OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

Analogues of light and gravity in the collective excitations of quantum magnets

Leilee Chojnacki[1]

Rico Pohle[2]

Han Yan[1,3]

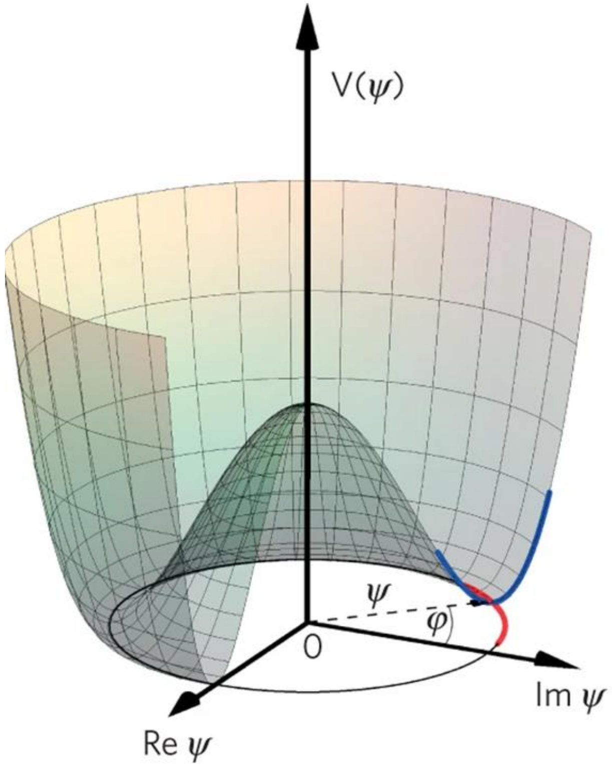
Yutaka Akagi[2]

Nic Shannon[1]

1: Theory of Quantum Matter Unit, OIST; 2: University of Tokyo; 3: Rice University

The Universe in Quantum Matter:
Some examples of low energy phenomenology exhibiting high energy parallels

Anderson-Higgs mechanism in Superconductors



P.W.Anderson *Phys. Rev.* **130**, 439–442 (1963)

D.Sherman et al. *Nature Phys.* **11**, 188–192 (2015)

A.Jain et al. *Nature Phys*. **13**, 633-637 (2017)

The Quantum Vacuum in a Superfluid

GUT in Standard Model
symmetry breaking phase transitions

$SO(10) \rightarrow SU(3)\times SU(2)\times U(1) \rightarrow SU(3)\times U(1)$

GUT in superfluid ^{3}He
symmetry breaking phase transitions

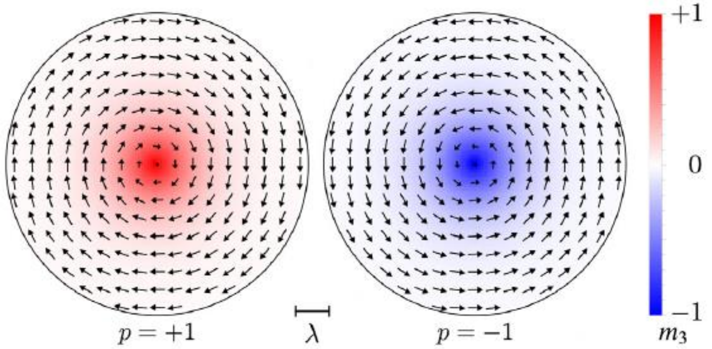
$SO(3)\times SO(3)\times U(1) \rightarrow U(1)\times U(1) \rightarrow SO(3)$

G.E.Volovik, “*The Universe in a Helium Droplet*”, Oxford University Press (2009)

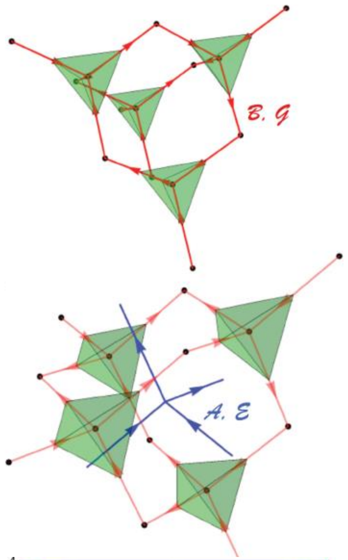
(2+1)D Electromagnetism in the XY model

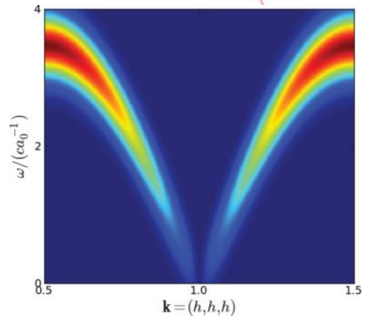
J.M.Kosterlitz, *J. Phys. C* **7**, 1046 (1974).

S.Dasguspta et al. *Phys. Rev. Lett.* **124**, 157203 (2020)



(3+1)D Electromagnetism in Quantum Spin Ice





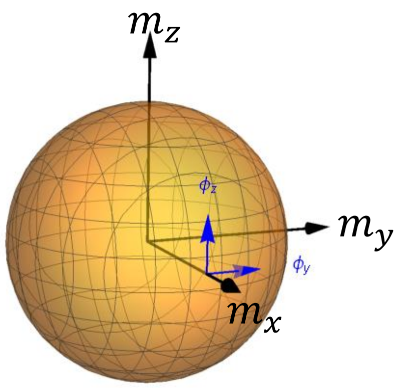
R. Moessner, S.L.Sondhi *Phys. Rev. B* **68**, 064411 (2003)

M. Hermele et al. *Phys. Rev. B* **69**, 064404 (2004).

Benton et al. *Phys Rev B* **86**.075154 (2012)

leilee.chojnacki@oist.jp

“Photons” in collinear Heisenberg antiferromagnets



2 Goldstone bosons $\vec{\phi}_{\circlearrowleft}, \vec{\phi}_{\circlearrowright}$
Spin-1 modes

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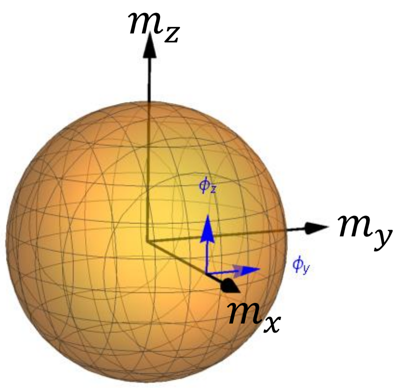

2 Goldstone bosons $\vec{\phi}_{\circlearrowleft}, \vec{\phi}_{\circlearrowright}$
Spin-1 modes

Low energy field theory for HAF

$$\mathcal{L} = \frac{1}{2}\left[\chi(\partial_t\vec{\phi})^2 - \rho(\partial_i\vec{\phi})^2\right]$$

c.f. EM in Feynman gauge

$$\mathcal{L} = \frac{1}{2}\left[\frac{1}{c^2}(\partial_t\vec{A})^2 - (\partial_i\vec{A})^2\right]$$

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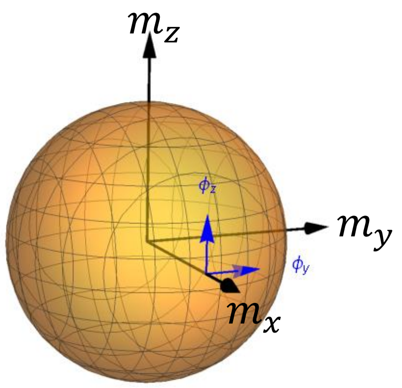
“Photons” in collinear Heisenberg antiferromagnets



2 Goldstone bosons $\vec{\phi}_{\circlearrowright}, \vec{\phi}_{\circlearrowleft}$
Spin-1 modes

Left circular polarization		Right circular polarization
$\vec{\phi}_{\circlearrowleft} = \vec{A}_{\circlearrowleft}$	$\overset{\hat{T}}{\longleftrightarrow}$	$\vec{\phi}_{\circlearrowright} = \vec{A}_{\circlearrowright}$
$-\partial_t\vec{\phi}_{\circlearrowleft} = \vec{E}_{\circlearrowleft}$	$\overset{\hat{T}}{\longleftrightarrow}$	$-\partial_t\vec{\phi}_{\circlearrowright} = \vec{E}_{\circlearrowright}$
$\vec{\nabla} \times \vec{\phi}_{\circlearrowleft} = \vec{B}_{\circlearrowleft}$	$\overset{\hat{T}}{\longleftrightarrow}$	$\vec{\nabla} \times \vec{\phi}_{\circlearrowright} = \vec{B}_{\circlearrowright}$

Low energy field theory for HAF

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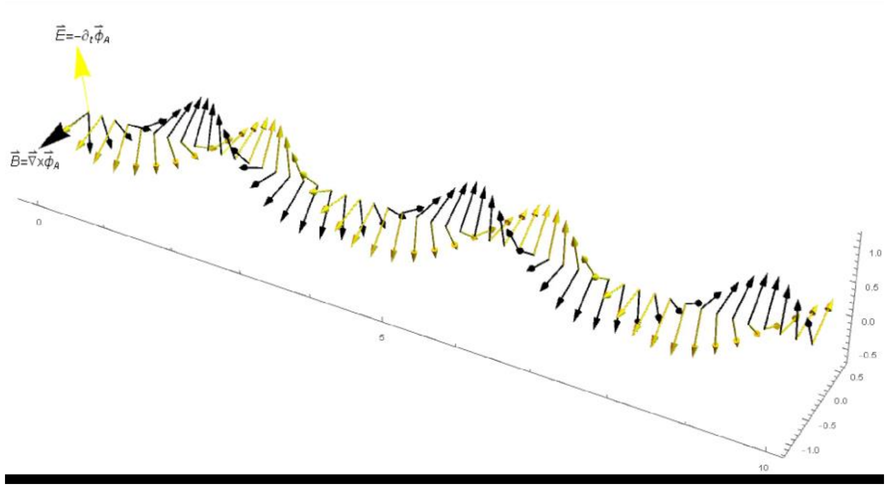

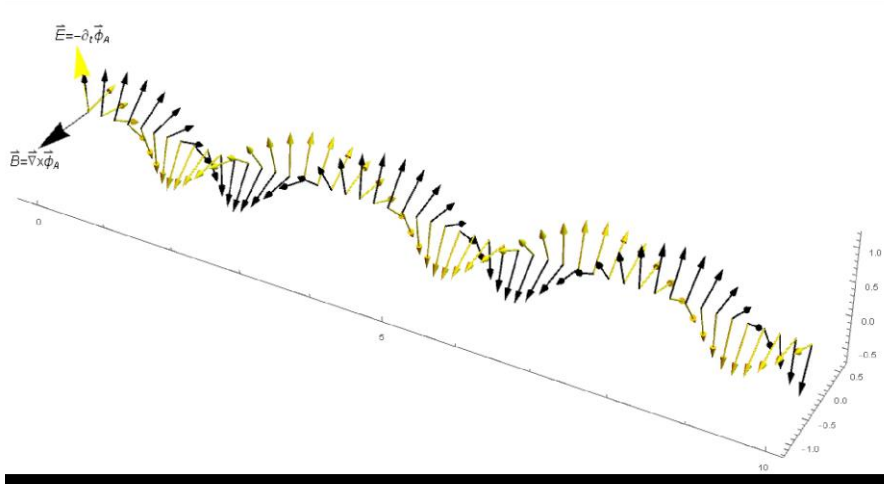

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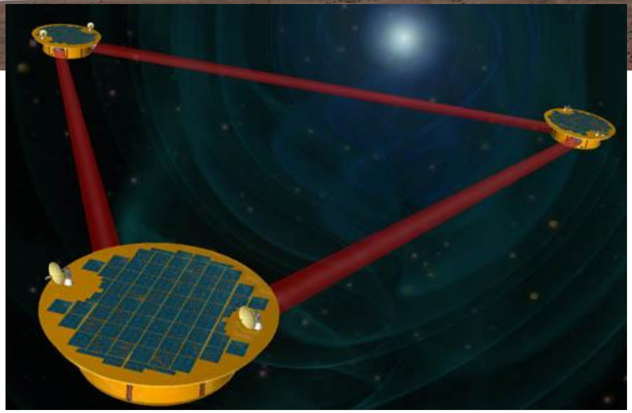
Detection of gravitational waves

Virgo (Europe)

LIGO Hanford (USA)

KAGRA (Japan)



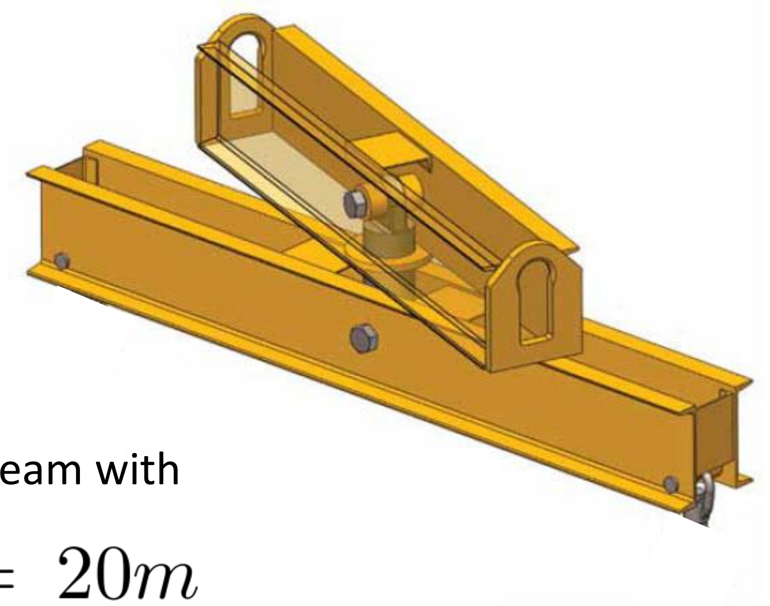
LISA (NASA)

Could we create ever detectable gravitational waves

in a lab?

A back of the envelope calculation…

Gravitational wave output of a uniform rod rotating about its centre



$$P_{GW} = \frac{2}{45}\frac{G}{c^4}M^2 l^4 \omega^6$$

Given that

$$\frac{G}{c^4} \approx 8 * 10^{-45}\frac{s^2}{kg\ m}$$

For a steel beam with

$$l = 20m$$

$$\omega = 28 rad/s$$

$$P_{GW} \approx 10^{-30} J/s$$

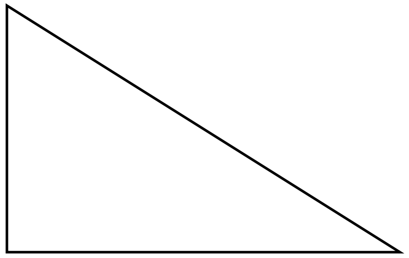
leilee.chojnacki@oist.jp

C. Misner, K. Thorne, J. Wheeler "Gravitation"


Can we find an analogue for gravitational waves in the context of magnetism?

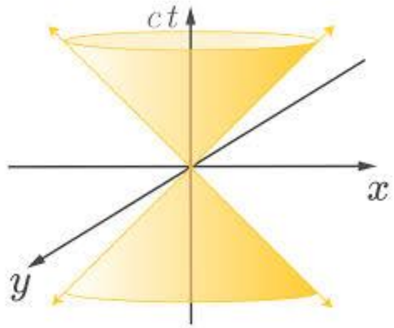
The metric and linearized gravity

Metric of distance in 3D space



$$ds^2 = dx^2 + dy^2 + dz^2$$

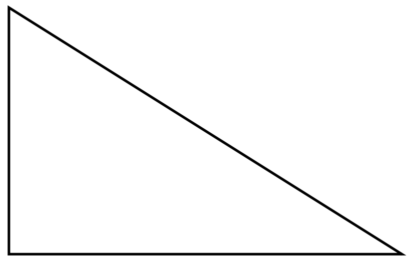
Metric of distances in space-time



$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

The metric and linearized gravity

Metric of distance in 3D space



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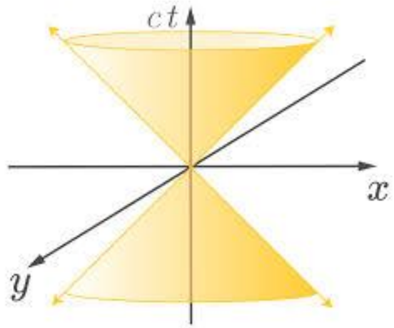
Metric of distances in space-time



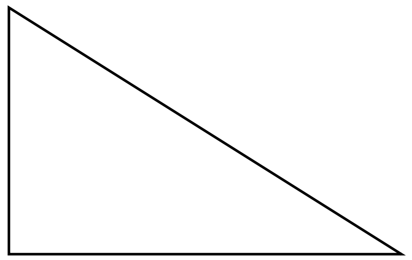
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Flat spacetime

$$ds^2 = g_{tt}dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2$$

The metric and linearized gravity

Metric of distance in 3D space



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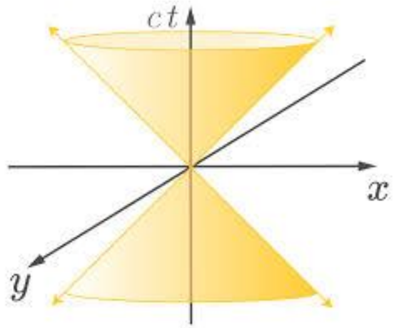


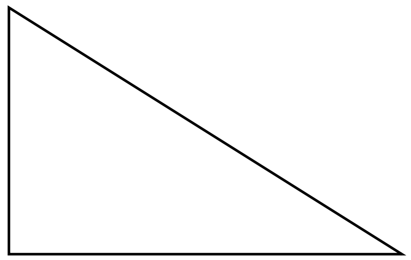
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

Flat spacetime $$ds^2 = g_{tt}dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2$$

Curved spacetime $g_{\mu\nu} \neq 0$ $\mu \neq \nu$ and $g'_{\mu\nu} = g_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}$

The metric and linearized gravity

Metric of distance in 3D space



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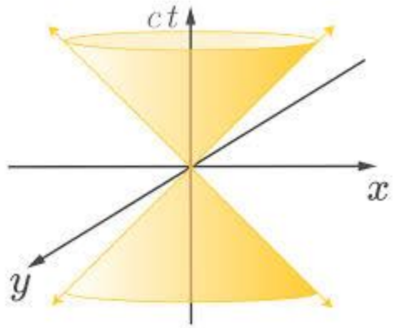
Metric of distances in space-time



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Linearized gravity $$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h'_{\mu\nu} = h_{\mu\nu} - \xi_{\mu,\nu} - \xi_{\nu,\mu}$$

Tensor field theory for linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Weak field limit (linearized gravity)

Effective field theory for linearized gravity

$$S_{LGR} = -\frac{c^4}{16\pi G} \int dx^4 \left[\partial^\nu h^{\mu\rho} \partial_\nu h_{\mu\rho} \right]$$

$$h^i_i = 0$$

$$h_{0\mu} = 0$$

$$\partial^i h_{ij} = 0$$

In harmonic gauge

Tensor field theory for linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

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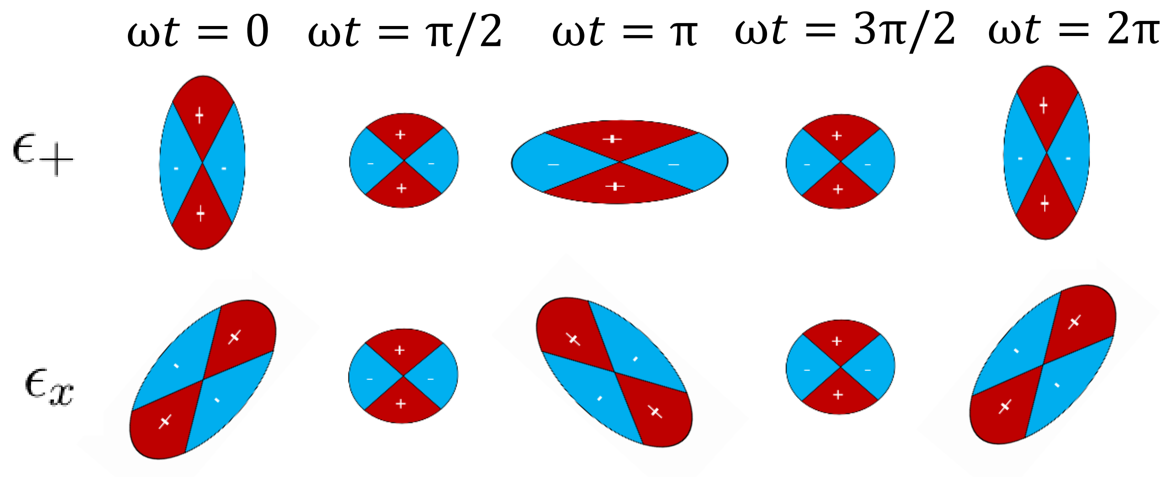
In harmonic gauge

with wave solutions

$$h_{\mu\nu}(t,x) = \sum_k \epsilon_{\mu\nu}(k) e^{ik^\alpha x_\alpha} + c.c.,$$

Dynamics only occurs in spatial components

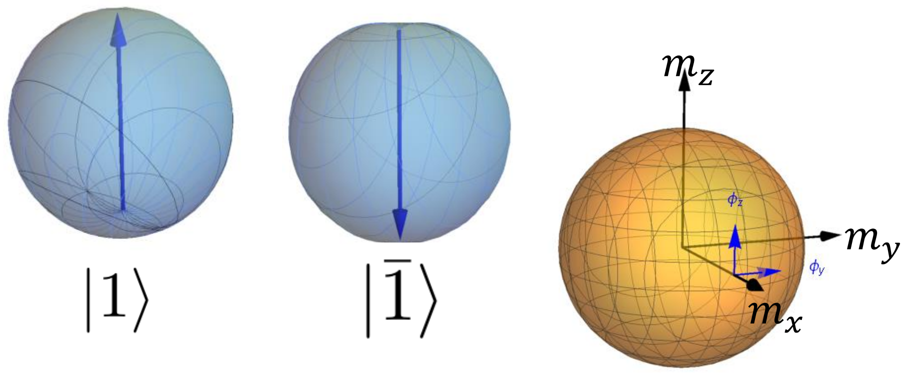
$$\epsilon_{ij} = \begin{pmatrix} \epsilon_+ & \epsilon_x & 0 \\ \epsilon_x & -\epsilon_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Effect of the gravitational polarizations

A microscopic model with tensor order

On site dipolar DoF



Order parameter space: $\boldsymbol{S_2}$

+

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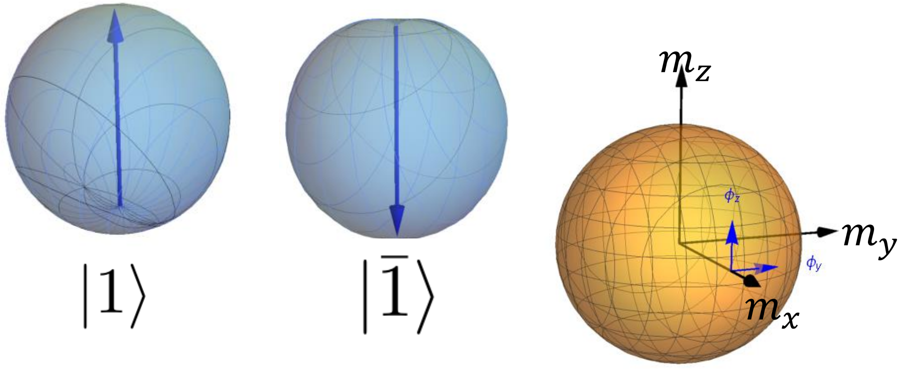
A microscopic model with tensor order

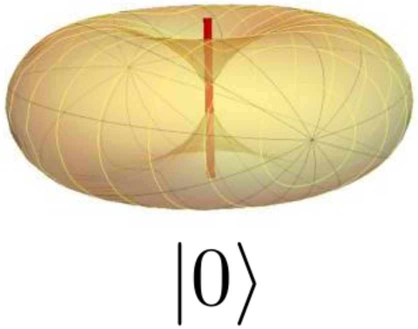
On site dipolar DoF



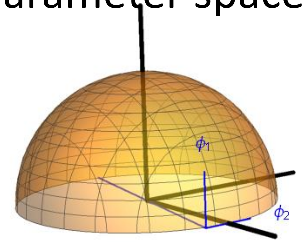
Order parameter space: $\boldsymbol{S_2}$

+

On site quadrupolar DoF



Order parameter space: $\boldsymbol{CP2}$



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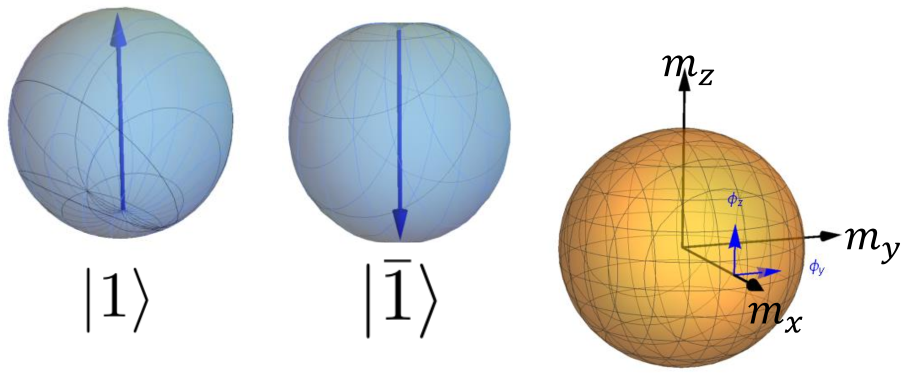
A microscopic model with tensor order

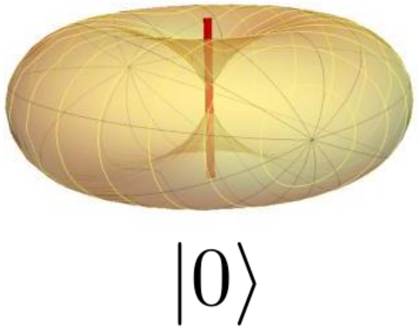
On site dipolar DoF



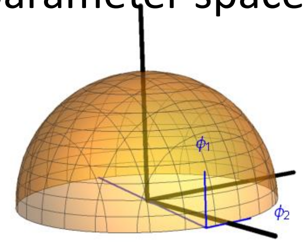
Order parameter space: $\boldsymbol{S_2}$

+

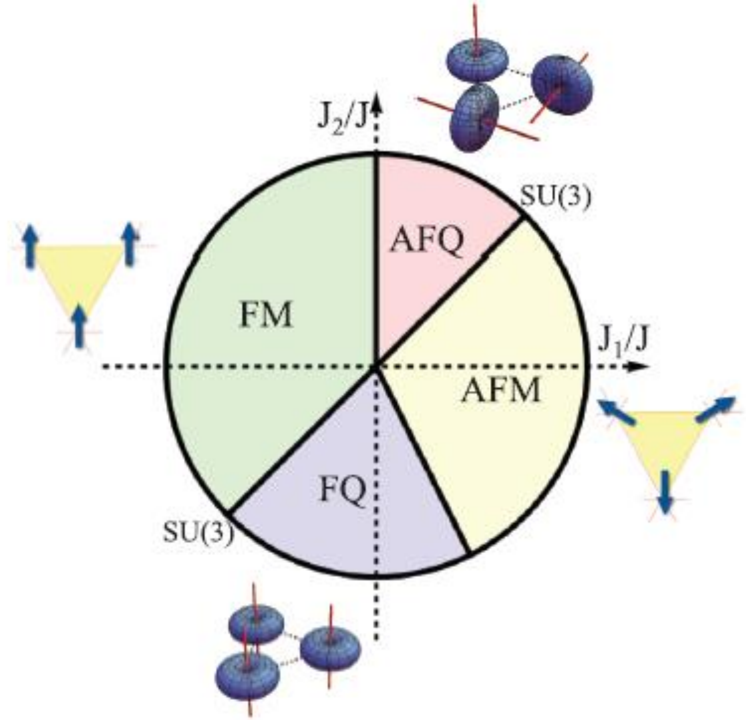
On site quadrupolar DoF



Order parameter space: $\boldsymbol{CP2}$



$$H_{BBQ} = \sum_{\langle ij \rangle} J_1 \vec{S}_i.\vec{S}_j + \sum_{\langle ij \rangle} J_2 (\vec{S}_i.\vec{S}_j)^2$$



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A. Smerald and N. Shannon, Phys. Rev. B **88**, 184430 (2013).
K. Remund et al, Phys. Rev. Research **4**, 033106 (2022).

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A microscopic model with tensor order

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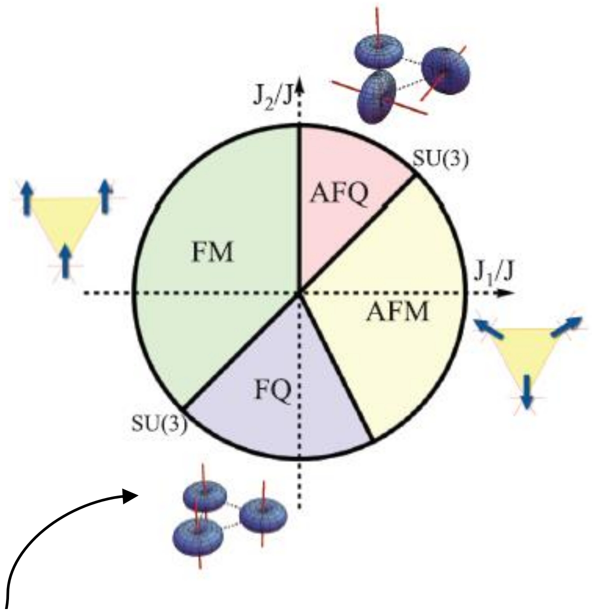

Ferroquadrupolar phase

$$J_2 < 0 \qquad J_1 << J_2$$

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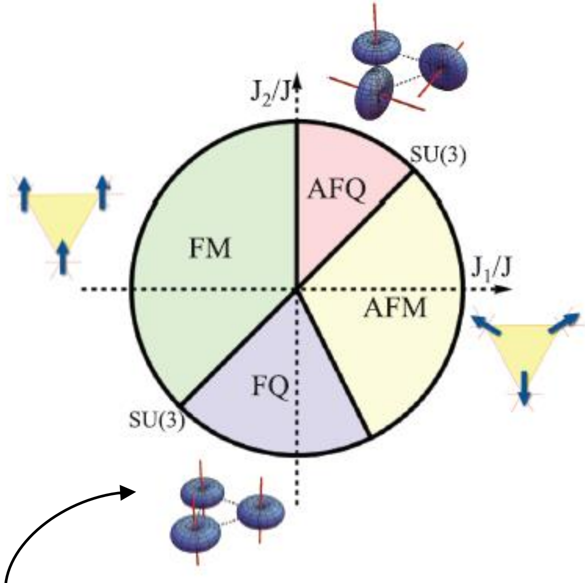
$$Q^{\alpha\beta} = S^\alpha S^\beta + S^\beta S^\alpha - \frac{2}{3} S(S+1)\delta^{\alpha\beta}$$



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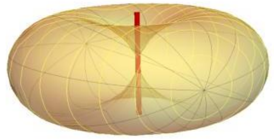
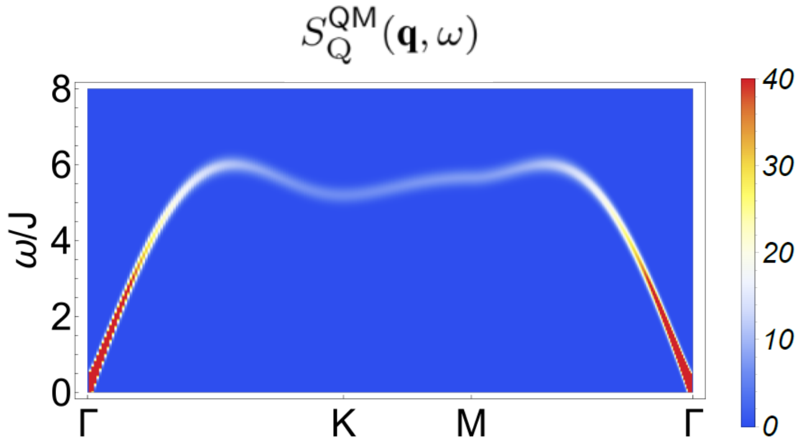
On site quadrupolar DoF





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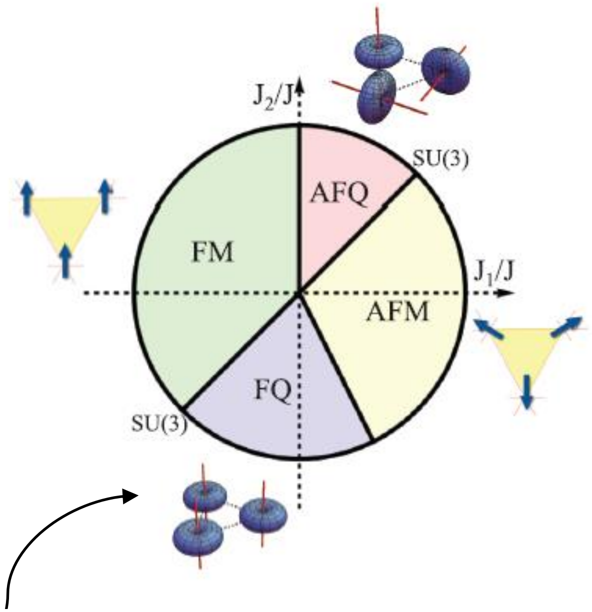

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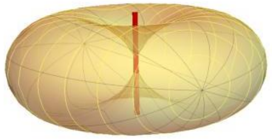
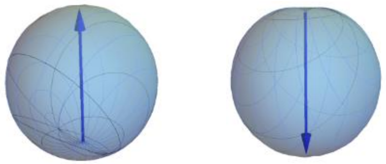
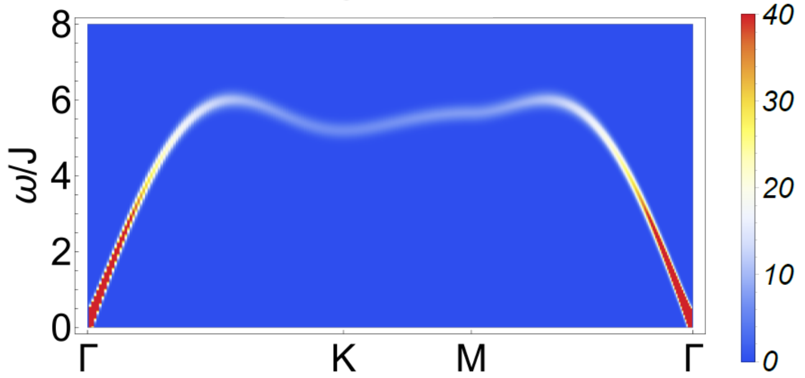
On site quadrupolar DoF



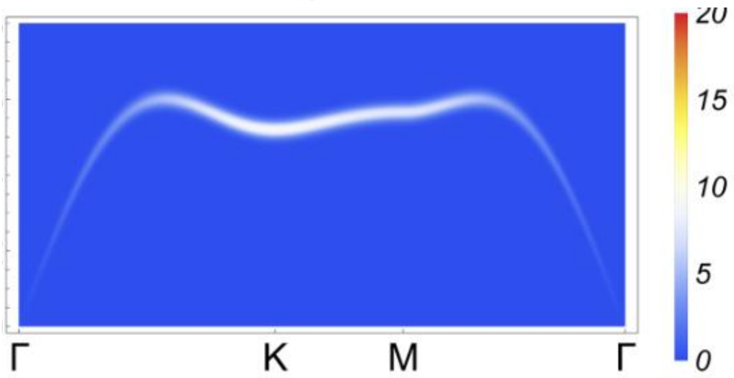
On site dipolar DoF



$S_Q^{QM}(\mathbf{q}, \omega)$



$S_S^{QM}(\mathbf{q}, \omega)$



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Hydrodynamic limit of ferroquadrupolar (spin nematic) phase

$$\mathcal{S}_{FQ} = -\frac{1}{2}\int dt dr^d \Big[\chi_\perp (\partial^t \tilde{Q}^{\alpha\beta}\partial_t \tilde{Q}_{\alpha\beta}) - \rho_s (\partial^i \tilde{Q}^{\alpha\beta}\partial_i \tilde{Q}_{\alpha\beta})\Big]$$

$$\tilde{Q}^i_i = 0$$

$$\tilde{Q}_{0\mu} = 0$$

$$\partial^i \tilde{Q}_{ij} = 0$$

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$$\tilde{Q}^i_i = 0$$

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$$\partial^i \tilde{Q}_{ij} = 0$$

Linearized vacuum action for gravity

$$\mathcal{S}_{LGR} = \frac{c^3}{16\pi G}\int dtdx^3 \left[\frac{1}{c^2}\partial^t h^{\alpha\beta}\partial_t h_{\alpha\beta} - \partial^i h^{\alpha\beta}\partial_i h_{\alpha\beta}\right]$$

$$h^i_i = 0$$

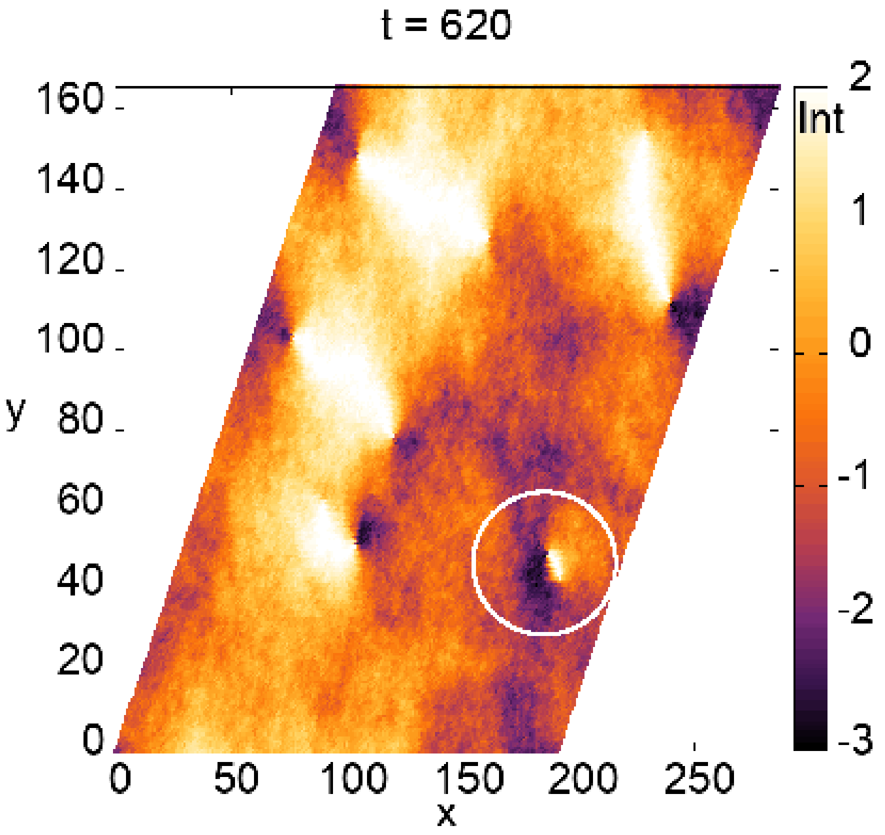
$$h_{0\mu} = 0$$

$$\partial^i h_{ij} = 0$$

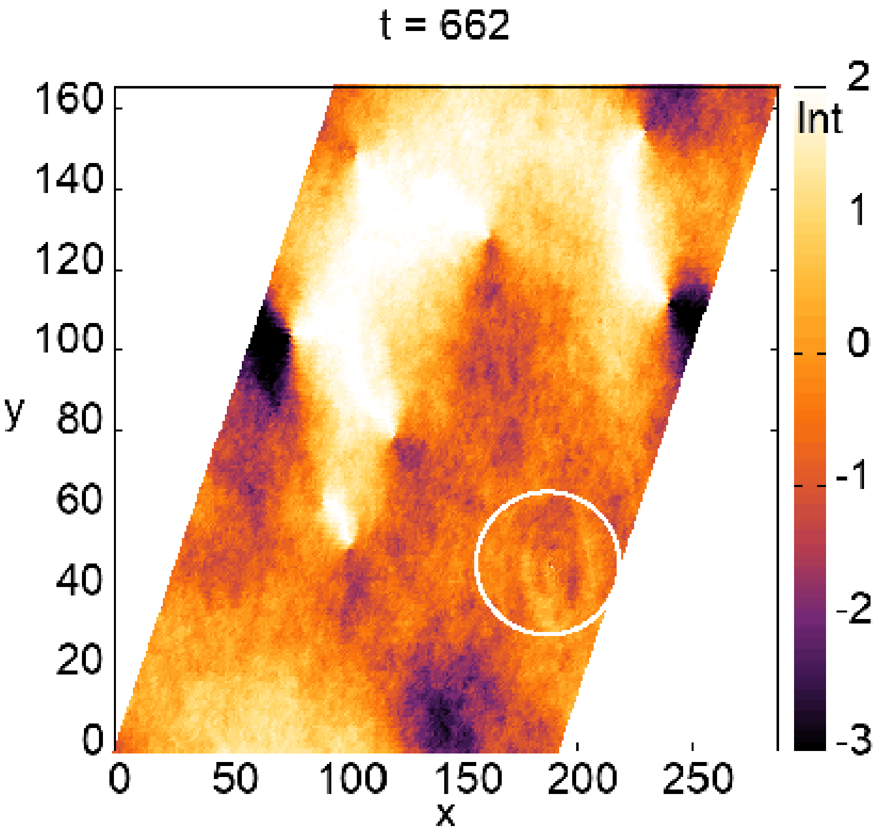
“Gravitational waves” in the quadrupolar excitations of a quantum magnet

Dynamical simulation of a **nematic ordered phase** (ferroquadrupolar) on 2D triangular lattice.

Annihilation of topological defects → Dissipation in the quadrupole channel



$$Q(r) = \sqrt{Q^{*}_{ab}(r) Q^{ab}(r)}$$



$$Q(r) = \sqrt{Q^{*}_{ab}(r) Q^{ab}(r)}$$

leilee.chojnacki@oist.jp

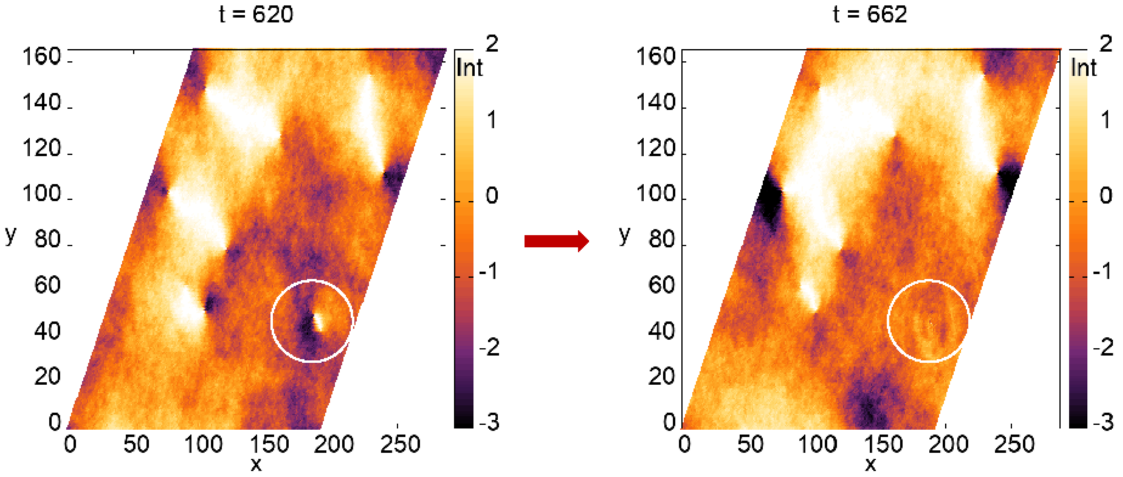
Conclusion

- From analysis of low energy field theory, we can build dictionaries ex. for electromagnetism to the Heisenberg antiferromagnet

$$\vec{\phi}_{\circlearrowright} \longleftrightarrow \vec{A}_{\circlearrowright}$$

- In the linear low energy limit, the excitations of linearized gravity are in 1 to 1 correspondence with ferroquadrupolar waves

$$h_{ij} \longleftrightarrow Q_{ij}$$



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Rico Pohle
University of Tokyo

Han Yan
Rice University

Yutaka Akagi
University of Tokyo

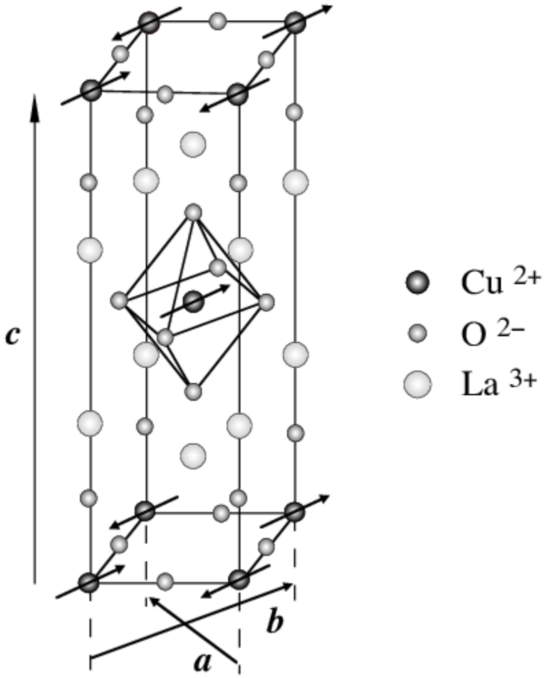
Nic Shannon
TQM, OIST

Thanks to the TQM unit!

And thank you for your attention!

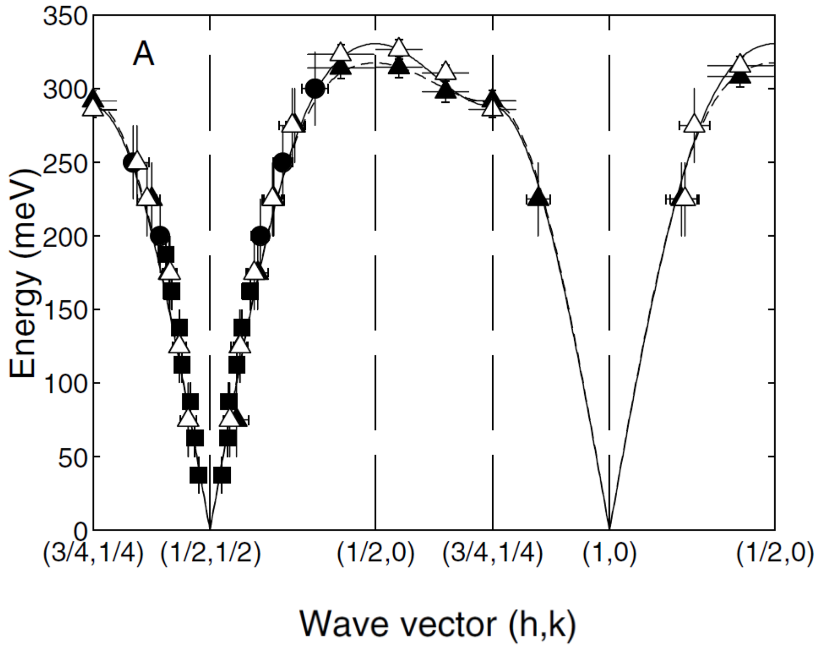
How to build low energy field theories for condensed matter systems

Ex. La_2CuO_4 Antiferromagnetic ordered phase



Effective description of
low energy ordered phase

In the limit $a \ll \lambda$



R.Coldea et al. Phys. Rev. Lett. 86.5377 (2001)

I. A. Zaliznyak.and J. M. Tranquada, "Neutron Scattering and Its Application to Strongly Correlated Systems", (2014)

Ordered phases of O(3) magnets

Most general vector field theory to Gaussian order

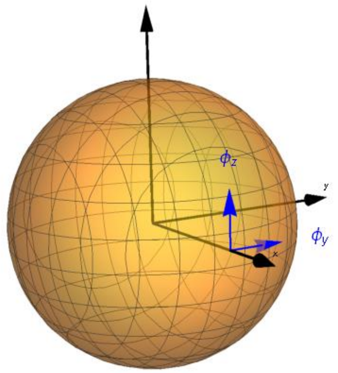
$$\mathcal{L} = a^m \partial_t \phi_m + c^m \partial_t^2 \phi_m + d^m \partial_r^2 \phi_m$$

More info see ex.:
H. Watanabe, H. Maruyama,
Phys. Rev. Lett. **108,** 251602 (2012)

Consider microscopic O(3) Heisenberg model

$$H = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j = \sum_{\delta} J \vec{S}_i \cdot \vec{S}_{i+\delta}$$

Phases on non-frustrated geometries



Ferromagnet

Magnetization

$$\vec{M} = \frac{1}{N} \sum_i^N \vec{m}_i$$

Antiferromagnet

Staggered Magnetization

$$\vec{M}_s = \frac{1}{N} \sum_i^N (-1)^i \vec{m}_i$$

And with equations of motion

$$\partial_t S_i^\gamma = \frac{1}{i\hbar} [S_i^\gamma, H]$$

$$= \frac{J}{\hbar} \sum_\delta S_i^\alpha S_{i+\delta}^\beta \epsilon_{\alpha\beta}^\gamma$$

Example: ordered phases of O(3) magnets

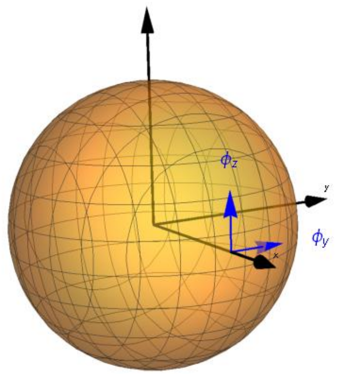
Case 1: The ferromagnet

Equations of motion to linear order $S^z \approx S$

$$\partial_t S_i^x \approx \frac{Js}{\hbar}\left(2S_i^y - S_{i+1}^y - S_{i-1}^y\right)$$
$$\partial_t S_i^y \approx -\frac{Js}{\hbar}\left(2S_i^x - S_{i+1}^x - S_{i-1}^x\right)$$
$$\partial_t S_i^z \approx 0$$

Hydrodynamic limit $\Longrightarrow$

$$\partial_t \phi_i^x \approx \frac{Js}{a^2\hbar}\boldsymbol{\nabla}^2\phi^y$$
$$\partial_t \phi_i^y \approx -\frac{Js}{a^2\hbar}\boldsymbol{\nabla}^2\phi^x$$



$$\mathcal{L} = \vec{A}\partial_t\vec{\phi} + \frac{\rho s^2}{2}\sum_{i\in x,y,z}(\partial_i\vec{\phi})^2$$

$$\omega \propto |\vec{k}|^2$$

Example: ordered phases of O(3) magnets

Case 2: The antiferromagnet

Equations of motion to linear order $S^z \approx S$

Sublattice A

$$\partial_t S_i^x \approx \frac{Js}{\hbar}\big(-2S_i^y - S_{i+1}^y - S_{i-1}^y\big)$$

$$\partial_t S_i^y \approx \frac{Js}{\hbar}\big(2S_i^x + S_{i+1}^x + S_{i-1}^x\big)$$

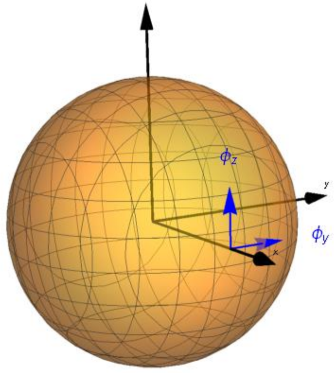
$$\partial_t S_i^z \approx 0$$

Sublattice B

$$\partial_t S_j^x \approx \frac{Js}{\hbar}\big(2S_j^y + S_{j+1}^y + S_{j-1}^y\big)$$

$$\partial_t S_j^y \approx -\frac{Js}{\hbar}\big(-2S_j^x - S_{j+1}^x - S_{j-1}^x\big)$$

$$\partial_t S_j^z \approx 0$$



Define

$$S^+ = S^x + iS^y$$

$$\partial_t S_i^+ \approx \frac{iJs}{\hbar}\big(2S_i^+ + S_{i+1}^+ + S_{i-1}^+\big)$$

$$\partial_t S_j^+ \approx \frac{-iJs}{\hbar}\big(2S_j^+ + S_{j+1}^+ + S_{j-1}^+\big)$$

$$\partial_t^2 S_i^+ \approx \Big(\frac{Js}{\hbar}\Big)^2\big(-2S_i^+ + S_{i+2}^+ + S_{i-2}^+\big)$$

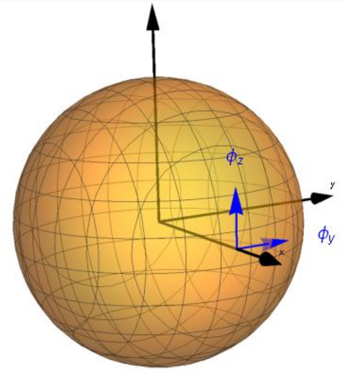
$$\partial_t^2 S_j^+ \approx \Big(\frac{Js}{\hbar}\Big)^2\big(-2S_j^+ + S_{j+2}^+ + S_{j-2}^+\big)$$

In the hydrodynamic limit

$$\mathcal{L} = \frac{1}{2}\Big[\chi(\partial_t\vec{\phi})^2 - \rho\sum_{i\in x,y,z}(\partial_i\vec{\phi})^2\Big]$$

$$\omega \approx c|\vec{k}|$$

Field theories for ordered phases of O(3) magnets



Without Lorentz invariance
Ferromagnet

Equations of motion to linear order $S^z \approx S$

Hydrodynamic limit

$$\partial_t \phi_i^x \approx \frac{Js}{\hbar} \nabla^2 \phi^y \qquad - S_{i-1}^y)$$

$$\partial_t \phi_i^y \approx -\frac{Js}{\hbar} \nabla^2 \phi^x \qquad - S_{i-1}^x)$$

Effective Lagrangian density

$$\mathcal{L} = \vec{A}\partial_t \vec{\phi} + \frac{\rho s^2}{2} \sum_{i \in x,y,z} (\partial_i \vec{\phi})^2$$

Dispersion relation

$$\omega \propto |\vec{k}|^2$$

Lorentz invariant
Antiferromagnet

Equations of motion to linear order $S^z \approx S$

Hydrodynamic limit

$$\partial_t^2 \phi^\alpha = \nabla^2 \phi^\alpha$$

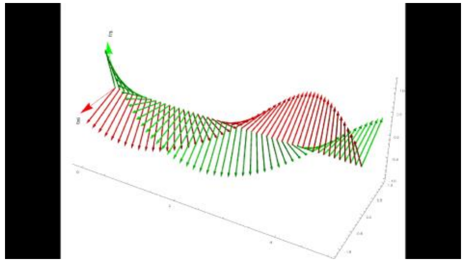
Effective Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left[\chi(\partial_t \vec{\phi})^2 - \rho \sum_{i \in x,y,z} (\partial_i \vec{\phi})^2\right]$$

Dispersion relation

$$\omega \approx c|\vec{k}| \qquad c = \sqrt{\frac{\chi}{\rho}}$$

Electromagnetism in vacuum



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{with} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad A_0 = 0$$

$$A_i \to A_i + \partial_i \lambda \qquad\qquad \partial_i A^i = 0$$

$$\mathcal{L} = \frac{1}{2} \sum_{i,j \in x,y,z} \left[(c\partial_t A_i)^2 + (\partial_i A_j)^2 \right]$$

Heisenberg antiferromagnet

$$\mathcal{L} = \frac{1}{2} \left[\chi (\partial_t \vec{\phi})^2 - \rho \sum_{i \in x,y,z} (\partial_i \vec{\phi})^2 \right] \quad \text{with} \quad \omega \approx c|\vec{k}| \qquad c = \sqrt{\frac{\chi}{\rho}}$$

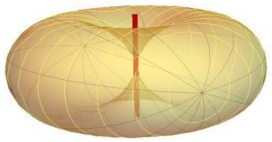
A convenient ground state description

Time reversal invariant basis

$$|x\rangle = i\frac{|1\rangle - |\bar{1}\rangle}{\sqrt{2}} \qquad |y\rangle = \frac{|1\rangle + |\bar{1}\rangle}{\sqrt{2}} \qquad |z\rangle = -i\,|0\rangle$$

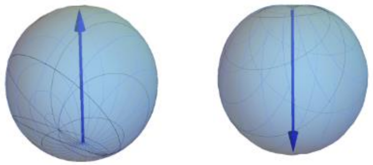
$$H_{BBQ} = \sum_{\langle ij\rangle} J|\vec{d}_i \cdot \vec{d}_j^{\dagger}|^2 + J$$

On site quadrupolar DoF



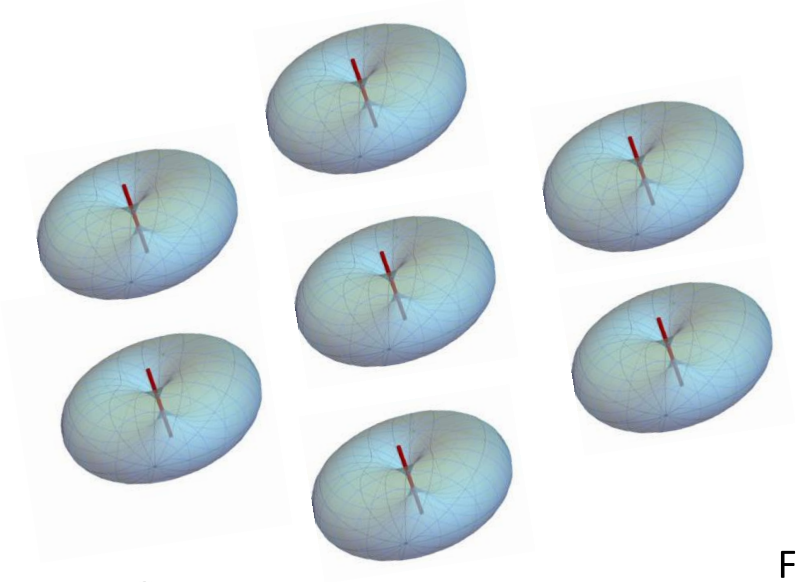
$\mathrm{Re}(\vec{d})$ or $\mathrm{Im}(\vec{d})$

On site dipolar DoF



$\vec{S} = 2\vec{u} \times \vec{v}$

Quadrupole waves in ferroquadrupolar phase



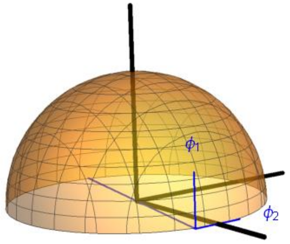
$$H_{BBQ} = \sum_{\langle ij \rangle} J|\vec{d}_i \cdot \vec{d}_j^{\dagger}|^2 + J$$

Ground state

$$d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Fluctuating ground state

$$d' = u = \begin{pmatrix} \phi_1 \\ \phi_2 \\ 1 \end{pmatrix}$$



2 Goldstone bosons (real) correspond to **tensor Quadrupolar modes**

Full description of quadrupolar fluctuations

$$d\tilde{Q} = \tilde{Q} - Q_{GS} = u^T \otimes u - d_{GS} \otimes d_{GS}$$

$$= \begin{pmatrix} \phi_1^2 & \phi_1\phi_2 & \phi_1 \\ \phi_1\phi_2 & \phi_2^2 & \phi_2 \\ \phi_1 & \phi_2 & 1-1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0 & 0 & \phi_1 \\ 0 & 0 & \phi_2 \\ \phi_1 & \phi_2 & 0 \end{pmatrix}$$

in the small fluctuation limit

Mapping of polarizations to strain-like components of Q_{ij}

$$d\tilde{Q}_1 = \begin{pmatrix} 0 & 0 & \phi_1 \\ 0 & 0 & 0 \\ \phi_1 & 0 & 0 \end{pmatrix} \quad d\tilde{Q}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \phi_2 \\ 0 & \phi_2 & 0 \end{pmatrix}$$

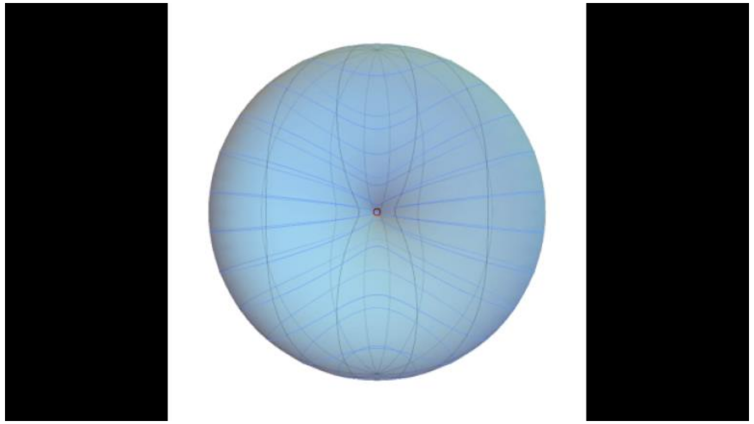
$$Q = Q_1 + Q_2 = A(d\tilde{Q}_1)A^T + B(d\tilde{Q}_2)B^T$$

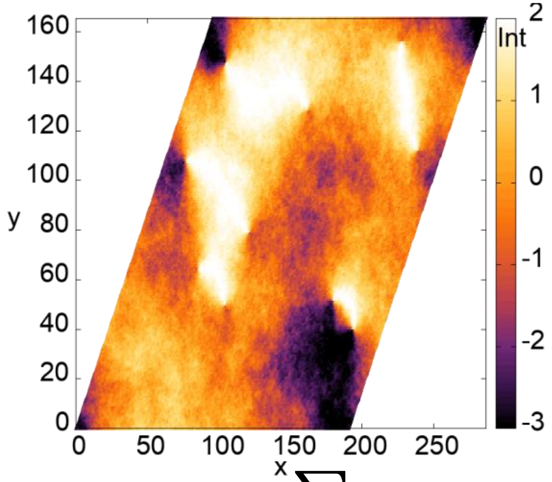
$$Q = \begin{pmatrix} \phi_1 & \phi_2 & 0 \\ \phi_2 & -\phi_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Equivalence of quadrupolar waves in Nematic order and Linearized Gravity

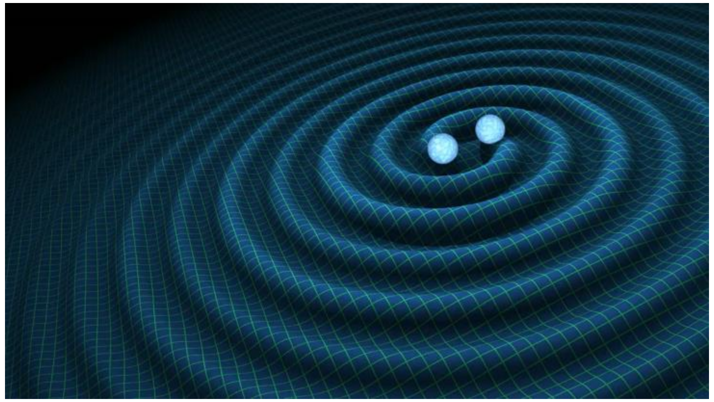
Mapping to strain-like components of Q_{ij}

$$Q = Q_1 + Q_2 = A(d\tilde{Q}_1)A^T + B(d\tilde{Q}_2)B^T \qquad Q = \begin{pmatrix} \phi_1 & \phi_2 & 0 \\ \phi_2 & -\phi_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & -g & -\frac{1}{2g} \\ 0 & g & -\frac{1}{2g} \\ 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

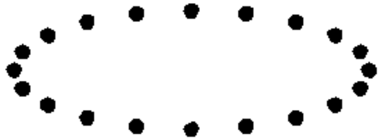




$$Q(r) = \sum_{a,b} Q^{ab}(r)$$



animation by R. Hurt, Caltech/JPL, LIGO (2015)

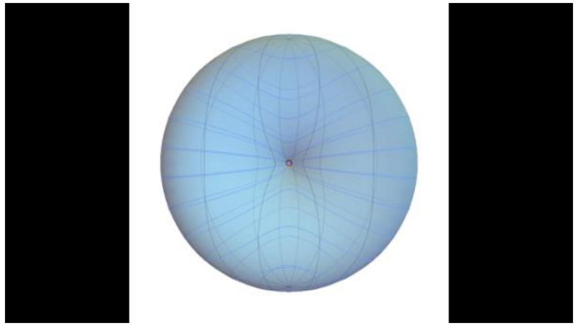


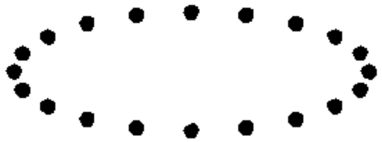
$$Q = \begin{pmatrix} \phi_1 & \phi_2 & 0 \\ \phi_2 & -\phi_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Longleftrightarrow \epsilon_{ij} = \begin{pmatrix} \epsilon_+ & \epsilon_x & 0 \\ \epsilon_x & -\epsilon_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

https://www.ligo-india.in/outreach/detecting-gravitational-waves/

SU(3) transformation on the FQ excitations

Mapping to a strain-like form of Q





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Using a subset of the Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_3^{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3^{xz} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \lambda_3^{yz} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

And defining

$$\Lambda_{rso} = \hat{O} = (\lambda_6, \lambda_4, \lambda_1)$$

$$\Lambda_{rsd} = \hat{D} = (\lambda_3^{yz}, \lambda_3^{xz}, \lambda_3^{xy})$$

We can express transformation as

$$Q = d\tilde{Q}\left[\sum_{i,j\neq k} \delta_{ij}\Lambda_{rso}^{j} \otimes \Lambda_{rso}^{k} + \epsilon_{ijk}\Lambda_{rso}^{j} \otimes \Lambda_{rsd}^{k}\right]$$

$$Q = \begin{pmatrix} -\phi_1 + \phi_2 & \phi_1 + \phi_2 & 0 \\ \phi_1 + \phi_2 & \phi_1 - \phi_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$