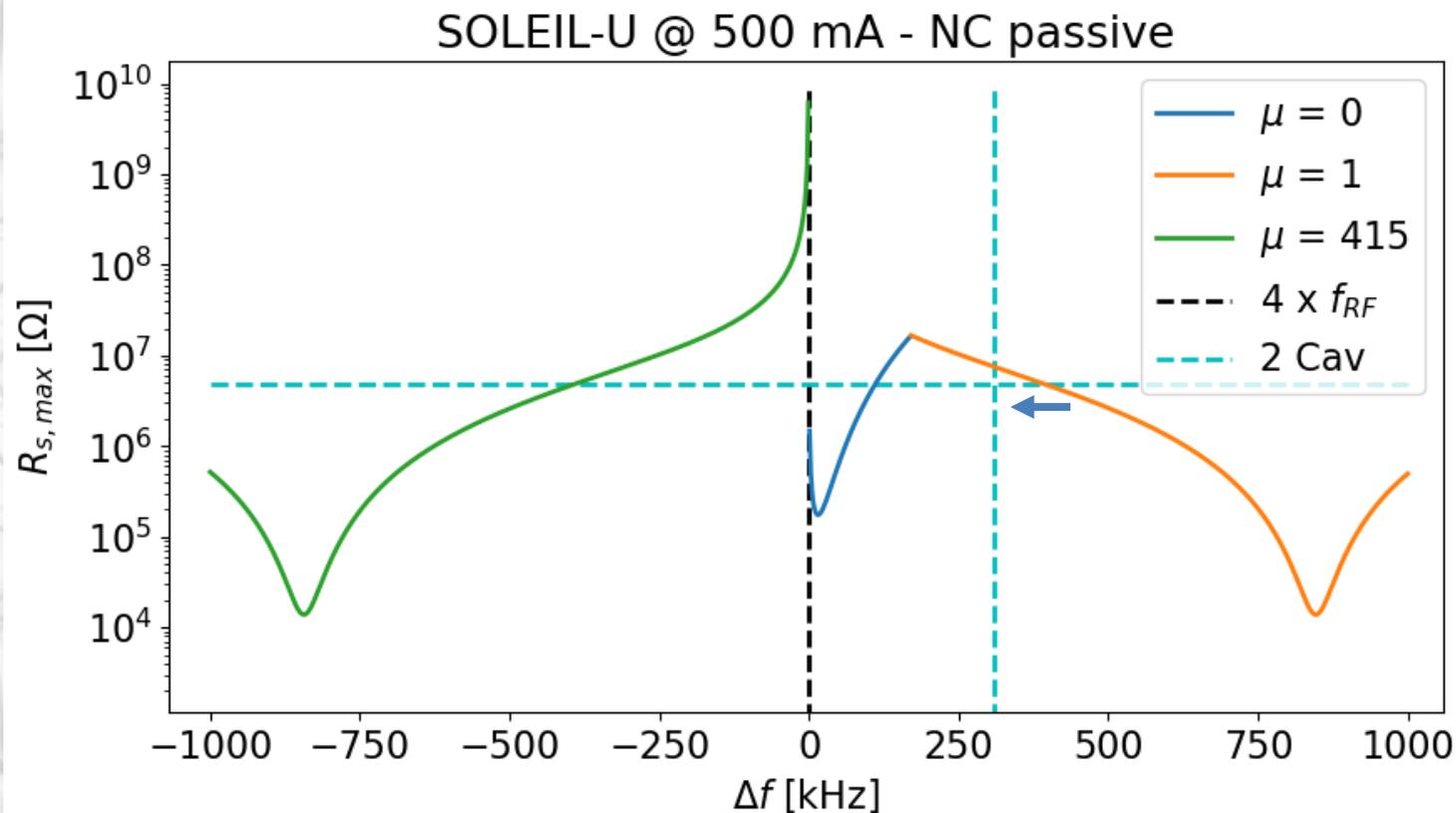
In agreement with the analytical estimate, the tracking show that $N_{cav} = 3$ is unstable for the full tuning range.

If Q is increased to 100 000, the beam is stable as expected from the LCBI formula and bunch lengthening is possible.

SOLEIL-U @ 500 mA - NC passive

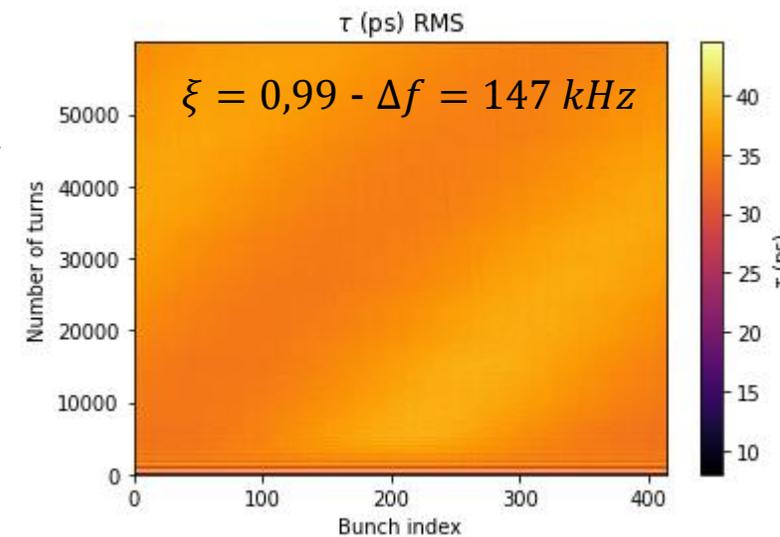
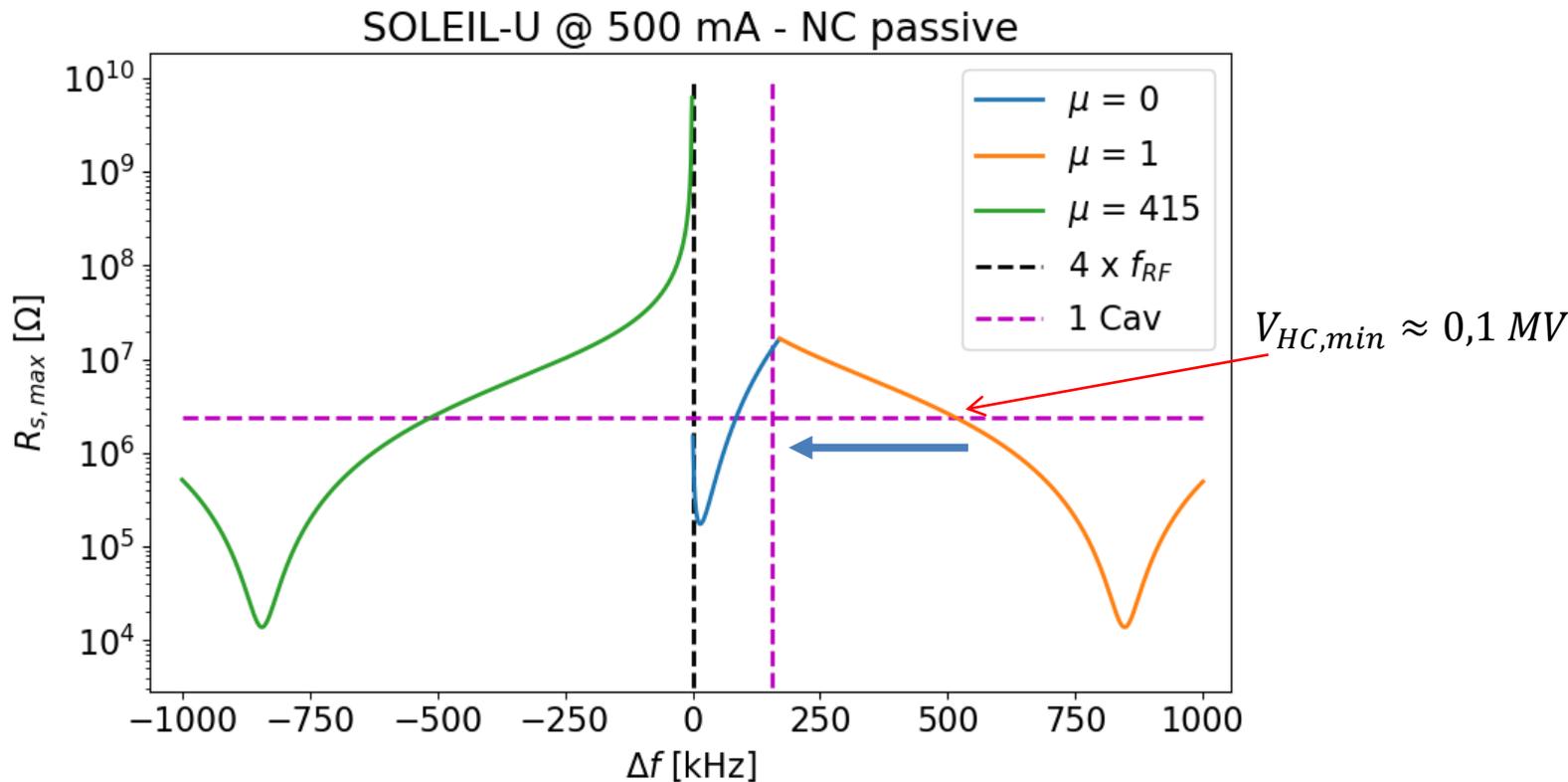
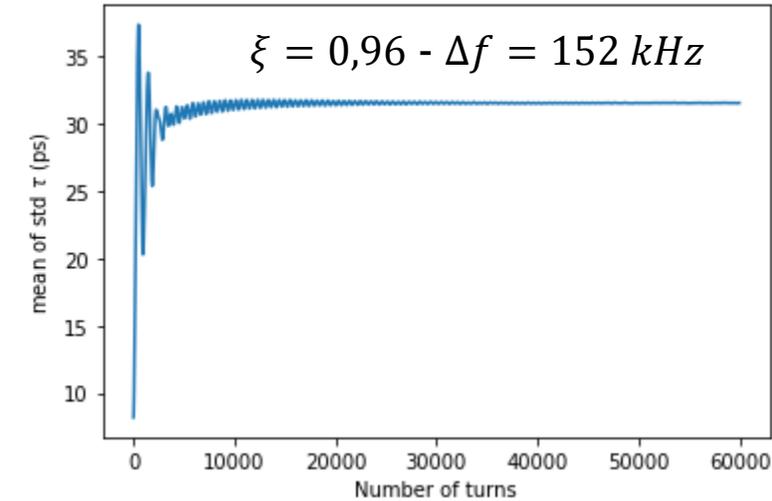


For $N_{cav} = 2$, the allowed tuning range is small, and the available range is unstable due to the PTBL instability:



$N_{cav} = 1$ allows bunch lengthening up to 31 ps, $\xi = 0,96$, then it is limited because of the PTBL instability. But:

- The parking position of the HC, $V_{HC} \approx 0 \text{ MV}$, would still be unstable
- The dissipated power in single cavity would be too high: $P_c \approx 37,6 \text{ kW}$



Fixed parameters:

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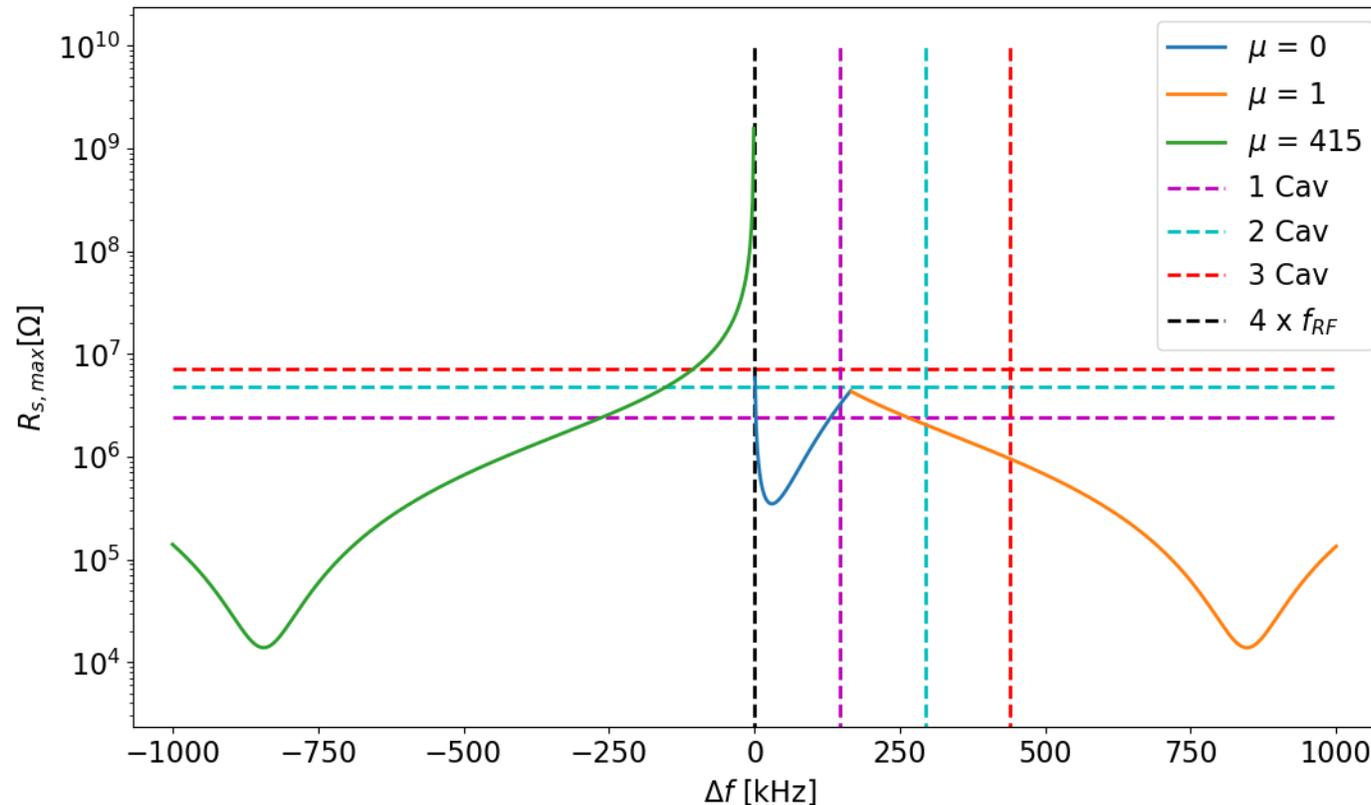
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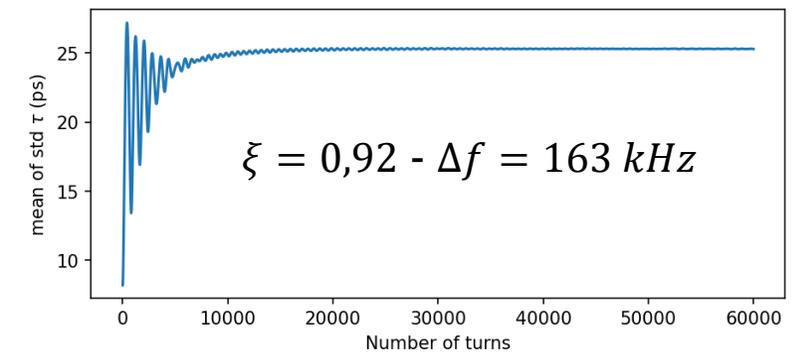
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The situation seems even worse for the NC active as the QL goes down to 13 500 instead of 27 000 which lower the threshold.

Feedback would be absolutely essential to be able to operate such a system.



Stable setting found for $N_{cav} = 1$



1. Passive SC : Super3HC ($m = 3$ or 4)

- At high current, $m = 3$ allows for larger Touschek lifetime factors (up to 6,1) than $m = 4$ (up to 4,3), as the PTBL is stronger for $m = 4$.
- For single bunch mode, $m = 4$ seems to provide more bunch lengthening than $m = 3$ but other collective effects should have a high impact.
- Important sensitivity to transient beam loading: to keep a Touschek lifetime factor larger than 3 the gap length should not exceed 9 bunches w/ 3HC and 4 bunches w/ 4HC.

2. Passive NC : ESRF harmonic 2-cell cavity ($m = 4$)

- Tuning range is strongly limited by AC Robinson for $N_{cav} = 2/3$.
- Would need a dedicated FB system, for at least $\mu = 1$, to (maybe) allow stable operation.
- PTBL is the limiting factor for bunch lengthening. For now, no idea of how efficient a FB would be against this instability.
- $N_{cav} = 1$ is the only stable option w/o FB but is not possible because of dissipated power.
- Ideally, we should consider other cavity types with lower R/Q but we are limited by available HC systems at harmonics of 352 MHz.

3. Active NC : ESRF harmonic 2-cell cavity ($m = 4$)

- The situation is even more dire than for passive NC as the Q_L is reduced by half.
- Would also need a dedicated FB system, for at least $\mu = 1$, to (maybe) allow stable operation.

Thank you for your attention!

Also ... two post-doc positions are open at SOLEIL:

- General beam dynamics and lattice design
 - Collective effects

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