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Three-Body Non-Leptonic B Decays

within QCD-factorization

(with a focus on : Huber, Vos, Virto, 2007.08881)

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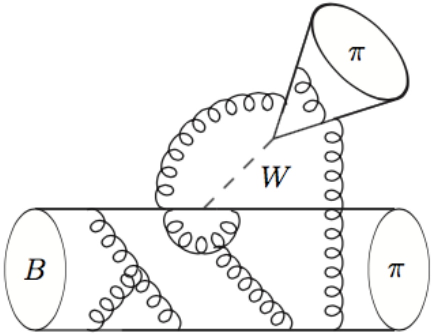
Status and prospects of Non-leptonic B meson decays – May 31st, 2022 (Siegen)

Motivations

- Huge multiplicity of final states (2-body + multi-body), large data sets
- Important input in CKM studies (mostly angles)
- CP violation (SM and new physics)
- Non-trivial hadronic dynamics $\Rightarrow$ Perturbative and non-perturbative QCD methods



Non-leptonic B-decay Amplitudes

- Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\rm eff} = -\mathcal{L}_{QED+QCD} + \sum_i C_i(\mu)\, \mathcal{O}_i(\mu)$$

- C_i – Wilson coefficients (UV physics) $\rightarrow$ perturbation theory

  Known to NNLL: Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06.

- $\mathcal{O}_i$ – Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

- Amplitudes:

$$\mathcal{A}(B \rightarrow M_1 M_2 \cdots) = \sum_i C_i \, \langle M_1 M_2 \cdots | \mathcal{O}_i | B \rangle$$

The problem is to compute the **operator matrix elements (MEs)**

$\longrightarrow$ non-perturbative, process dependent (non-universal)

Direct CP Violation

$$\mathcal{A}(\bar{B} \to f) \equiv \mathcal{A}_f = \underbrace{\lambda_u}_{\sim e^{i\gamma}} \underbrace{(T_f^u - P_f)}_{\mathcal{A}^u} + \underbrace{\lambda_c}_{\simeq \text{real}} \underbrace{(T_f^c - P_f)}_{\mathcal{A}^c} \qquad \lambda_p = V_{pb} V^\star_{p\{d,s\}}$$

$$T_f^p = \sum_{1,2} C_i^p \langle f | Q_i^p | \bar{B} \rangle \quad \text{(current-current operators)}$$

$$P_f = \sum_{3,\dots,6} C_i \langle f | Q_i^p | \bar{B} \rangle \quad \text{(penguin operators)}$$

- In the SM, C_i contain <u>no phases</u>.
- We write $\mathcal{A}^p = |\mathcal{A}^p| e^{i\delta_p}$. Then:

$$\mathcal{A}_{\rm CP} \equiv \frac{|\mathcal{A}_f| - |\bar{\mathcal{A}}_{\bar{f}}|}{|\mathcal{A}_f| + |\bar{\mathcal{A}}_{\bar{f}}|} \propto \left| \frac{\lambda_u \mathcal{A}^u}{\lambda_c \mathcal{A}^c} \right| \cdot \sin\gamma \cdot \sin(\delta_c - \delta_u)$$

- Look for relative strong phases in interfering amplitudes

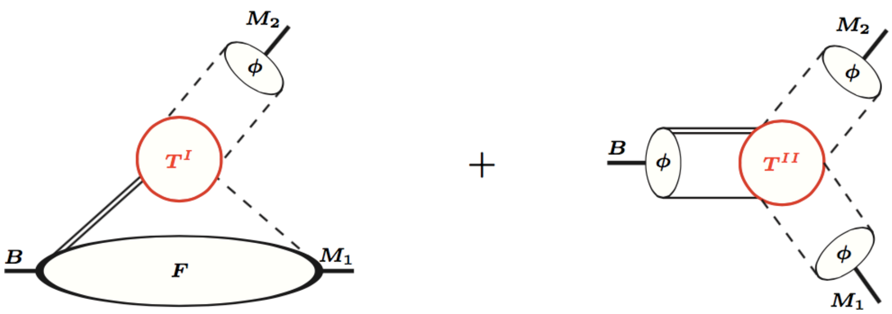
How to deal with MEs

1. Isolate contributions sensitive to IR physics
   - Scale separation – Factorization – Effective Field Theory
2. Parametrize them by a few ”universal” quantities
   - **Form Factors :** $F^{BM} \sim \langle M|\bar{q}\,\Gamma\,b|B\rangle$
   - **LCDAs :** $\phi_M(u) \sim \int dt\, e^{-it(p\cdot n)u}\, \langle M(p)|\bar{q}(tn)[tn,0]\,\not{n}\gamma_5\, q(0)|0\rangle$
   - **Decay constants :** $f_M \sim \langle M|\bar{q}\,\Gamma\,q|0\rangle$
   - $\cdots$
3. Then, either:
   - Calculate them
   - Extract them from experiment
   - Build observables where they cancel out

Factorization formula for $B \to M_1 M_2$

To leading power in the heavy-quark limit Beneke, Buchalla, Neubert, Sachrajda '99

$$\langle M_1 M_2 | \mathcal{O} | B \rangle = F^{BM_1} \int du\, T^I(u) \phi_{M_2}(u) + \int d\omega\, du\, dv\, T^{II}(\omega, u, v) \phi_B(\omega) \phi_{M_1}(u) \phi_{M_2}(v)$$



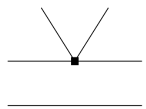
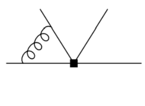
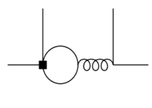
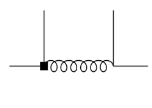
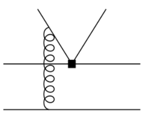
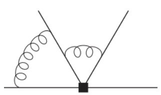
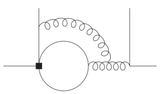
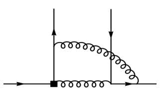
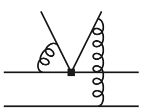
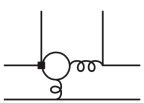
- Vertex corrections: $T^I(u) = 1 + \mathcal{O}(\alpha_s)$
- Spectator scattering: $T^{II}(\omega, u, v) = \underbrace{\mathcal{O}(\alpha_s)}_{real} + \mathcal{O}(\alpha_s^2/\pi)$ – (power supp. if M_1 heavy)
- Strong phases are perturbative [$\mathcal{O}(\alpha_s)$] or power suppressed [$\mathcal{O}(\Lambda/m_b)$].
- $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ – But ... $\alpha_s(m_b)/\pi \sim \Lambda/m_b$!!

Perturbative "matching" calculation

Two hard-scattering kernels for each operator insertion: T^I (vertex), T^{II} (spectator)

$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \simeq F^{BM_1} T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: "Tree", "Penguin".

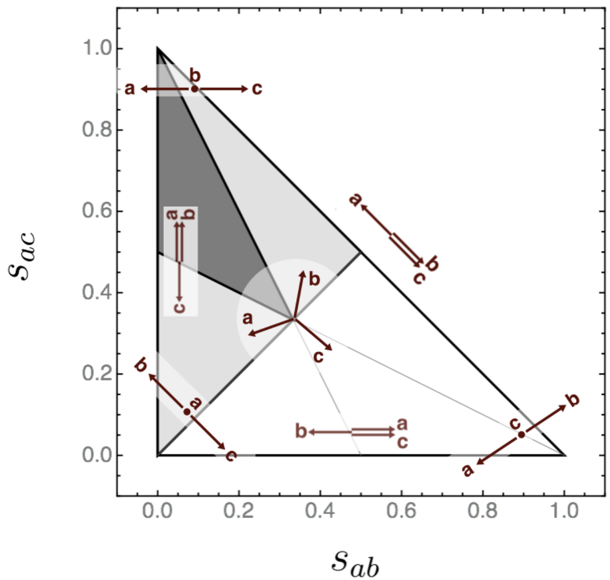
	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04		 		
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	  Kim, Yoon '11, Bell, Beneke, Huber, Li '15,'20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

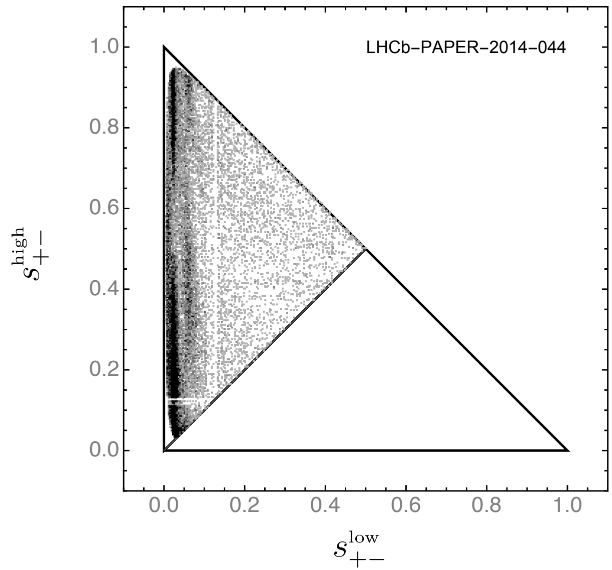
Three-body B decays

- Model-independent treatment of vector resonances:
  - $B \to \rho\ell\nu \quad \longrightarrow \quad B \to [\pi\pi]\ell\nu$
  - $B \to K^*\ell\ell \quad \longrightarrow \quad B \to [K\pi]\ell\ell$
  - Finite-width effects, interference (S-wave pollution, etc.)
- More complicated kinematics $\longrightarrow$ more observables
- Larger phase space: different kinematic regimes, different theory descriptions
- Kinematic distributions $\longrightarrow$ tests of EFT expansions & Factorization
- E-dependent rescattering effects $\longrightarrow$ large strong phases
  $\longrightarrow$ Large localized CP asymmetries
- Huge data sets
- Many applications: CKM parameters, tests of factorization, New Physics, spectroscopy, meson-meson scattering,...

Three-body decays – kinematics $\bar{B} \to M_a(p_a)M_b(p_b)M_c(p_c)$

- Two independent invariants, *e.g.* $s_{ab} = \frac{(p_a+p_b)^2}{m_B^2}$ and $s_{ac} = \frac{(p_a+p_c)^2}{m_B^2}$





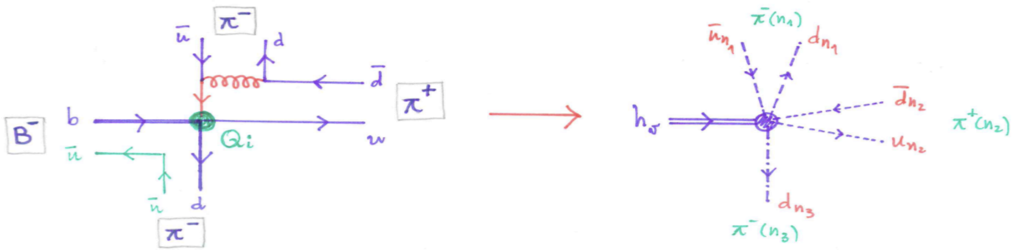
- Different kinematic regions with different factorization properties.

- Can also trade s_{ac} by angle θ_c : $2s_{ac} = (1 - s_{ab})(1 - \cos\theta_c)$, and do PWE :

$$\mathcal{A}(s_{ab}, s_{ac}) = \sum_{\ell=0}^{\infty}(2\ell+1)\mathcal{A}^{(\ell)}(s_{ab})P_\ell(\cos\theta_c)$$

Central region

⋆ Three collinear directions n_1, n_2, n_3, disconnected at the leading power.



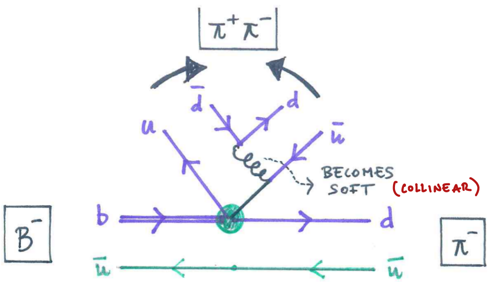
$$\langle \pi^- \pi^+ \pi^- | \mathcal{O}_i | \bar{B} \rangle = F^{B\to\pi} \int du\, dv\, T_i^I(u,v)\, \phi_\pi(u)\, \phi_\pi(v)$$
$$+ \int d\omega\, du\, dv\, dy\, T_i^{II}(\omega,u,v,y)\, \phi_B(\omega)\, \phi_\pi(u)\, \phi_\pi(v)\, \phi_\pi(y)$$

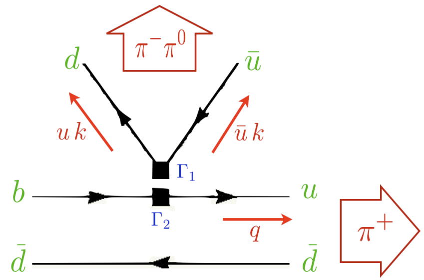
- Power $(1/m_b^2)$ & α_s suppressed with respect to two-body.
- At leading order/power/twist all convolutions are finite $\rightarrow$ factorization ✓
- Some pieces proven at NLO: Factorization of $B \to \pi\pi$ form factors **Böer, Feldmann, van Dyk '16** and 2π LCDAs **Diehl, Feldmann, Kroll, Vogt '99**
- $A_{\rm CP} = \mathcal{O}(\alpha_s(m_b)/\pi) + \mathcal{O}(\Lambda/m_b)$ – Like two-body !
- But this region might not exist for $m_B = 5$ GeV **Kränkl, Mannel, JV '15**

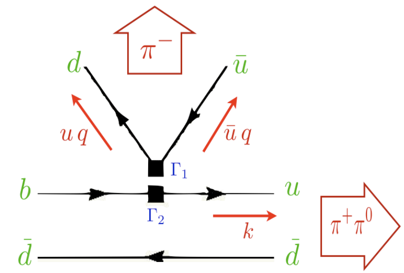
Edges

Kränkl, Mannel, JV ’15

- Breakdown of factorization at resonant edges requires new NP functions.
- 3-body decay resembles 2-body, but with new $(\pi\pi)$ "compound object":







- Operators are the same as in 2-body, but final states different:

$$\langle \pi^-_{\bar n}\,\pi^+_{\bar n}\,\pi^-_n\,|\mathcal{O}|B\rangle = F^{B\to\pi}\int du\; T_1(u,\zeta,s)\,\phi_{\pi\pi}(u,\zeta,s) + F^{B\to\pi\pi}(\zeta,s)\int du\; T_2(u,\zeta,s)\,\phi_\pi(u)$$
$$+\int d\omega\, du\, dv\; T_i^{II}(\omega,u,v,y)\,\phi_B(\omega)\,\phi_\pi(u)\,\phi_{\pi\pi}(v,\xi,s)$$

- New non-perturbative input: (Contains non-perturbative strong phases!!)
  - Generalized Distribution Amplitudes (GDAs) Diehl, Polyakov, Gousset, Pire, Grozin...
  - Generalized Form Factors (GFFs) Faller, Feldmann, Khodjamirian, Mannel, van Dyk...

Generalized Distribution Amplitudes

Polyakov 1999; Kränkl, Mannel, JV 2015

- Definition: $[k_{12} = k_1 + k_2\ ;\ s = k_{12}^2\ ;\ k_1 = \zeta k_{12}\ ;\ k_2 = (1-\zeta)k_{12}]$

$$\phi^q_{\pi\pi}(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization (local correlator):

$$\int dz\, \phi_{\pi\pi}(z,\zeta,s) = (2\zeta - 1)\, F_\pi(s) \quad \text{(pion vector FF)}$$

- Double Gegenbauer + Partial Wave Expansion:

$$\Phi_{\parallel}^{I=1}(u,\zeta,k^2) = 6u\bar{u} \sum_{n=0,2,\cdots}^{\infty} \sum_{\ell=1,3,\cdots}^{n+1} B^{\parallel}_{n\ell}(k^2)\, C_n^{3/2}(u-\bar{u})\, \beta_\pi(k^2) P_\ell^{(0)}(\cos\theta_\pi)$$

where $B^{\parallel}_{01}(k^2) = F_\pi(k^2)$

- $I = 0$ involves n odd and ℓ even (but normalization is zero).

$B \to \pi[\pi\pi]$: Leading order formula

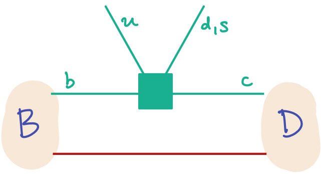
Klein, Mannel, Vos, Virto '17

$$\mathcal{A}(s_\pm^{\rm low}, s_\pm^{\rm high}) = \frac{G_F}{\sqrt{2}} \Big\{ \big[\lambda_u(a_2 - a_4^u) - \lambda_c a_4^c\big]\, m_B^2 f_+(s_\pm^{\rm low})\,(1 - s_\pm^{\rm low} - 2 s_\pm^{\rm high})\, F_\pi(s_\pm^{\rm low})$$

$$+ \big[\lambda_u(a_1 + a_4^u) + \lambda_c a_4^c\big]\, f_\pi\, m_\pi\, \big[F_t^{I=0}(s_\pm^{\rm low}, s_\pm^{\rm high}) + F_t^{I=1}(s_\pm^{\rm low}, s_\pm^{\rm high})\big] \Big\}$$

- $\lambda_{u,c}$ are (complex) CKMs, and $a_{1,2,4}$ are (real) WCs
- F_π is the vector pion form factor
- $F_t^{I=0}$ and $F_t^{I=1}$ are isoscalar and isovector $B \to \pi\pi$ form factors

- Note the interplay of weak and strong phases



We consider $\bar{B}^0 \to D^+ L^-$ with $L^- = \{\pi^-, \rho^-, \pi^-\pi^0, K^-, K^{*-}, K^-\pi^0, \ldots\}$

$$\mathcal{L}_{\rm eff}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{ux}^* V_{cb} \left(C_1 Q_1 + C_2 Q_2\right) + h.c., \quad \text{with } x = d, s \ .$$

$$Q_1 = (\bar{c}\gamma^\mu P_L T^a b)\,(\bar{x}\gamma_\mu P_L T^a u)\ , \quad Q_2 = (\bar{c}\gamma^\mu P_L b)\,(\bar{x}\gamma_\mu P_L u)\ .$$

$$\mathcal{A}(\bar{B}^0 \to D^+ L^-) = \frac{4G_F}{\sqrt{2}} V_{ux}^* V_{cb} \left[C_1 \langle Q_1\rangle + C_2 \langle Q_2\rangle\right]$$

$$\langle Q_i\rangle \equiv \langle D^+ L^- | Q_i | \bar{B}^0\rangle\ .$$

We are mostly interested in $L^- = \pi^-\pi^0$ and $L^- = K^-\pi^0$, but will also reproduce the cases $L^- = \pi^-, K^-$ and $L^- = \rho^-, K^{*-}$ in order to validate the general approach.

Heavy-to-Heavy decays – SCET Matching

$B(p) \to D(q)\, M(k_1)\, \pi(k_2)$

$k \equiv k_1 + k_2$

$k^2 \sim \lambda^2$

$k^+ \sim \lambda^2$; $k^- \sim 1$

$\uparrow$ $n \cdot k$; $\uparrow$ $\bar{n} \cdot k$

We match the QCD operators Q_i onto SCET operators $\mathcal{O}_k(t)$ via

$$Q_i = \sum_k \int d\hat{t}\, C_{ik}(\hat{t})\, \mathcal{O}_k(t)\,,$$

where $\hat{t} = t\,k^-$ and the SCET operators read

$$\mathcal{O}_1(t) \quad = \quad [\bar{h}_{v'} \not{n} P_L h_v][\bar{\chi}_n^{(d)}(t\bar{n}) \frac{\not{\bar{n}}}{2} P_L \chi_n^{(u)}(0)]\,,$$

$$\mathcal{O}_2(t) \quad = \quad [\bar{h}_{v'} \not{n} P_R h_v][\bar{\chi}_n^{(d)}(t\bar{n}) \frac{\not{\bar{n}}}{2} P_L \chi_n^{(u)}(0)]\,.$$

Here h_v is an HQET field which satisfies $\not{v} h_v = h_v$ (and similar for $h_{v'}$), while χ is a collinear SCET field with equation of motion $\not{n}\chi = 0$.

In momentum space:

$$\Big(H_{i1}(u),\ H'_{i1}(u)\Big) = \int d\hat{t}\, e^{iutk^-} \Big(C_{i1}(\hat{t}),\ C_{i2}(\hat{t})\Big)$$

with $H_{i1}^{(\prime)}(u)$ known to NNLO in α_s Huber, Kränkl, Li.

Heavy-to-Heavy decays – Matrix Elements

Matrix elements of SCET operators are factorized:

$$\begin{aligned}\langle Q_i \rangle &= \sum_k \int d\hat{t}\, C_{ik}(\hat{t})\, \langle \mathcal{O}_k(t) \rangle \\ &= \int d\hat{t} \left(C_{i1}(\hat{t})\; C_{i2}(\hat{t}) \right) \begin{pmatrix} \langle D^+ | \bar{h}_{v'} \not{n} P_L h_v | \bar{B}^0 \rangle \\ \langle D^+ | \bar{h}_{v'} \not{n} P_R h_v | \bar{B}^0 \rangle \end{pmatrix} \langle L^- | \bar{\chi}_n^{(d)}(t\bar{n}) \frac{\not{\bar{n}}}{2} P_L \chi_n^{(u)}(0) | 0 \rangle \,.\end{aligned}$$

The individual factors in this equation can be expressed in terms of non-perturbative objects:

Form Factors:

HQET vs QCD currents.

$$\begin{pmatrix} \langle D^+ | \bar{h}_{v'} \not{n} P_L h_v | \bar{B}^0 \rangle \\ \langle D^+ | \bar{h}_{v'} \not{n} P_R h_v | \bar{B}^0 \rangle \end{pmatrix} = \begin{pmatrix} C_{FF}^{D} & C_{FF}^{ND} \\ C_{FF}^{ND} & C_{FF}^{D} \end{pmatrix}^{-1} \begin{pmatrix} \langle D^+ | \bar{c} \not{n} P_L b | \bar{B}^0 \rangle \\ \langle D^+ | \bar{c} \not{n} P_R b | \bar{B}^0 \rangle \end{pmatrix} ,$$

LCDAs:

SCET vs QCD currents.

$$\begin{aligned}\langle L^-(k) | \bar{\chi}_n^{(d)}(t\bar{n}) \frac{\not{\bar{n}}}{2} P_L \chi_n^{(u)}(0) | 0 \rangle &= C_{q\bar{q}}^{-1} \langle L^-(k) | \bar{d}_n(t\bar{n}) \frac{\not{\bar{n}}}{2} P_L u_n(0) | 0 \rangle \\ &\equiv C_{q\bar{q}}^{-1}\, \hat{\Phi}_L(k,t) = C_{q\bar{q}}^{-1}\, k^- \int_0^1 du\, e^{iutk^-}\, \hat{\Phi}_L(k,u) \,,\end{aligned}$$

Fourier Transform.

Finally

BBNS 2000

$$\langle Q_i \rangle = k^- \, F_n^{B\to D} \int_0^1 du \; T_i(u) \; \hat{\Phi}_L(k,u) \; .$$

At order α_s^0, the hard functions are given by

$$T_1(u) = \mathcal{O}(\alpha_s) \; , \quad T_2(u) = 1 + \mathcal{O}(\alpha_s) \; ,$$

At NNLO: $(T_i(u) \equiv \hat{T}_i(u) + \hat{T}_i'(u))$

Huber, Krank, Li 2016

$$\left(\hat{T}_i(u) \; \hat{T}_i'(u) \right) = \left(H_{i1}(u) \; H_{i1}'(u) \right) \begin{pmatrix} C_{FF}^{D} & C_{FF}^{ND} \\ C_{FF}^{ND} & C_{FF}^{D} \end{pmatrix}^{-1} C_{q\bar{q}}^{-1} \; .$$

In terms of "canonical" LCDAs:

Huber, Vos, Virto 2020

$$\hat{\Phi}_P(k,u) = \frac{if_P}{4} \, \Phi_P(u) \; , \qquad \hat{\Phi}_{\pi\pi}(k_1,k_2,u) = -\frac{1}{2\sqrt{2}} \, \Phi_{\pi\pi}(u,k^2,\theta_\pi) \; ,$$

$$\hat{\Phi}_V(k,u) = \frac{f_V}{4} \, \Phi_V(u) \; , \qquad \hat{\Phi}_{K\pi}(k_1,k_2,u) = -\frac{1}{2\sqrt{2}} \, \Phi_{K\pi}(u,k^2,\theta_\pi) \; ,$$

with $P = \{\pi, K\}$ and $V = \{\rho, K^*\}$

We can now write down the amplitudes

$$\mathcal{A}(\bar{B} \to D^+ L^-) = \frac{4G_F}{\sqrt{2}} V_{ux}^* V_{cb}\; k^-\; F_n^{B\to D} \int_0^1 du\; (C_1 T_1(u) + C_2 T_2(u)) \hat{\Phi}_L(k, u)\;.$$

With the previous considerations, we have:

$$\begin{aligned}
\mathcal{A}(\bar{B} \to D^+ P^-) &= i\, \frac{G_F}{\sqrt{2}} V_{ux}^* V_{cb}\; (m_B^2 - m_D^2)\; F_0^{B\to D}(m_P^2)\, f_P\; a_1(D^+ P^-)\;, \\
\mathcal{A}(\bar{B} \to D^+ V^-) &= \frac{G_F}{\sqrt{2}} V_{ux}^* V_{cb}\; (m_B^2 - m_D^2)\; F_0^{B\to D}(m_V^2)\, f_V^{||}\; a_1(D^+ V^-)\;, \\
\mathcal{A}(\bar{B} \to D^+ M^- \pi^0) &= -\, G_F\, V_{ux}^* V_{cb}\; (m_B^2 - m_D^2)\; F_0^{B\to D}(k^2)\; a_1(D^+ M^- \pi^0)\;.
\end{aligned}$$

- $a_1(D^+ L^-)$ correspond to the same coefficients as in (BBNS 2000, Huber, Kränkl, Li 2016) for the cases $L = P, V$, which we generalize here,

$$a_1(D^+ L^-) = \int_0^1 du\; (C_1 T_1(u) + C_2 T_2(u))\; \Phi_L(u)\;,$$

- For $L = M\pi$ the dimeson LCDAs $\Phi_L(u, k^2, \theta_\pi)$ depend on the two variables k_1, k_2.
- The amplitudes for $B \to DP$ and $B \to DV$ agree with the literature (BBNS 2000, Huber, Kränkl, Li 2016), after accounting for the phase redefinition in the vector-meson state.

Dimeson LCDAs

Normalization (Local limit):

$$\int_0^1 du\, \Phi_P(u) = \int_0^1 du\, \Phi_V(u) = 1\,.$$

$$\begin{aligned}\int_0^1 du\, \Phi_{\pi\pi}(u,k^2,\theta_\pi) &= \cos\theta_\pi\, \beta_\pi(k^2) F_\pi(k^2)\,,\\ \int_0^1 du\, \Phi_{K\pi}(u,k^2,\theta_\pi) &= \cos\theta_\pi\, \frac{\sqrt{\lambda_{K\pi}(k^2)}}{2k^2} f_+^{K\pi}(k^2) + \frac{\Delta m_{K\pi}^2}{2k^2} f_0^{K\pi}(k^2)\,,\end{aligned}$$

with the local form factors in the timelike region:

$$\begin{aligned}\langle \pi^-(k_1)\pi^0(k_2)|\bar{d}\gamma_\mu u|0\rangle &= -\sqrt{2}\, F_\pi(k^2)\,\bar{k}_\mu\,,\\ \langle K^-(k_1)\pi^0(k_2)|\bar{s}\gamma_\mu u|0\rangle &= -\frac{f_+^{K\pi}(k^2)}{\sqrt{2}}\,\bar{k}_\mu - \frac{\Delta m_{K\pi}^2}{\sqrt{2}k^2}\, f_0^{K\pi}(k^2)\, k_\mu\,,\end{aligned}$$

[Note: $\int \Phi_{K\pi} \to \int \Phi_{K\pi}$ when $m_K \to m_\pi$]

Dimeson LCDAs

Normalization (Local limit):

Descotes-Genon, Khodjamirian, Virto, 2019

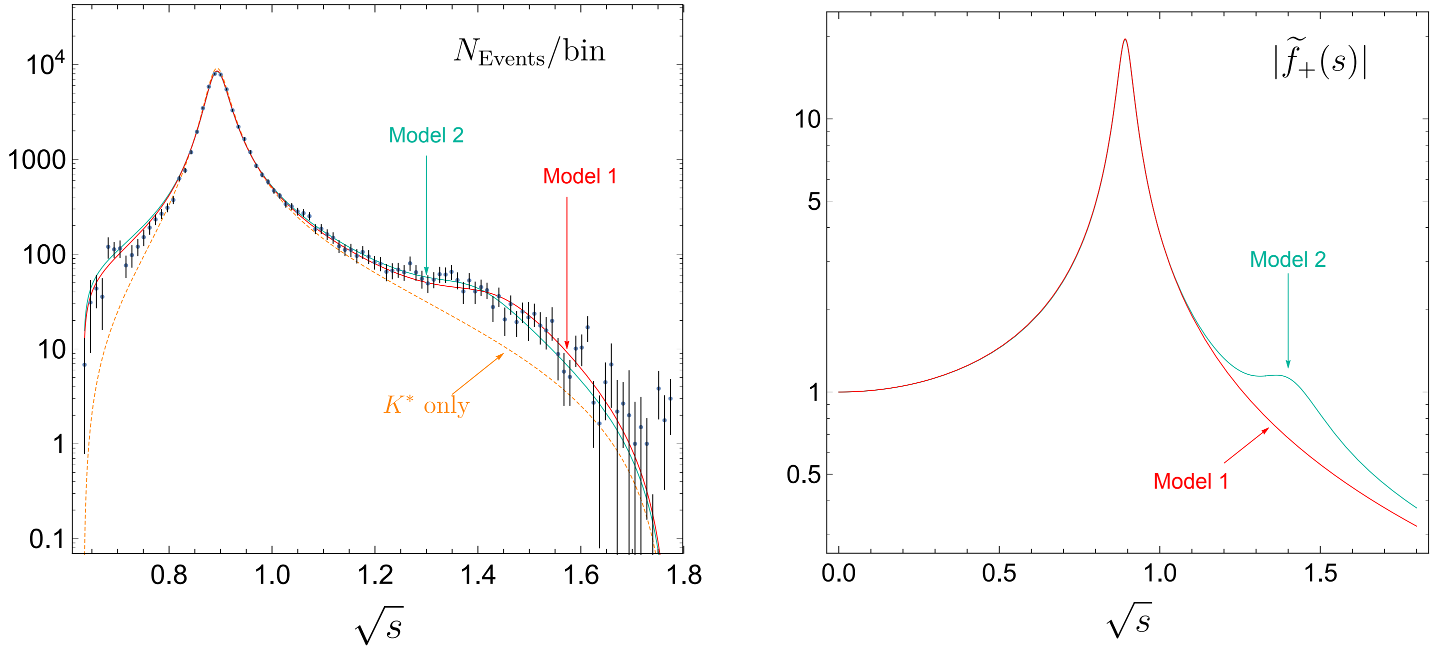


Figure 1: Left: $\tau \to K_S \pi^- \nu_\tau$ spectrum from Belle'2007, and the corresponding curves from Model 1 and Model 2 (two solid lines) as well as the isolated contribution from the $K^*(892)$ (dashed). Right: Normalized form factor $\widetilde{f}_+(s)$ in the two models that fit well the spectrum.

Dimeson LCDAs

Gegenbauer + Partial-wave expansions:

$$\Phi_L(u) = 6u\bar{u} \sum_{n=0}^{\infty} \alpha_n^L \, C_n^{3/2}(u-\bar{u}) \, ,$$

with $\alpha_0^P = \alpha_0^V = 1$. For $L = \{\rho, \pi\pi\}$, $\alpha^L_{n\,\mathrm{odd}} = 0$ due to C-parity in the isospin limit.

For the dimeson:

$$\alpha_n^{\pi\pi}(k^2, \theta_\pi) = \sum_{\ell=\mathrm{odd}}^{n+1} B_{n\ell}^{\pi\pi}(k^2) \, P_\ell(\cos\theta_\pi) \qquad (n \text{ even}) \, ,$$

$$\alpha_n^{K\pi}(k^2, \theta_\pi) = \sum_{\ell=0}^{n+1} B_{n\ell}^{K\pi}(k^2) \, P_\ell(\cos\theta_\pi) \qquad (\text{all } n) \, .$$

The normalization fixes:

$$B_{01}^{\pi\pi}(k^2) = \beta_\pi(k^2) F_\pi(k^2) \, , \quad B_{00}^{K\pi}(k^2) = \frac{\Delta m^2_{K\pi}}{2k^2} f_0^{K\pi}(k^2) \, , \quad B_{01}^{K\pi}(k^2) = \frac{\sqrt{\lambda_{K\pi}(k^2)}}{2k^2} f_+^{K\pi}(k^2) \, .$$

The other coefficients $B^L_{n\ell}(k^2)$ are mostly unknown.

Convolutions and amplitudes at NNLO

Convolution of matching coefficients and Gegenbauer polynomials can be done once:

$$a_1(D^+L^-) = \int_0^1 du\ \left(C_1 T_1(u) + C_2 T_2(u)\right) \Phi_L(u)$$

$$\int_0^1 du\, T_i(u,\mu)\, 6u\bar{u}\, C_n^{3/2}(u-\bar{u}) = \mathcal{V}_{in}(\mu)$$

$\mathcal{V}_{in}(\mu)$ are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO) from Huber, Kränkl, Li.

Then,

$$a_1(D^+L^-) = \sum_{n\geq 0} \alpha_n^L \left[C_1(\mu)\mathcal{V}_{1n}(\mu) + C_2(\mu)\mathcal{V}_{2n}(\mu)\right] \equiv \sum_{n\geq 0} \alpha_n^L\, \mathcal{G}_n(\mu)$$

$$\mathcal{A}^{(\ell)}(k^2) = -\, G_F\, V_{ux}^* V_{cb}\, (m_B^2 - m_D^2)\, F_0^{B\to D} \sum_{n\geq\max(\ell-1,0)} B_{n\ell}^L(k^2)\, \mathcal{G}_n(\mu)\ .$$

Prefactors

"unknowns" (non-perturbative)

Convolutions (perturbative)

NNLO Numerics

(Can easily go to n>2)

NLO ≈ NNLO

$$\mathcal{G}_0(\mu_b) = 1.034_{\rm LO} + (0.026 + i\,0.020)_{\rm NLO} + (0.013 + i\,0.027)_{\rm NNLO} = 1.07 + i\,0.047$$

$$\mathcal{G}_1(\mu_b) = (-0.013 + i\,0.030)_{\rm NLO} + (-0.044 + i\,0.018)_{\rm NNLO} = -0.057 + i\,0.048$$

$$\mathcal{G}_2(\mu_b) = (0.0023 - i\,0.0017)_{\rm NLO} + (0.0017 - i\,0.0054)_{\rm NNLO} = 0.0040 - i\,0.0071$$

For $\alpha_{1,2}$: NNLO very important !

$$\begin{aligned}
|a_1(D^+L^-)|^2 &= |\alpha_0^L|^2 \Big\{1.07_{\rm LO} \\
&+ \left[0.053 - 0.026\,\mathrm{Re}\,\hat\alpha_1^L - 0.062\,\mathrm{Im}\,\hat\alpha_1^L + 0.0047\,\mathrm{Re}\,\hat\alpha_2^L + 0.0034\,\mathrm{Im}\,\hat\alpha_2^L\right]_{\rm NLO} \\
&+ \left[0.029 - 0.091\,\mathrm{Re}\,\hat\alpha_1^L - 0.040\,\mathrm{Im}\,\hat\alpha_1^L + 0.0036\,\mathrm{Re}\,\hat\alpha_2^L + 0.011\,\mathrm{Im}\,\hat\alpha_2^L\right]_{\rm NNLO}\Big\} \\
&= 1.15|\alpha_0^L|^2 \left\{1 - 0.10\,\mathrm{Re}\,\hat\alpha_1^L - 0.09\,\mathrm{Im}\,\hat\alpha_1^L + 0.007\,\mathrm{Re}\,\hat\alpha_2^L + 0.014\,\mathrm{Im}\,\hat\alpha_2^L\right\}
\end{aligned}$$

with $\hat\alpha_i^L \equiv \alpha_i^L/\alpha_0^L$.

Mostly NNLO

$\sim 10\%$ ← $B \to DK\pi$

$\sim 1\%$ ← $a_1(D\pi\pi) \simeq 1.15\,|\alpha_0^{\pi\pi}|^2 \pm 1\%$

$\sim F_\pi(k^2)$

Modelling the dimeson system

Assumption: The $M\pi$ system couples to the current through a set of single resonances: (model described in Descotes-Genon, Khodjamirian, Virto 2019, consistent with τ-decay models by Belle.)

$$B_{n0}^{M\pi}(s) = \sum_{R_0} \frac{m_{R_0} f_{R_0} g_{R_0 M\pi} e^{i\varphi_{R_0}}}{\sqrt{2}[m_{R_0}^2 - s - i\sqrt{s}\,\Gamma_{R_0}(s)]} \alpha_n^{R_0},$$

$$B_{n1}^{M\pi}(s) = \frac{\sqrt{\lambda_{M\pi}(s)}}{s} \sum_{R} \frac{m_R f_R g_{RM\pi} e^{i\varphi_R}}{\sqrt{2}[m_R^2 - s - i\sqrt{s}\,\Gamma_R(s)]} \alpha_n^{R}.$$

Parameters of the model.

- $\pi\pi$ data well described by $J^P = 1^-$ resonances (ρ, ρ', ρ'')

  Belle 2008;Cheng,Khodjamirian, Virto 2017

- $K\pi$ data may require both $J^P = 1^-$ $(K^*(892), K^*(1410))$ and $J^P = 0^+$ $(K_0^*(800), K_0^*(1430))$

  Belle 2007; Descotes-Genon, Khodjamirian, Virto 2019

- For $n = 0$ these models agree with the models for the pion and $K\pi$ form factors which give excellent fits to τ decay rate distributions. Belle 2007,2008

Finite-width effects

Identify α_n^V with Gegenb. coeffs of V LCDAs

Narrow-width limit

$$\left|a_1(D^+V^-(\to M^-\pi^0))\right|^2 = \frac{\lambda_{M\pi}(s)\cos^2\theta_\pi}{2s^2}\left[\frac{g_{VM\pi}^2\, f_V^2\, m_V^2}{(m_V^2-s)^2+s\,\Gamma_V(s)^2}\right]|a_1(D^+V^-)|^2$$

$$\frac{g_{VM\pi}^2 f_V^2 m_V^2}{(m_V^2-s)^2+s\Gamma_V(s)^2} \xrightarrow{\Gamma_V^{\rm tot}\to 0} \frac{48\pi^2 f_V^2 m_V^6}{\lambda_{M\pi}^{3/2}(m_V^2)}\mathcal{B}(V\to M\pi)\,\delta(s-m_V^2)$$

Integrating over the angle θ_π and over the invariant squared mass of the dimeson we have

$$\Gamma(\bar{B}\to D^+M^-\pi^0) \xrightarrow{\Gamma_V^{\rm tot}\to 0} \Gamma(\bar{B}\to D^+V^-)\,\mathcal{B}(V\to M\pi)$$

The model has the correct narrow-width limit, and allows us to go beyond it.

Finite-width effects

Beyond the narrow-width limit

We define a "resonance observable":

$$\Gamma_{[R]} \equiv \int_{(m_R-\delta)^2}^{(m_R+\delta)^2} ds\, \frac{d\Gamma(\bar{B}\to D^+M^-\pi^0)}{ds}$$

$$= \sum_\ell c_\ell \int_{(m_R-\delta)^2}^{(m_R+\delta)^2} ds\, \frac{\sqrt{\lambda_{BD}(s)\,\lambda_{M\pi}(s)}}{64(2\pi)^3 s m_B^3} |\mathcal{A}^{(\ell)}(s)|^2 = \sum_\ell \Gamma_{[R]}^{(\ell)}$$

with $c_\ell = \int_{-1}^1 dx\, P_\ell(x)^2 = 2/(2\ell+1)$.

With this we define a measure of the width effect:

$$\mathcal{W}_R^{(\ell)} = \frac{\Gamma_{[R]}^{(\ell)}}{\Gamma_{[R]\,,\mathrm{NWL}}^{(\ell)}}$$

E.g., neglecting $B_{n\ell}$ for $n \geq 2$

$$\mathcal{W}_\rho^{(1)} = \int_{(m_\rho-\delta)^2}^{(m_\rho+\delta)^2} ds\, \frac{\lambda_{BD}^{1/2}(s)}{\lambda_{BD}^{1/2}(m_\rho^2)} \frac{[\beta_\pi(s)]^3\,|F_\pi(s)|^2}{24\pi^2 f_\rho^2\,\mathcal{B}(\rho\to\pi\pi)} \xrightarrow{\Gamma_\rho^{\rm tot}\to 0} 1$$

Finite-width effects: $B \to D\rho \to D\pi\pi$

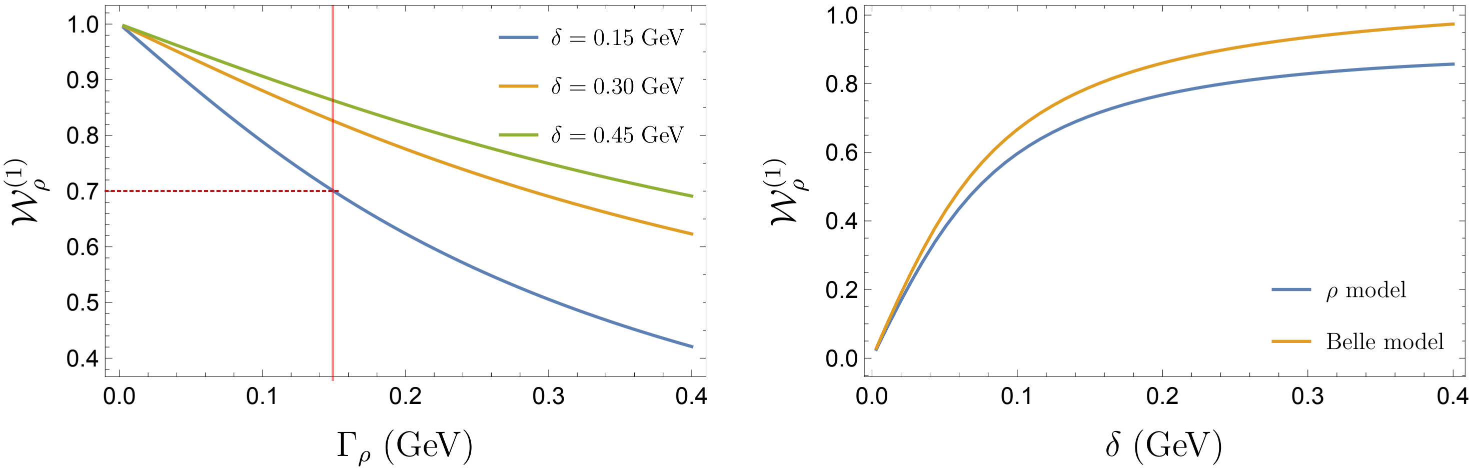


Figure 2: Study of finite-width and bin-size effects on $\mathcal{W}_\rho$. Left: Corrections to the narrow-width limit of the ρ model. The vertical band indicates the physical width $\Gamma_\rho = (149.1 \pm 0.8)$ MeV. Right: ρ model as specified in the text and the Belle model as function of the bin size δ.

Finite-width effects $B \to DK^* \to DK\pi$

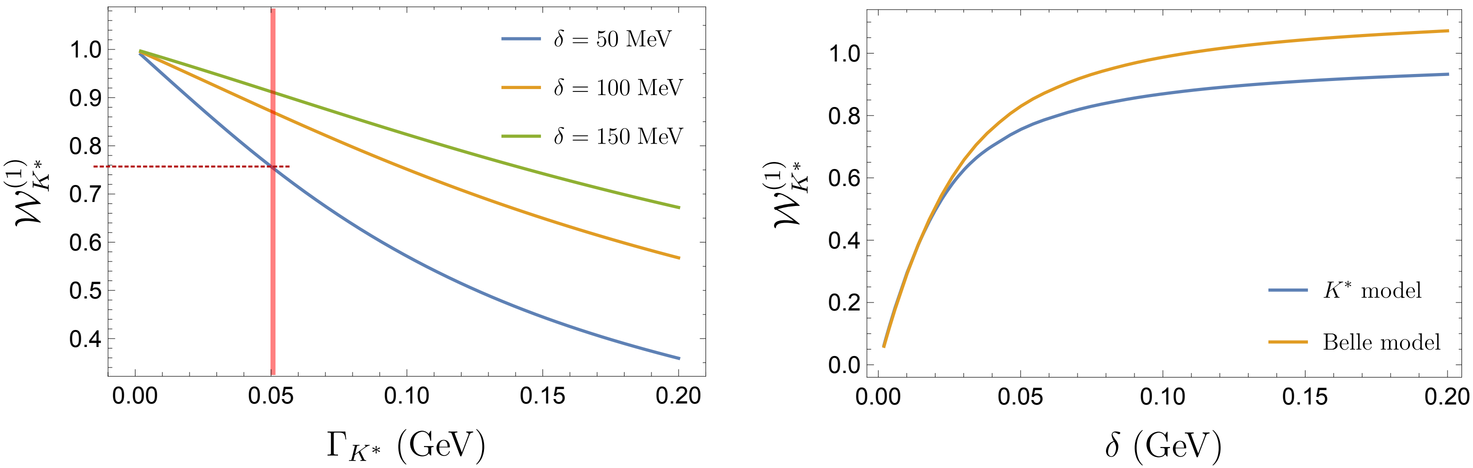


Figure 3: Study of finite-width and bin-size effects on $\mathcal{W}_{K^*}$. Left: Corrections to the narrow-width limit of the K^* model. The vertical band indicates the physical width $\Gamma_{K^*} = (50.8 \pm 0.9)$ MeV. Right: K^* model and the Belle model as specified in the text as a function of the bin size δ.

Probing Higher-Order QCD effects

We find that the following ratios are of some interest: $(z \equiv \cos\theta_\pi)$

$$\mathcal{R}_{MM'}[z_1, z_2; z_1', z_2'](k^2) \equiv \frac{\displaystyle\int_{z_1}^{z_2} dz\, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2\, dz}}{\displaystyle\int_{z_1'}^{z_2'} dz\, \frac{d\Gamma(\bar{B} \to D^+ M'^- \pi^0)}{dk^2\, dz}} \,.$$

- For $M = M'$, all prefactors (such as the $B \to D$ form factor and CKM elements) cancel, *i.e.*,

$$\mathcal{R}_{MM}[z_1, z_2; z_1', z_2'](k^2) = \frac{\displaystyle\int_{z_1}^{z_2} dz\, \left|a_1(D^+ M^- \pi^0)\right|^2}{\displaystyle\int_{z_1'}^{z_2'} dz\, \left|a_1(D^+ M^- \pi^0)\right|^2}$$

and may help to access higher Gegenbauer functions and other higher-order QCD effects.

- $\mathcal{R}_{K\pi}$ is proportional to $|V_{us}/V_{ud}|$. It may be interesting as a probe of $\alpha_1^{K\pi}$.

Probing Higher-Order QCD effects

Examples:

1. Isospin limit $\Rightarrow$ $z \leftrightarrow -z$ symmetry: (No even partial waves)

$$A_{\rm FB}^{\pi\pi}(k^2) = \mathcal{R}_{\pi\pi}[0,1;-1,1](k^2) - \mathcal{R}_{\pi\pi}[-1,0;-1,1](k^2) = 0$$ ("Null" test)

2. Up to $n = 2$ in the Gegenbauer expansion:

$$\mathcal{R}_{\pi\pi}[-1/2,1/2,-1,1](k^2) = \frac{1}{8} - 0.28\,\mathrm{Re}\left[\frac{B_{23}^{\pi\pi}(k^2)}{B_{01}^{\pi\pi}(k^2)}\,\frac{\mathcal{G}_2(\mu_b)}{\mathcal{G}_0(\mu_b)}\right] + \mathcal{O}((\mathcal{G}_2/\mathcal{G}_0)^2)$$

$\leftarrow F_\pi(k^2)$

where we have used that $\mathcal{G}_2/\mathcal{G}_0 \simeq 0.4\%$ is small.

3. $S-$ and $P-$wave dominance:

$f_0^{K\pi}$ $f_+^{K\pi}$

$$A_{\rm FB}^{K\pi}(k^2) = \mathcal{R}_{K\pi}[0,1;-1,1](k^2) - \mathcal{R}_{K\pi}[-1,0;-1,1](k^2) \simeq \frac{2\mathrm{Re}(B_{00}B_{01}^*)}{2|B_{00}|^2 + 2/3|B_{01}|^2}$$

$$+\frac{2\mathrm{Re}\left[\left(2(B_{00}^*)^2 - 2/3(B_{01}^*)^2\right)\mathcal{G}_0^*\left(B_{00}(B_{11}\mathcal{G}_1 + B_{21}\mathcal{G}_2) - B_{01}(B_{10}\mathcal{G}_1 + B_{20}\mathcal{G}_2)\right)\right]}{|\mathcal{G}_0|^2\left(2|B_{00}|^2 + 2/3|B_{01}|^2\right)^2}$$

NLO probes B_{11}, B_{20}, B_{21} through S-P-wave interference.

Summary

- Three-body B decays can be studied within QCDF

  Beneke 2006; Stewart 2006; Kränkl, Mannel, Virto 2015

- Duality between interesting features and complications
  - Generalized Form factors and LCDAs depend on 3 variables — Polyakov 1999, Faller et al 2013, ...
  - More difficult to assess — Cheng, Khodjamirian, Virto 2017, Descotes-Genon, Khodjamirian, Virto 2019
  - Leading structure related to measurable form factors in timelike region (*e.g.* τ-decay).

    Belle 2007, 2008; Cheng, Khodjamirian, Virto 2017, Descotes-Genon, Khodjamirian, Virto 2019

- Rigorous study of the simplest case: $B \to DMM'$ for $s_{MM'}$ small. — Huber, Vos, Virto 2020
  - NNLO factorization $\simeq B \to DV$
  - Finite-width effects: $\mathcal{W}^{(1)}_{\rho,K*} \sim 0.8 - 0.9.$ ← $\delta \sim 2\Gamma$
  - Partial angular integrations potentially constraining $B_{n\ell}(k^2)$ as function of k^2.

→ Can we learn about $B_{n\ell}(k^2)$ with $n>0$??

Extra

$B \to \pi\pi$ form factors from 2π-LCDAs

- Correlation function

$$\Pi^5(p^2, k^2, q^2, q\cdot\bar{k}) = i\int d^4x\, e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{\bar{u}(x) i m_b\gamma_5 b(x), \bar{b}(0) i m_b \gamma_5 d(0)\}|0\rangle$$

- Unitarity relation

$$2\mathrm{Im}\Pi^5 = (2\pi)\delta(p^2 - m_B^2)\underbrace{\langle\pi^+(k_1)\pi^0(k_2)|\bar{u} i m_b\gamma_5 b|\bar{B}(p)\rangle}_{\sqrt{q^2}F_t(q^2,k^2,q\cdot k)}\underbrace{\langle\bar{B}(p)|\bar{b} i m_b\gamma_5 d|0\rangle}_{m_B^2 f_B} + \cdots$$

$$= (2\pi)\,\delta(p^2 - m_B^2)\, m_B^2 f_B\,\sqrt{q^2}\, F_t(q^2, k^2, q\cdot k) + \cdots$$

- Dispersion relation + LCOPE + Borel + duality

$$m_B^2 f_B\sqrt{q^2}\, F_t(q^2, k^2, q\cdot\bar{k})\, e^{-m_B^2/M^2} = \Pi^5_{OPE}(M^2, q^2, k^2, q\cdot\bar{k})$$

- In this case:

$$\Pi^5_{OPE}(M^2, q^2, k^2, q \cdot \bar{k}) = \frac{m_b^2}{\sqrt{2}} \int_{u_0}^1 \frac{du}{u^2}\, e^{-s(u)/M^2}\, (m_b^2 - q^2 + u^2 k^2)\, \Phi_{\parallel}^{l=1}(u, q \cdot \bar{k}, k^2)$$

- **SUM RULE :**

$$\sqrt{q^2} F_t(q^2, k^2, \zeta) = \frac{m_b^2}{\sqrt{2} m_B^2 f_B} \int_{u_0}^1 \frac{du}{u^2}\, e^{\frac{m_B^2 - s(u)}{M^2}}\, (m_b^2 - q^2 + u^2 k^2)\, \Phi_{\parallel}^{l=1}(u, \zeta, k^2)$$

- **Gegenbauer + Partial Wave Expansions :**

$$\sqrt{q^2} F_t^{(\ell)}(q^2, k^2) = -\frac{6 m_b^2}{\sqrt{2} f_B m_B^2} \frac{\beta_\pi(k^2)}{\sqrt{2\ell+1}} \sum_{\binom{n=\ell-1}{n\ \text{even}}}^{\infty} B^{\parallel}_{n\ell}(k^2) \int_{u_0}^1 \frac{du}{u}\, \bar{u}\, e^{\frac{m_B^2 - s(u)}{M^2}}\, (m_b^2 - q^2 + u^2 k^2)\, C_n^{3/2}(u - \bar{u})$$

- $B^{\parallel}_{01}(k^2) = F_\pi(k^2)$ – but for the sum rule we need higher moments.
- Narrow-ρ dominance on $\Phi_\parallel$ leads to $B \to \rho$ form factor from ρ-LCDA. ✓

  [$\Phi_\parallel \longleftrightarrow \phi_\rho$ Polyakov '98, K. Vos, JV w.i.p]

$B \to \pi\pi$ form factors from B-meson LCDAs

Cheng, Khodjamirian, JV, 1701.01633

- Correlation function

$$F_\mu(k,q) = i \int d^4x e^{ik\cdot x} \langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), m_b \bar{u}(0)\gamma_5 b(0)\}|\bar{B}^0(q+k)\rangle$$

- Unitarity relation

$$2\mathrm{Im}F_\mu(k,q) = m_b \int d\tau_{2\pi} \underbrace{\langle 0|\bar{d}\gamma_\mu u|\pi(k_1)\pi(k_2)\rangle}_{F_\pi^\star(k^2)} \underbrace{\langle \pi(k_1)\pi(k_2)|\bar{u}\gamma_5 b|\bar{B}^0(q+k)\rangle}_{F_t(k^2,q^2,\cos\theta_\pi)} + \cdots$$

$$= q_\mu \frac{s\sqrt{q^2}[\beta_\pi(s)]^2}{4\sqrt{6}\pi\sqrt{\lambda}} F_\pi^\star(k^2)\, F_t^{(\ell=1)}(k^2,q^2) + \cdots$$

Corollary : $F_\pi^\star(s)\, F_t^{(\ell=1)}(s,q^2)$ is real for all $s < 16m_\pi^2 \Rightarrow$

Phase($F_{P-wave}^{B\to\pi\pi}$) = Phase(vector pion form factor)

Important for CP violation!!! [See also Kang, Kubis, Hanhart, Meissner '13]

- Dispersion relation + LCOPE + Borel + duality

$$-\int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \frac{s\sqrt{q^2}\,[\beta_\pi(s)]^2}{4\sqrt{6}\pi^2\sqrt{\lambda}} F_\pi^\star(s)\, F_t^{(1)}(s,q^2) = f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma,q^2)/M^2} \times \right.$$

$$\left. \times \left[\frac{\sigma}{\bar\sigma}\phi_+^B(\sigma m_B) - \frac{\sigma}{\bar\sigma}\left[\phi_+^B(\sigma m_B) - \phi_-^B(\sigma m_B)\right] - \frac{1}{\bar\sigma m_B}\bar\Phi_\pm^B(\sigma m_B)\right] + \Delta A_0^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right\}$$

- ρ-dominance + zero-width limit:

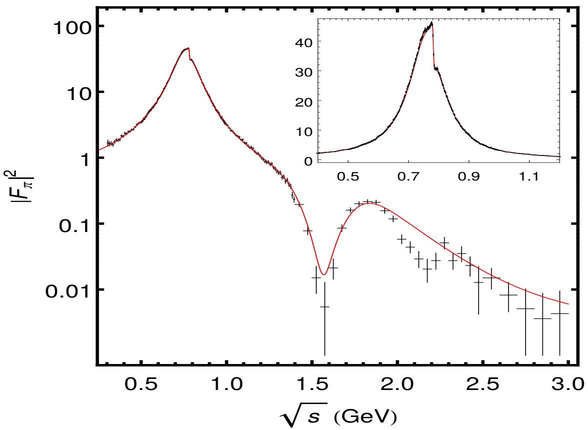
$$F_\pi^\star(s) \simeq \frac{f_\rho g_{\rho\pi\pi} m_\rho/\sqrt{2}}{m_\rho^2 - s + i\sqrt{2}\Gamma_\rho(s)} \quad , \quad F_t^{(1)}(s,q^2) \simeq -\frac{\beta_\pi(s)\sqrt{\lambda}}{\sqrt{3q^2}} \frac{m_\rho g_{\rho\pi\pi} A_0^{B\rho}(q^2)}{m_\rho^2 - s - i\sqrt{2}\Gamma_\rho(s)}$$

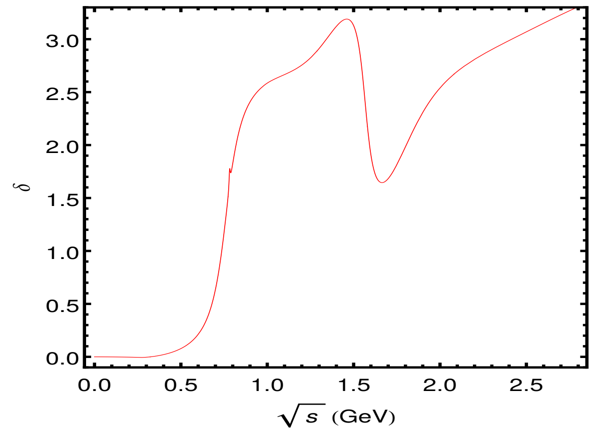
$$LHS = 2f_\rho m_\rho A_0^{B\rho}(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \underbrace{\left[\frac{\sqrt{s}\,\Gamma_\rho(s)/\pi}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right]}_{\xrightarrow{\Gamma_\rho \to 0} \delta(s - m_\rho^2)} \xrightarrow{\Gamma_\rho \to 0} 2f_\rho m_\rho A_0^{B\rho}(q^2)\, e^{-s/m_\rho^2}$$

hep-ph/0611193 ✓

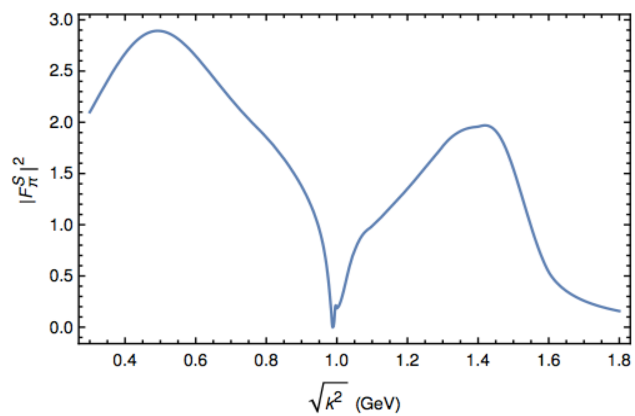
Pion form factors

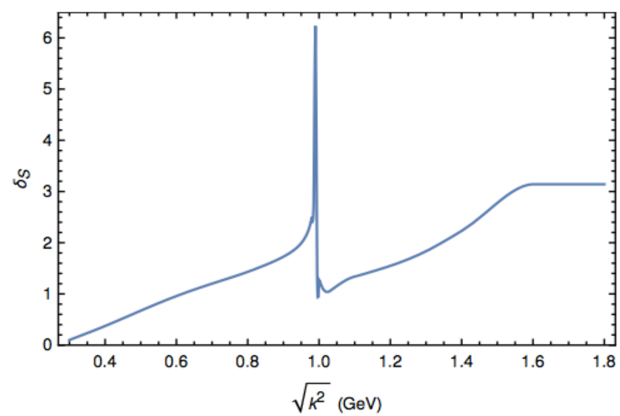
- $F_\pi(s)$: Data ($e^+e^- \to \pi\pi(\gamma)$ [BaBar] or $\tau \to \pi\pi\nu_\tau$ [Belle])





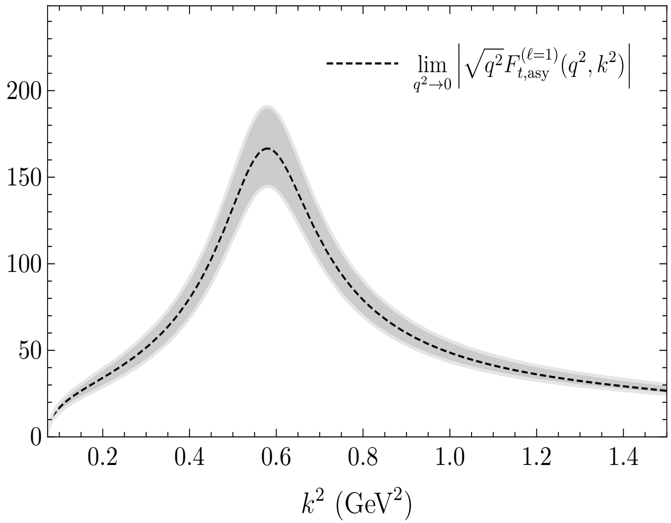
- $F_\pi^S(s)$: Dispersive methods [e.g. 1309.3564]



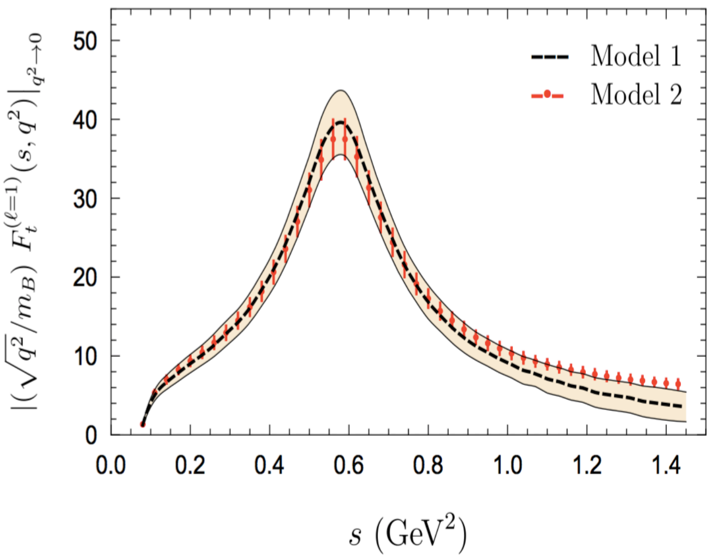


$B \to \pi\pi$ form factor ($F_t^{\ell=1}$)

Cheng, Khodjamirian, JV, 1709.00173

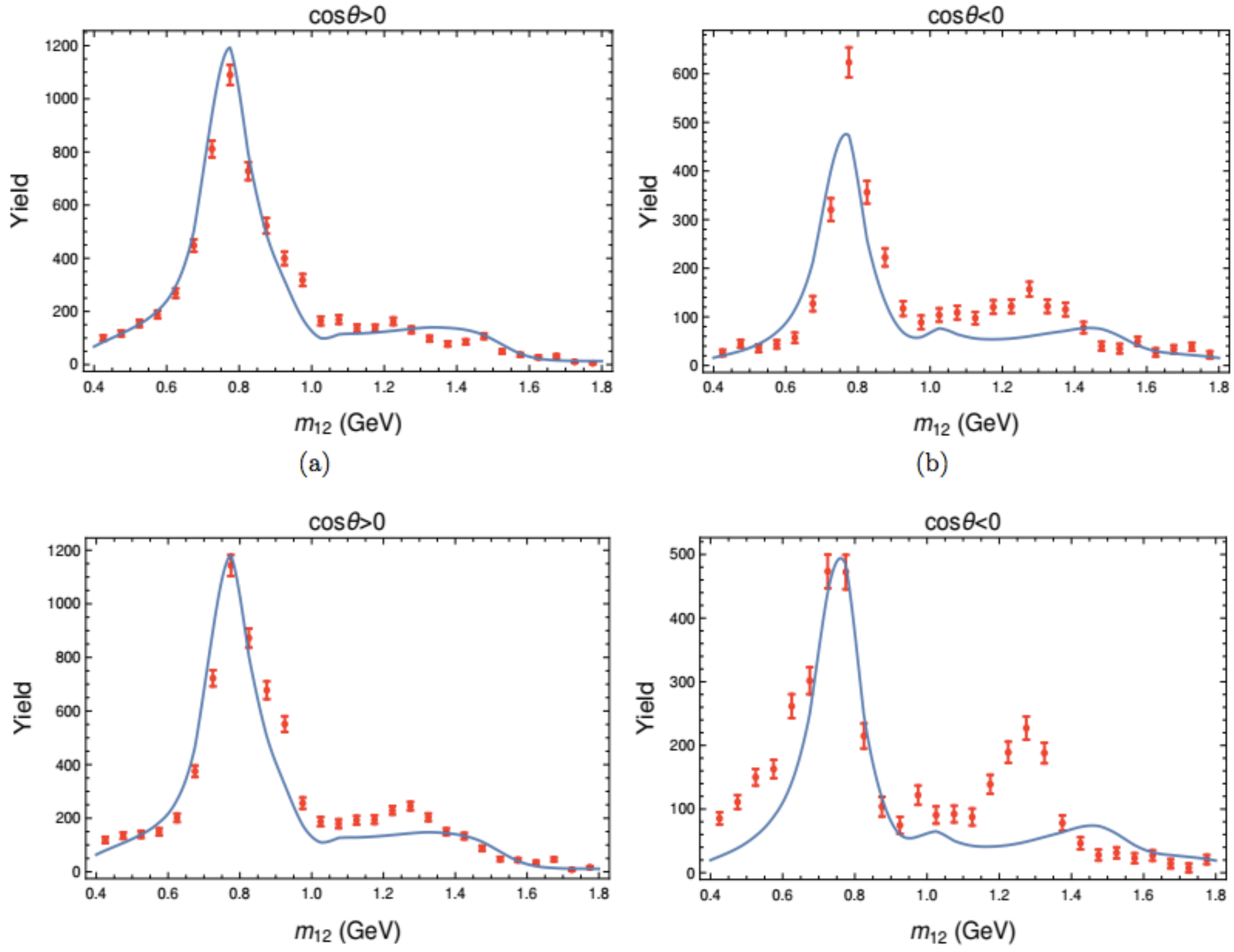


Cheng, Khodjamirian, JV, 1701.01633



- Both approaches give consistent results
- Corrections to narrow-ρ approximation at the level of 10 - 20 %

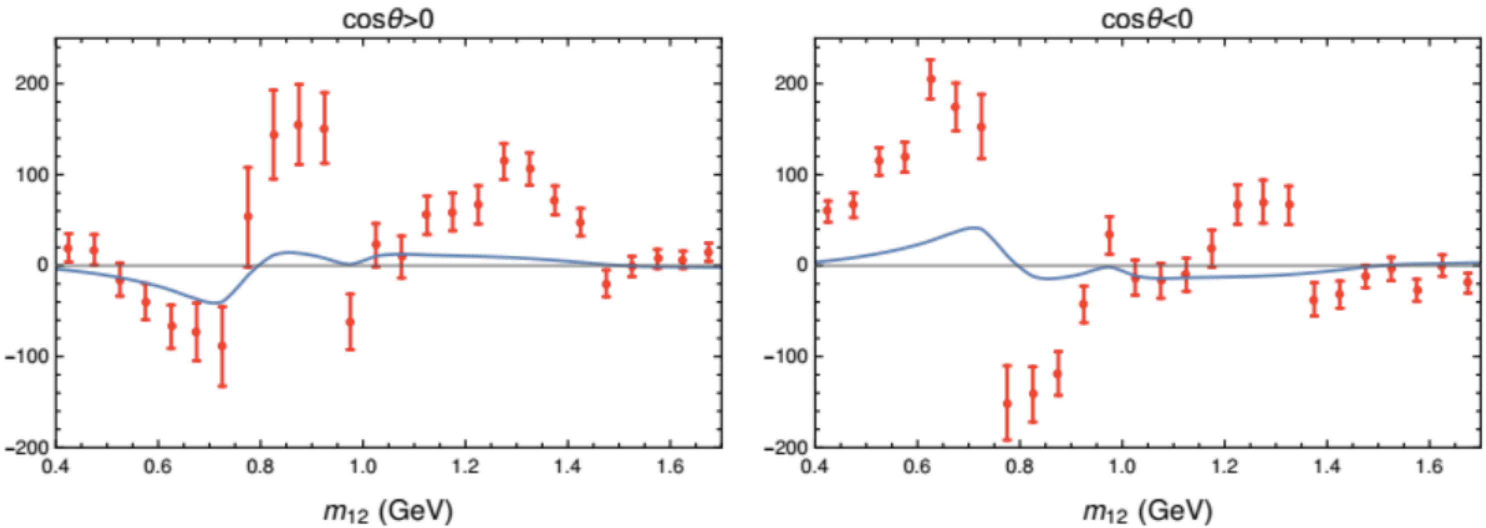
Klein, Mannel, Vos, Virto 2017



(c) (d)

$B^\pm \to \pi^\mp\pi^\pm\pi^\pm$ – CP violation

Klein, Mannel, Vos, Virto 2017



- Probably need to understand much better $F_t^{I=0}$ and phase of F_π.
- Only a first exploratory analysis.

Main theory objects

PERTURBATIVE:

Hard-scattering kernels (T_I, T_{II}) : Same as two-body!! (just matching coefficients)

NON-PERTURBATIVE:

Quasi-two-body		Three-body
$B \to \rho$ form factors	$\longleftarrow \cdots \longrightarrow$	$B \to \pi\pi$ form factors
$\langle \rho \| \bar{q}(x)\Gamma b(0) \| \bar{B} \rangle$		$\langle \pi\pi \| \bar{q}(x)\Gamma b(0) \| \bar{B} \rangle$
ρ-LCDAs...	$\longleftarrow \cdots \longrightarrow$	2π-LCDAs...
$\langle \rho \| \bar{q}(x)\Gamma q(0) \| 0 \rangle_{x^2 \to 0}$		$\langle \pi\pi \| \bar{q}(x)\Gamma q(0) \| 0 \rangle_{x^2 \to 0}$
...and its normalization f_ρ	$\longleftarrow \cdots \longrightarrow$	...and its normalization $F_\pi(s)$

This is **always** an improvement w.r.t. quasi-two-body decays:

$$\mathcal{A}(B^- \to \pi^-[\pi^+\pi^-]) = F^{B\to\pi}\ T_1 \star \phi_{\pi\pi} + F^{B\to\pi\pi}\ T_2 \star \phi_\pi$$

$$\Big\downarrow \quad \rho \text{ dominance + zero-width limit}$$

$$\mathcal{A}(B^- \to \pi^-\rho) = F^{B\to\pi}\ T_1 \star \phi_\rho + F^{B\to\rho}\ T_2 \star \phi_\pi$$

This limit can be checked analytically.

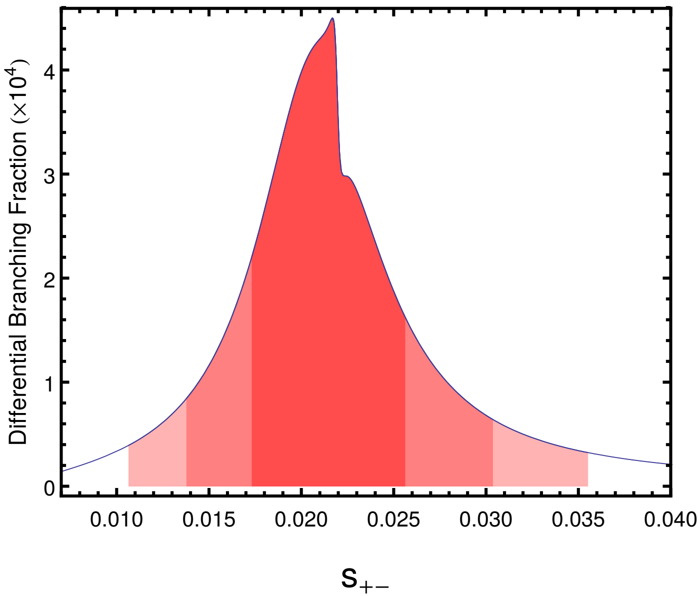
- Factorization is at the same level of theoretical rigour for quasi-two-body and 3-body.
- Any model for $\phi_{\pi\pi}$ and $F^{B\to\pi\pi}$ satisfying axiomatic constraints and compatible with data (e.g. $e^+e^- \to \pi\pi$) replaces any notion of "ρ".

⋆ Leading order amplitude:

$$\mathcal{A}|_{s_{+-}\ll 1} = \frac{G_F}{\sqrt{2}}\left[4m_B^2 f_0(s_{+-})(2\zeta-1)F_\pi(s_{+-})(a_2+a_4) + f_\pi m_\pi (a_1-a_4) F_t(\zeta, s_{+-})\right]$$

⋆ Integrating around the ρ:

$$BR(B^- \to \rho\,\pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-}\; \frac{\tau_B\, m_B |\mathcal{A}|^2}{32(2\pi)^3}$$



with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2/m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6}$ $(n = 0.5)$

$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6}$ $(n = 1)$

$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6}$ $(n = 1.5)$

$BR(B^+ \to \rho\pi^+)_{\text{EXP}} = (8.3 \pm 1.2) \cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_{\text{QCDF}} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$