Cross section of muon pair production by photons

Alexander Sandrock

February 10, 2022

The following formulae are based on the bremsstrahlung cross section of [1]. The pair production cross section follows from it by crossing symmetry and can be written as

$$\frac{d\sigma}{dx} = 4Z^2\alpha\left(r_e\frac{m}{\mu}\right)^2\Phi(\delta)\left\{x^2 + (1-x)^2 + \frac{2}{3}x(1-x)\right\}, \tag{1}$$

where $\delta = m_\mu^2/(2Ex(1-x))$ is the minimum momentum transfer and

$$\Phi(\delta) = \underbrace{\ln\frac{\frac{\mu}{m}BZ^{-1/3}}{1 + BZ^{-1/3}\sqrt{e}\delta/m_e}}_{\Phi_0} - \underbrace{\ln\frac{D_n}{1 + \delta(D_n\sqrt{e}-2)/\mu}}_{\Delta_n}, \tag{2}$$

with $D_n = \ln(1.54A^{0.27})$ the elastic nuclear formfactor correction and $B \approx 183$ the radiation logarithm.

The contribution of atomic electrons can be taken into account by replacing $\Phi(\delta)$ with

$$\Phi(\delta) + \frac{1}{Z}\left\{\ln\frac{\mu/\delta}{\delta\mu/m^2 + \sqrt{e}} - \ln\left(1 + \frac{1}{\delta\sqrt{e}B'Z^{-2/3}/m}\right)\right\} \tag{3}$$

with $B' \approx 1440$ the inelastic radiation logarithm, the effect of the inelastic nuclear formfactor by the substitution $\Delta_n \to (1-1/Z)\Delta_n$ for $Z \leq 1$; obviously, for $Z = 1$, there are no nuclear levels to excite quasielastically, thus there is no inelastic nuclear formfactor.

The formulae given above approximate the no-screening limit for $\delta \to \mu/\sqrt{e}$, i. e. $\Phi(\delta) \to \ln\mu/\delta - 1/2$. With sufficient accuracy, the limits of x are thus given by

$$\frac{1}{2}\left(1 - \sqrt{1 - 2\sqrt{e}\frac{\mu}{E}}\right) \leq x \leq \frac{1}{2}\left(1 + \sqrt{1 - 2\sqrt{e}\frac{\mu}{E}}\right); \tag{4}$$

for relativistic muons, this can be approximated by

$$\frac{\mu\sqrt{e}}{2E} \leq x \leq 1 - \frac{\mu\sqrt{e}}{2E}. \tag{5}$$

References

[1] S. R. Kelner, R. P. Kokoulin, and A. A. Petrukhin. *About Cross Section for High-Energy Muon Bremsstrahlung.* Preprint MEPhI 024-95. Moscow, 1995.