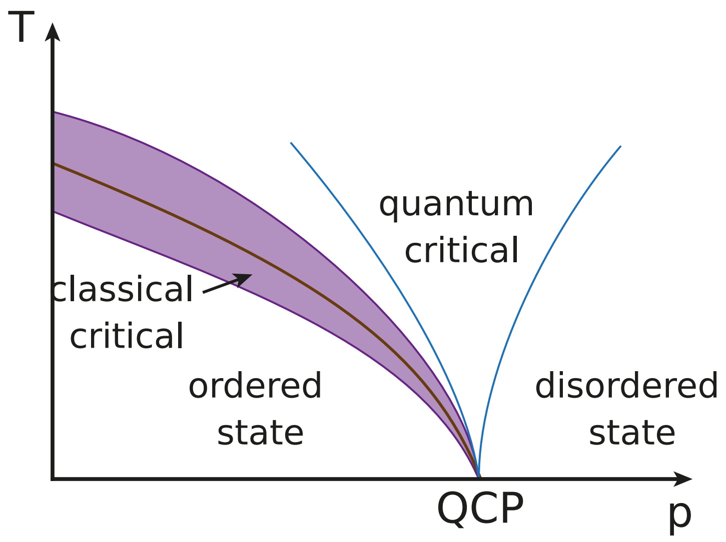
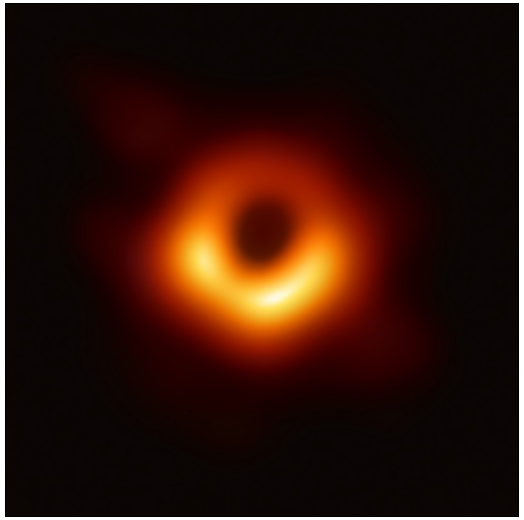
The Dirac-SYK Superconductor

Veronika Stangier, TKM

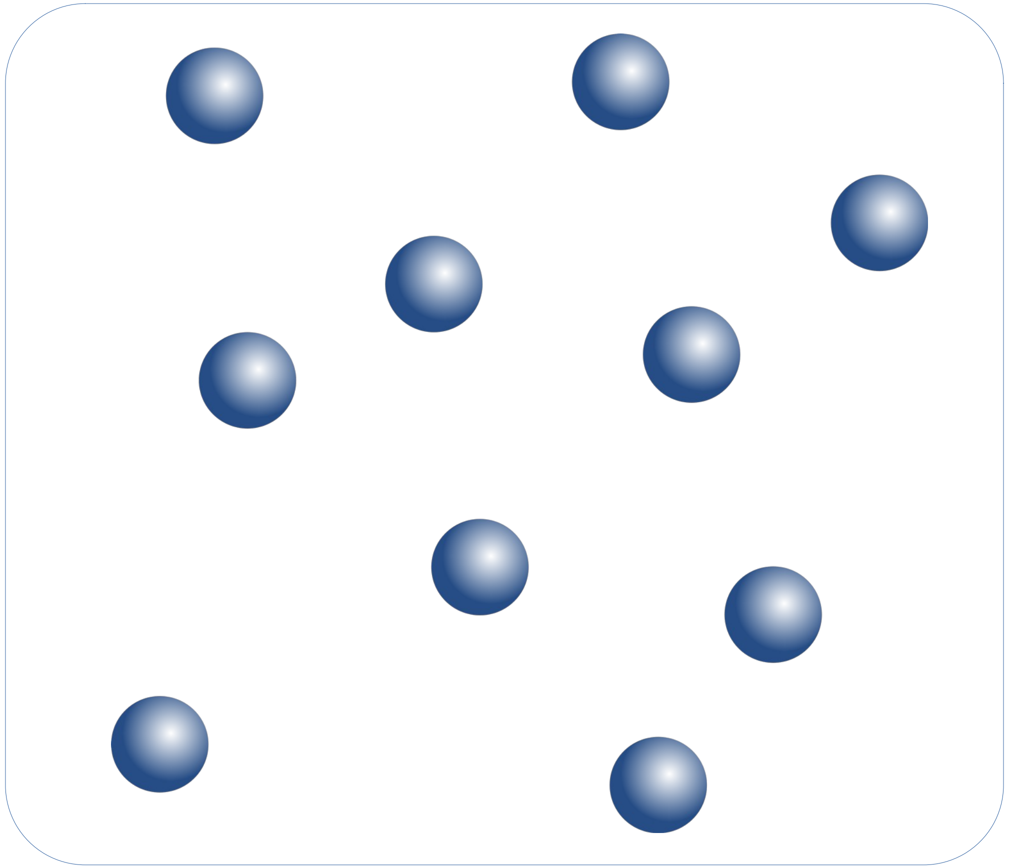




(Don't ask)

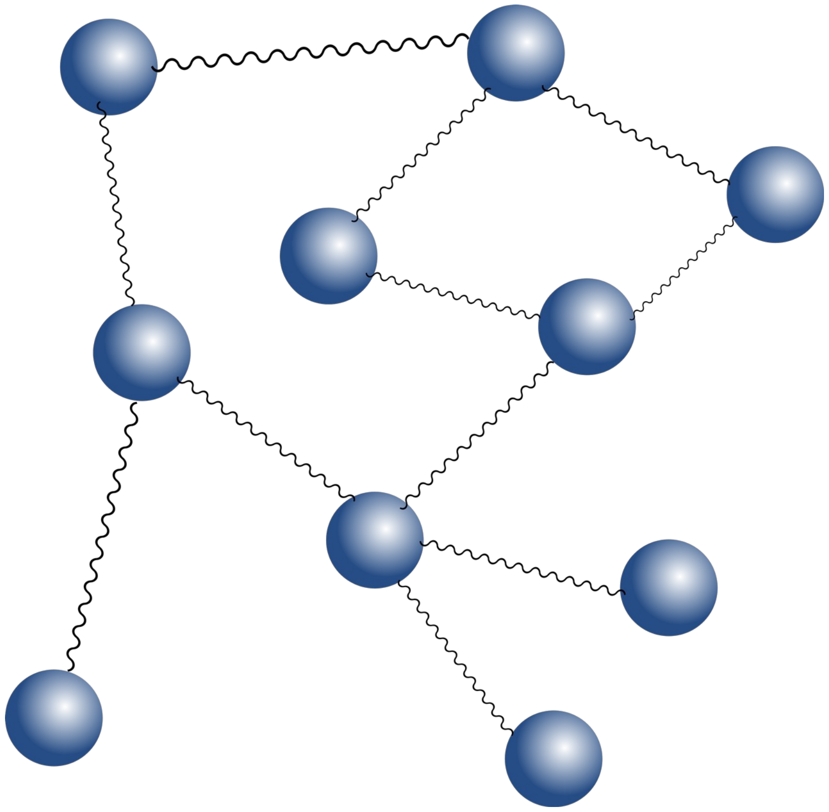
The TKM

Fermi Liquid Theory



Spin $:\vec{\sigma}$, Charge: e, Momentum: k
Mass: m, Magnetic moment: μ

Fermi Liquid Theory



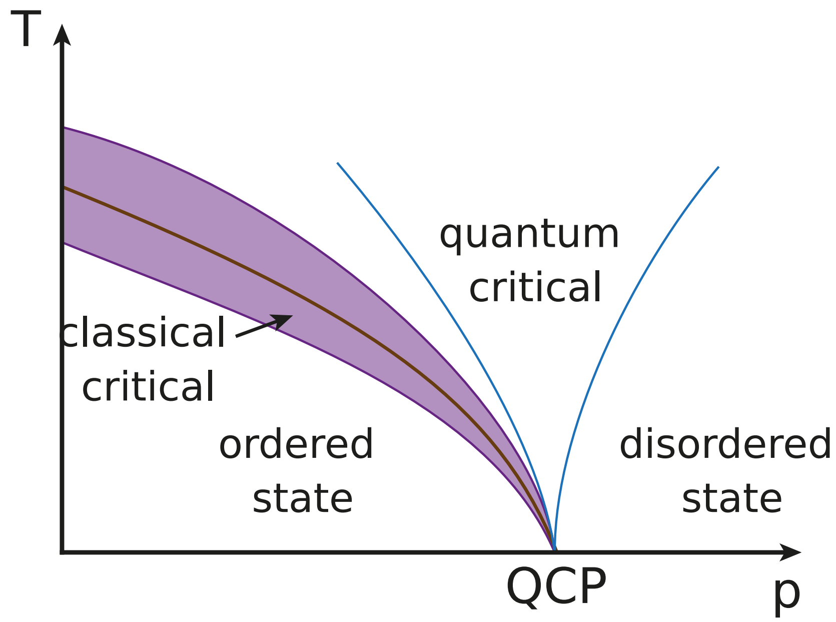
Spin :$\overrightarrow{\sigma}$, Charge: e, Momentum: k
Mass: m^*, Magnetic moment: μ^*

Resistivity :$\rho \sim T^2$

Specific heat:$c_V \sim T$

Background

[1]



Strange Metals ($Sr_3Ru_2O_7$) [2]

Cuprates [3]

- Breakdown of the quasi-particle picture
- Anomalous transport properties
- Often: Superconductivity around QCP

[1] https://commons.wikimedia.org/wiki/File:QuantumPhaseTransition.png
[2] C. Mousatov, S. Hartnoll, E. Berg, PNAS 117 (6) 2852-2857 (2020)
[3] L. Taillefer, arxiv:1003.2972

Background

Karlsruhe Institute of Technology

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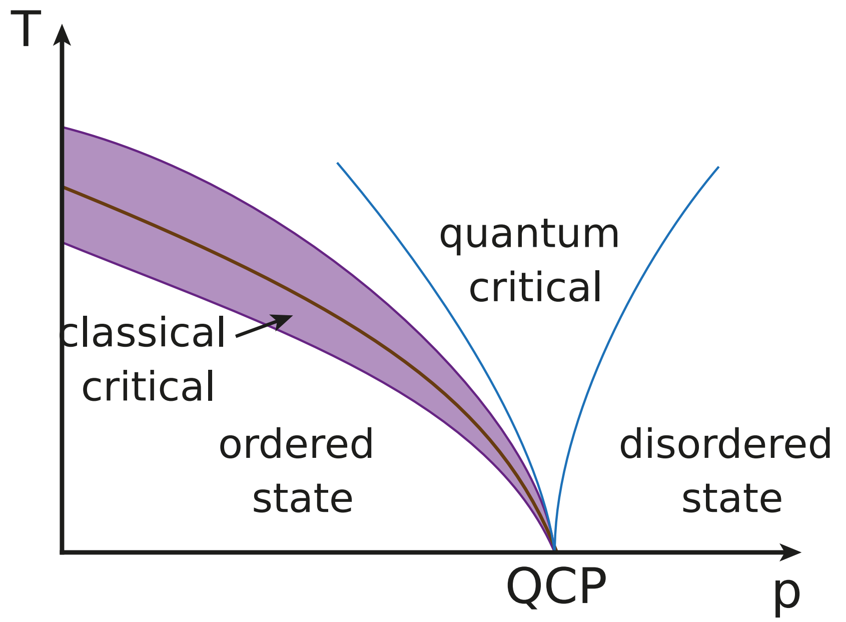

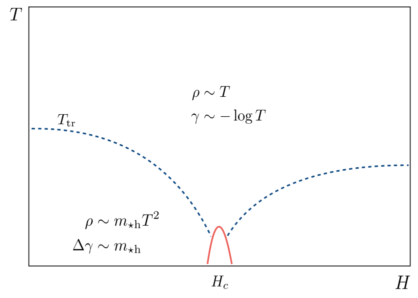

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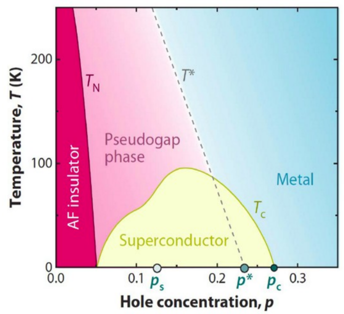

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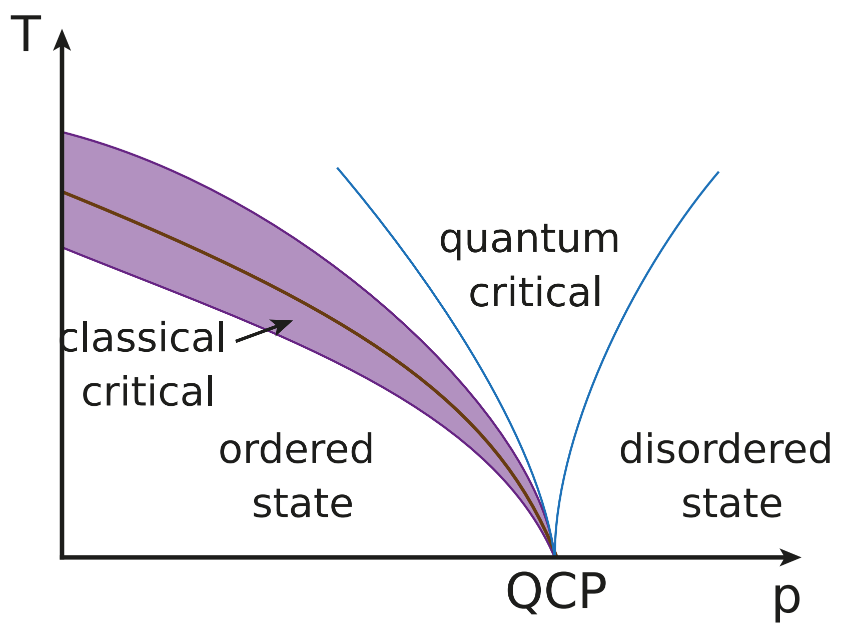

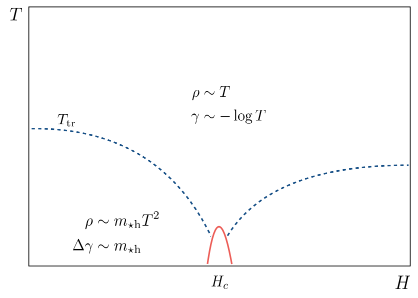

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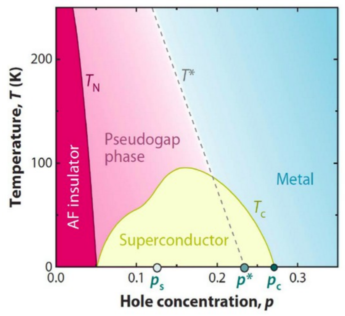

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The Yukawa-SYK model

$$H = -\sum_{i\sigma} \mu c^\dagger_{i\sigma} c_{i\sigma} + \frac{1}{2}\sum_k \left(\pi_k^2 + \omega_0^2 \phi_k^2\right) + \frac{\sqrt{2}}{N} \sum_{ijk,\sigma} g_{ij,k} c^\dagger_{i\sigma} c_{j\sigma} \phi_k$$

- ✔ Exactly solvable
- ✔ Critical solutions
- ✔ Superconducting
- ✖ Finite dimensions

I. Esterlis, J. Schmalian. Phys. Rev. B 100.11 (2019): 115132

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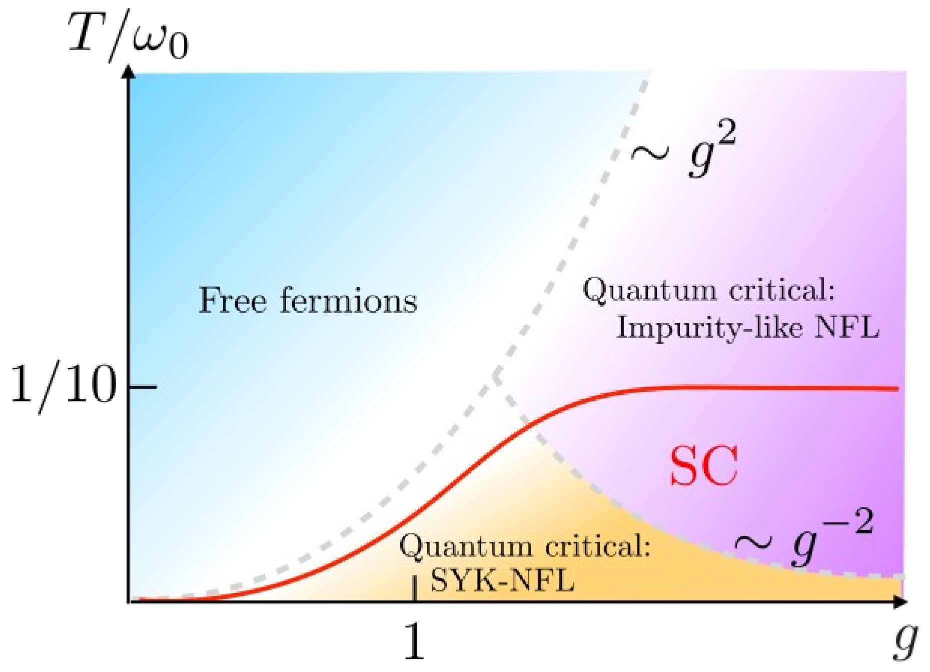

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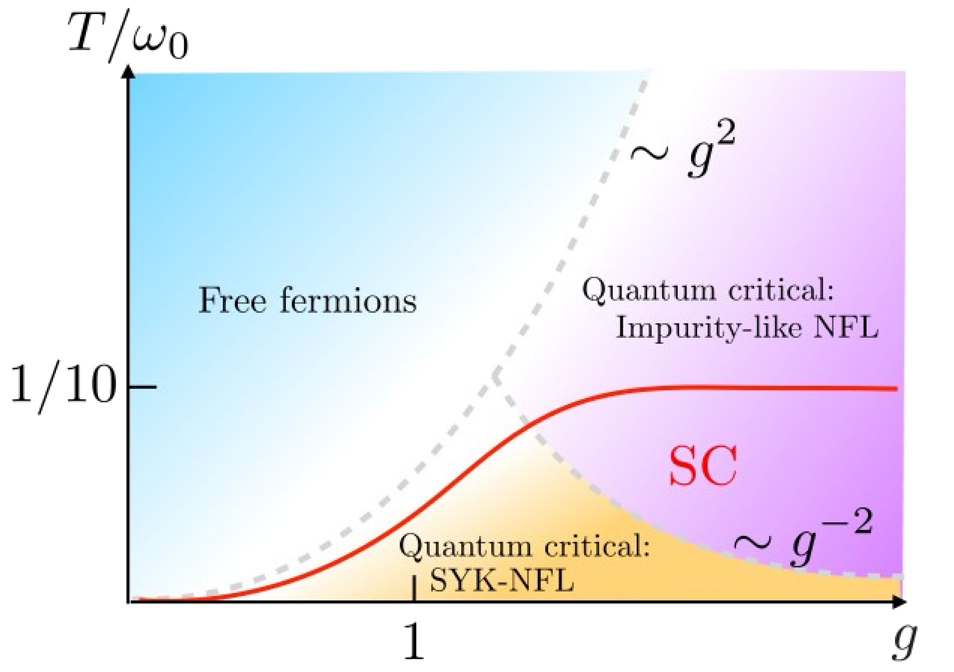

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The Dirac-SYK model – normal state

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_{i\sigma} \partial\!\!\!/ \psi_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \phi_k \left(m_0^2 - b\partial_\mu^2 \right) \phi_k + \frac{1}{N} \sum_{ijk} g_{ijk} \bar{\psi}_{i\sigma} \hat{\Gamma}_{\sigma\sigma'} \psi_{j\sigma'} \phi_k$$

Disorder average

$$\overline{F} = -k_B T \overline{\log Z}$$

Bi-local fields

$$\mathrm{G}(x,x') = \tfrac{1}{N} \textstyle\sum_i \psi_i(x)\, \psi_i^\dagger(x')$$

$$D(x,x') = \tfrac{1}{M} \textstyle\sum_k \phi_k(x)\, \phi_k(x')$$

Critical Solutions

$$\left(\boldsymbol{G}^{-1}\right)(x',x) = -\left(\left(\boldsymbol{G}_0^{-1}\right)(x,x') - \boldsymbol{\Sigma}(x,x')\right)$$

$$D(x',x) = \frac{1}{\left(D_0^{-1}\right)(x,x') - \Pi(x,x')}$$

$$\widehat{\Sigma}(x',x) = -g^2 n_s \xi \left[\widehat{\Gamma}\widehat{G}(x,x')\widehat{\Gamma}'\right] D(x,x')$$

$$\Pi(x',x) = -g^2 \mathrm{Tr}\left[\hat{\Gamma}\hat{G}(x',x)\hat{\Gamma}\hat{G}(x,x')\right]$$

J. Kim, E. Altman, X. Cao. Phys. Rev. B 103.8 (2021): L081113.

The Dirac-SYK model – normal state

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_{i\sigma} \partial\!\!\!/ \psi_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \phi_k \left(m_0^2 - b\partial_\mu^2 \right) \phi_k + \frac{1}{N} \sum_{ijk} g_{ijk} \bar{\psi}_{i\sigma} \hat{\Gamma}_{\sigma\sigma'} \psi_{j\sigma'} \phi_k$$

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Critical Solutions
(for large g)

$$G(k) \sim k_\mu \gamma^\mu k^{2\Delta+D-1}$$

$$D(q) \sim q^{-4\Delta+D}$$

$$\Sigma(k) \sim -k_\mu \gamma^\mu k^{-2\Delta+D-1}$$

$$\Pi(q) \sim q^{4\Delta-D}$$

- ✓ Exactly solvable
- ✓ Critical solutions
- ✗ Superconducting
- ✓ Finite dimensions

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- ✔ Exactly solvable
- ✔ Critical solutions
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Disorder Average

$$S_{int} = \frac{1}{N}\sum_{ijk} g_{ijk}\bar{\psi}_{i\sigma}\hat{\Gamma}_{\sigma\sigma'}\psi_{j\sigma'}\phi_k$$

Gaussian Unitary Ensemble

$$g^*_{ijk} = g_{jik}$$

Gaussian Orthogonal Ensemble

$$g_{ijk} = g_{jik}$$

Disorder Average

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➡ No superconductivity

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Disorder Average

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Gaussian Unitary Ensemble

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➡ No superconductivity

Gaussian Orthogonal Ensemble

$$g_{ijk} = g_{jik}$$

➡ Maybe superconductivity?

The Dirac-SYK model - superconducting state

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_{i\sigma} \partial\!\!\!/ \psi_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \phi_k \left(m_0^2 - b\partial_\mu^2 \right) \phi_k + \frac{1}{N} \sum_{ijk} g_{ijk} \bar{\psi}_{i\sigma} \hat{\Gamma}_{\sigma\sigma'} \psi_{j\sigma'} \phi_k$$

Additional bi-local fields:

$$\mathrm{F}(x,x') = \tfrac{1}{N} \textstyle\sum_i \psi_i(x)\, \psi_i(x')$$

$$\widehat{\Phi}(x',x) = g^2 n_s \xi \left(\widehat{\Gamma}^T \widehat{F}^\dagger (x',x)\, \widehat{\Gamma} \right) D(x,x') \, .$$

Linearized Gap Equation

$$\hat{\Phi}(k) = C \int_q \left(\hat{\Gamma}^T \hat{G}_n^T (-q)\, \hat{\Phi}(q)\, \hat{G}_n(q)\, \hat{\Gamma} \right) D(q-k)$$

The Dirac-SYK model - superconducting state

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_{i\sigma} \not{\partial} \psi_{i\sigma} + \frac{1}{2} \sum_{k=1}^{M} \phi_k \left(m_0^2 - b\partial_\mu^2 \right) \phi_k + \frac{1}{N} \sum_{ijk} g_{ijk} \bar{\psi}_{i\sigma} \hat{\Gamma}_{\sigma\sigma'} \psi_{j\sigma'} \phi_k$$

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→ Solvable by tuning fermion-to-boson ratio

The Dirac-SYK superconductor

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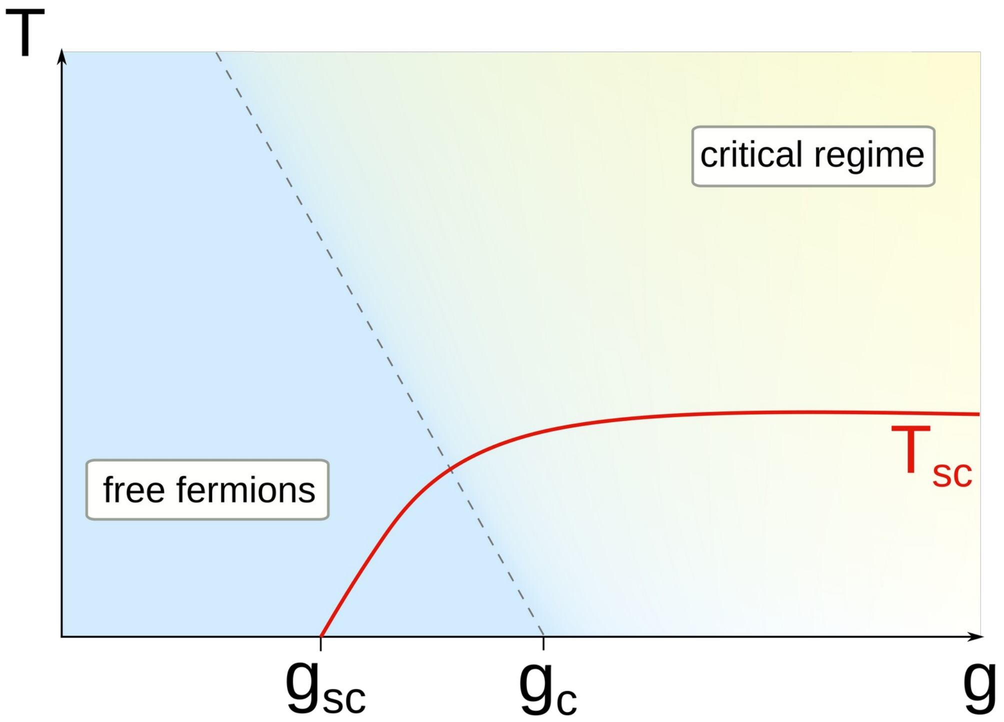

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