Numerical Investigation of Isospin Breaking Effects in 1+1D QED

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Introduction

- Brief reminder of continuum QCD
- What is Lattice Field Theory?
- Why is Lattice Field Theory interesting?
- How are Lattice calculations performed?
- Overview of the Schwinger Model (1+1D QED)
- Isospin Splitting in the Schwinger Model

QCD in the Continuum

Continuum QCD

QCD-Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}\underbrace{G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{Gluons}} + \underbrace{\bar{\psi}(iD_\mu\gamma^\mu - m)\psi}_{\text{(Anti)fermions}}$$

Gluon field strength tensor

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^a_{bc} A^b_\mu A^c_\nu$$

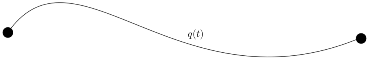
Covariant derivative

$$D_\mu = \partial_\mu + g A^a_\mu \frac{\lambda^a}{2i}$$

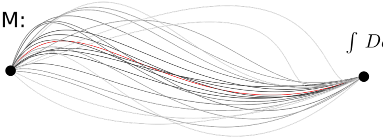
SU(3) gauge symmetry
Peculiar feature: Negative β function (asymptotic freedom)
$\Rightarrow$ Interaction between particles gets **stronger** with increasing distance

Path integrals: From classical to quantum

Mechanics:

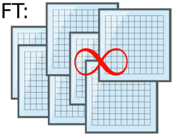


QM:

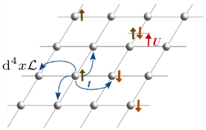


$\int Dq(t)e^{\mathrm{i}\int \mathrm{d}t\, L}$

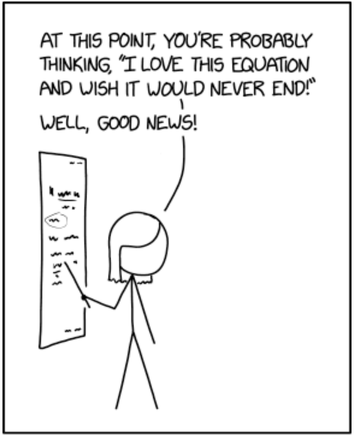
QFT:



$\int \mathcal{D}\phi(x)e^{\mathrm{i}\int \mathrm{d}^4x\mathcal{L}}$

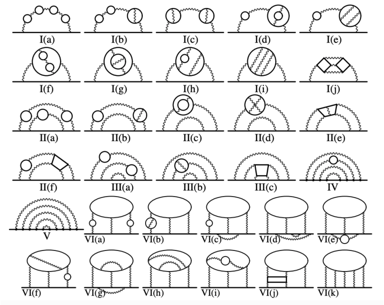


The usual approach: Perturbation theory



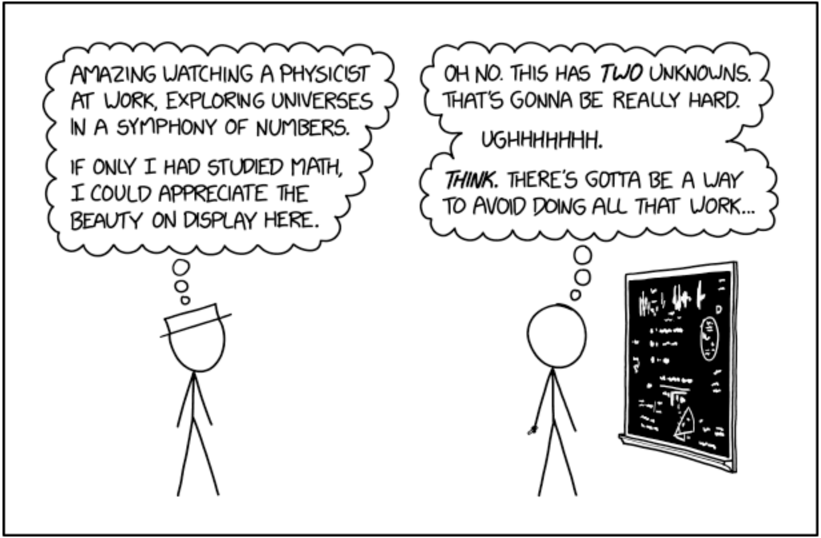
TAYLOR SERIES EXPANSION IS THE WORST.

- Treat theory as free theory $+$ small perturbation (basically a series expansion)
- Works well for weak coupling (QED, high energies in QCD)



Lattice QCD

The general idea



From the continuum to the lattice...

Path integral

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \exp\big(iS[\psi, \bar{\psi}, A]\big)$$

Wick rotation

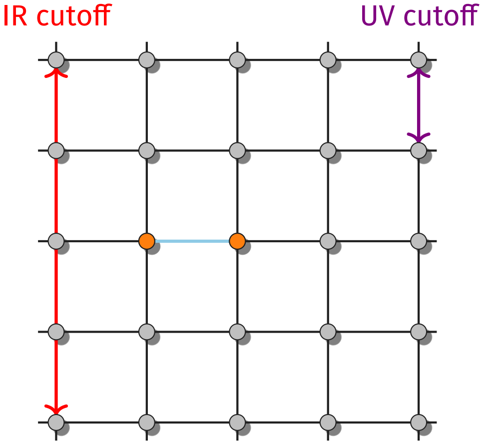
$$t \to i\tau \quad \Rightarrow \quad \mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \exp\big(-S[\psi, \bar{\psi}, A]\big)$$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \mathcal{O}[\psi, \bar{\psi}, A] \exp\big(-S[\psi, \bar{\psi}, A]\big)$$

Analogous to system from statistical mechanics

From the continuum to the lattice...

- Infinite degrees of freedom impractical to implement on computers
- Instead **discretize** spacetime on **finite** lattice
- Gluons live on links, quarks/antiquarks on lattice sites
- Lattice regularization through cutoffs
  - Finite spacetime of spatial size L: $\Lambda_{IR} = \frac{2\pi}{L}$
  - Discrete lattice of spacing a: $\Lambda_{UV} = \frac{2\pi}{a}$



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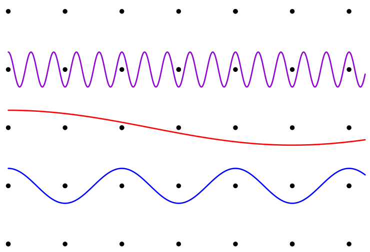

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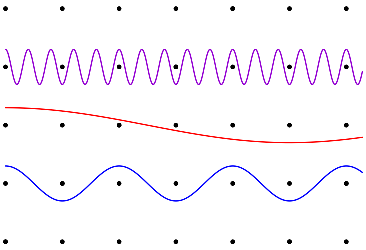
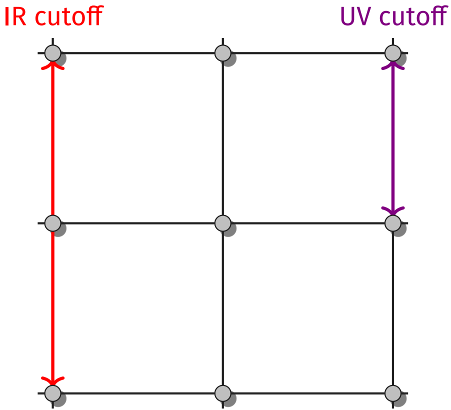


- To recover continuum theory
  - Continuum limit $a \to 0$
  - Infinite volume limit $L \to \infty$

...and back to the continuum



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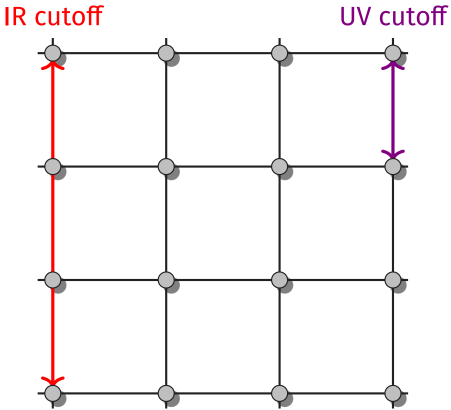

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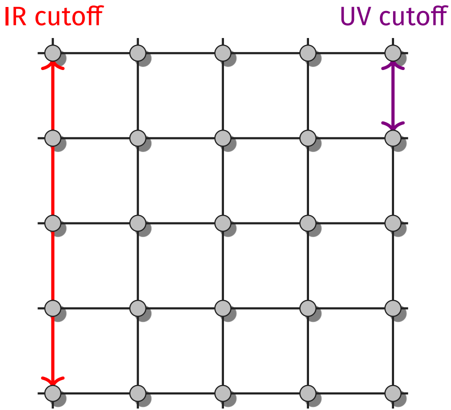

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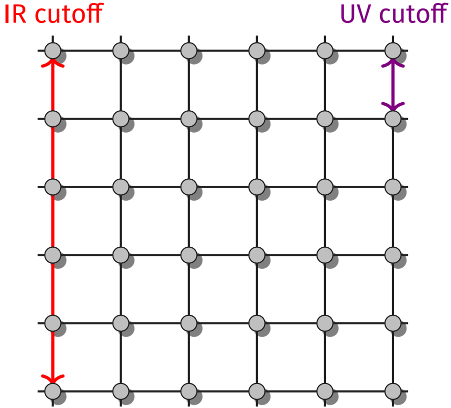
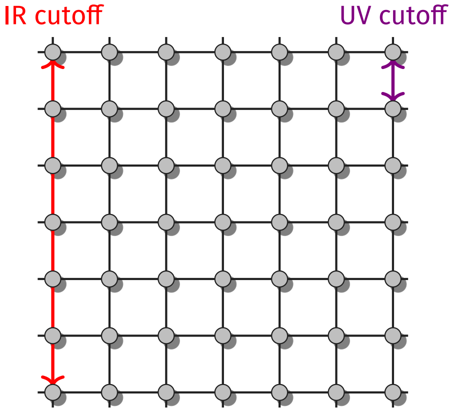


Lattice needs to be fine enough to capture relevant details!

...and back to the continuum



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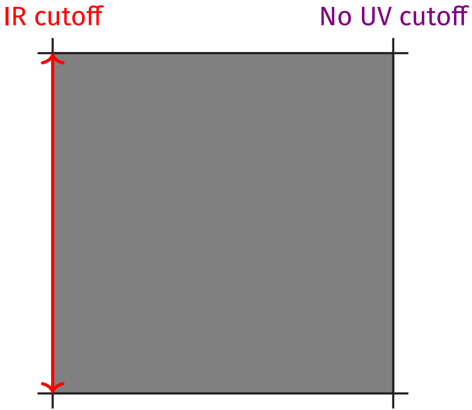


Lattice needs to be fine enough to capture relevant details!

The Schwinger Model: An Overview

The Schwinger model

1+1D quantum electrodynamics

Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4}\underbrace{F_{\mu\nu}F^{\mu\nu}}_{\text{Photons}} + \underbrace{\bar{\psi}_{\text{f}}(iD_\mu\gamma^\mu - m_{\text{f}})\psi_{\text{f}}}_{\text{(Anti)fermions}}$$

EM field strength tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

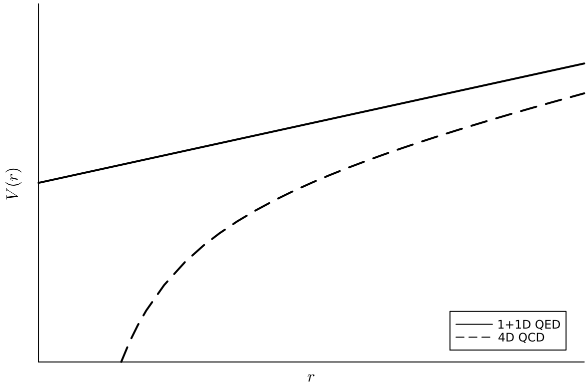
Covariant derivative

$$D_\mu = \partial_\mu - ieA_\mu$$

$U(1)$ gauge symmetry

Peculiar feature: Confinement of fermions $\Rightarrow$ Toy model for QCD

The Schwinger model



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Bosonized Schwinger model

The two-flavor light field Lagrangian in the strong coupling limit $(\mu \gg m_\mathrm{f})$

$$\mathcal{L}^{\text{light}} = \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2\pi} M^2 N_M \left[\cos\sqrt{2\pi}\chi\right]$$

Mass

$$M = \left(e^\gamma \mu^{1/2} \sqrt{m_u^2 + m_d^2 + 2 m_u m_d}\right)^{2/3}$$

Schwinger mass

$$\mu = e\sqrt{\frac{2}{\pi}}$$

- Resulting Lagrangian resembles sine-Gordon model
- Three solutions
  - soliton (π^+)
  - antisoliton (π^-)
  - lighter breather (π^0)

Isospin Splitting in the Schwinger Model

Isospin

QCD

- Isospin-dubletts
  $u = |\frac{1}{2}, \frac{1}{2}\rangle \quad d = |\frac{1}{2}, -\frac{1}{2}\rangle$
- Pions differ
  $\pi^+ = u\bar{d} = |1, 1\rangle \quad \pi^- = d\bar{u} = |1, -1\rangle$
  $$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) = |1, 0\rangle$$
- NLO contributions

$$M_{\pi^0}^2 - M_{\pi^\pm}^2 \propto \delta m^2$$

where $\delta m = m_u - m_d$

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where $\delta m = m_u - m_d$

Schwinger model

- Low energy conformal sector
- Mass splitting

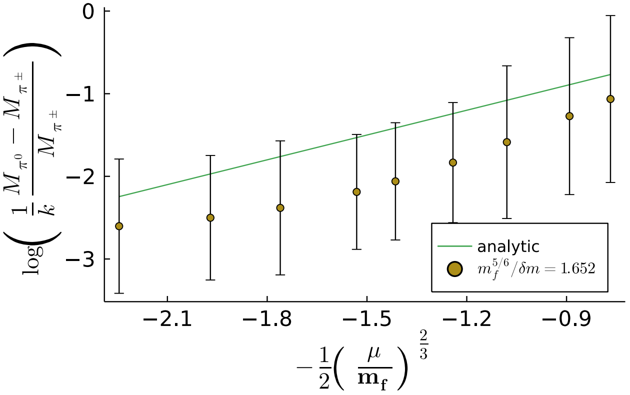
$$M_{\pi^0} - M_{\pi^\pm} \propto \delta m e^{-\left(\frac{\mu}{m_\mathrm{f}}\right)^{\frac{2}{3}}}$$

μ: Schwinger mass
m_f: fermion mass

Georgi, 2020

Numerical results

$$\frac{M_{\pi^0} - M_{\pi^\pm}}{M_{\pi^\pm}} = k(m_{\mathrm{f}}) \exp\left(-\frac{1}{2}\left(\frac{\mu}{m_{\mathrm{f}}}\right)^{\frac{2}{3}}\right)$$



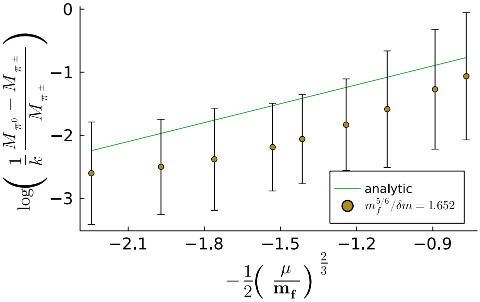
μ: Schwinger mass

m_{f}: fermion mass

Conclusion

Conclusion

- Lattice QCD: a non-perturbative approach to QCD
- Discretize spacetime on finite lattice
- Schwinger model: toy model with confined fermions
- Pions via bosonization
- Isospin breaking exponentially suppressed in the Schwinger model



Thank you for your attention!

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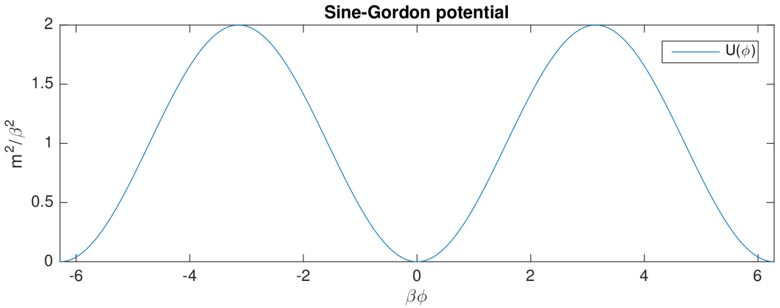
Backup

sine-Gordon Model

Scalar field theory

Lagrangian

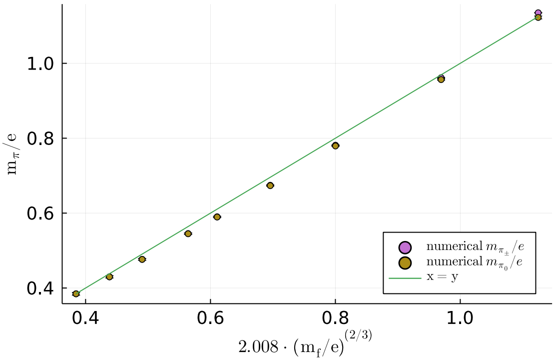
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{m}{\beta^2}(1 - \cos\beta\phi)$$



M_π vs. m_f

Masses of pions from solition and breather solutions in the standard sine-Gordon model

$$\frac{M_\pi}{e} = 2.008\ldots \left(\frac{m_\mathrm{f}}{e}\right)^{2/3}$$



Prefactor

- Lagrangian mass term

$$\mathcal{L} = m_{\mathrm{f}}(O_{+1} + O^*_{+1}) + \delta m(O_{-1} + O^*_{-1})$$

- Correlators

$$\langle 0|T(O_{\pm 1}(x)O^*_{\pm 1}(0))|0\rangle = \frac{\xi m}{2\pi^2}(e^{\kappa_0} \pm e^{-\kappa_0})\frac{1}{\sqrt{-x^2 + i\varepsilon}}$$

with $\kappa_0 = K_0\left(m\sqrt{-x^2 + i\varepsilon}\right)$

- Asymptotic behavior of κ_0

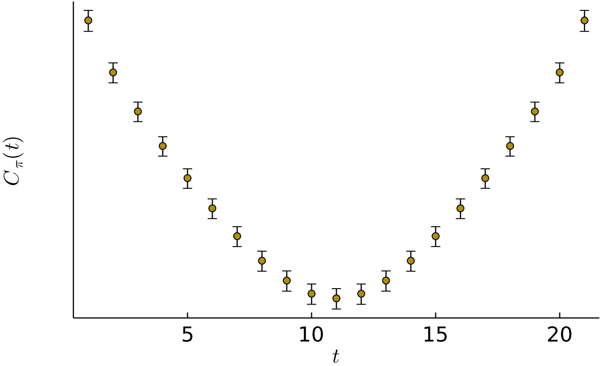
$$\kappa_0 \xrightarrow{x\to\infty} -i\sqrt{\frac{\pi^3}{8mx}}e^{-i(mx-\frac{\pi}{4})}$$

- Mass scale is $(m_{\mathrm{f}}\mu)^{\frac{1}{3}}$
- From mass perturbation theory $M_\pi \propto m_f^{\frac{2}{3}}$
- Overall pion mass to leading order as $\delta m \xrightarrow{m_f\to 0} 0$

$$M_\pi \propto m_f^{\frac{2}{3}}\left(1 + \frac{2}{3}\left(\frac{\pi}{2}\right)^{\frac{1}{4}}\frac{\delta m}{m_f^{\frac{5}{6}}\mu^{\frac{1}{6}}}e^{-\frac{1}{2}\left(\frac{\mu}{m_f}\right)^{\frac{2}{3}}}\right)$$

Pion masses on the lattice

$$c_\pi(t) = \langle 0|\, \pi(x,t)\bar{\pi}(y,0)\, |0\rangle \approx C e^{-\boldsymbol{M_\pi} t}$$



10% splitting