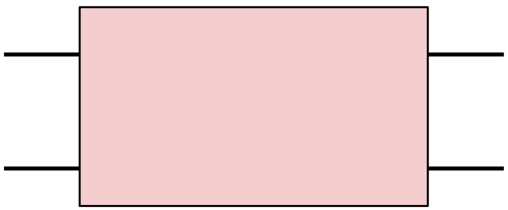
Non-Adiabatic Holonomic Quantum Computing in Integrated Quantum Optics

Vera Neef, **Julien Pinske, Matthias Heinrich, Stefan Scheel, and Alexander Szameit**

Institute for Physics, University of Rostock, Germany

Quantum Computing in Optics



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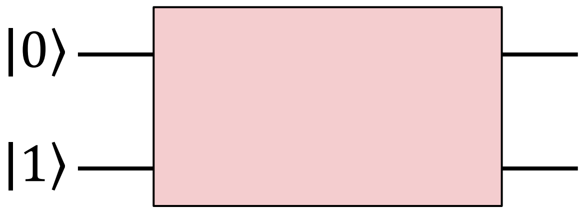
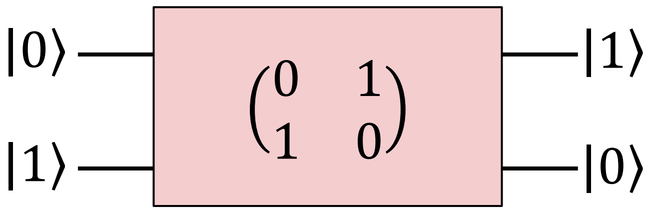


Pauli-X


$|0\rangle$
$|1\rangle$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$|1\rangle$
$|0\rangle$

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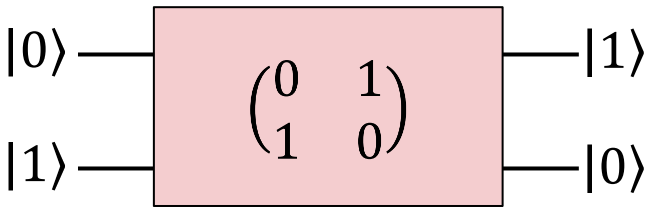
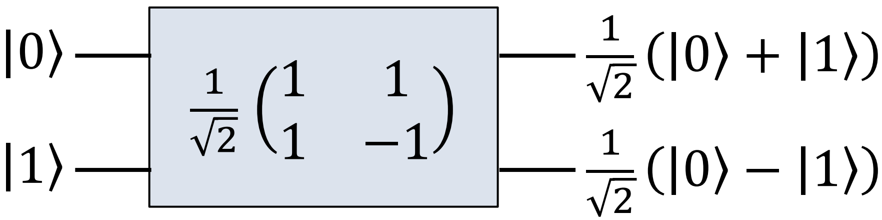


Hadamard



Quantum Computing in Optics

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$$|0\rangle \;\rightarrow\; \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \;\rightarrow\; \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

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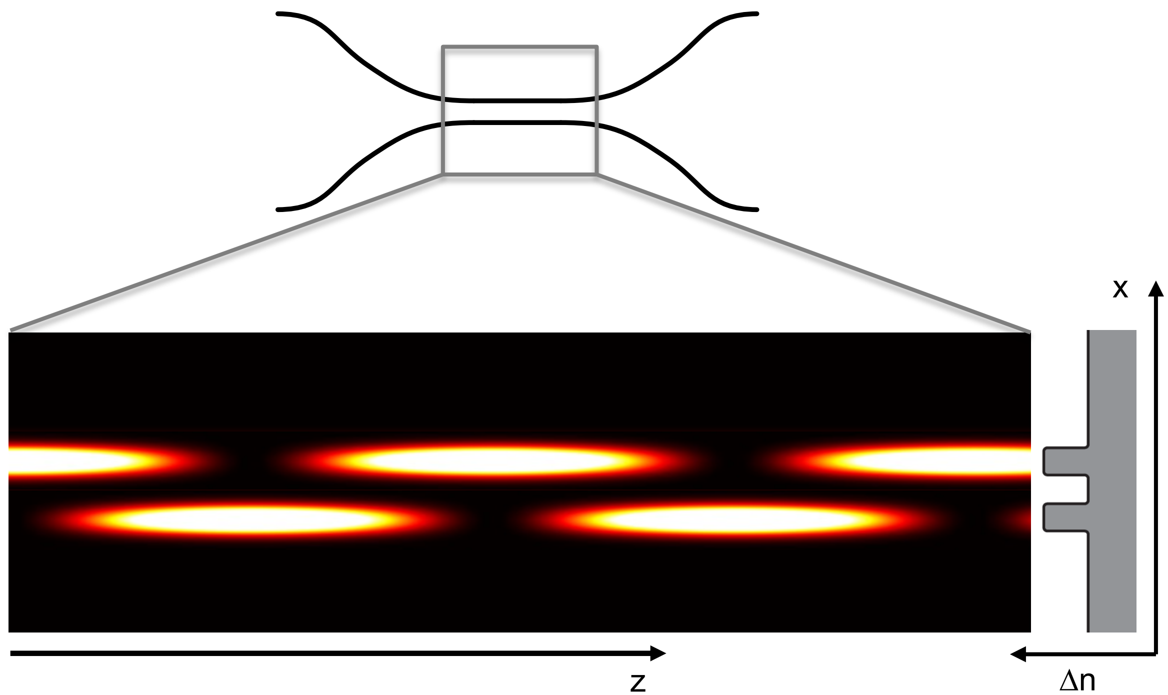

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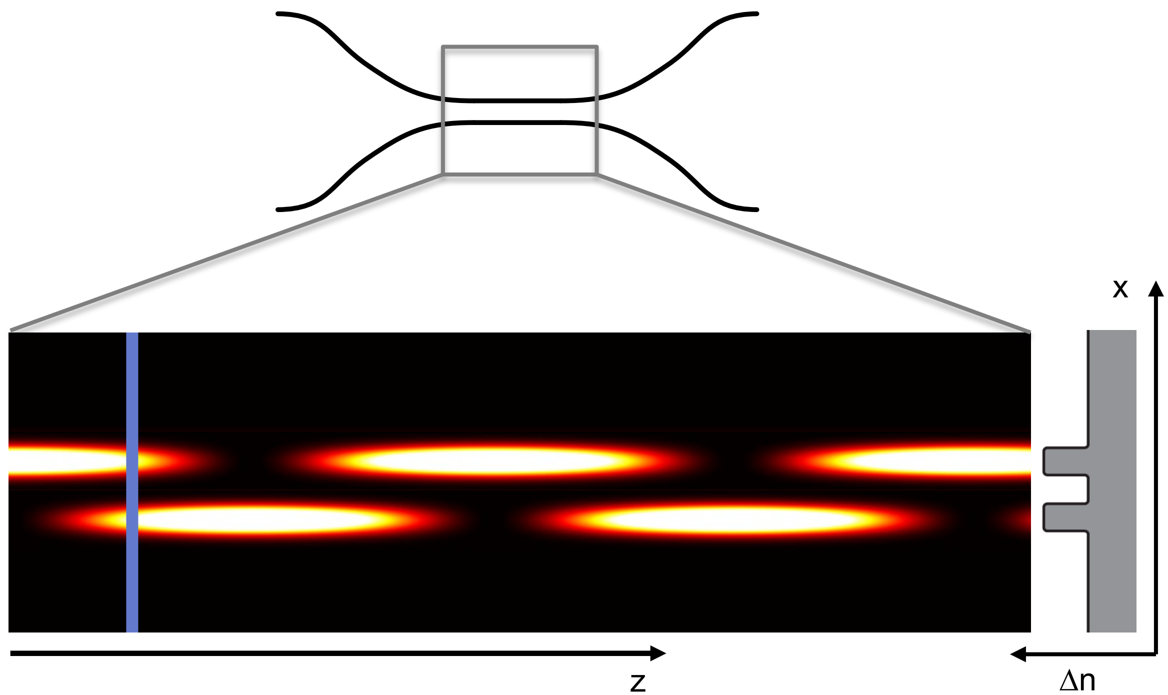

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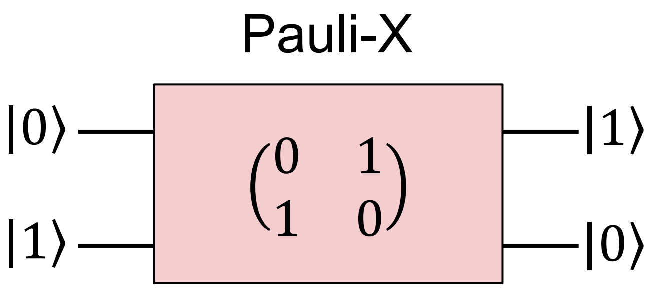

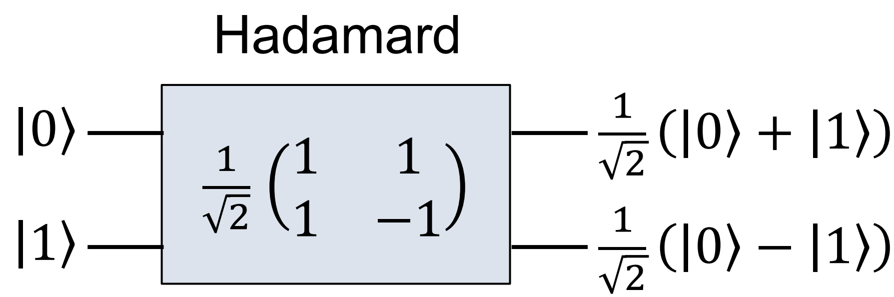

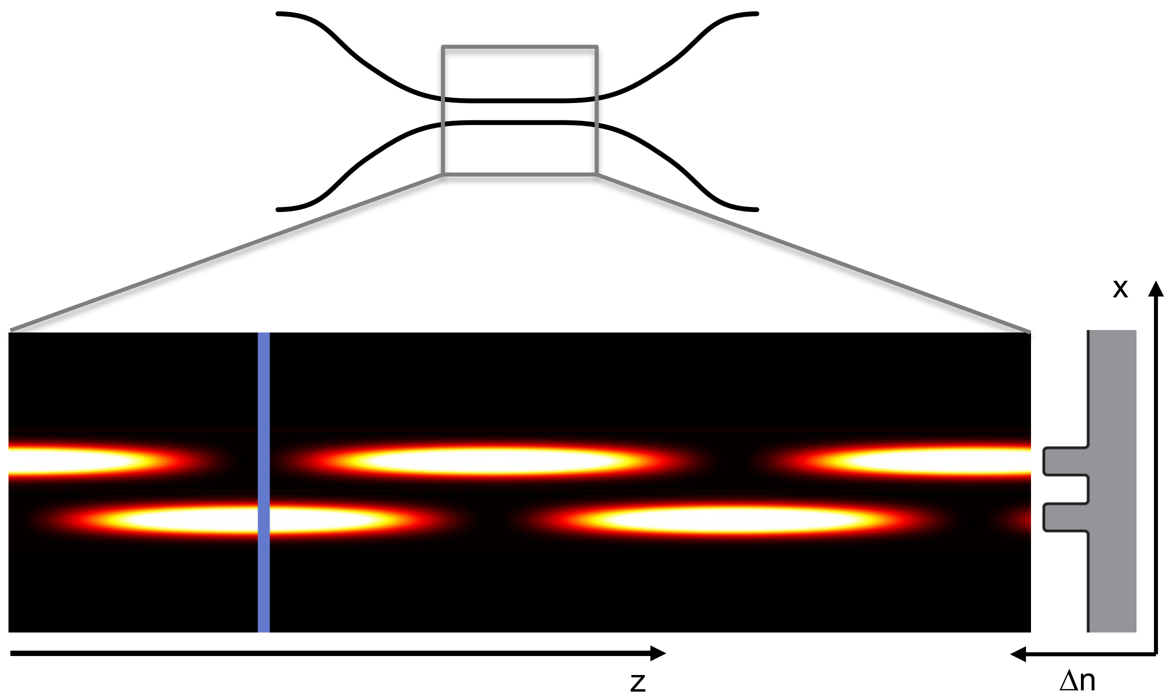

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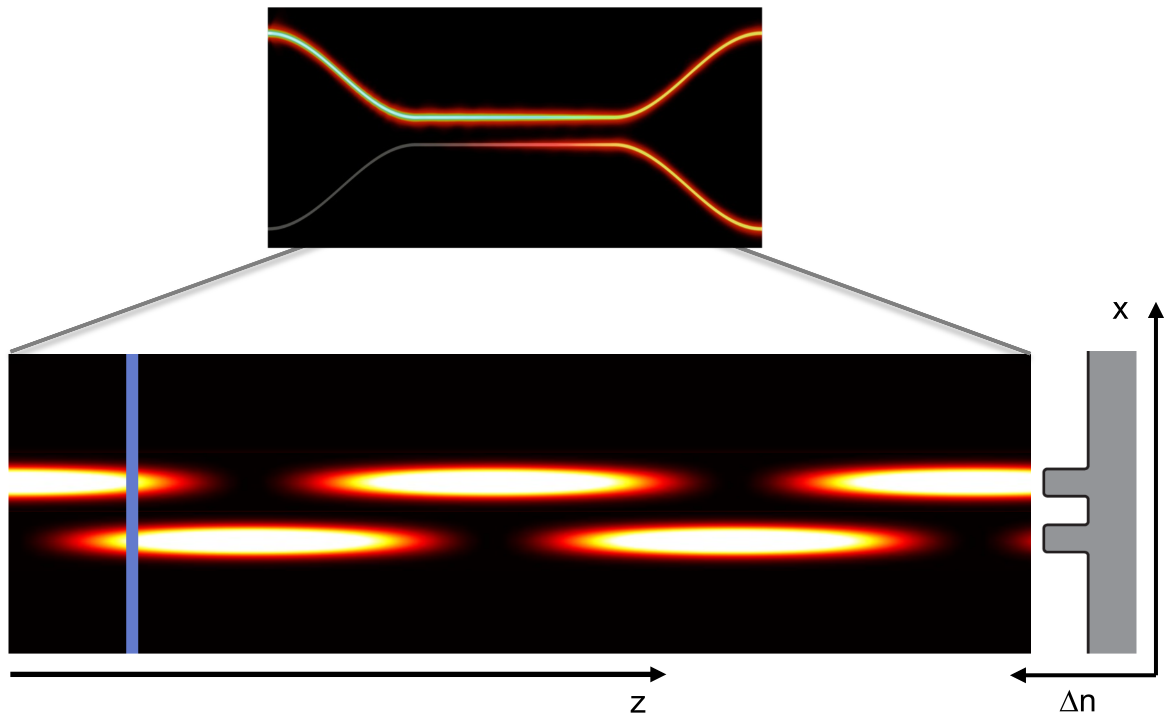

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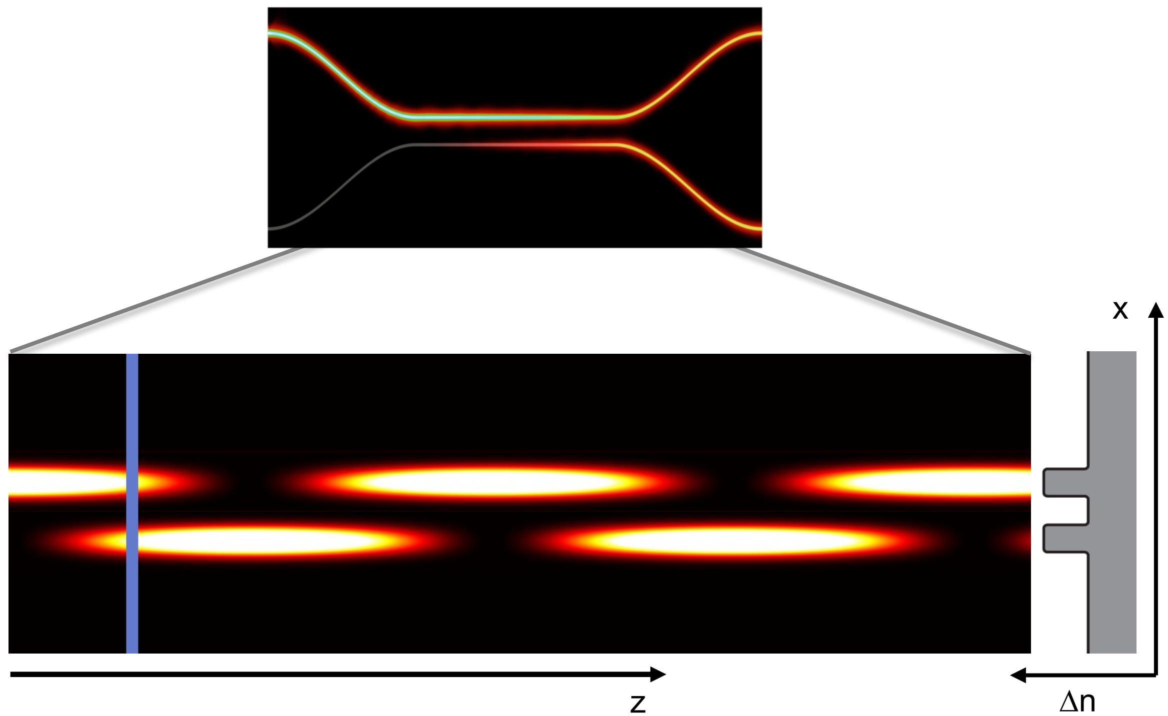

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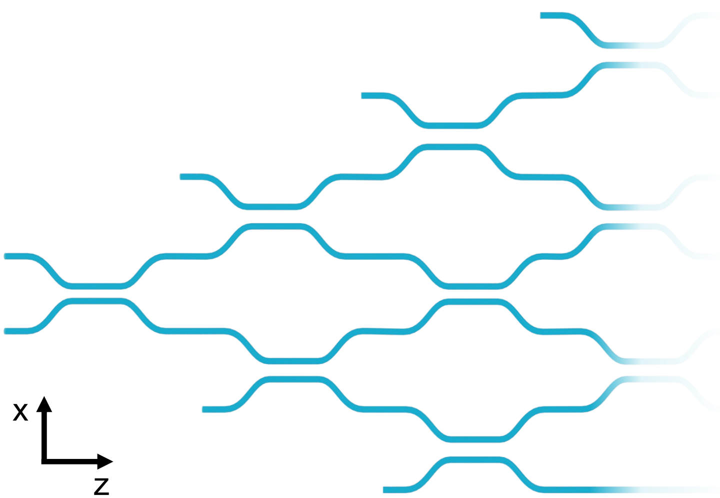

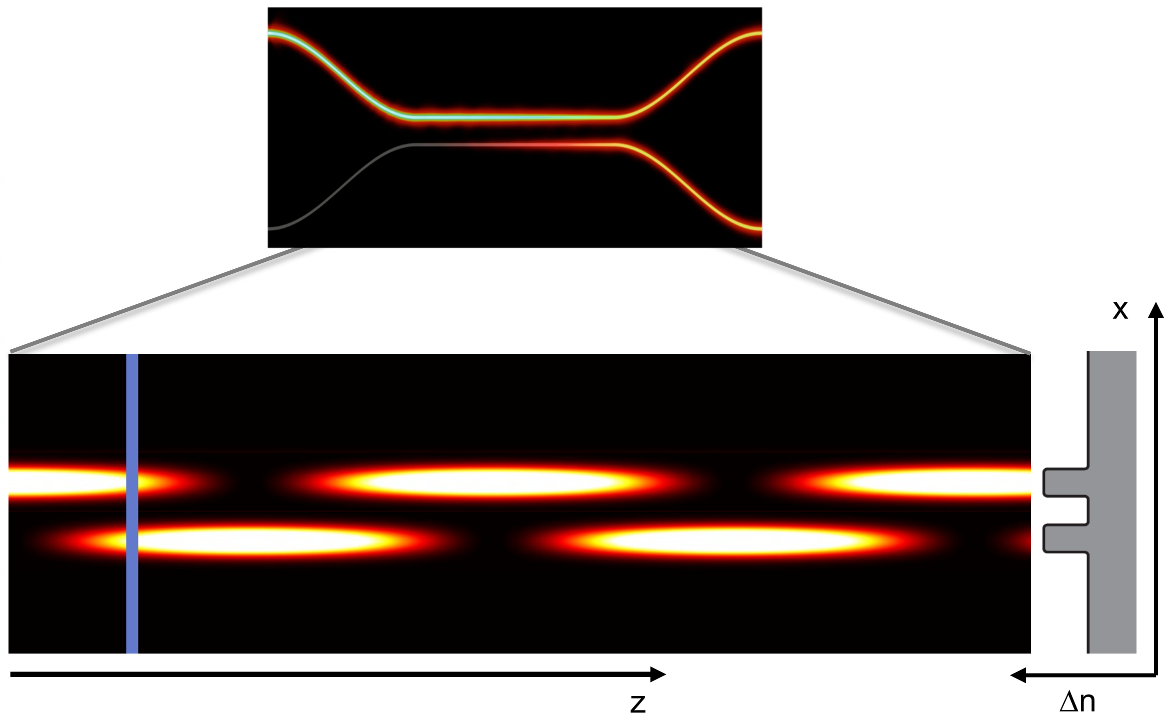

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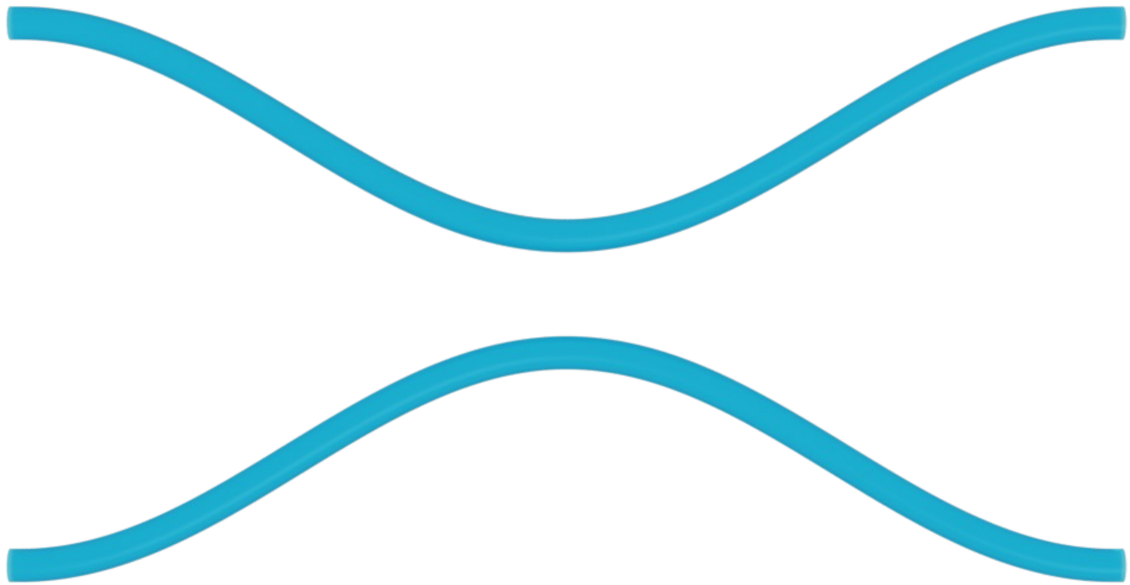
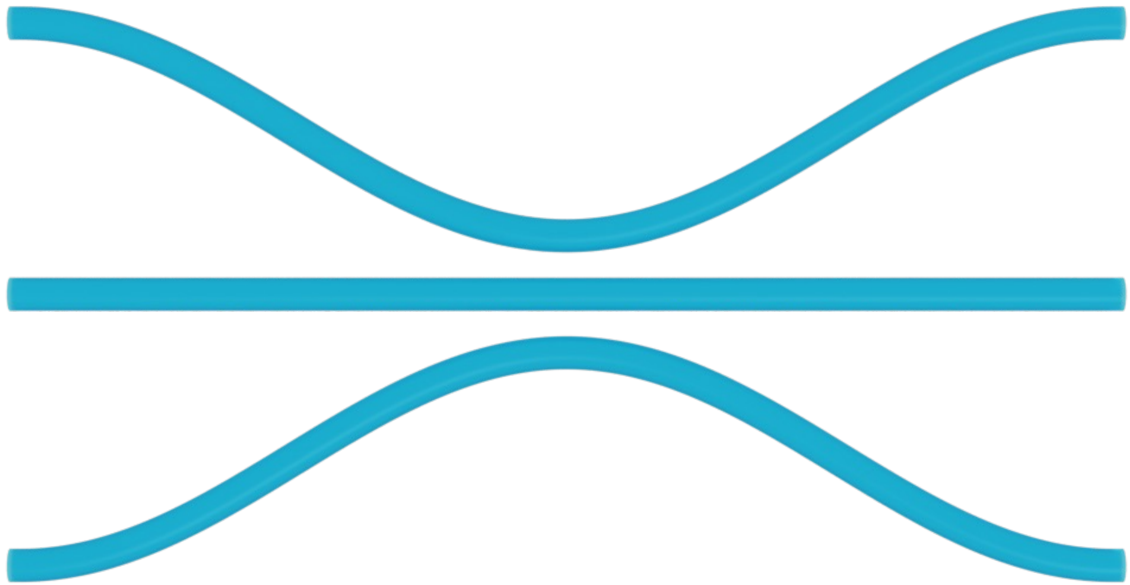
Directional waveguide coupler



Motivation

Non-adiabatic, non-Abelian holonomy



Topologically-Protected Single-Qubit Quantum Gate

Outline

Motivation

Introduction to Holonomies

Experimental Implementation

Realization of Holonomic Quantum Gates and an Algorithm

Conclusion

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Vera Neef vera.neef@uni-rostock.de

Phases in quantum mechanics

Berry. *Proc. R. Soc. Lond. A* **392**, 45–57 (1984). Wilczek, Zee. *Phys. Rev. Lett*. **52**, 2111-2114 (1984).

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Phases in quantum mechanics

Dynamical

passage of time

$$|\Psi_{\text{out}}\rangle = \mathrm{e}^{\mathrm{i}\varphi(t)}\,|\Psi_{\text{in}}\rangle$$

Geometric

of Hilbert space and path

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Phases in quantum mechanics

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Geometric

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Abelian

1D

$$|\Psi_{\text{out}}\rangle = \mathrm{e}^{\mathrm{i}\alpha}\,|\Psi_{\text{in}}\rangle$$

Non-Abelian

ND

$$|\Psi_{\text{out}}\rangle = \hat{U}\,|\Psi_{\text{in}}\rangle$$

Holonomy

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A Recipe for Holonomies

1. Suitable subspace of the Hilbert-space with non-trivial topology

Wilczek, Zee. *Phys. Rev. Lett*. **52**, 2111-2114 (1984).

Anandan. *Phys. Lett. A*. **133**, 171-175 (1988).

A Recipe for Holonomies

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1. Suitable subspace of the Hilbert-space with non-trivial topology

Geometric subspace $\mathcal{H}_{\text{geo}}$
Mean energy zero

$$\left\langle \Phi_j(t) \middle| \hat{H} \middle| \Phi_k(t) \right\rangle = 0$$

Hamiltonian

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2. Tunable parameter space

Coupling space $\{\kappa_1, \kappa_2, \dots\}$

$$|\Phi_k(t)\rangle = |\Phi_k(\boldsymbol{\kappa}(t))\rangle$$

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Non-Adiabatic Holonomy

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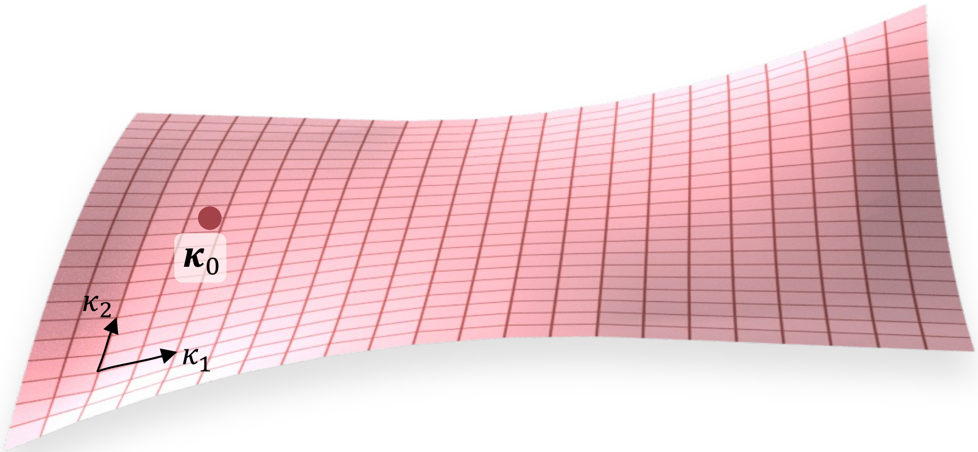
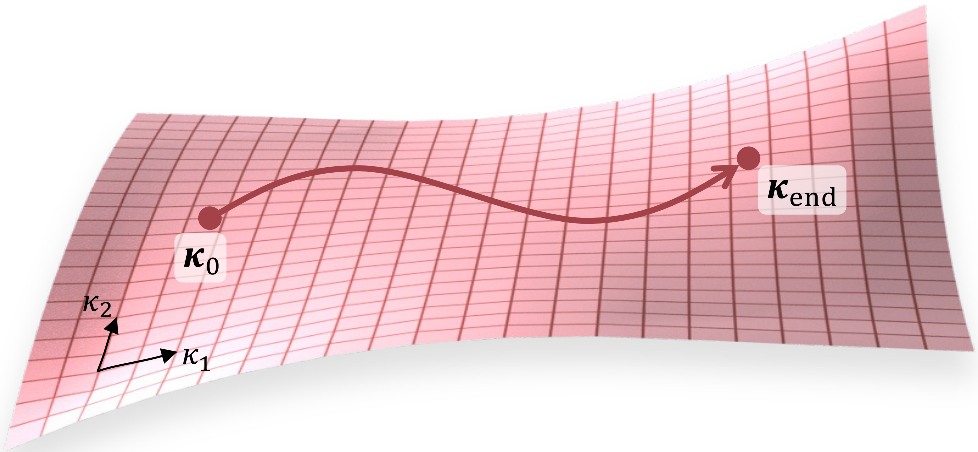
Holonomies in Quantum Optics



Johansson, *et al*. *Phys. Rev. A*. **86**, 062322 (2012).



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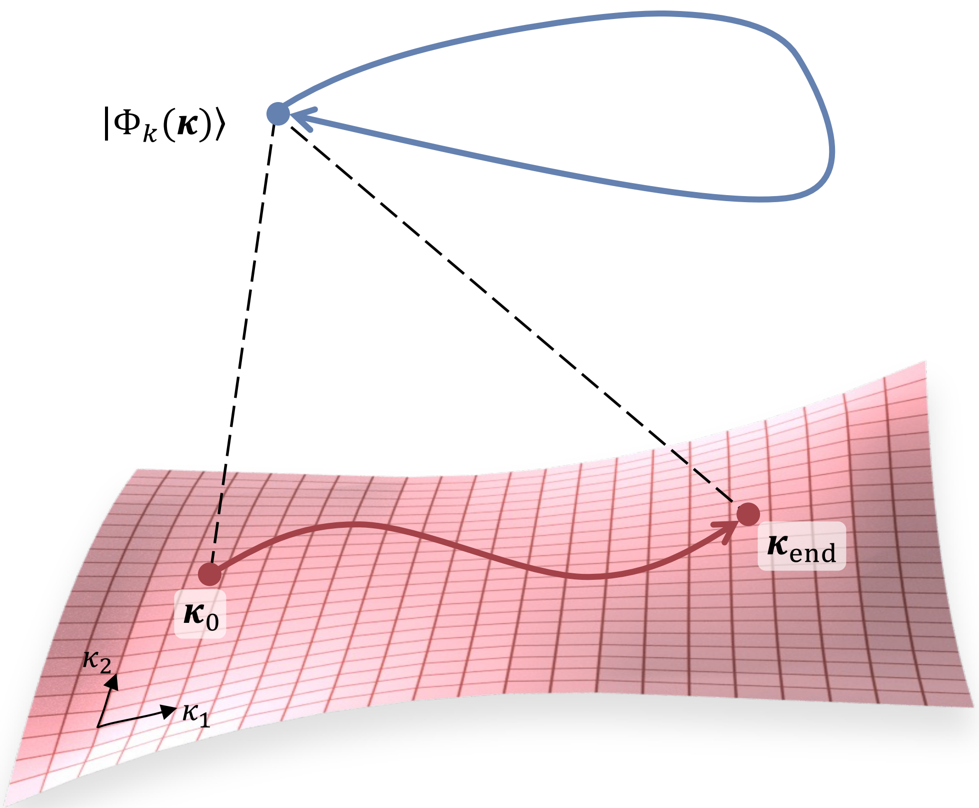

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Holonomies in Quantum Optics

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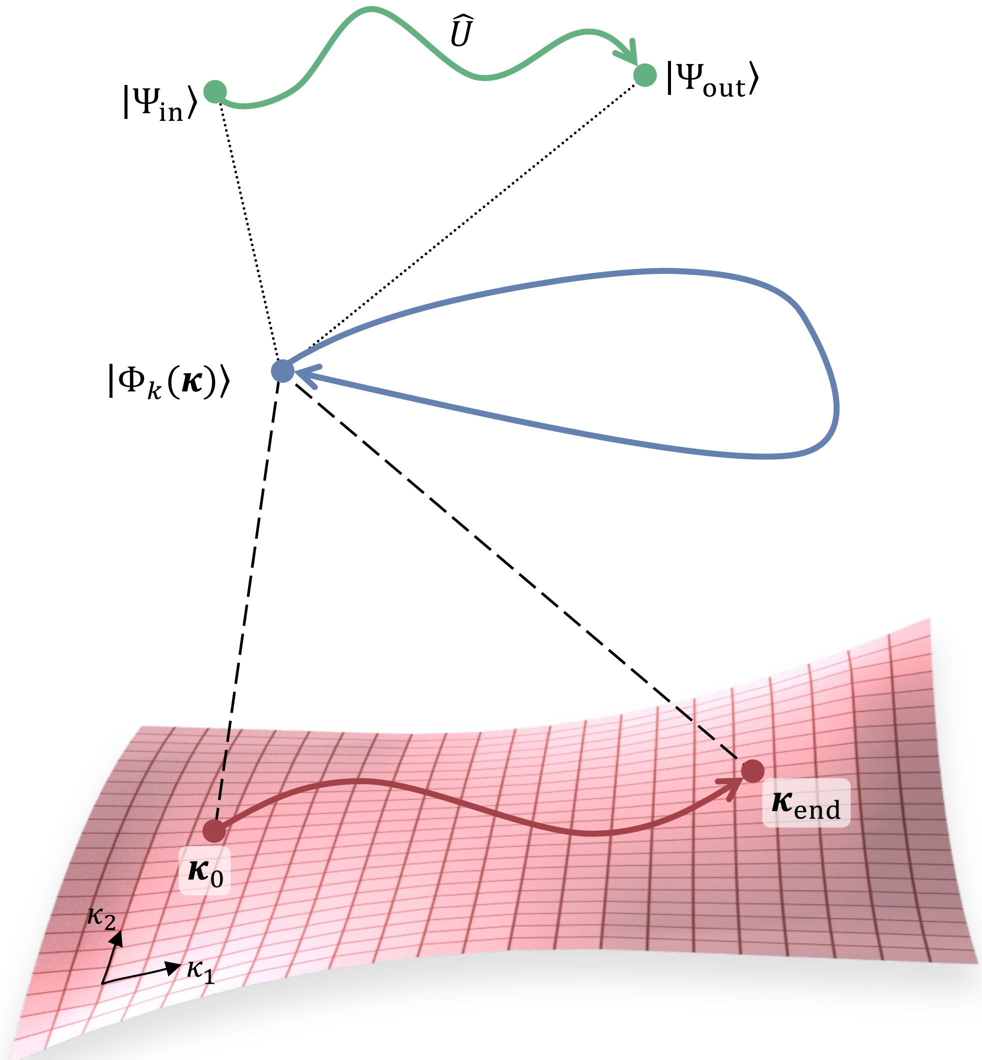

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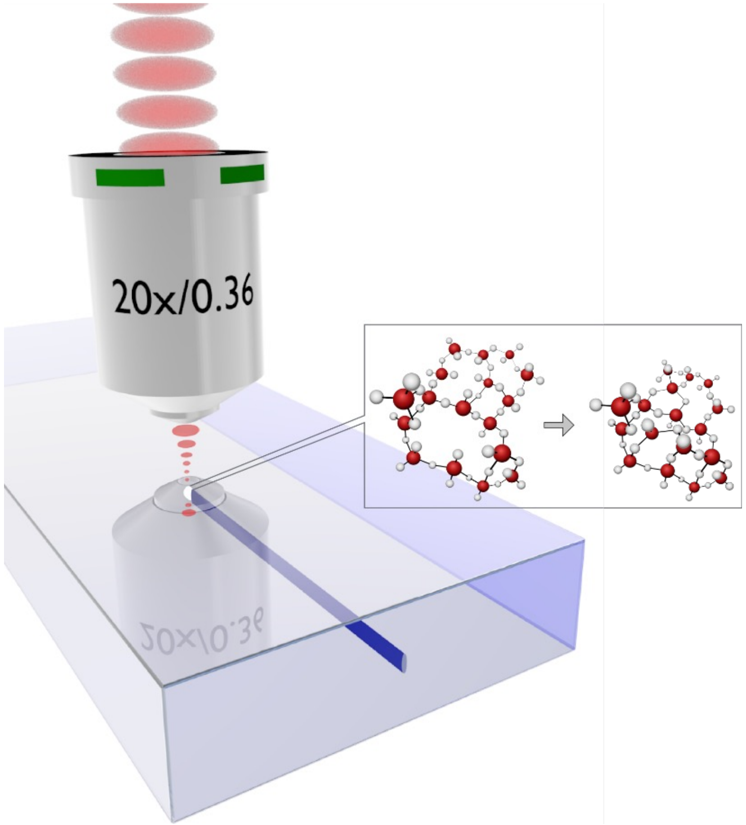
Experimental Platform

Femtosecond-laser-written waveguides



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Experimental Platform

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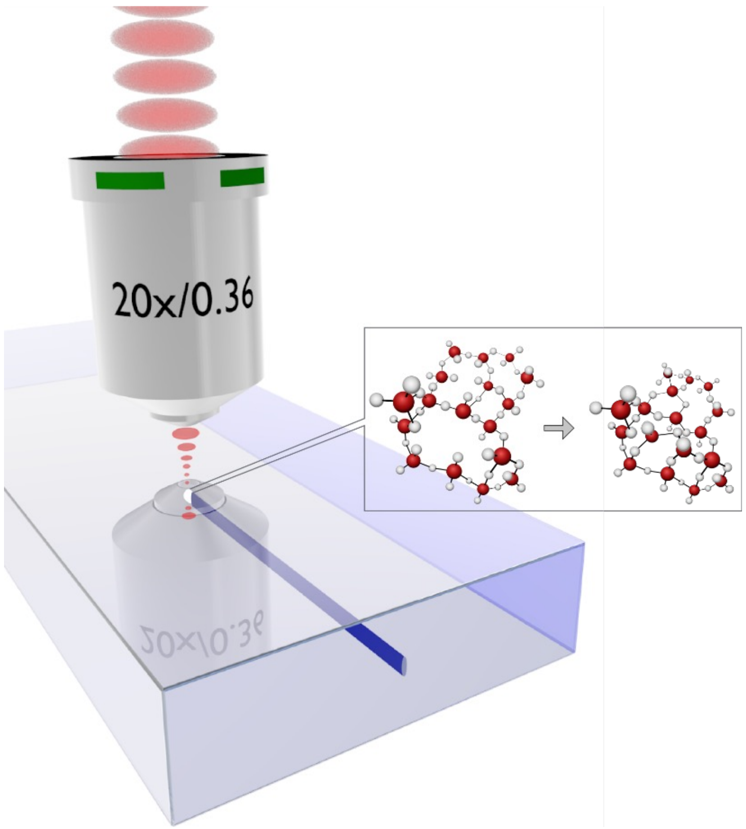
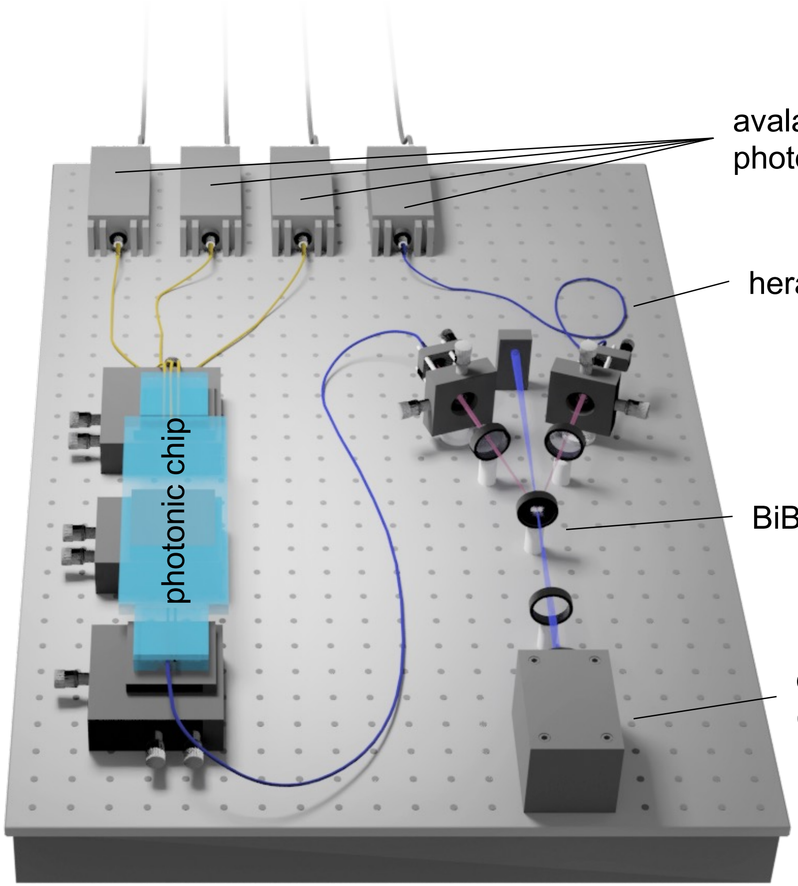


Heralded single-photon measurement



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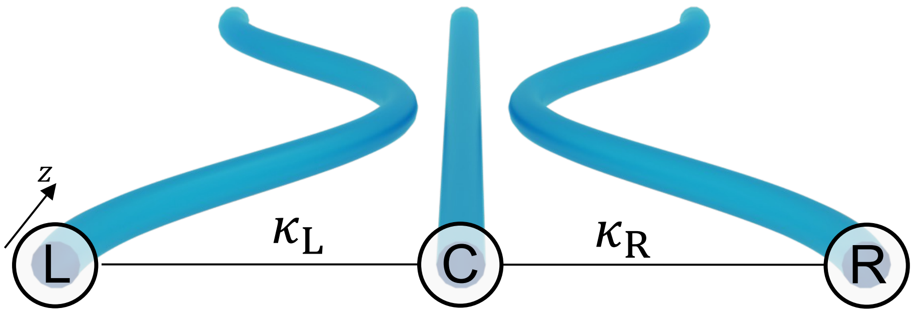
A Single-Photon Tritter

$\mathcal{H}_{\text{geo}}$ is two-dimensional

→ Two-dimensional holonomy $\hat{U}$



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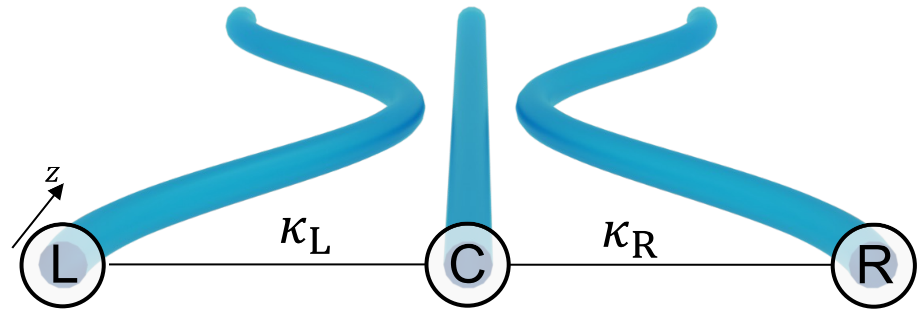

Parametrization

envelope

$$\kappa_{\mathrm{L}}(z) = \Omega(z)\, g_{\mathrm{L}}$$
$$\kappa_{\mathrm{R}}(z) = \Omega(z)\, g_{\mathrm{R}}$$

constant weights

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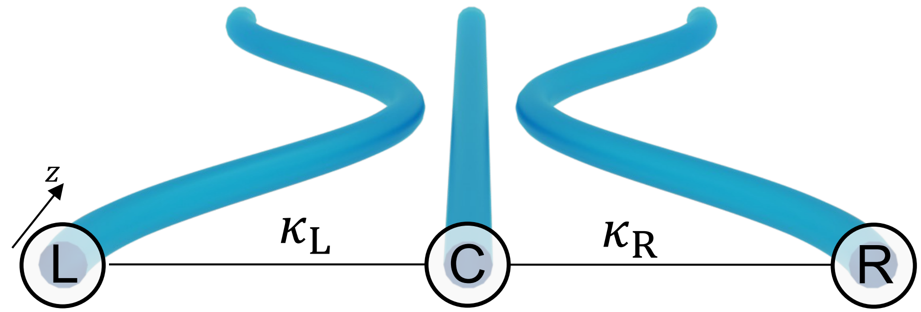

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Cyclicity condition

$|\Phi_k(z_0)\rangle = |\Phi_k(z_{\mathrm{end}})\rangle$

$$\rightarrow \int_{z_{\mathrm{i}}}^{z_{\mathrm{f}}} \Omega(z)\mathrm{d}z = \pi$$

$$\rightarrow \hat{U} = \hat{U}(g_{\mathrm{L}}, g_{\mathrm{R}})$$

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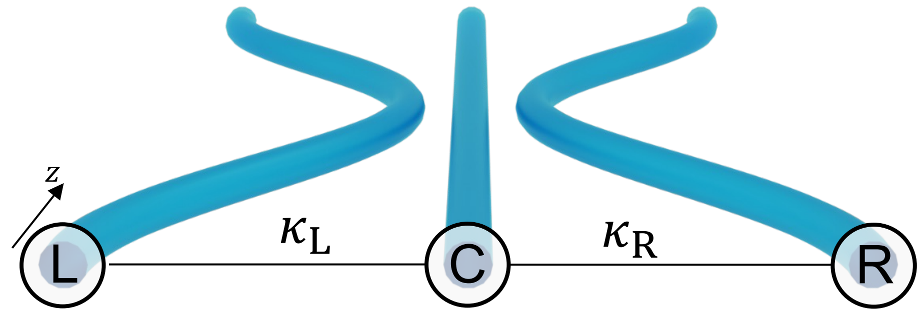

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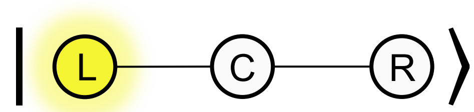
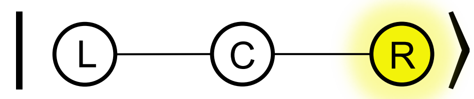
Qubit

$|0\rangle_{\mathrm{logic}} =$ 

$|1\rangle_{\mathrm{logic}} =$ 

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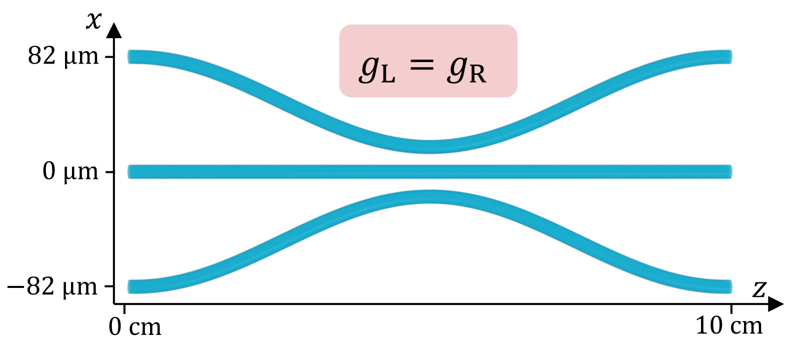

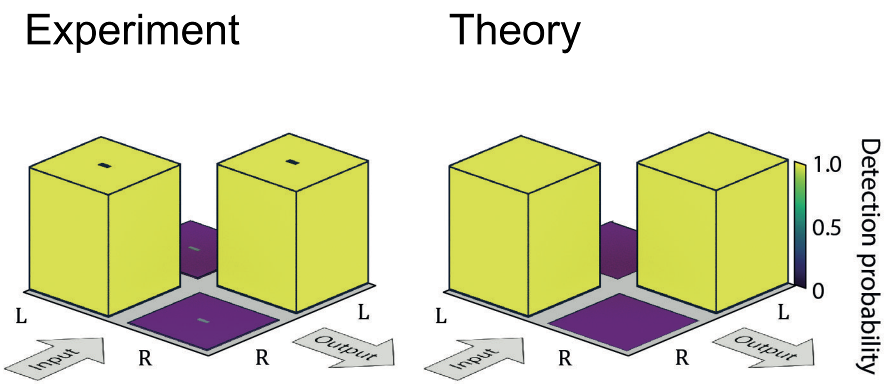

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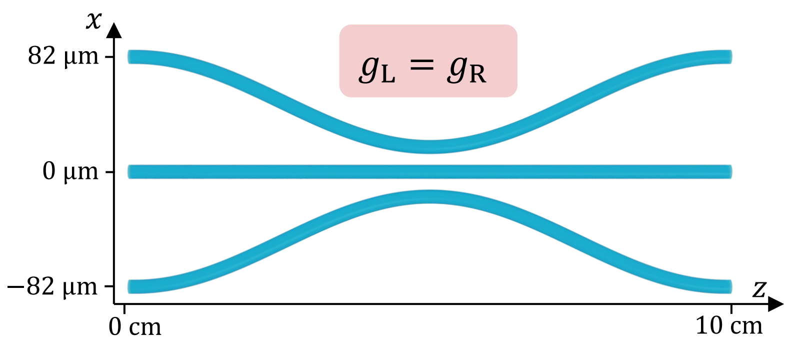
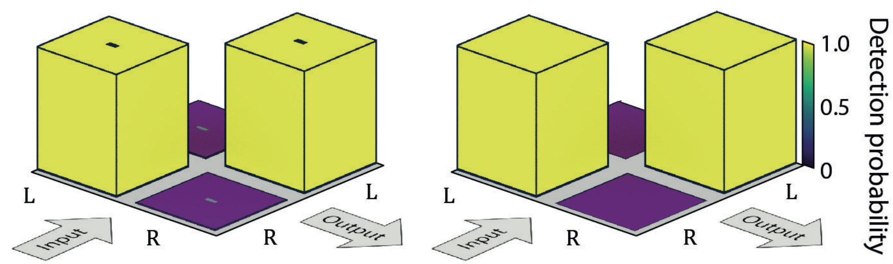
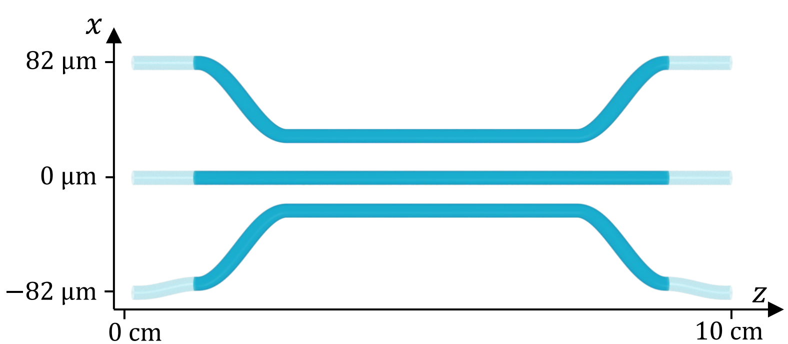


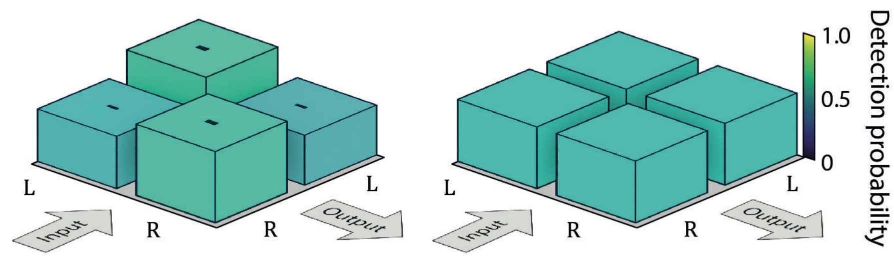
Experiment

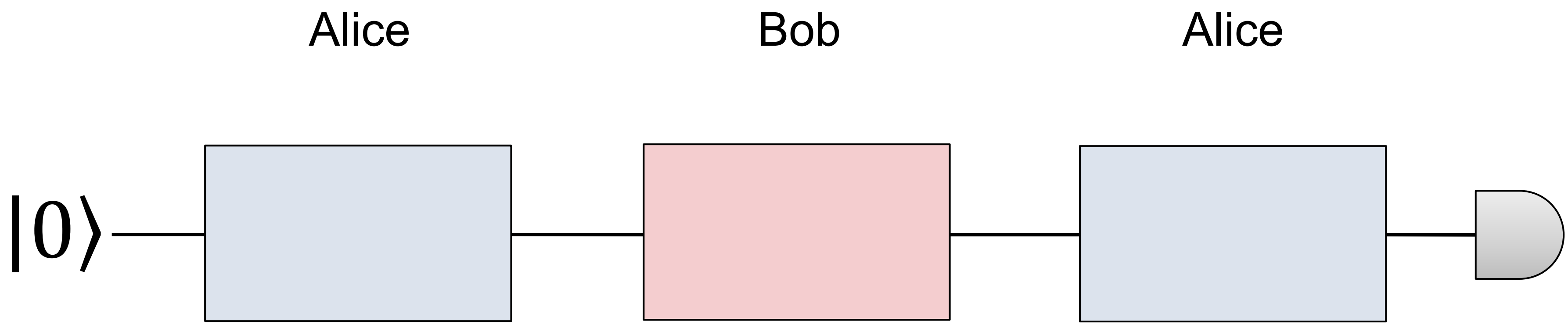
Theory



Hadamard

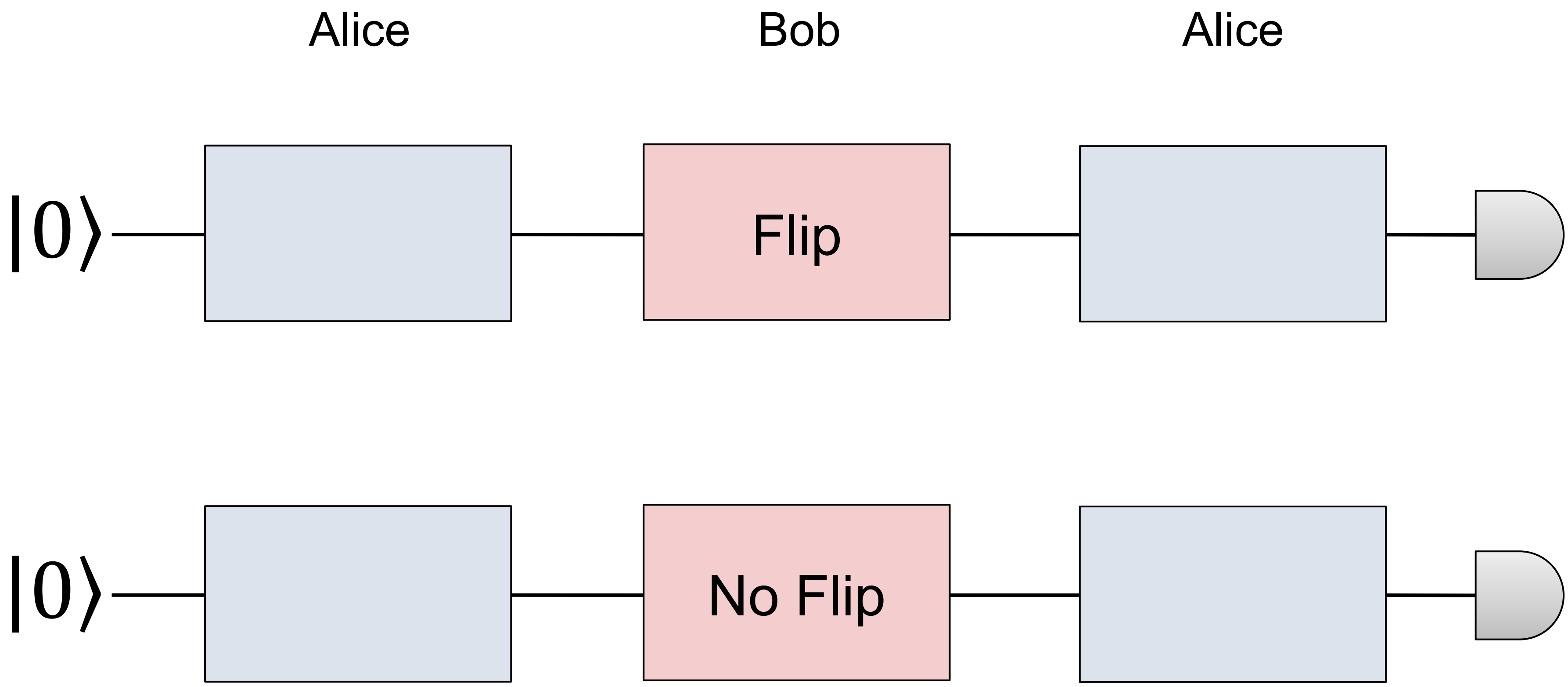






Müller, Greinert. *arXiv* **2108.06271** (2021).

The Coin-Flip Algorithm



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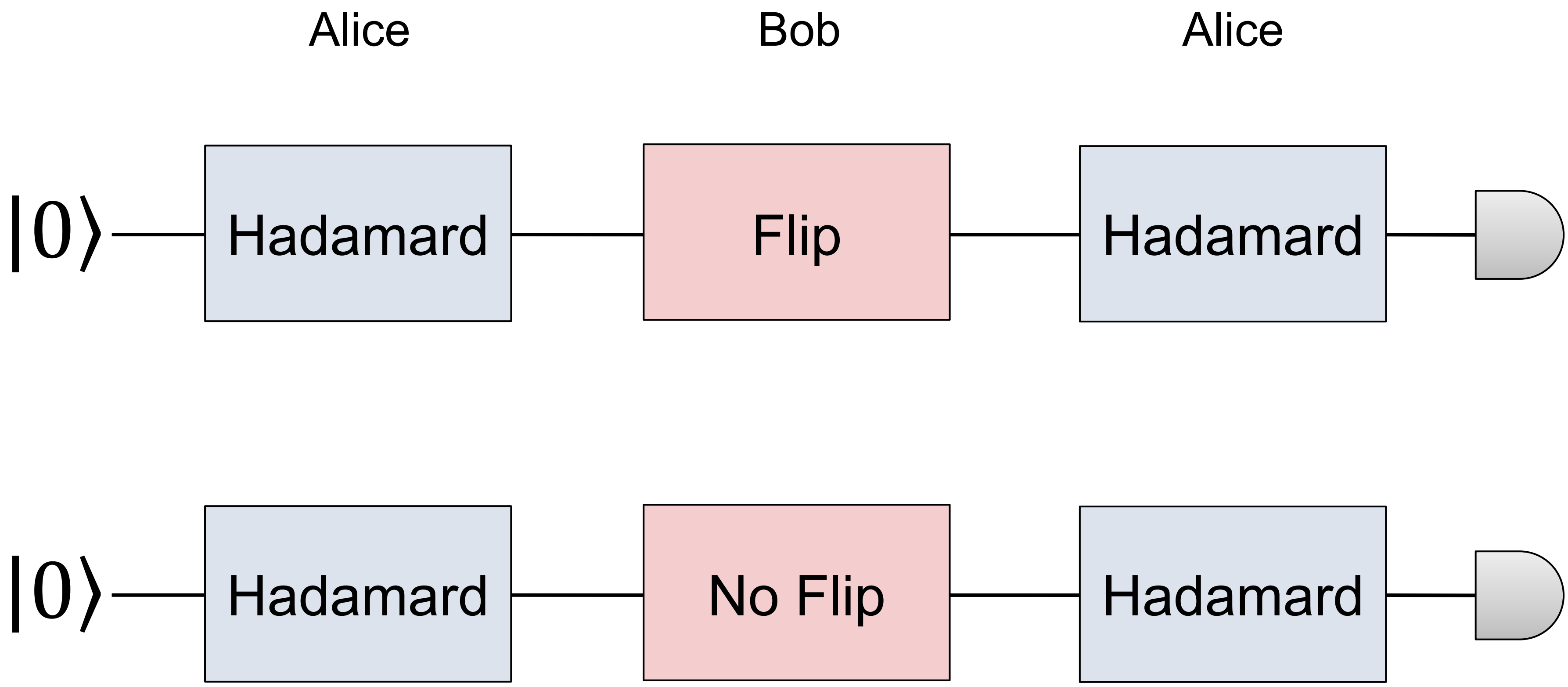

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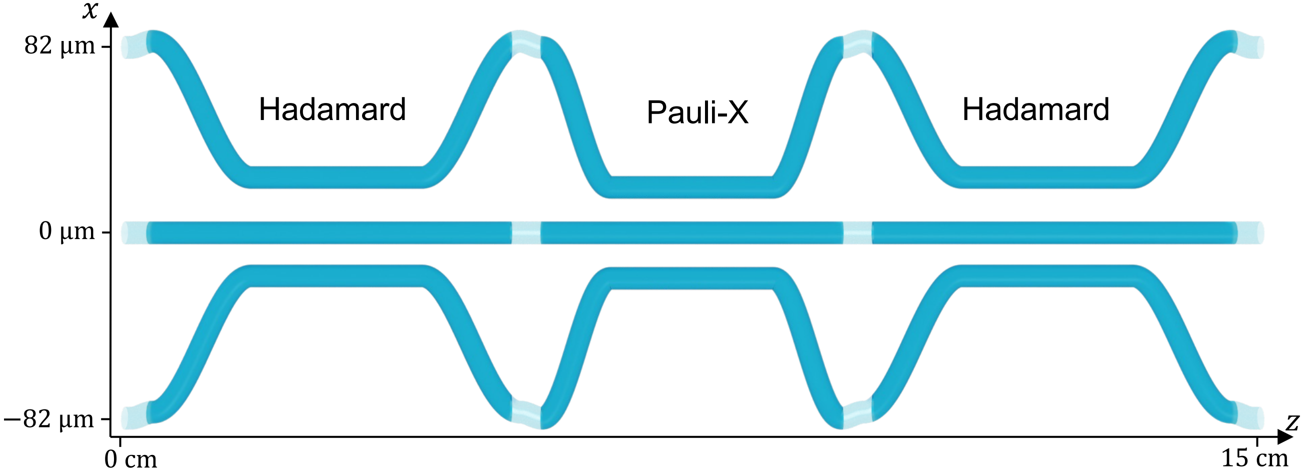
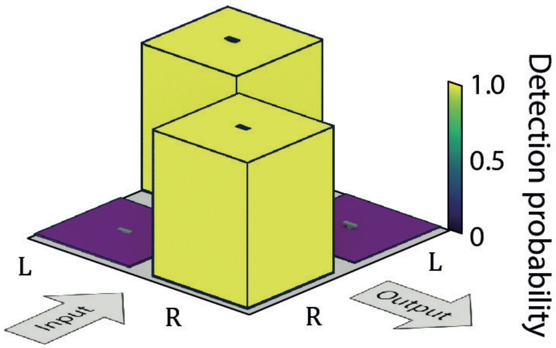


Bob flips the coin



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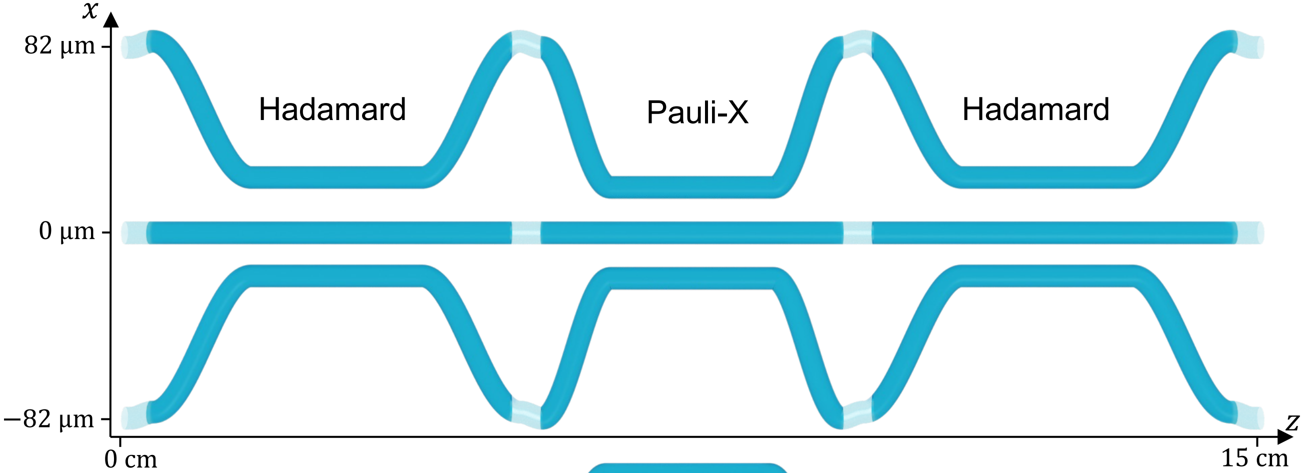
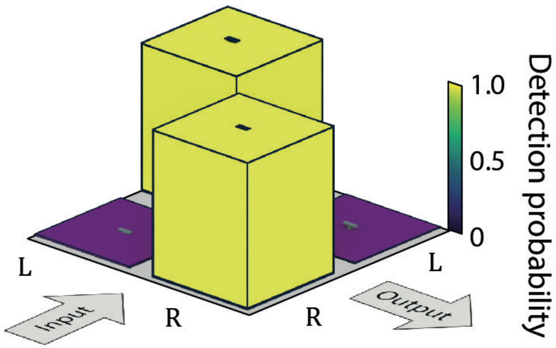
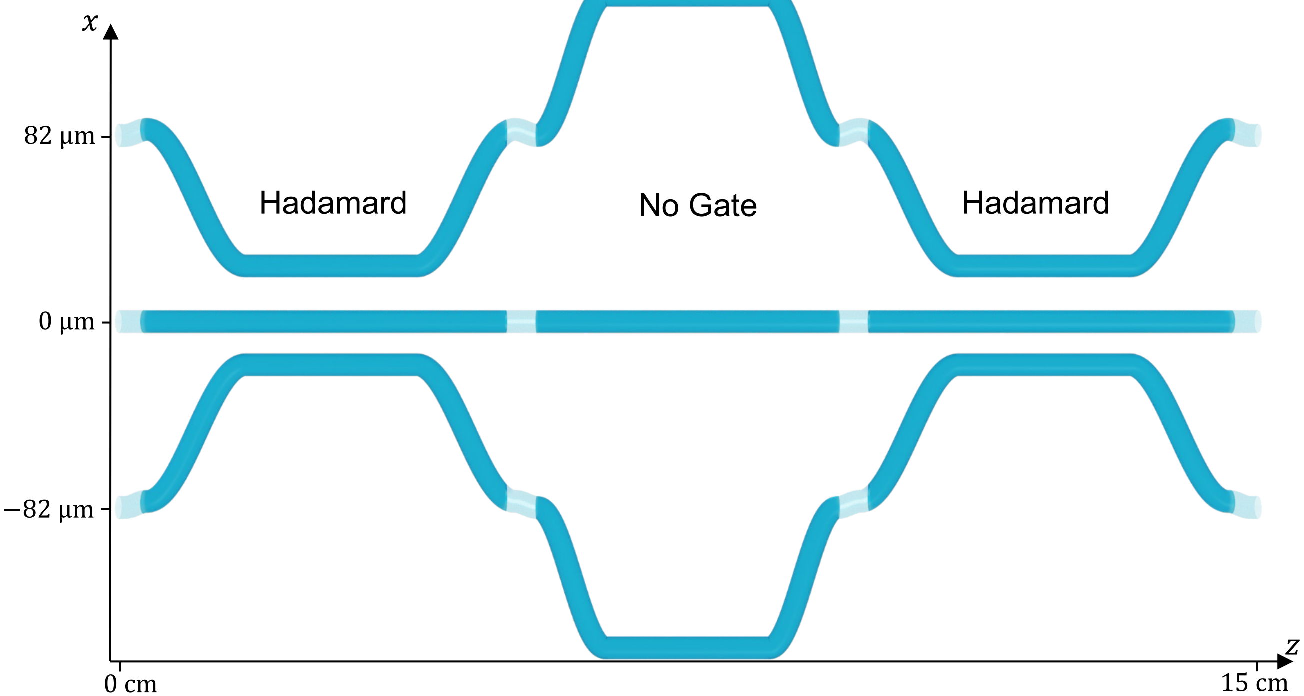
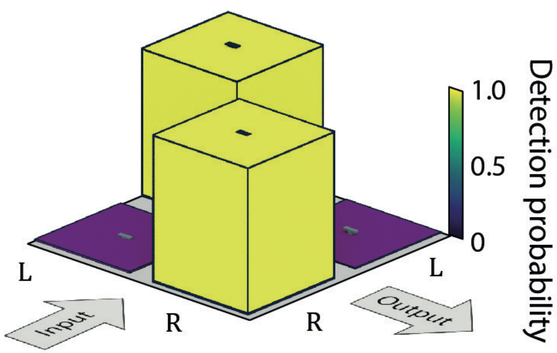


Bob flips the coin





Bob doesn’t flip the coin



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Motivation

Introduction to Holonomies

Experimental Implementation

Realization of Holonomic Quantum Gates and an Algorithm

Conclusion

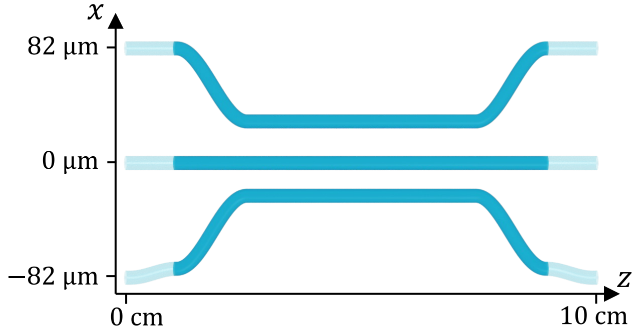
Conclusion

- First quantum optical realization of non-adiabatic holonomies

Conclusion

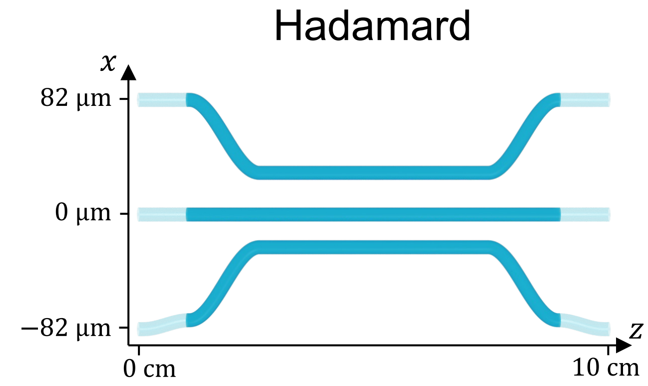
- First quantum optical realization of non-adiabatic holonomies
- Realization of gates with high fidelities

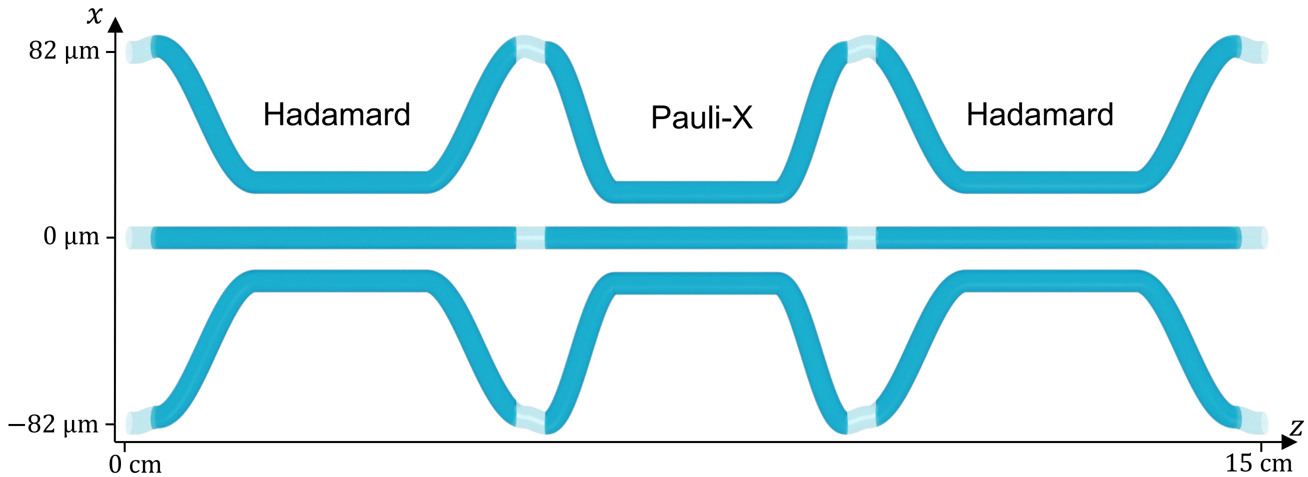
Hadamard

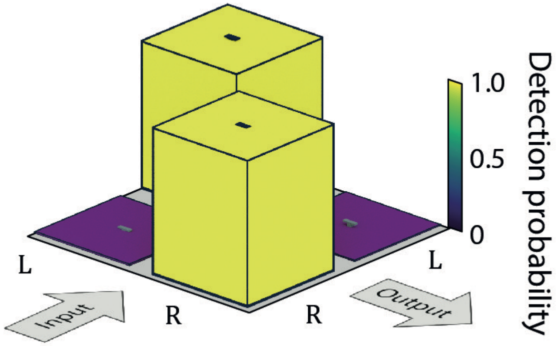


Conclusion

- First quantum optical realization of non-adiabatic holonomies
- Realization of gates with high fidelities
- Realization of a single-qubit algorithm



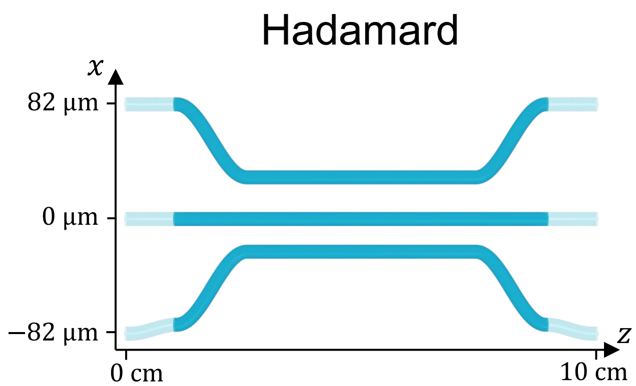


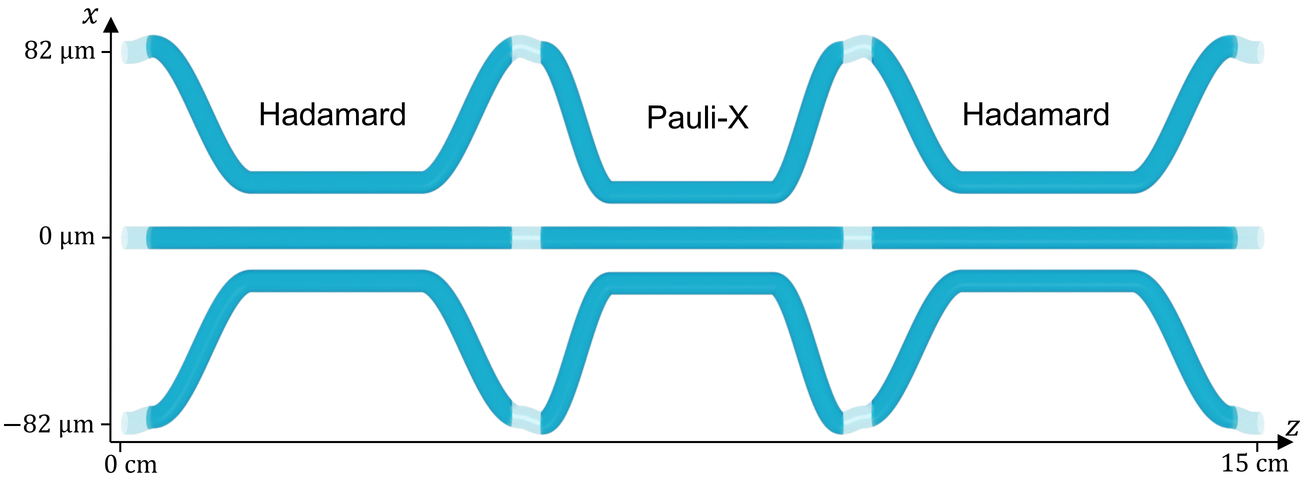


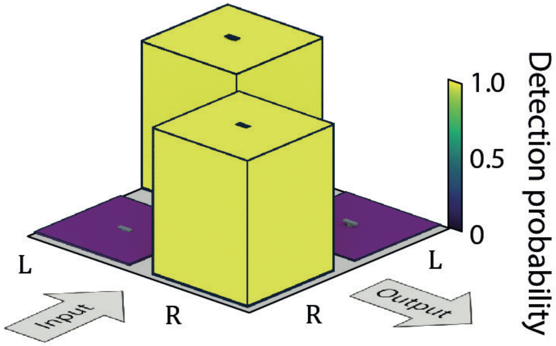
Conclusion

- First quantum optical realization of non-adiabatic holonomies
- Realization of gates with high fidelities
- Realization of a single-qubit algorithm

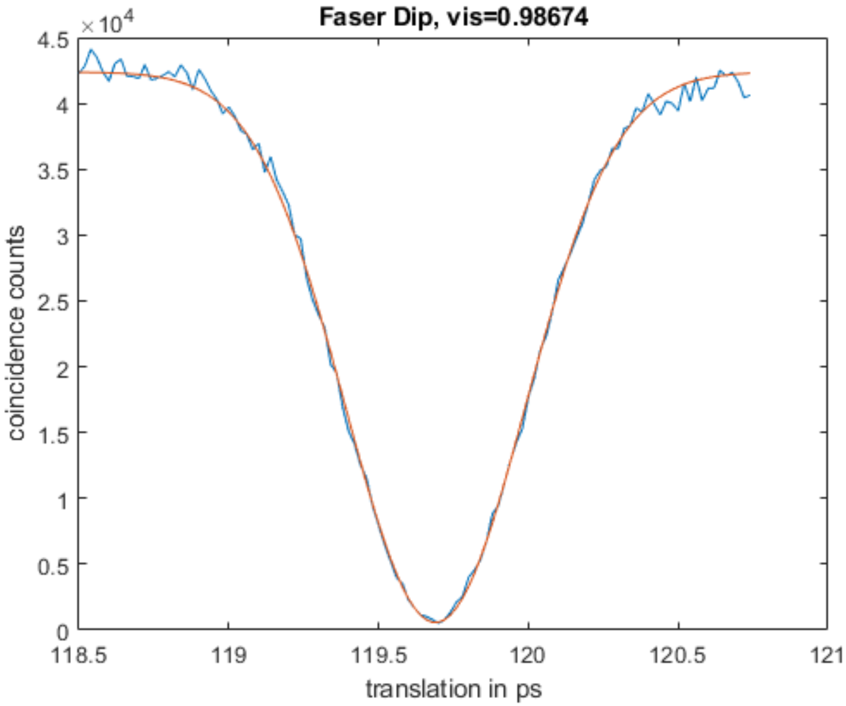
→ Universal non-adiabatic, holonomic quantum computing

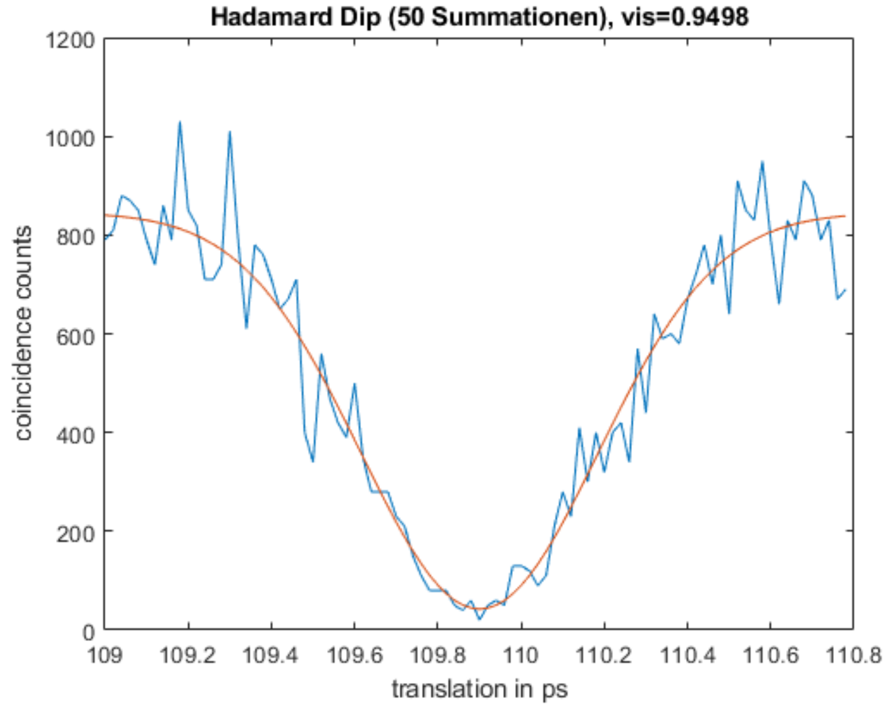






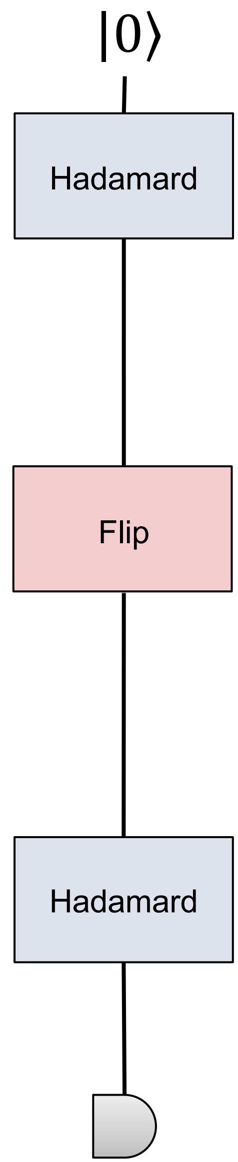
Hong-Ou-Mandel Effect





Hong, Ou, Mandel. *Phys. Rev. Lett*. **59**, 2044–2046 (1987).

App.: Coin-Flip Algorithm



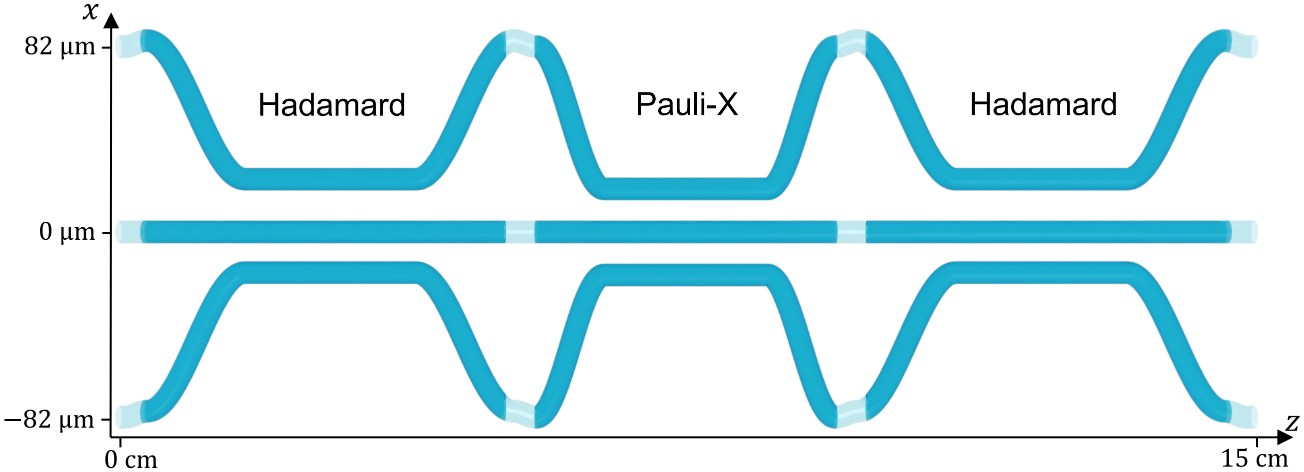
$$|0\rangle$$

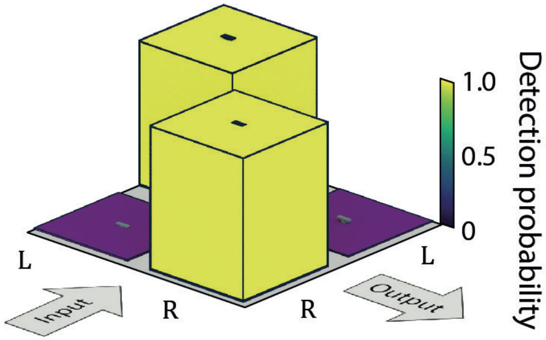
$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{\sqrt{2}}|1\rangle + \frac{1}{\sqrt{2}}|0\rangle$$

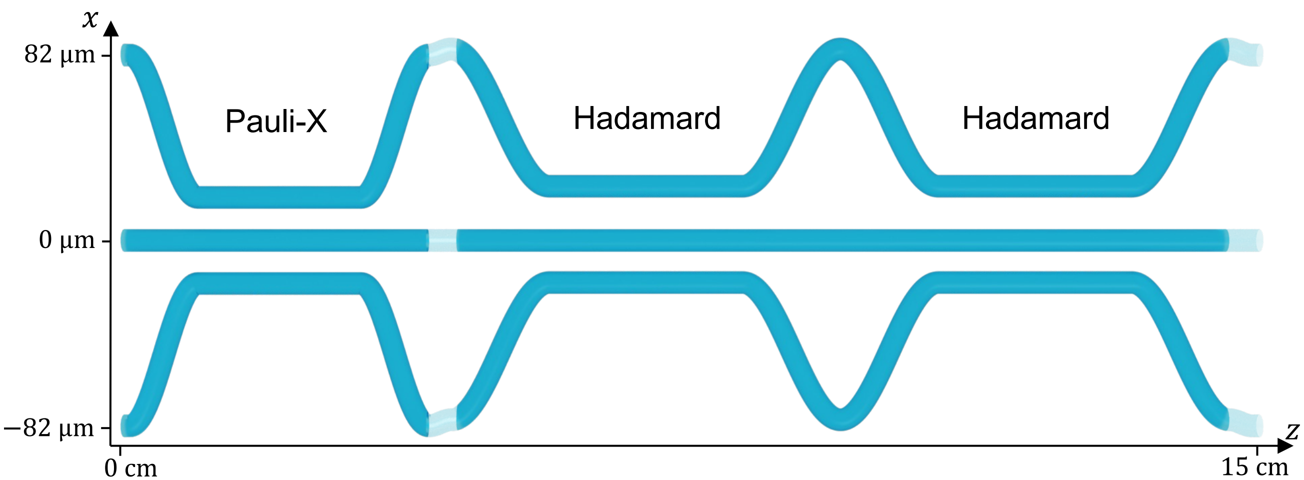
$$|0\rangle$$

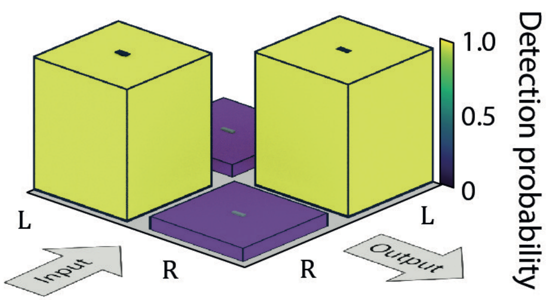
App.: Verifying the non-Abelian Character





$$\left[\left(\hat{h}\hat{x}\right), \hat{h}\right] \neq 0$$





App.: A Photonic Chip

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Three non-Adiabatic Holonomic Gates

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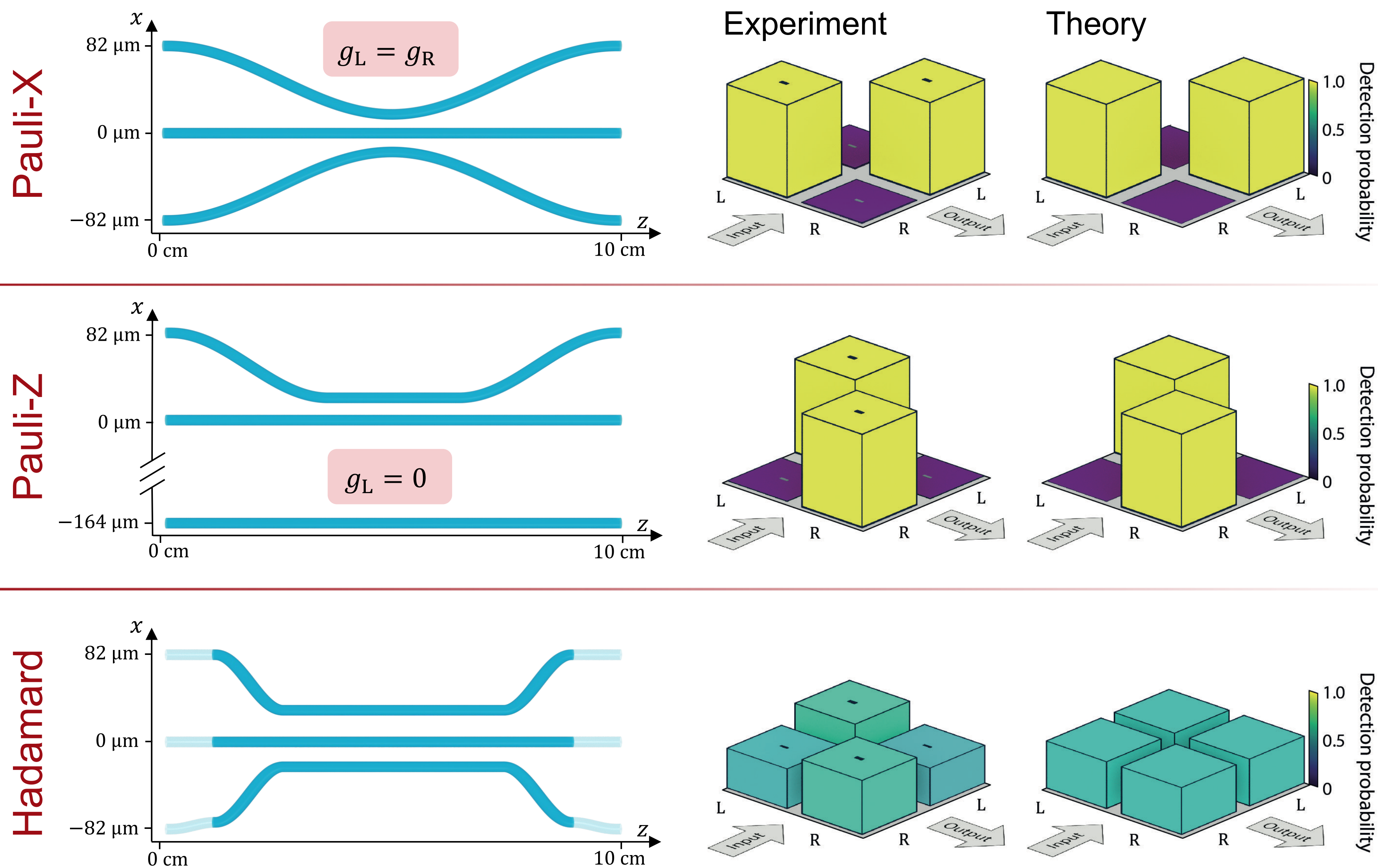

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Appendix

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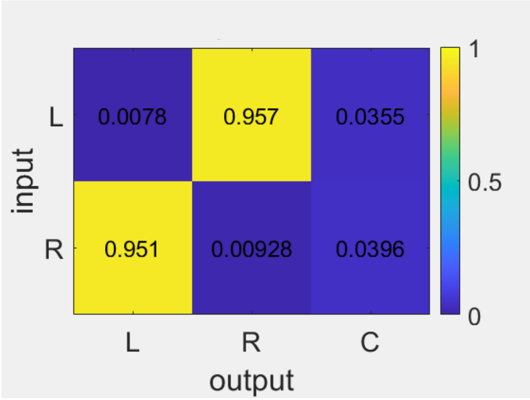
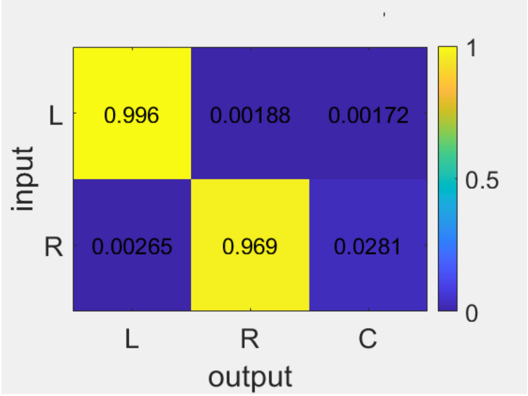
$$\text{Fidelity} = \left(\sum_i \sqrt{p_i q_i} \right)^2$$

Pauli-X



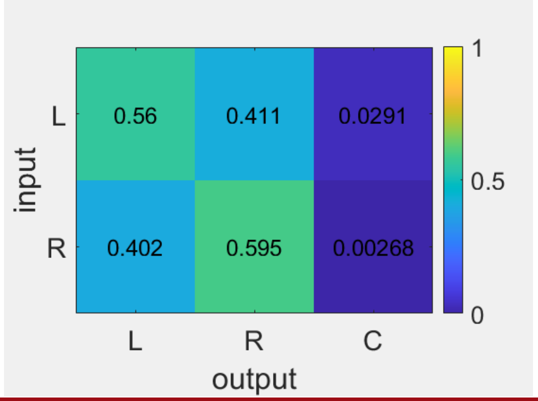
$$\text{Fidelity} = 99.11\%$$

Pauli-Z



$$\text{Fidelity} = 99.77\%$$

Hadamard



$$\text{Fidelity} = 99.23\%$$

Non-adiabatic holonomic condition: $\left\langle \Psi_k(z) \middle| \hat{H}(z) \middle| \Psi_j(z) \right\rangle = 0$

Parametrization: $\kappa_\mathrm{L}(z) = \Omega(z)\ \sin\frac{\theta}{2}\ \ \mathrm{e}^{\mathrm{i}\phi}$

$$\kappa_\mathrm{R}(z) = \Omega(z)\ \cos\frac{\theta}{2}$$

Cyclicity condition: $\delta = \int_{z_\mathrm{i}}^{z_\mathrm{f}} \Omega(z)\mathrm{d}z = \pi$

Holonomy: $\begin{pmatrix} \cos\theta & -\mathrm{e}^{-\mathrm{i}\phi}\sin\theta \\ -\mathrm{e}^{\mathrm{i}\phi}\sin\theta & -\cos\theta \end{pmatrix}$

App.: Non-Adiabatic Operators and Gauge Field

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$$\Phi_1^\dagger(z) = \sin(\theta/2)\mathrm{e}^{\mathrm{i}\varphi} a_{\mathrm{R}}^\dagger - \cos(\theta/2) a_{\mathrm{L}}^\dagger,$$

$$\Phi_2^\dagger(z) = \mathrm{e}^{\mathrm{i}\delta(z)} \big(\cos\delta(z) B^\dagger - \mathrm{i}\sin\delta(z) a_{\mathrm{C}}^\dagger \big),$$

$$B^\dagger(z_0) = \sin(\theta/2)\mathrm{e}^{-\mathrm{i}\varphi} a_{\mathrm{L}}^\dagger + \cos(\theta/2) a_{\mathrm{R}}^\dagger$$

App.: Non-Adiabatic Operators and Gauge Field

Universität Rostock Traditio et Innovatio

$$\Phi_1^\dagger(z) = \sin(\theta/2)\mathrm{e}^{\mathrm{i}\varphi} a_\mathrm{R}^\dagger - \cos(\theta/2) a_\mathrm{L}^\dagger,$$

$$\Phi_2^\dagger(z) = \mathrm{e}^{\mathrm{i}\delta(z)} \left(\cos\delta(z) B^\dagger - \mathrm{i}\sin\delta(z) a_\mathrm{C}^\dagger \right),$$

$$B^\dagger(z_0) = \sin(\theta/2)\mathrm{e}^{-\mathrm{i}\varphi} a_\mathrm{L}^\dagger + \cos(\theta/2) a_\mathrm{R}^\dagger$$

$$(A_\nu)_{ki} = \left\langle \Phi_k \middle| \frac{\partial}{\partial \kappa^\nu} \middle| \Phi_i \right\rangle$$

$$\boldsymbol{A} = \mathrm{i}\,\Omega(z) |b\rangle\langle b|$$

$$|b\rangle = \kappa_\mathrm{L} |1_\mathrm{L}\rangle + \kappa_\mathrm{R} |1_\mathrm{R}\rangle$$

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Abelian U(1)

Berry phase: $\alpha(\gamma)$

$$\exp\{\mathrm{i}\,\alpha(\gamma)\} = \exp\left\{\mathrm{i}\oint_\gamma A_\nu \mathrm{d}\kappa^\nu\right\}$$

Abelian gauge field: $[A_\nu, A_\mu] = 0$

Single eigenstate: $|D(\boldsymbol{\kappa})\rangle$

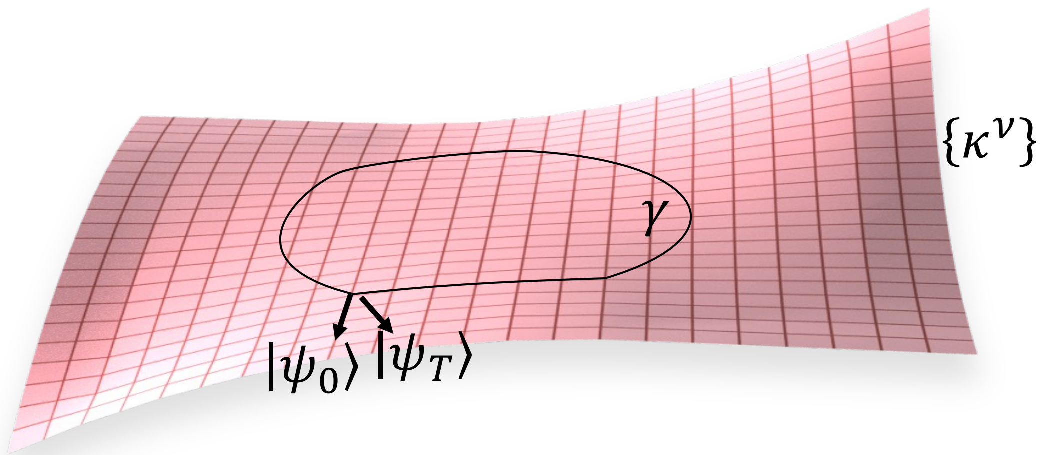
Non-Abelian U(N)

Non-Abelian holonomy: $\hat{U}(\gamma)$

$$\hat{U}(\gamma) = \mathcal{P}\exp\left\{\oint_\gamma \hat{A}_\nu \mathrm{d}\kappa^\nu\right\}$$

Non-Abelian gauge field: $[\hat{A}_\nu, \hat{A}_\mu] \neq 0$

Degenerate eigenspace: $\{|D_i(\boldsymbol{\kappa})\rangle\}_{i=1}^N$



Berry. *Proc. R. Soc. Lond. A* **392**, 45–57 (1984).
Wilczek, Zee. *Phys. Rev. Lett*. **52,** 2111–2114 (1984).

App.: Non-Adiabatic Holonomies

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Adiabatic U(N)

Degenerate eigenspace:

$$\mathcal{H}_{\text{eig}} = \{|\Psi_i(\boldsymbol{\kappa})\rangle\}_{i=1}^{N}$$

$$\hat{H}(z)|\Psi_i(z)\rangle = 0$$

Non-Adiabatic U(M)

Geometric Subspace:

$$\mathcal{H}_{\text{geo}} = \{|\Phi_i(\boldsymbol{\kappa})\rangle\}_{i=1}^{M}$$

$$\left\langle \Phi_i(z) \middle| \hat{H}(z) \middle| \Phi_j(z) \right\rangle = 0$$

$$M \geq N$$