Topological diagrammatic approach and its unification with SU(3) irreducible representations

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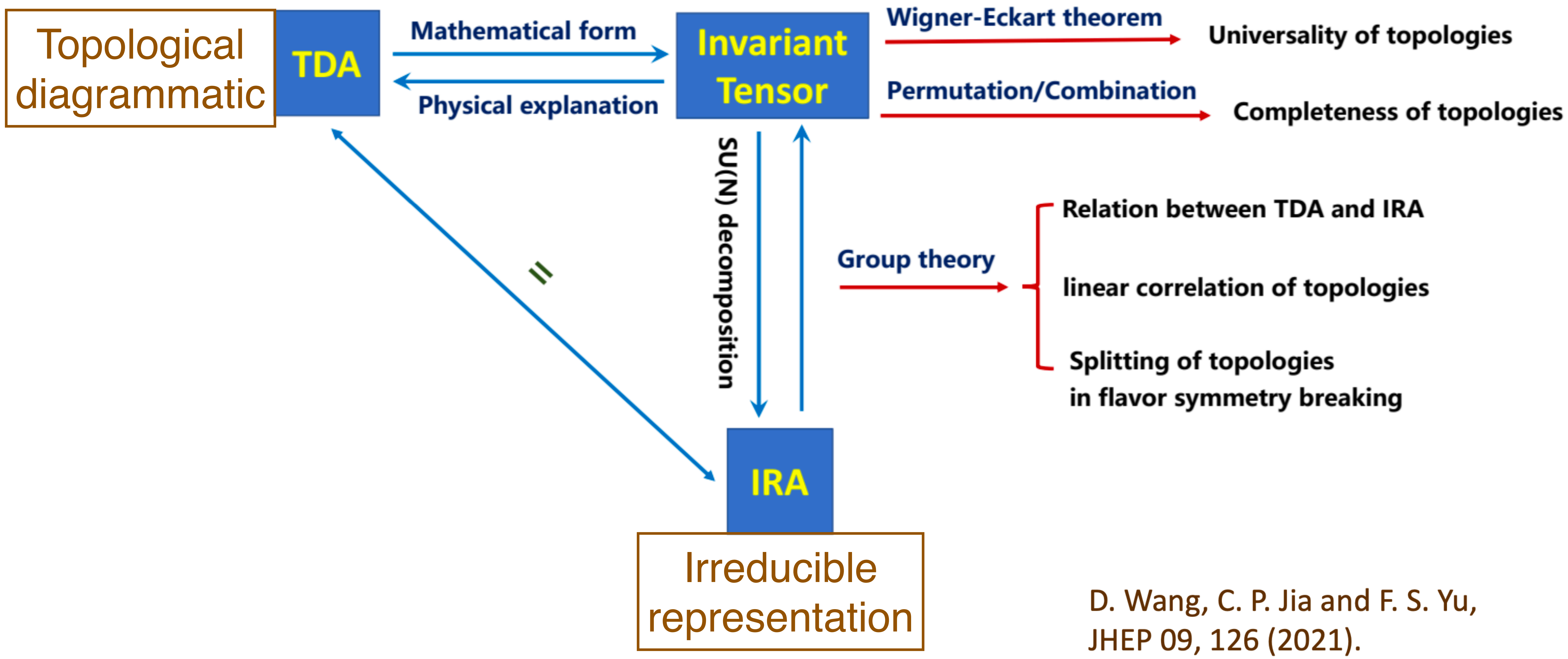
Theoretical methods for non-leptonic weak decays

Theoretical approaches	Advantages	Disadvantages
QCDF, PQCD, SCET	(Almost) first-principle for dynamics, very predictive for B decays	Difficult for power corrections and non-perturbative contributions
Final-state interaction	Dynamics for non-perturbations	Suffer very large theoretical uncertainties
SU(3) irreducible representations	Based on approximate flavor symmetry, no additional assumptions	No link to dynamics
Topological diagrams	Include non-perturbations, successful for charm phenomenologies	Mathematical foundation is not clear

FSI = QCD = Topological diagrams = SU(3) irreducible representations ???

Topological diagrams = Irreducible representations

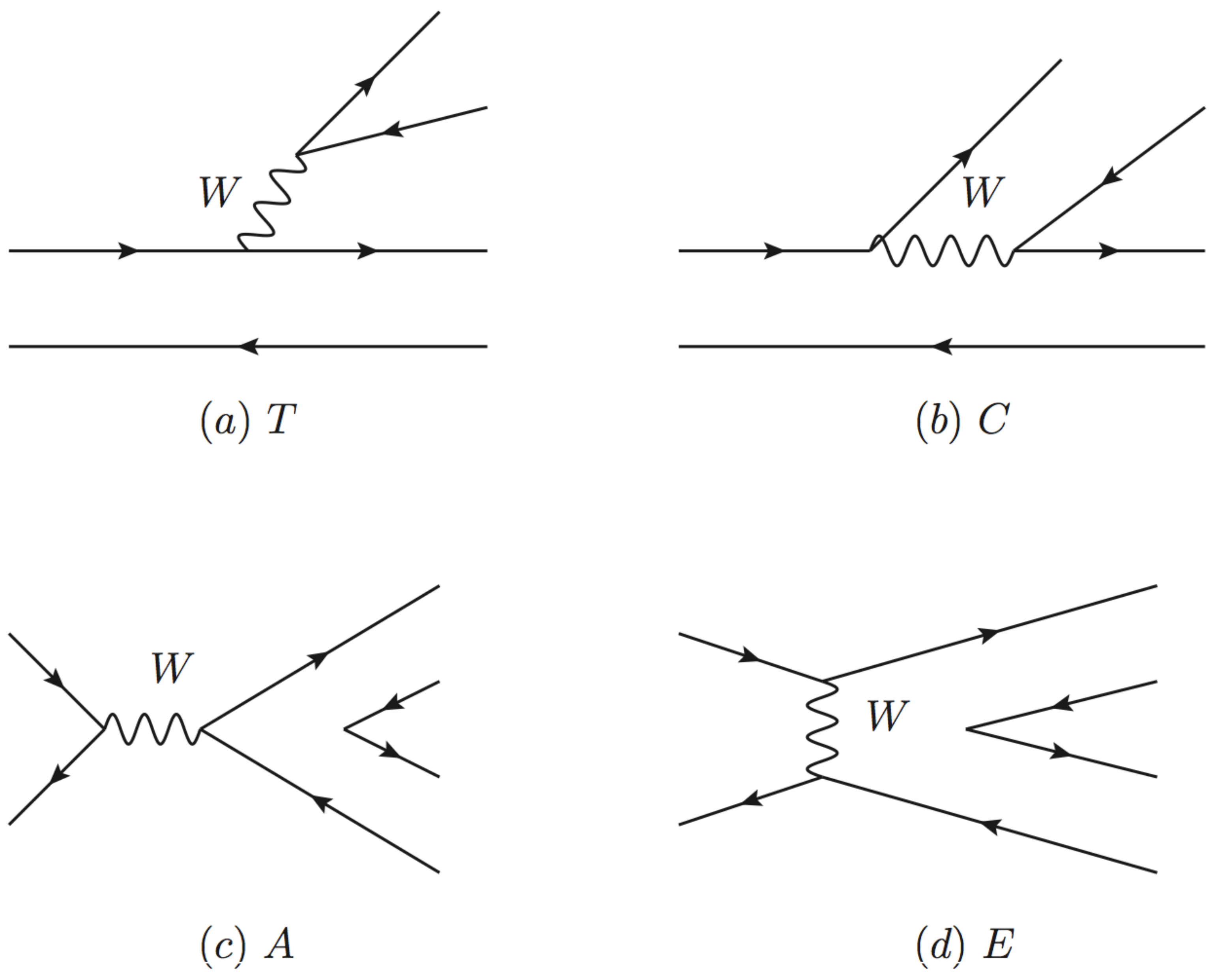
- The Equivalence was firstly pointed out by [X.G.He, W.Wang, 2018]
- The invariant tensors are the bridge between the two approaches.
- One topological diagram is found independent.



D. Wang, C. P. Jia and F. S. Yu, JHEP 09, 126 (2021).

Topological Diagrams

- Decaying amplitudes are classified according to the **weak flavour flows**
- **All the strong interaction effects** are included. Therefore, non-perturbative contributions are all considered.
- Amplitudes extracted from data



Chau,'86; Chau,Cheng,'87, '89; Chiang, Gronau, Rosner, Suprun '04; Cheng, Chiang,'10;
And many many other references

- Always in the flavour **SU(3) symmetry** limit
- W-annihilation diagrams are important for CP violation of B decays

SU(3) irreducible representation approach

- Zeppendfeld, 1981
  - First SU(3) relations for B decays
  - with reduced amplitudes
- Savage and Wise, 1989
  - First **tensor contraction** formulae
  - SU(3) irreducible representation

$$b(c) \to q_1\bar{q}_2 q_3, \quad q_i = u, d, s$$

$$\boxed{3 \otimes \overline{3} \otimes 3 = 3_p \oplus 3_t \oplus \overline{6} \oplus 15}$$

$$\begin{aligned}
\mathcal{A}_t^{\rm IRA} \; = \; & A_3^T B_i (H_{\overline{3}})^i (M)_k^j (M)_j^k + C_3^T B_i (M)_j^i (M)_k^j (H_{\overline{3}})^k + B_3^T B_i (H_3)^i (M)_k^k (M)_j^j + D_3^T B_i (M)_j^i (H_{\overline{3}})^j (M)_k^k \\
& + A_6^T B_i (H_6)_k^{ij} (M)_j^l (M)_l^k + C_6^T B_i (M)_j^i (H_6)_k^{jl} (M)_l^k + B_6^T B_i (H_6)_k^{ij} (M)_j^k (M)_l^l \\
& + A_{15}^T B_i (H_{\overline{15}})_k^{ij} (M)_j^l (M)_l^k + C_{15}^T B_i (M)_j^i (H_{\overline{15}})_l^{jk} (M)_k^l + B_{15}^T B_i (H_{\overline{15}})_k^{ij} (M)_j^k (M)_l^l .
\end{aligned}$$

- Widely applications

Everything is tensor

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V^*_{cq_1} V_{uq_2}\left(\sum_{q=1}^{2} C_i(\mu)O_i(\mu)\right) - V^*_{cb}V_{ub}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)\right)\right]$$

$$\boxed{\mathcal{H} = \mathcal{H}^k_{ij}(\bar{q}^i q_k)(\bar{q}^j c)}$$

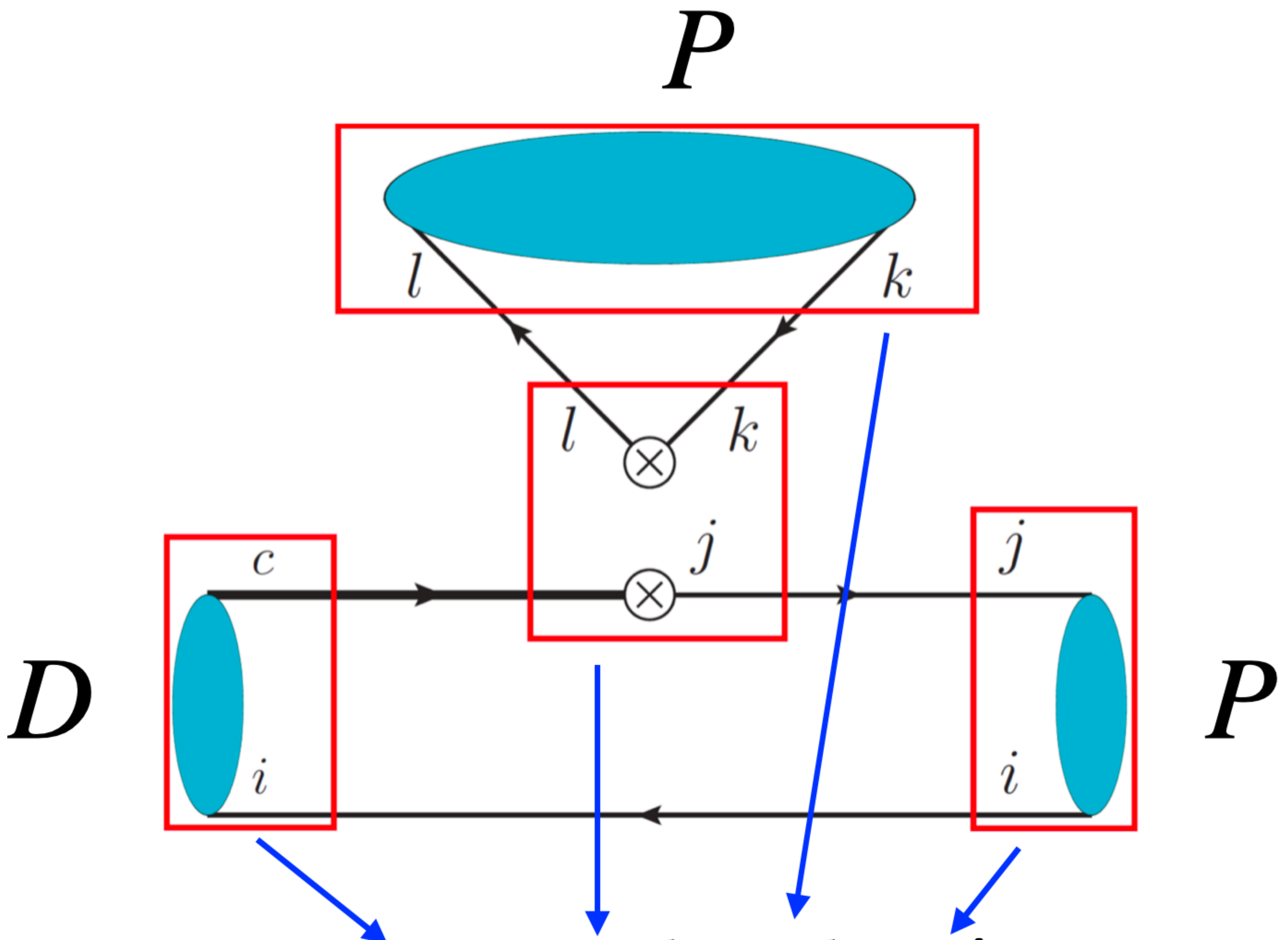
$$q_{i,j,k} = u, d, s$$

$$D^i = (D^0, D^+, D^+_s)$$

$$(P)^i_j = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\sqrt{2/3}\eta_8 \end{pmatrix} + \frac{1}{\sqrt{3}}\begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}$$

X.G.He, W.Wang, 2018; D.Wang, C.P.Jia, FSY, 2021

Topological diagrams



$$A = \quad T\, D^i\, H^k_{lj}\, P^l_k\, P^j_i \quad + \ldots$$

color-favored tree-emitted amplitude, T

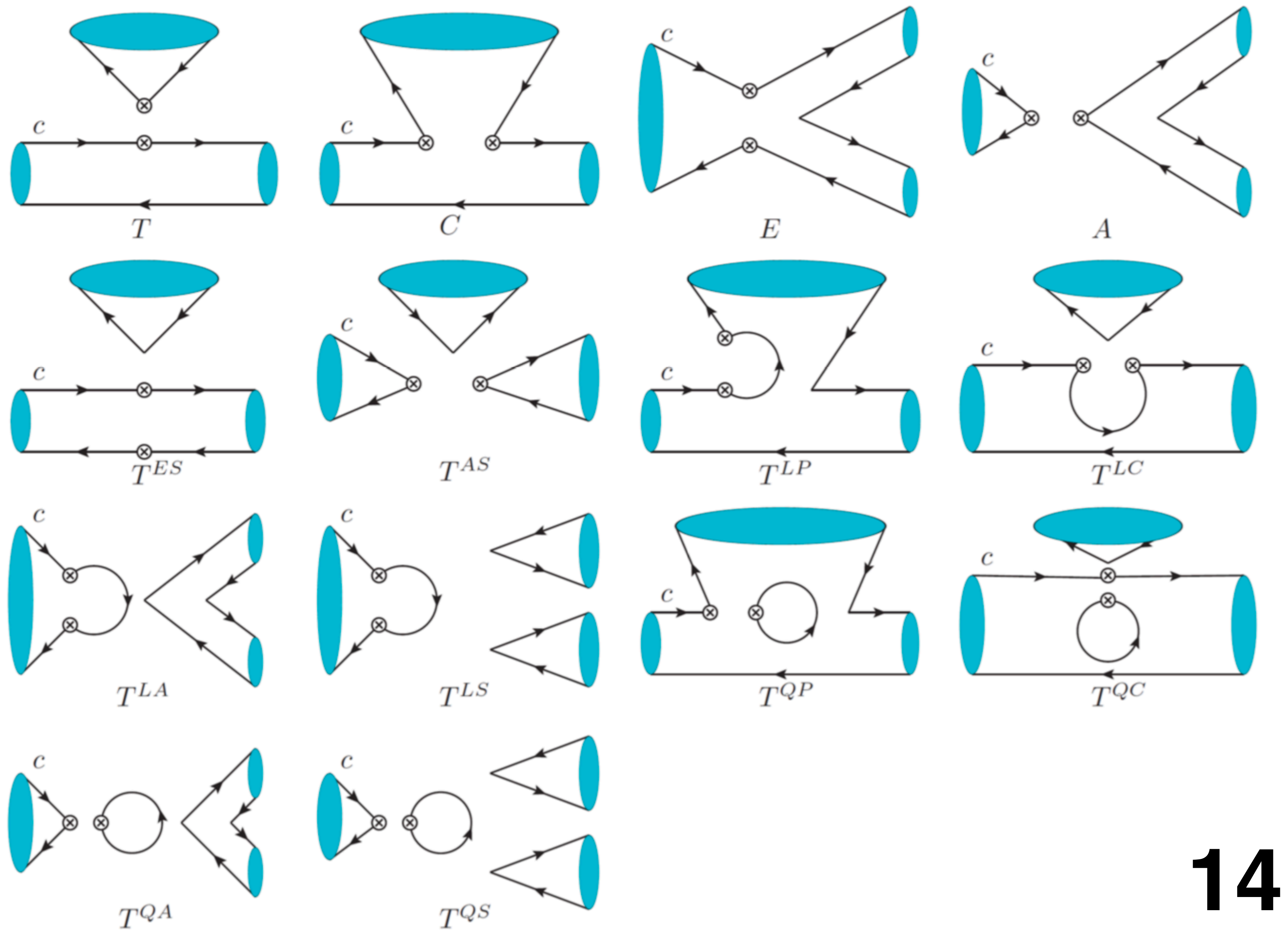
$$\mathcal{H} = \mathcal{H}^k_{ij}(\bar{q}^i q_k)(\bar{q}^j c)$$

- Under the SU(3) flavor symmetry
- Tensor indices are all contracted
- **Completeness of topological diagrams:**
    - For $D \to PP$: $A^4_4 - 2(A^3_3 - 1) = 14$ diagrams
    - For $D \to PV$: $A^4_4 = 24$ diagrams
- With the complete set of diagrams, we can then discuss the independence of diagrams

D.Wang, C.P.Jia, FSY, 2021

Topological diagrams

$$A = TD^i\mathcal{H}^k_{lj}(P)^j_i(P)^l_k + CD^i\mathcal{H}^k_{jl}(P)^j_i(P)^l_k + ED^i\mathcal{H}^j_{il}(P)^k_j(P)^l_k + AD^i\mathcal{H}^j_{li}(P)^k_j(P)^l_k$$
$$+ T^{ES}D^i\mathcal{H}^l_{ij}(P)^j_l(P)^k_k + T^{AS}D^i\mathcal{H}^l_{ji}(P)^j_l(P)^k_k + T^{LP}D^i\mathcal{H}^l_{kl}(P)^j_i(P)^k_j + T^{LC}D^i\mathcal{H}^l_{jl}(P)^j_i(P)^k_k$$
$$+ T^{LA}D^i\mathcal{H}^l_{il}(P)^k_j(P)^j_k + T^{LS}D^i\mathcal{H}^l_{il}(P)^j_j(P)^k_k + T^{QP}D^i\mathcal{H}^l_{lk}(P)^j_i(P)^k_j + T^{QC}D^i\mathcal{H}^l_{lj}(P)^j_i(P)^k_k$$
$$+ T^{QA}D^i\mathcal{H}^l_{li}(P)^k_j(P)^j_k + T^{QS}D^i\mathcal{H}^l_{li}(P)^j_j(P)^k_k.$$

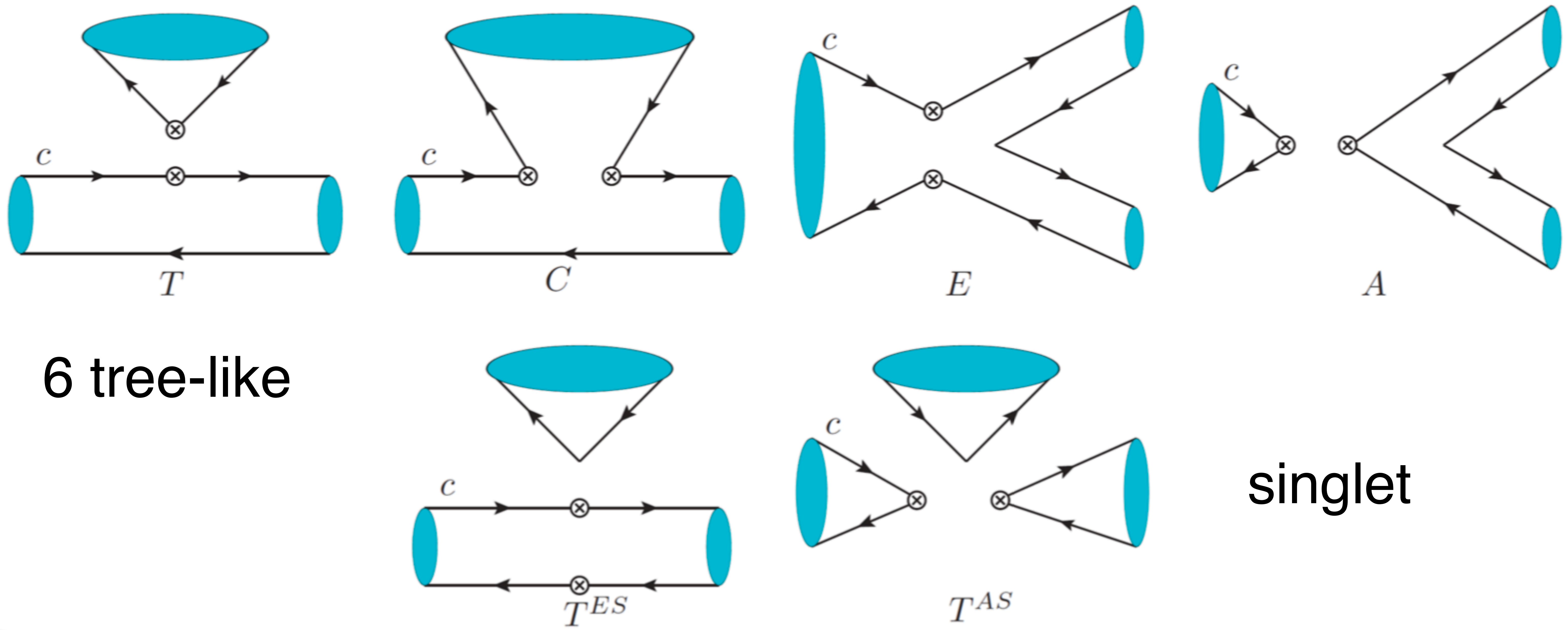


Before: directly draw all possible diagrams

Now: systematically obtain all the diagrams

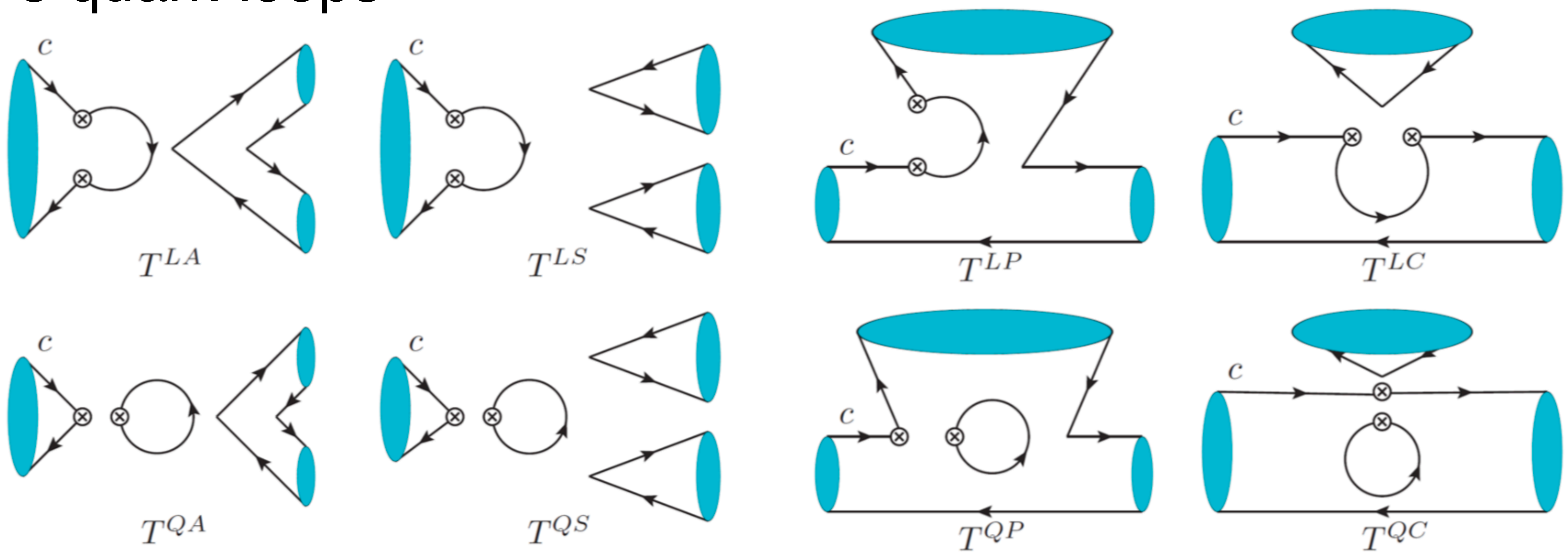
14 topological diagrams with tree operators

6 tree-like



singlet

8 quark-loops



D.Wang, C.P.Jia, FSY, 2021

SU(3) decomposition

$$\mathcal{H} = \mathcal{H}^k_{ij}(\bar{q}^i q_k)(\bar{q}^j c)$$

$$\boxed{3 \otimes \overline{3} \otimes 3 = 3_p \oplus 3_t \oplus \overline{6} \oplus 15}$$

$$\boxed{\mathcal{H}^k_{ij} = \delta^k_j\left(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\right) + \delta^k_i\left(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\right) + \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk} + \mathcal{H}(15)^k_{ij}}$$

$$\begin{aligned}
A = \; & a_3^p D^i\mathcal{H}(3_p)_i(P)^j_k(P)^k_j + b_3^p D^i\mathcal{H}(3_p)_i(P)^k_k(P)^j_j + c_3^p D^i\mathcal{H}(3_p)_k(P)^k_i(P)^j_j + d_3^p D^i\mathcal{H}(3_p)_k(P)^j_i(P)^k_j \\
& + a_3^t D^i\mathcal{H}(3_t)_i(P)^j_k(P)^k_j + b_3^t D^i\mathcal{H}(3_t)_i(P)^k_k(P)^j_j + c_3^t D^i\mathcal{H}(3_t)_k(P)^k_i(P)^j_j + d_3^t D^i\mathcal{H}(3_t)_k(P)^j_i(P)^k_j \\
& + a_6 D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_l(P)^l_k + b_6 D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_k(P)^l_l + c_6 D^i\mathcal{H}(\overline{6})^k_{jl}(P)^j_i(P)^l_k \\
& + a_{15} D^i\mathcal{H}(15)^k_{ij}(P)^j_l(P)^l_k + b_{15} D^i\mathcal{H}(15)^k_{ij}(P)^j_k(P)^l_l + c_{15} D^i\mathcal{H}(15)^k_{jl}(P)^j_i(P)^l_k.
\end{aligned}$$

14 SU(3) irreducible representations

same rule of tensor contraction to the topological diagrams, so the same number of amplitudes

Equivalence

$$
\begin{aligned}
A = &\, TD^i\mathcal{H}^k_{lj}(P)^j_i(P)^l_k + CD^i\mathcal{H}^k_{jl}(P)^j_i(P)^l_k + ED^i\mathcal{H}^j_{il}(P)^k_j(P)^l_k + AD^i\mathcal{H}^j_{li}(P)^k_j(P)^l_k \\
&+ T^{ES}D^i\mathcal{H}^l_{ij}(P)^j_l(P)^k_k + T^{AS}D^i\mathcal{H}^l_{ji}(P)^j_l(P)^k_k + T^{LP}D^i\mathcal{H}^l_{kl}(P)^j_i(P)^k_j + T^{LC}D^i\mathcal{H}^l_{jl}(P)^j_i(P)^k_k \\
&+ T^{LA}D^i\mathcal{H}^l_{il}(P)^k_j(P)^j_k + T^{LS}D^i\mathcal{H}^l_{il}(P)^j_j(P)^k_k + T^{QP}D^i\mathcal{H}^l_{lk}(P)^j_i(P)^k_j + T^{QC}D^i\mathcal{H}^l_{lj}(P)^j_i(P)^k_k \\
&+ T^{QA}D^i\mathcal{H}^l_{li}(P)^k_j(P)^j_k + T^{QS}D^i\mathcal{H}^l_{li}(P)^j_j(P)^k_k.
\end{aligned}
$$

topological approach

$$
\boxed{\mathcal{H}^k_{ij} = \delta^k_j\left(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\right) + \delta^k_i\left(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\right) + \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk} + \mathcal{H}(15)^k_{ij}}
$$

$$
\begin{aligned}
A = &\, a^p_3D^i\mathcal{H}(3_p)_i(P)^j_k(P)^k_j + b^p_3D^i\mathcal{H}(3_p)_i(P)^k_k(P)^j_j + c^p_3D^i\mathcal{H}(3_p)_k(P)^k_i(P)^j_j + d^p_3D^i\mathcal{H}(3_p)_k(P)^j_i(P)^k_j \\
&+ a^t_3D^i\mathcal{H}(3_t)_i(P)^j_k(P)^k_j + b^t_3D^i\mathcal{H}(3_t)_i(P)^k_k(P)^j_j + c^t_3D^i\mathcal{H}(3_t)_k(P)^k_i(P)^j_j + d^t_3D^i\mathcal{H}(3_t)_k(P)^j_i(P)^k_j \\
&+ a_6D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_l(P)^l_k + b_6D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_k(P)^l_l + c_6D^i\mathcal{H}(\overline{6})^k_{jl}(P)^j_i(P)^l_k \\
&+ a_{15}D^i\mathcal{H}(15)^k_{ij}(P)^j_l(P)^l_k + b_{15}D^i\mathcal{H}(15)^k_{ij}(P)^j_k(P)^l_l + c_{15}D^i\mathcal{H}(15)^k_{jl}(P)^j_i(P)^l_k.
\end{aligned}
$$

SU(3) decomposition

topological diagrams = SU(3) irreducible representations

Equivalence is obvious: H_{ij}^{k} is decomposed or not

$$T \times D^i \mathcal{H}_{lj}^{k} (P)_i^j (P)_k^l = T \times D^i (P)_i^j (P)_k^l \times$$

$$\Big[\delta_j^k \big(\frac{3}{8}\mathcal{H}(3_t)_l - \frac{1}{8}\mathcal{H}(3_p)_l\big) + \delta_l^k \big(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\big) + \varepsilon_{ljm}\mathcal{H}(\overline{6})^{mk} + \mathcal{H}(15)_{lj}^{k}\Big]$$

Relations of parameters

Linear combinations

$$a_6 = \frac{E-A}{4}, \quad b_6 = \frac{T^{ES}-T^{AS}}{4}, \quad c_6 = \frac{-T+C}{4},$$

$$a_{15} = \frac{E+A}{8}, \quad b_{15} = \frac{T^{ES}+T^{AS}}{8}, \quad c_{15} = \frac{T+C}{8},$$

$$a_3^t = \frac{3}{8}E - \frac{1}{8}A + T^{LA}, \quad a_3^p = -\frac{1}{8}E + \frac{3}{8}A + T^{QA},$$

$$b_3^t = \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{LS}, \quad b_3^p = -\frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{QS},$$

$$c_3^t = -\frac{1}{8}T + \frac{3}{8}C - \frac{1}{8}T^{ES} + \frac{3}{8}T^{AS} + T^{LC}, \quad c_3^p = \frac{3}{8}T - \frac{1}{8}C + \frac{3}{8}T^{ES} - \frac{1}{8}T^{AS} + T^{QC},$$

$$d_3^t = \frac{3}{8}T - \frac{1}{8}C - \frac{1}{8}E + \frac{3}{8}A + T^{LP}, \quad d_3^p = -\frac{1}{8}T + \frac{3}{8}C + \frac{3}{8}E - \frac{1}{8}A + T^{QP}.$$

Tree operators and penguin operators separated

$$\mathcal{H}_{ij}^{k} = \delta_j^k\left(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\right) + \delta_i^k\left(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\right) + \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk} + \mathcal{H}(15)_{ij}^{k}$$

D.Wang, C.P.Jia, FSY, 2021

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}}\left[\sum_{q=d,s} V_{cq_1}^* V_{uq_2}\left(\sum_{q=1}^{2} C_i(\mu)O_i(\mu)\right) - V_{cb}^* V_{ub}\left(\sum_{i=3}^{6} C_i(\mu)O_i(\mu) + C_{8g}(\mu)O_{8g}(\mu)\right)\right]$$

Difference with He&Wang2018

$$\bar{H}_k^{ij} = \frac{1}{8}(H_{\overline{15}})_k^{ij} + \frac{1}{4}(H_6)_k^{ij} - \frac{1}{8}(H_{\overline{3}})^i\delta_k^j + \frac{3}{8}(H_{\overline{3}'})^j\delta_k^i$$

X.G.He, W.Wang, 2018

$$\mathcal{A} = V_{ub}V_{uq}^*\mathcal{A}_u + V_{tb}V_{tq}^*\mathcal{A}_t$$

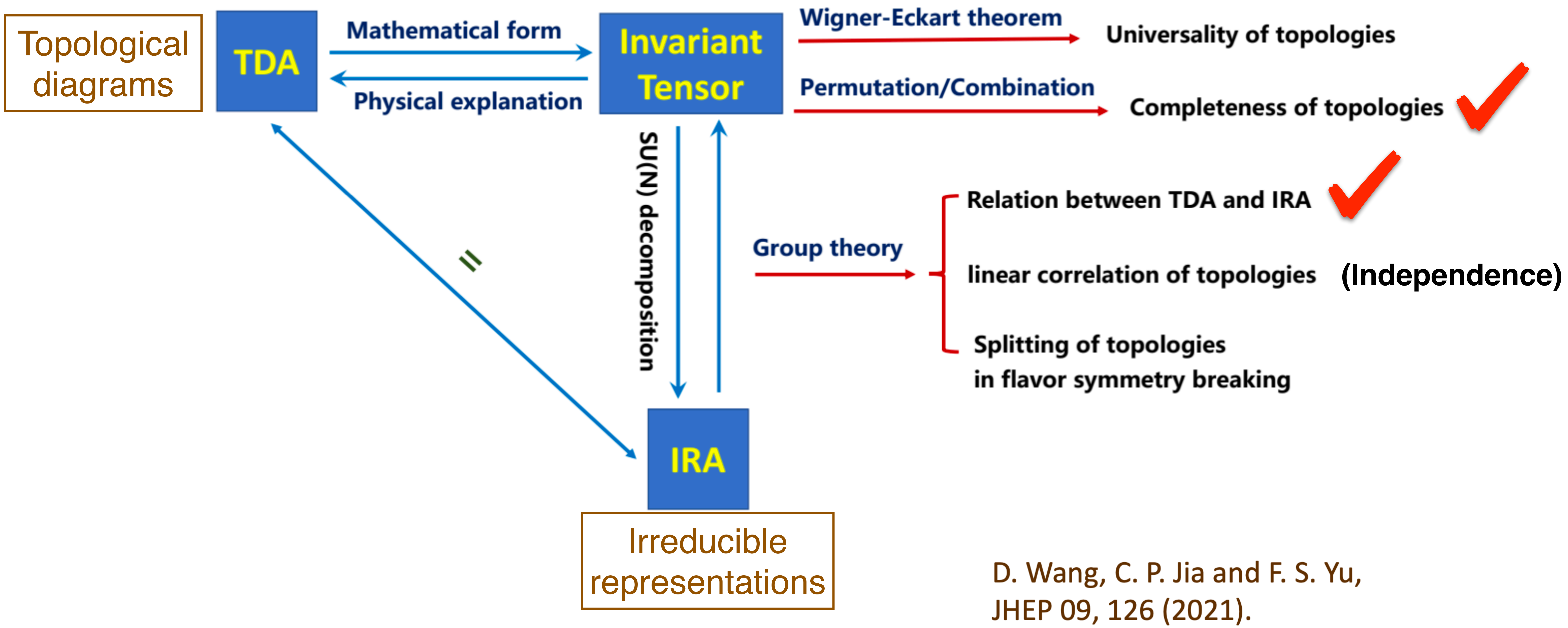
X.G.He, Y.J.Shi, W.Wang, 2018

u and c loops c and t loops

Topological diagrams = Irreducible representations

Advantages:

a) **Mathematical foundation of topological approach is clear now**



D. Wang, C. P. Jia and F. S. Yu, JHEP 09, 126 (2021).

Topological diagrams = Irreducible representations

Advantages:

a) Mathematical foundation of topological approach is clear now

b) Are all topological diagrams independent with each other?

NO !! X.G.He & W.Wang found one amplitude is not independent [He, Wang, 2018]

channel	IRA
$B^-\to\pi^0\pi^-$	$4\sqrt{2}C_{15}^T$
$B^-\to\pi^-\eta_8$	$\sqrt{\frac{2}{3}}(A_6^T+3A_{15}^T+C_3^T-C_6^T+3C_{15}^T)$
$B^-\to\pi^-\eta_1$	$\frac{1}{\sqrt{3}}(2A_6^T+6A_{15}^T+3B_6^T+9B_{15}^T+2C_3^T+C_6^T+3C_{15}^T+3D_3^T)$
$B^-\to K^0K^-$	$A_6^T+3A_{15}^T+C_3^T-C_6^T-C_{15}^T$
$\overline{B}^0\to\pi^+\pi^-$	$2A_3^T-A_6^T+A_{15}^T+C_3^T+C_6^T+3C_{15}^T$
$\overline{B}^0\to\pi^0\pi^0$	$2A_3^T-A_6^T+A_{15}^T+C_3^T+C_6^T-5C_{15}^T$

$$\boxed{\begin{aligned} C_6^{T\prime} &= C_6^T - A_6^T \\ B_6^{T\prime} &= B_6^T + A_6^T \end{aligned}}$$

one irreducible amplitude is not independent

One topological diagram is not independent !!

One diagram not independent

Now the reason:

$$H_{ij}^{k} \quad \rightarrow \quad \boxed{3_i \times 3_j} \times \overline{3}^k$$

$$= \left(\boxed{6_{\{ij\}} + \overline{3}_{[ij]}} \right) \times \overline{3}^k$$

i,j :symmetric

i,j :anti-symmetric

$$\varepsilon_{ijl}\, \overline{3}^l \quad \times \overline{3}^k$$

$$= \varepsilon_{ijl}\, \overline{6}^{lk} + \ldots$$

l,k : symmetric

One diagram not independent

$$\boxed{\mathcal{H}^k_{ij} = \delta^k_j\Big(\frac{3}{8}\mathcal{H}(3_t)_i - \frac{1}{8}\mathcal{H}(3_p)_i\Big) + \delta^k_i\Big(\frac{3}{8}\mathcal{H}(3_p)_j - \frac{1}{8}\mathcal{H}(3_t)_j\Big) + \epsilon_{ijl}\mathcal{H}(\overline{6})^{lk} + \mathcal{H}(15)^k_{ij}}$$

$$\boxed{H(\overline{6})^k_{ij} \;=\; \varepsilon_{ijl}\; H(\overline{6})^{lk}}$$

i,j : anti-sym

l,k : symmetric

Property of SU(3)

$$a_6 D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_l(P)^l_k = a_6 D^i\epsilon_{ijm}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k = a_6\boxed{D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k},$$

$$b_6 D^i\mathcal{H}(\overline{6})^k_{ij}(P)^j_k(P)^l_l = b_6 D^i\epsilon_{ijm}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l = b_6\boxed{D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l},$$

$$\begin{aligned} c_6 D^i\mathcal{H}(\overline{6})^k_{jl}(P)^j_i(P)^l_k &= \frac{1}{2}c_6\,\epsilon^{pqi}\epsilon_{jlm}D_{[pq]}\mathcal{H}(\overline{6})^{km}(P)^j_i(P)^l_k \\ &= c_6\Big[-D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k + D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l \\ &\quad + \cancel{D_{[jl]}\mathcal{H}(\overline{6})^{ki}(P)^j_i(P)^l_k}\Big], \\ &= c_6\Big[-\boxed{D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_l(P)^l_k} + \boxed{D_{[jm]}\mathcal{H}(\overline{6})^{km}(P)^j_k(P)^l_l}\Big]. \end{aligned}$$

$$T_i = \epsilon_{ijk}T^{[jk]}/2 \qquad \text{and} \qquad \epsilon_{ijk}\epsilon^{lmn} = \begin{vmatrix} \delta^l_i & \delta^m_i & \delta^n_i \\ \delta^l_j & \delta^m_j & \delta^n_j \\ \delta^l_k & \delta^m_k & \delta^n_k \end{vmatrix}$$

$$a'_6 = a_6 - c_6, \qquad b'_6 = b_6 + c_6$$

D.Wang, C.P.Jia, FSY, 2021

One diagram not independent

$H(\overline{6})^k_{ij} = \varepsilon_{ijl}\, H(\overline{6})^{lk} \Rightarrow$ one of a_6, b_6, c_6 not independent

$$a_6 = E - A, \qquad b_6 = SE - SA, \qquad c_6 = -T + C,$$

$$a_{15} = E + A, \qquad b_{15} = SE + SA, \qquad c_{15} = T + C,$$

$$a_3^t = \frac{3}{8}E - \frac{1}{8}A + T^{LA}, \qquad a_3^p = -\frac{1}{8}E + \frac{3}{8}A + T^{QA},$$

1. If data of $\eta(\eta')$ included, one of *T, C, E, A, SE, SA* is **not independent**

2. If data of $\eta(\eta')$ included, but SE, SA neglected, all of T,C,E,A are independent

3.If data of $\eta(\eta')$ excluded, and SE, SA excluded, one of T,C,E,A is **not independent**

Topological diagrams = Irreducible representations

Advantages:

a) Mathematical foundation of topological approach is clear now

b) One topological diagram is not independent due to SU(3) structure

c) How to systematically consider the flavor symmetry breaking effects?

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$$

$$m_u = m_d = m_s \qquad \mathcal{H}_{\cancel{\mathrm{SU}(3)_F}} = (m_s - m_d)\bar{s}s$$

SU(3) flavor symmetry:

$$\mathcal{A}^{\rm TDA}_{D_\gamma\to P_\alpha P_\beta} = T(D_\gamma)_i(H)^{lj}_k(P_\alpha)^i_j(P_\beta)^k_l + C(D_\gamma)_i(H)^{jl}_k(P_\alpha)^i_j(P_\beta)^k_l + E(D_\gamma)_i(H)^{il}_j(P_\alpha)^j_k(P_\beta)^k_l$$
$$+ A(D_\gamma)_i(H)^{li}_j(P_\alpha)^j_k(P_\beta)^k_l + T^{LP}(D_\gamma)_i(H)^{kl}_l(P_\alpha)^i_j(P_\beta)^j_k$$

Linear SU(3) breaking:

Explicit expression for strange quark

$$\mathcal{A}^{{\rm TDA},\cancel{{\rm SU}(3)_F}}_{D_\gamma\to P_\alpha P_\beta} = T(D_\gamma)_i(H)^{lj}_k(P_\alpha)^i_j(P_\beta)^k_l + T_1(D_\gamma)_i(H)^{l3}_k(P_\alpha)^i_3(P_\beta)^k_l + T_2(D_\gamma)_i(H)^{lj}_3(P_\alpha)^i_j(P_\beta)^3_l$$
$$+T_3(D_\gamma)_3(H)^{lj}_k(P_\alpha)^3_j(P_\beta)^k_l + C(D_\gamma)_i(H)^{jl}_k(P_\alpha)^i_j(P_\beta)^k_l + C_1(D_\gamma)_i(H)^{j3}_k(P_\alpha)^i_j(P_\beta)^k_3$$
$$+C_2(D_\gamma)_i(H)^{jl}_3(P_\alpha)^i_j(P_\beta)^3_l + C_3(D_\gamma)_3(H)^{jl}_k(P_\alpha)^3_j(P_\beta)^k_l + E(D_\gamma)_i(H)^{il}_j(P_\alpha)^j_k(P_\beta)^k_l$$
$$+E_1(D_\gamma)_i(H)^{i3}_j(P_\alpha)^j_k(P_\beta)^k_3 + E_2(D_\gamma)_i(H)^{il}_3(P_\alpha)^3_k(P_\beta)^k_l + E_3(D_\gamma)_i(H)^{il}_j(P_\alpha)^j_3(P_\beta)^3_l$$
$$+A(D_\gamma)_i(H)^{li}_j(P_\alpha)^j_k(P_\beta)^k_l + A_1(D_\gamma)_3(H)^{l3}_j(P_\alpha)^j_k(P_\beta)^k_l + A_2(D_\gamma)_i(H)^{li}_3(P_\alpha)^3_k(P_\beta)^k_l$$
$$+A_3(D_\gamma)_i(H)^{li}_j(P_\alpha)^j_3(P_\beta)^3_l + T^{LP}_{\rm break}(D_\gamma)_i(H)^{k3}_3(P_\alpha)^i_j(P_\beta)^j_k + \alpha\leftrightarrow\beta,$$

Reproduce the results of [Muller, Nierste, Schacht, 2015]

D.Wang, C.P.Jia, FSY, 2021

Topological diagrams = Irreducible representations

Advantages:

a) Mathematical foundation of topological approach is clear now

b) One topological diagram is not independent due to SU(3) structure

c) Systematically consider the flavor symmetry breaking effects

d) Why topological diagrams are universal to all processes in the SU(3) symmetry?

$$A(D^0 \to K^-\pi^+) = T + E$$
$$A(D^0 \to \overline{K}^0\pi^0) = (C - E)/\sqrt{2}$$
$$A(D^+ \to \overline{K}^0\pi^+) = T + C$$
$$A(D_s^+ \to \overline{K}^0 K^+) = C + A$$

No reason not to be universal under the SU(3) symmetry

Good, but not good enough !

d) Why topological diagrams are universal to all processes in the SU(3) symmetry?

Topology: $A = \boxed{t\, D^q H^k_{ij} P^n_m P^s_r} + \dots$

SU(3) irreducible representation: $A = a_{\bar{\mathbf{6}}} \boxed{D^t H(\bar{\mathbf{6}})^k_{ij} P^n_m P^s_r} + b_{\mathbf{15}}\, D^t H(\mathbf{15})^k_{ij} P^n_m P^s_r + \dots$

Wigner-Eckart theorem: $\boxed{\langle j\, m | T^{(k)}_q | j'\, m' \rangle} = \boxed{\langle j'\, m'\, k\, q | j\, m \rangle} \langle j \| T^{(k)} \| j' \rangle$

amplitude

CG coefficients

process related

reduced matrix elements

independent on processes

Topological diagrams = Irreducible representations

Advantages:

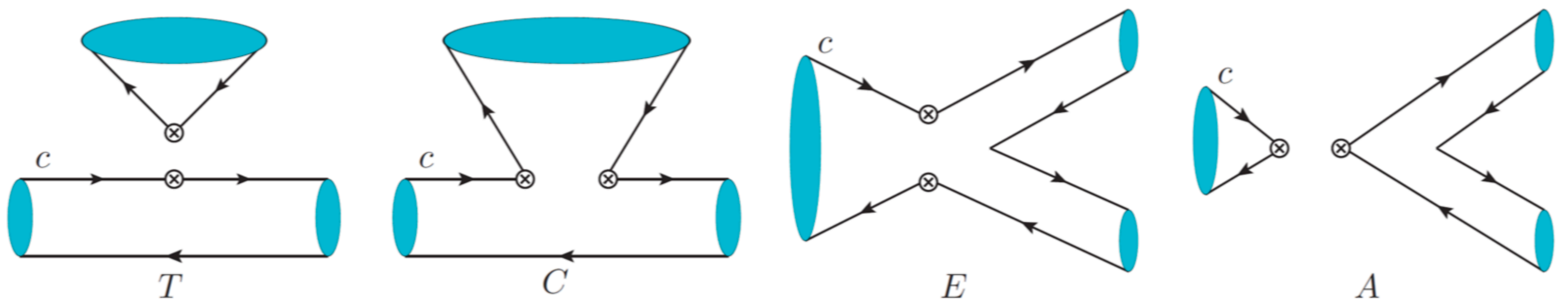
a) Mathematical foundation of topological approach is clear now

b) One topological diagram is not independent due to SU(3) structure

c) Systematically consider the flavor symmetry breaking effects

d) Wigner-Eckart theorem —> Topological diagrams are universal

e) Why each topological diagram includes all strong-interacting effects? Even FSI effects?



No gluons in the diagrams.

Good, but not good enough !

e) Why each topological diagram includes all strong-interacting effects? Even FSI effects?

Topology: $$A = t\, D^q H^k_{ij} P^n_m P^s_r + \dots$$

$$\Updownarrow$$

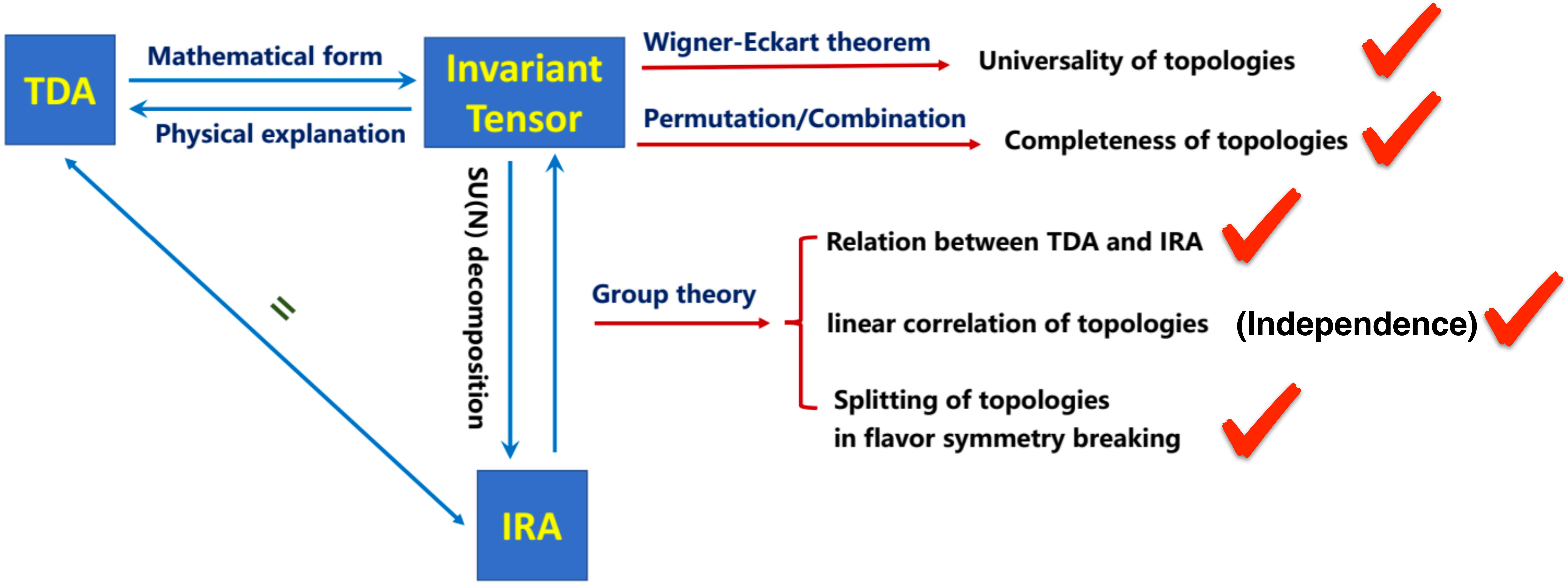
SU(3) irreducible representation: $$A = a_{\overline{6}} D^t H(\overline{\mathbf{6}})^k_{ij} P^n_m P^s_r + b_{15} D^t H(\mathbf{15})^k_{ij} P^n_m P^s_r + \dots$$

QCD corrections = flavor SU(3) singlet operators

- The SU(3) structure is not changed by QCD corrections
- Irreducible amplitudes include all the strong interaction !

Topological diagrams = Irreducible representations

Advantages:



D. Wang, C. P. Jia and F. S. Yu,
JHEP 09, 126 (2021).

QCD = Topological diagrams

QCD = Short-distance contributions of topological diagrams

Topological diagrams = SU(3) irreducible representations

$$T = 2C_6^T + 4C_{15}^T, \quad C = 4C_{15}^T - 2C_6^T, \quad A = 2A_6^T + 4A_{15}^T, \quad E = 4A_{15}^T - 2A_6^T,$$

$$T_P = -A_6^T - A_{15}^T + C_3^T - C_6^T - C_{15}^T, \quad T_{PA} = A_3^T + A_6^T - A_{15}^T, \quad T_{AS} = 4B_{15}^T - 2B_6^T,$$

$$T_{ES} = 2B_6^T + 4B_{15}^T, \quad T_{SS} = B_3^T + B_6^T - B_{15}^T, \quad T_S = -B_6^T - B_{15}^T + C_6^T - C_{15}^T + D_3^T.$$

Topological diagrams = QCDF

$$T = A_{M_1M_2}\left[\alpha_1 + \frac{3}{2}\alpha^u_{4,EW} - \frac{3}{2}\alpha^c_{4,EW}\right], \qquad C = A_{M_1M_2}\left[\alpha_2 + \frac{3}{2}\alpha^u_{3,EW} - \frac{3}{2}\alpha^c_{3,EW}\right],$$

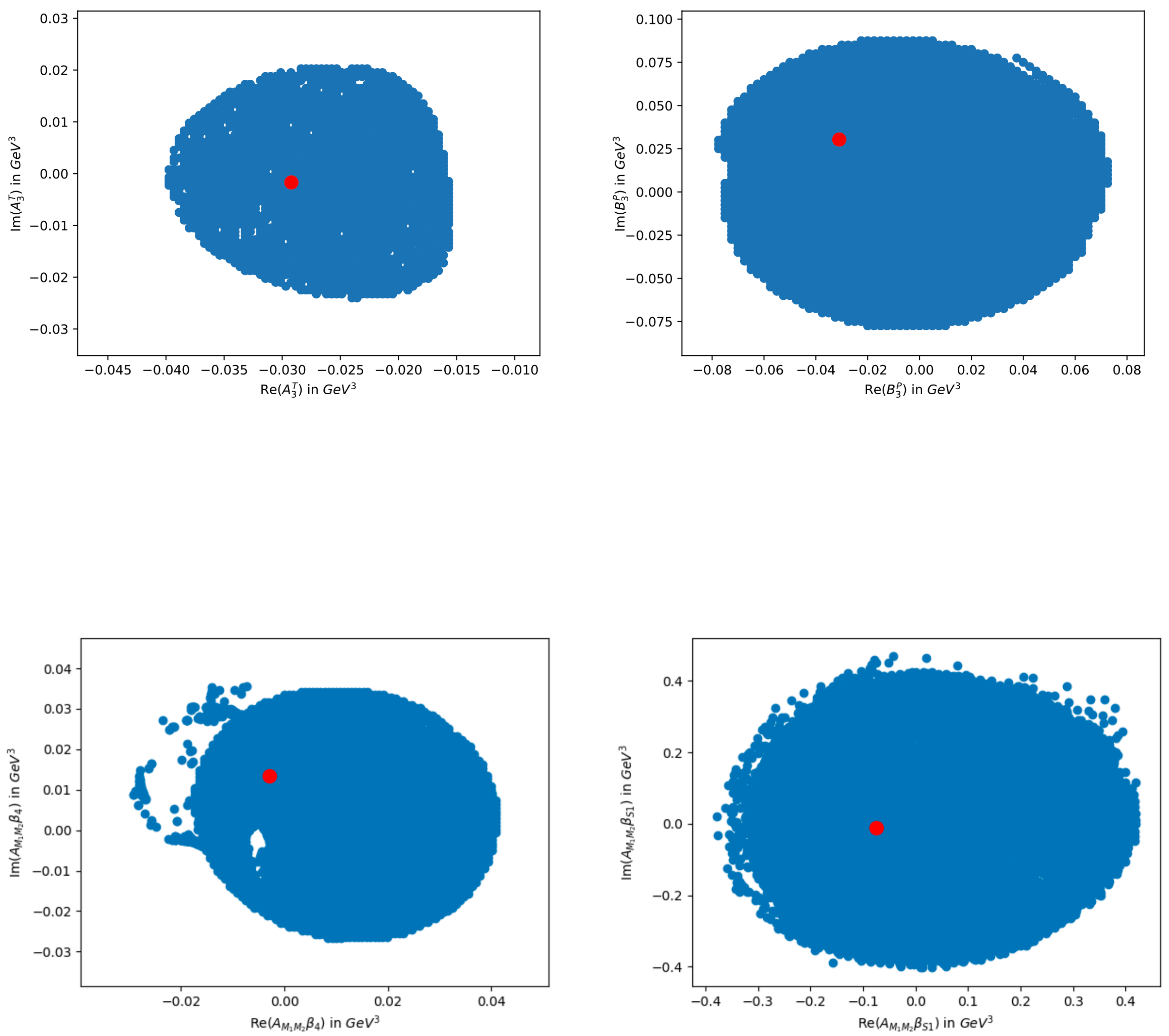
$$E = A_{M_1M_2}\left[\beta_1 + \frac{3}{2}b^u_{4,EW} - \frac{3}{2}b^c_{4,EW}\right], \qquad A = A_{M_1M_2}\left[\beta_2 + \frac{3}{2}\beta^u_{3,EW} - \frac{3}{2}\beta^c_{3,EW}\right],$$

$$T_{AS} = A_{M_1M_2}\left[\beta_{S1} + \frac{3}{2}b^u_{S4,EW} - \frac{3}{2}b^c_{S4,EW}\right], \quad T_{ES} = A_{M_1M_2}\left[\beta_{S2} + \frac{3}{2}\beta^u_{S3,EW} - \frac{3}{2}\beta^c_{S3,EW}\right]$$

$$A_{M_1M_2} = M_B^2\, F_0^{B\to M_1}(0)\, f_{M_2}$$

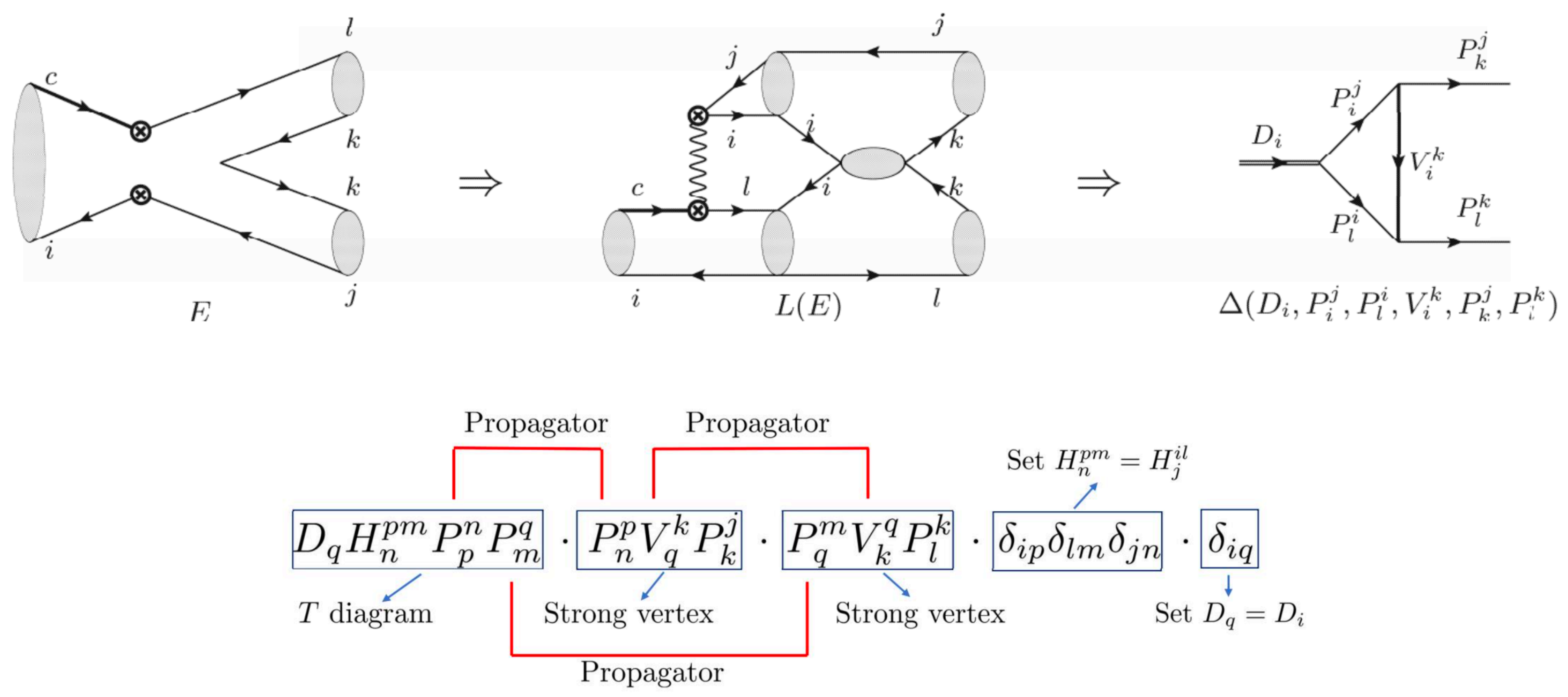
Doing a global fit

[T.Huber, Tetlalmatzi-Xolocotzi, 2021]



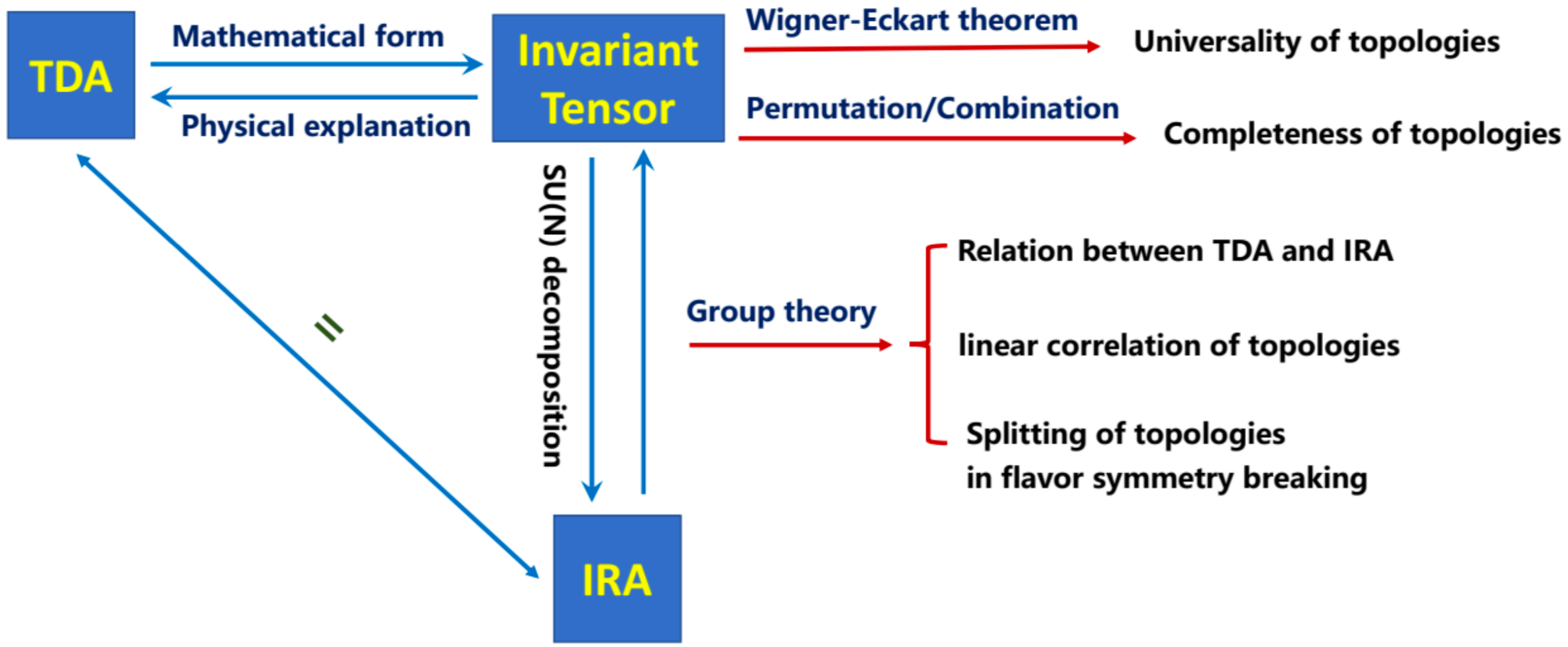
Final-state interaction = Topological diagrams

FSI = Long-distance contributions of topological diagrams [D.Wang, 2021]



- Topological diagram $\Longrightarrow$ Quark diagram $\Longrightarrow$ Triangle diagram

Summary

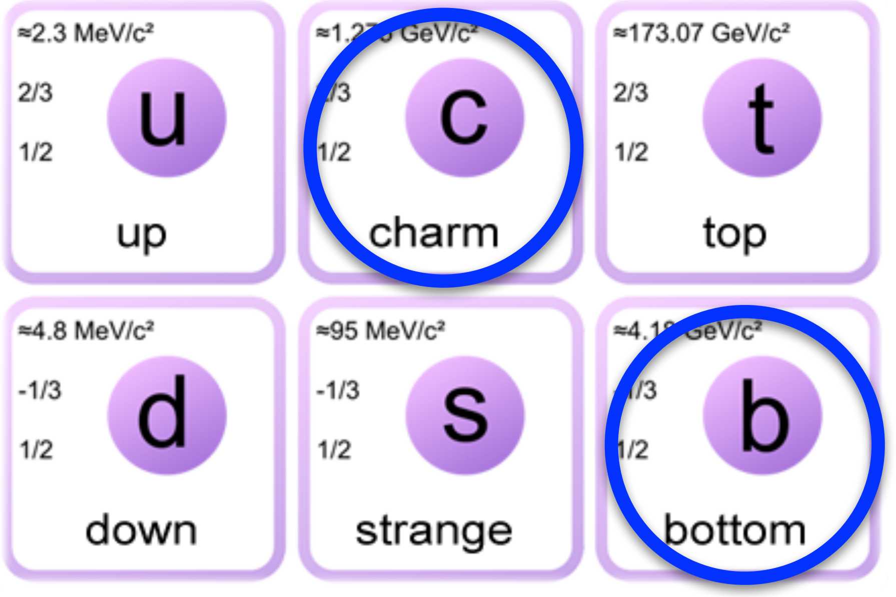


FSI = QCD = Topological diagrams = SU(3) irreducible representations

Thank you!

Backups

Heavy Flavor Physics



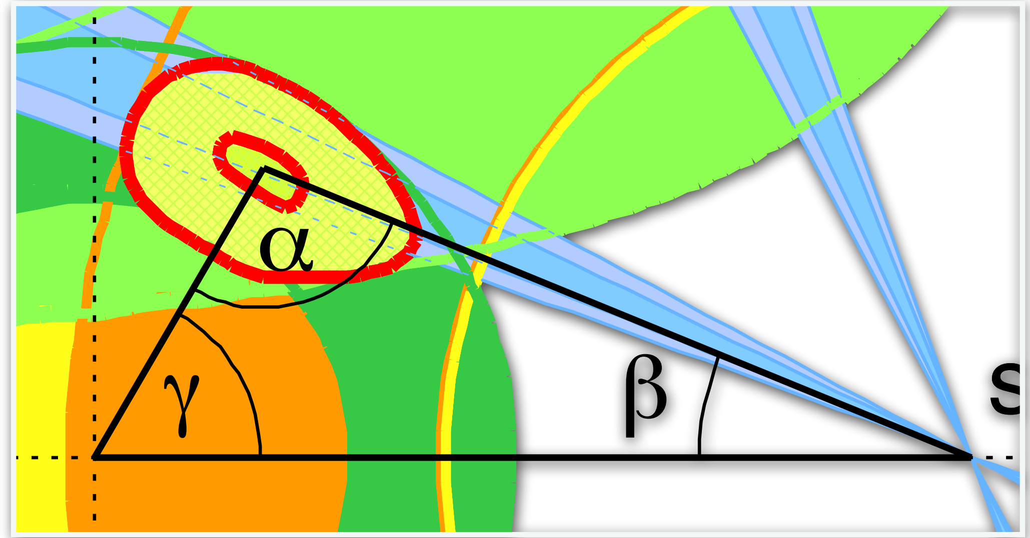
- 1980-2000, To test the SM (KM mechanism)
- 2000-now, To precisely test the SM and search for NP

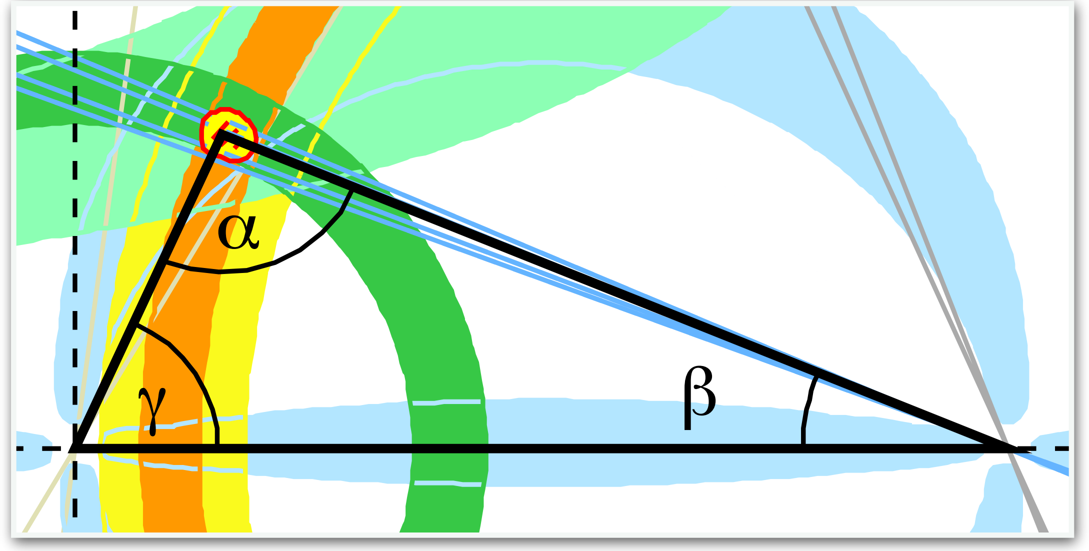
CP violation:

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- SM: KM mechanism, the only one phase in the SM
- BSM: Matter-antimatter in the Universe requires more CPV, needs new CPV source

2003

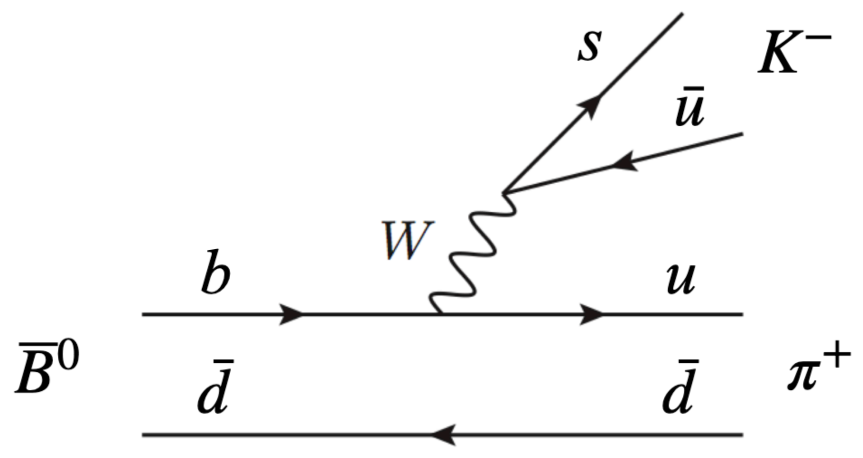


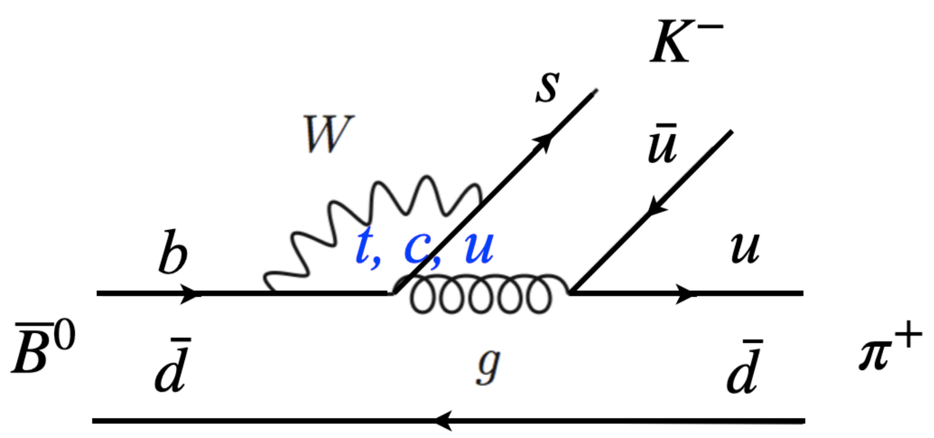


2019

Direct CPV in hadronic weak decays

- Distinguishable for the KM mechanism from the superweak interaction [Wolfenstein, 1964]





$$A_{CP} = -\frac{2|a_1 a_2| \sin(\delta_2 - \delta_1) \sin(\phi_2 - \phi_1)}{|a_1|^2 + |a_2|^2 + 2|a_1 a_2| \cos(\delta_2 - \delta_1) \cos(\phi_2 - \phi_1)}$$

$$A_f = |a_1| e^{i(\delta_1 + \phi_1)} + |a_2| e^{i(\delta_2 + \phi_2)}$$
$$\overline{A}_{\overline{f}} = |a_1| e^{i(\delta_1 - \phi_1)} + |a_2| e^{i(\delta_2 - \phi_2)}$$

CPV conditions:

1. At least two amplitudes
2. with different weak phases
3. with different strong phases $\longrightarrow$ Need QCD dynamics !

Most significant processes in exp charm

1. **First observation of charm CPV** LHCb, 2019

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

《物理世界》2019年十大突破

2. **First observation of double-charm baryon** LHCb, 2017

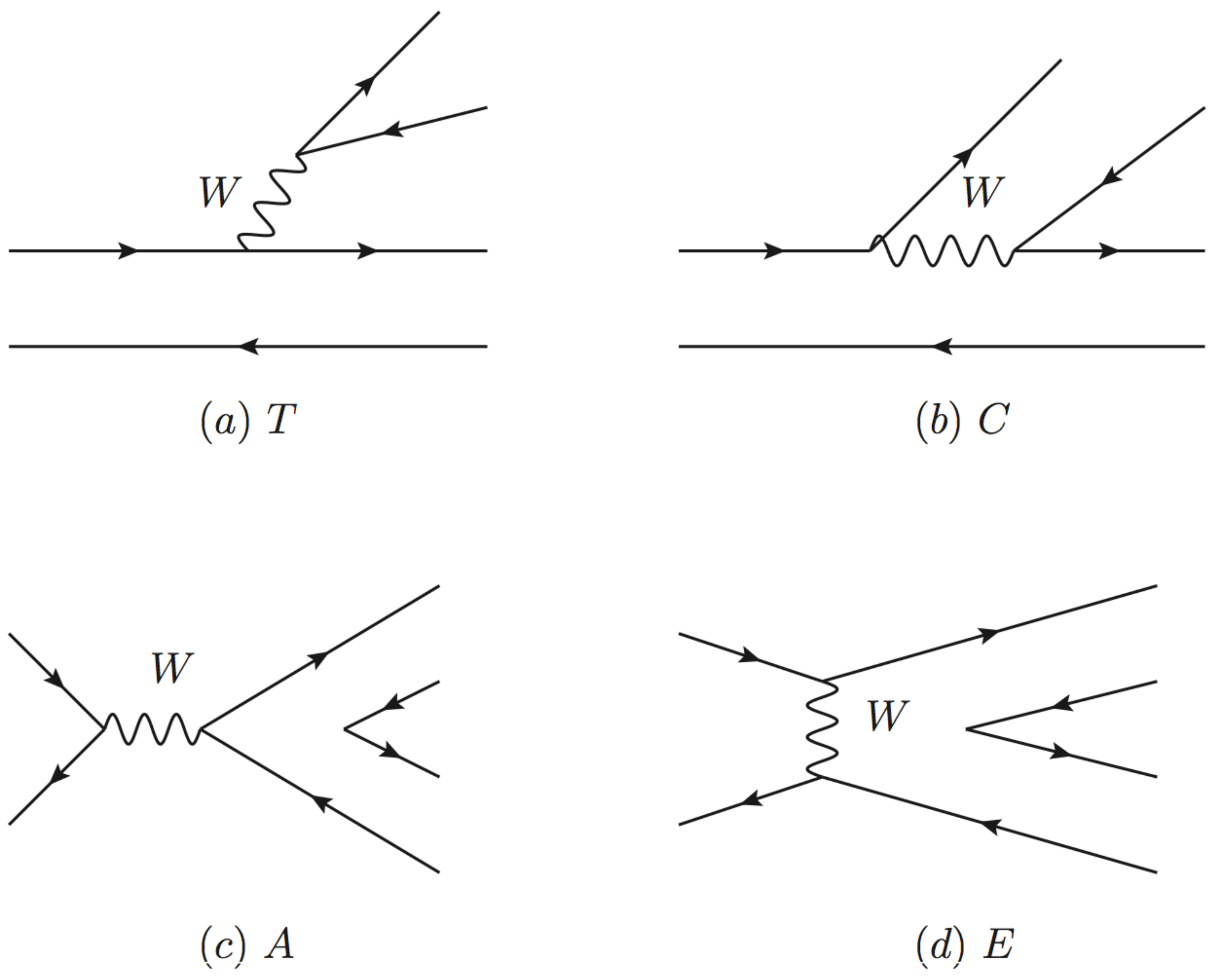
$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

2017年度中国科学十大进展

Topological diagrammatic approach plays an important role in the predictions

Topological Amplitudes

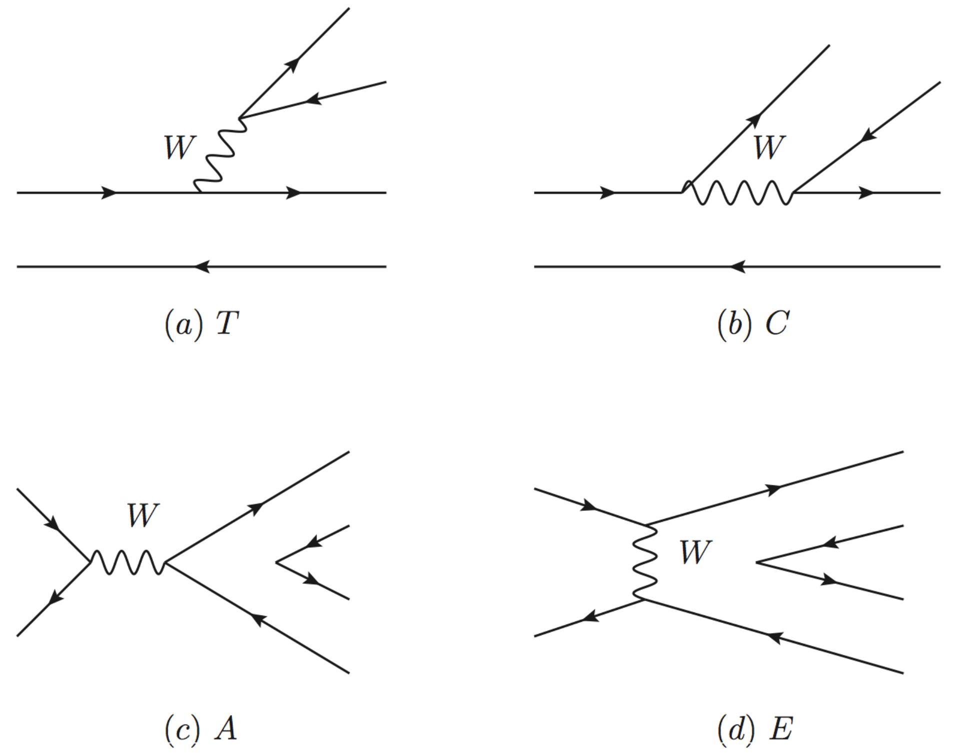
- Decaying amplitudes are classified according to the **weak flavour flows**
- **All the strong interaction effects** are included. Therefore, non-perturbative contributions are all considered.
- Amplitudes extracted from data



Chau,'86; Chau,Cheng,'87; Bhattacharya, Rosner, '08; Cheng, Chiang,'10

- Always in the flavour **SU(3) symmetry** limit

Topological Amplitudes



Cheng, Chiang,'10

$$T = 3.14 \pm 0.06, \qquad C = (2.61 \pm 0.08)e^{-i(152\pm1)^\circ},$$

$$E = (1.53^{+0.07}_{-0.08})e^{i(122\pm2)^\circ}, \qquad A = (0.39^{+0.13}_{-0.09})e^{i(31^{+20}_{-33})^\circ}$$

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ (%)	$\mathcal{B}_{\rm fit}$ (%)
D^0	$K^-\pi^+$	$V_{cs}^* V_{ud}(T+E)$	3.91 ± 0.08	3.91 ± 0.17
	$\bar{K}^0\pi^0$	$\frac{1}{\sqrt{2}} V_{cs}^* V_{ud}(C-E)$	2.38 ± 0.09	2.36 ± 0.08
	$\bar{K}^0\eta$	$V_{cs}^* V_{ud}[\frac{1}{\sqrt{2}}(C+E)\cos\phi - E\sin\phi]$	0.96 ± 0.06	0.98 ± 0.05
	$\bar{K}^0\eta'$	$V_{cs}^* V_{ud}[\frac{1}{\sqrt{2}}(C+E)\sin\phi + E\cos\phi]$	1.90 ± 0.11	1.91 ± 0.09
D^+	$\bar{K}^0\pi^+$	$V_{cs}^* V_{ud}(T+C)$	3.07 ± 0.10	3.08 ± 0.36
D_s^+	$\bar{K}^0 K^+$	$V_{cs}^* V_{ud}(C+A)$	2.98 ± 0.17	2.97 ± 0.32
	$\pi^+\pi^0$	0	<0.037	0
	$\pi^+\eta$	$V_{cs}^* V_{ud}(\sqrt{2}A\cos\phi - T\sin\phi)$	1.84 ± 0.15	1.82 ± 0.32
	$\pi^+\eta'$	$V_{cs}^* V_{ud}(\sqrt{2}A\sin\phi + T\cos\phi)$	3.95 ± 0.34	3.82 ± 0.36

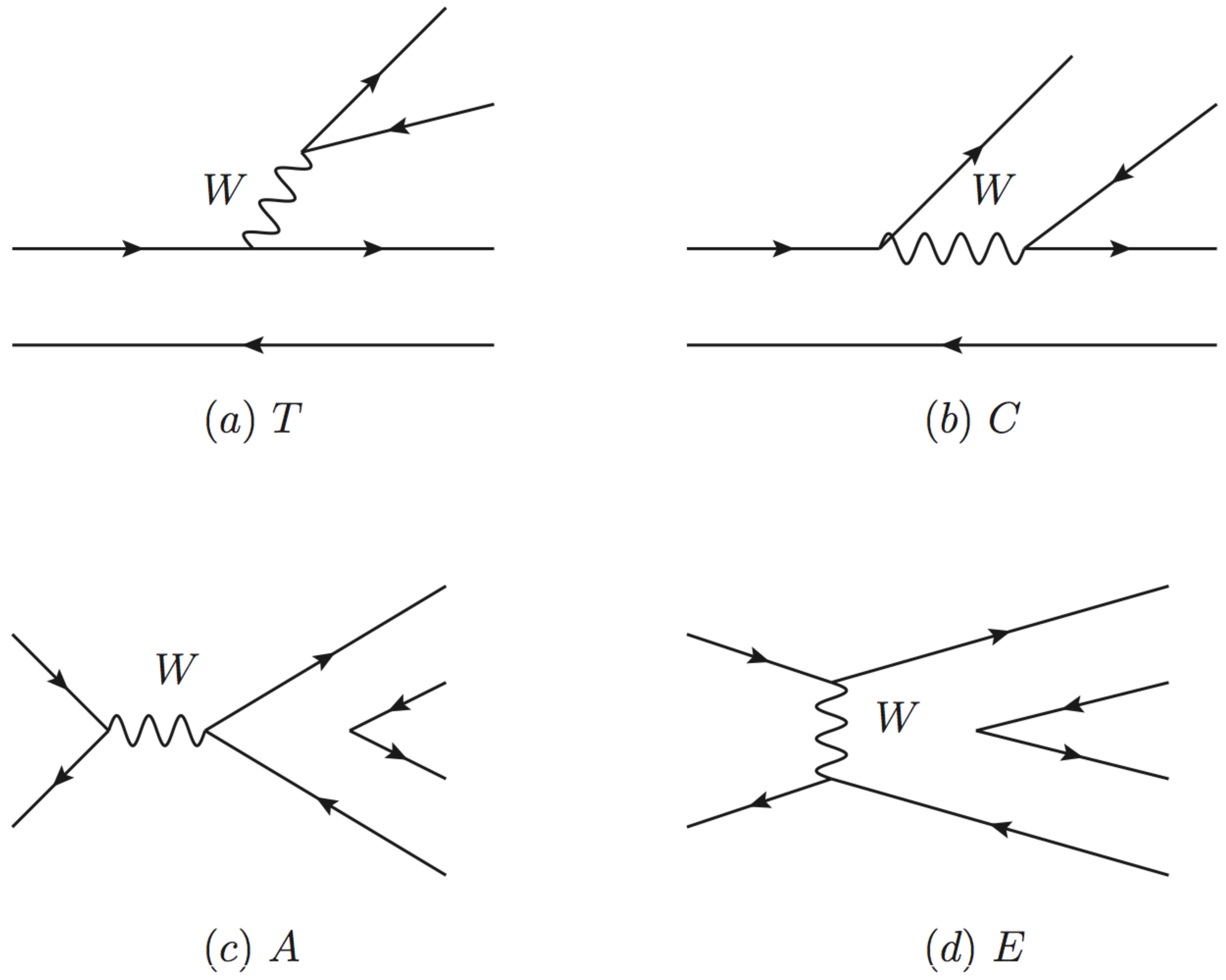
Under flavor SU(3) symmetry

Meson	Mode	Representation	$\mathcal{B}_{\rm exp}$ $(\times 10^{-3})$	$\mathcal{B}_{\rm theory}$ $(\times 10^{-3})$
D^0	$\pi^+\pi^-$	$V_{cd}^* V_{ud}(T'+E')$	1.45 ± 0.05	2.24 ± 0.10
	$\pi^0\pi^0$	$\frac{1}{\sqrt{2}} V_{cd}^* V_{ud}(C'-E')$	0.81 ± 0.05	1.35 ± 0.05
	$\pi^0\eta$	$-V_{cd}^* V_{ud} E'\cos\phi - \frac{1}{\sqrt{2}} V_{cs}^* V_{us} C'\sin\phi$	0.68 ± 0.07	0.75 ± 0.02
	$\pi^0\eta'$	$-V_{cd}^* V_{ud} E'\sin\phi + \frac{1}{\sqrt{2}} V_{cs}^* V_{us} C'\cos\phi$	0.91 ± 0.13	0.74 ± 0.02
	$\eta\eta$	$-\frac{1}{\sqrt{2}} V_{cd}^* V_{ud}(C'+E')\cos^2\phi + V_{cs}^* V_{us}(2E'\sin^2\phi - \frac{1}{\sqrt{2}}C'\sin 2\phi)$	1.67 ± 0.18	1.44 ± 0.08
	$\eta\eta'$	$-\frac{1}{2} V_{cd}^* V_{ud}(C'+E')\sin 2\phi + V_{cs}^* V_{us}(E'\sin 2\phi - \frac{1}{\sqrt{2}}C'\cos 2\phi)$	1.05 ± 0.26	1.19 ± 0.07
	K^+K^-	$V_{cs}^* V_{us}(T'+E')$	4.07 ± 0.10	1.92 ± 0.08
	$K^0\bar{K}^0$	$V_{cd}^* V_{ud} E'_s + V_{cs}^* V_{us} E'_d$ [a]	0.64 ± 0.08	0

SU(3) breaking effects

should be considered

Factorization-Assisted Topological-Amplitude Approach (FAT)



Li, Lu, **FSY**, ’12;
Qin, Li, Lu, **FSY**, ’14

- Dynamics In **factorization**:
  - **Short-distance**: Wilson coefficients
  - **Long-distance**: hadronic matrix elements

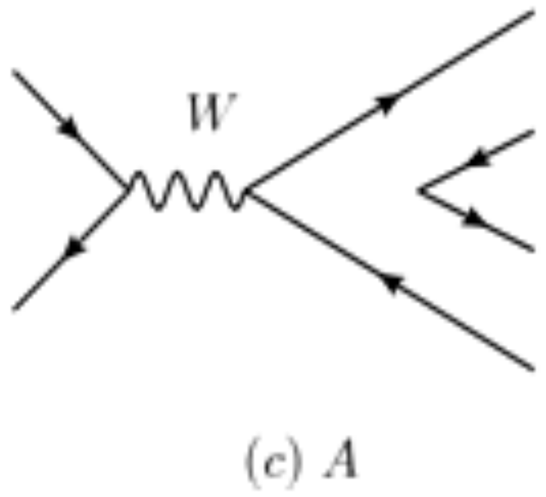
↓

Non-perturbative quantities

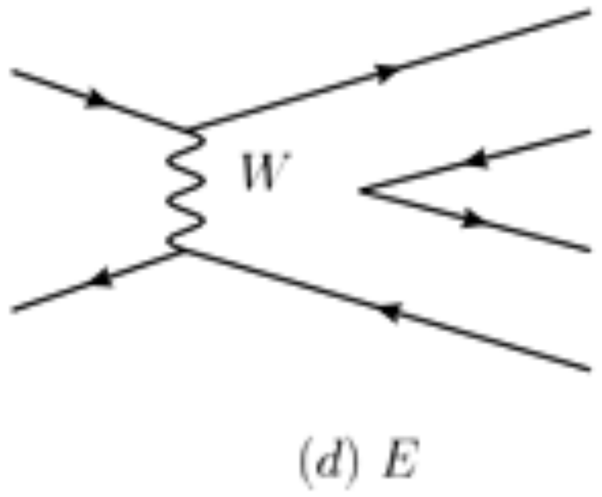
↓

Extracted from data

W-annihilation (A)
W-exchange (E)



(c) A



(d) E

$$\langle P_1 P_2 | \mathcal{H}_{\rm eff} | D \rangle_{E,A} = \frac{G_F}{\sqrt{2}} V_{\rm CKM} b^{E,A}_{q,s}(\mu) f_D m_D^2 \left(\frac{f_{P_1} f_{P_2}}{f_\pi^2} \right)$$

Li, Lu, **FSY**, ’12

A: $b^A_{q,s}(\mu) = C_1(\mu) \chi^A_{q,s} e^{i\phi^A_{q,s}}$

E: $b^E_{q,s}(\mu) = C_2(\mu) \chi^E_{q,s} e^{i\phi^E_{q,s}}$

SU(3) breaking effects

nonperturbative contributions

Modes	Br(exp)	Br(this work)	$A_{CP}^{SM} \times 10^{-3}$
$D^0 \to \pi^+\pi^-$	1.45 ± 0.05	1.43	0.58
$D^0 \to K^+K^-$	4.07 ± 0.10	4.19	-0.42
$D^0 \to K^0\bar{K}^0$	0.320 ± 0.038	0.36	1.38
$D^0 \to \pi^0\pi^0$	0.81 ± 0.05	0.57	0.05
$D^0 \to \pi^0\eta$	0.68 ± 0.07	0.94	-0.29
$D^0 \to \pi^0\eta'$	0.91 ± 0.13	0.65	1.53
$D^0 \to \eta\eta$	1.67 ± 0.18	1.48	0.18
$D^0 \to \eta\eta'$	1.05 ± 0.26	1.54	-0.94
$D^+ \to \pi^+\pi^0$	1.18 ± 0.07	0.89	0
$D^+ \to K^+\bar{K}^0$	6.12 ± 0.22	5.95	-0.93
$D^+ \to \pi^+\eta$	3.54 ± 0.21	3.39	-0.26
$D^+ \to \pi^+\eta'$	4.68 ± 0.29	4.58	1.18
$D_S^+ \to \pi^0K^+$	0.62 ± 0.23	0.67	0.39
$D_S^+ \to \pi^+K^0$	2.52 ± 0.27	2.21	0.84
$D_S^+ \to K^+\eta$	1.76 ± 0.36	1.00	0.70
$D_S^+ \to K^+\eta'$	1.8 ± 0.5	1.92	-1.60

$$\boxed{\Delta A_{CP}^{SM} = -1 \times 10^{-3}}$$

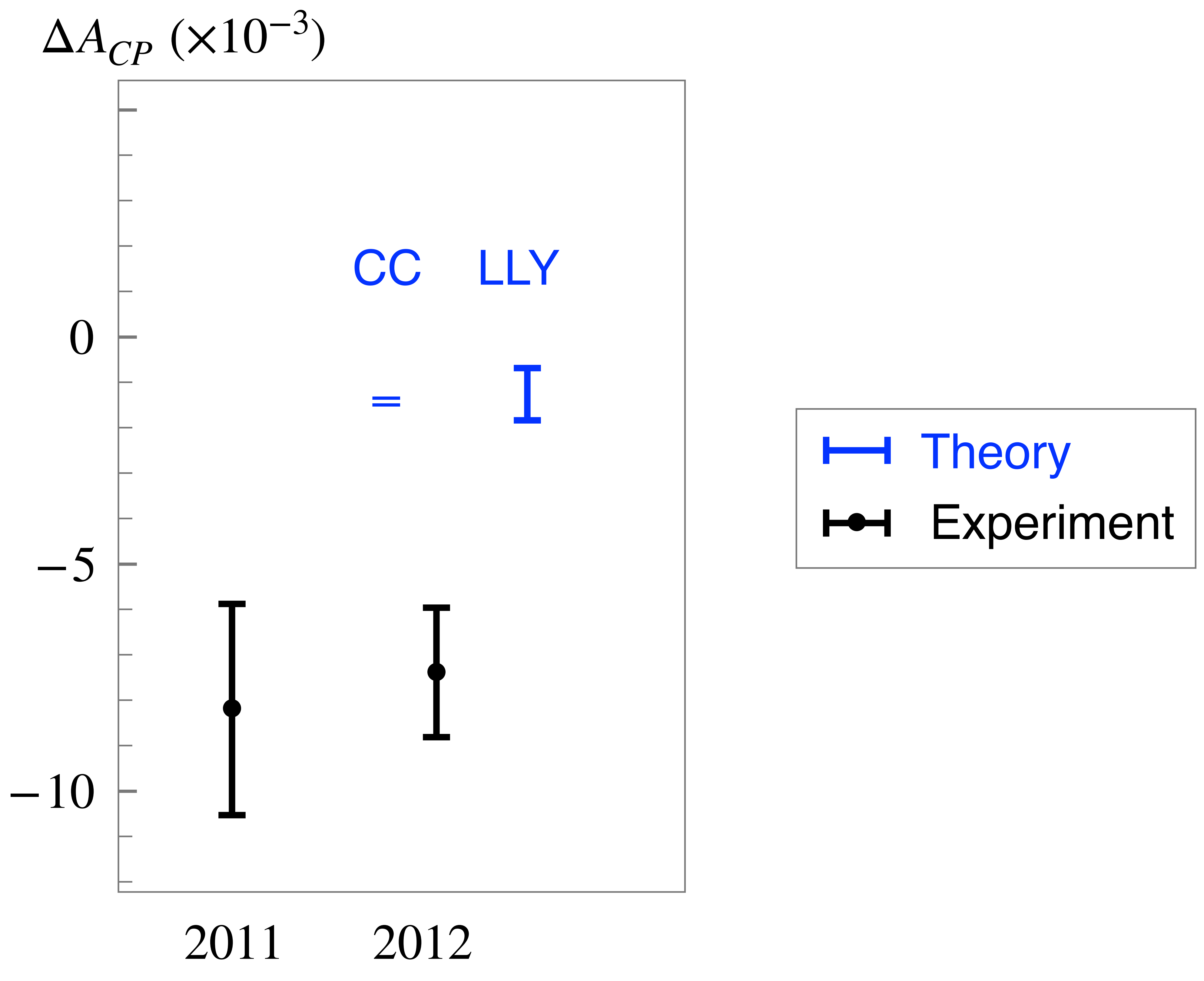
1. Understand QCD dynamics @ 1GeV by Branching Ratios

2. then predict charm CPV

@ BESIII & CLEO

Li, Lu, **FSY**, ’12

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$$



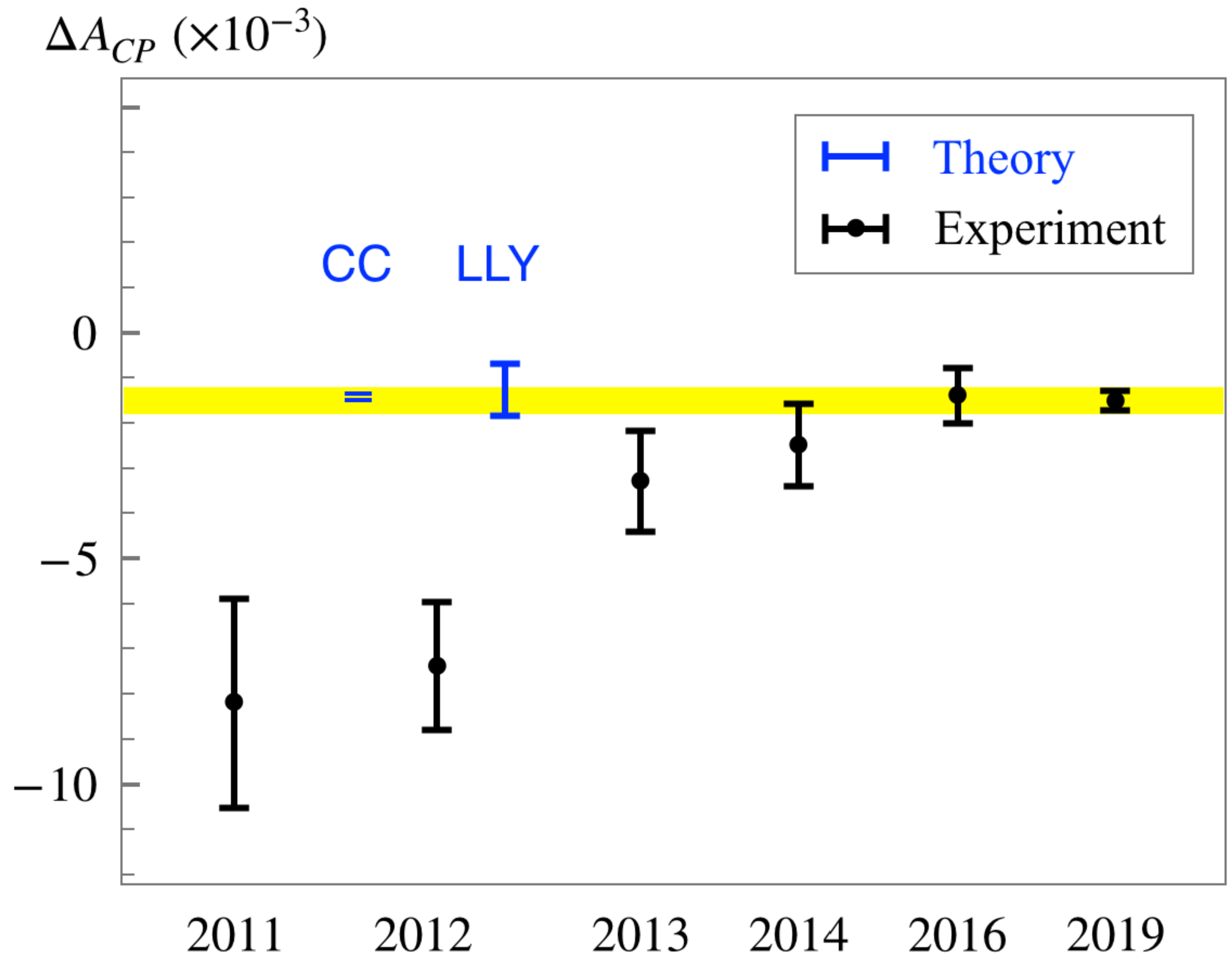
Th: the only prediction of O(10^{-3})

CC: H.Y.Cheng, C.W.Chiang, PRD2012

LLY: H.n.Li, C.D.Lu, **FSY**, PRD2012

EXP: LHCb 2011; CDF 2012; Belle 2012

$$\Delta A_{CP} = A_{CP}(D^0 \to K^+K^-) - A_{CP}(D^0 \to \pi^+\pi^-)$$



Saur, **FSY**, Sci.Bull.2020

Th: the only predictions of $O(10^{-3})$

CC: topological approach

H.Y.Cheng, C.W.Chiang, 2012

LLY: factorization-assisted topology (FAT)

H.n.Li, C.D.Lu, FSY, 2012

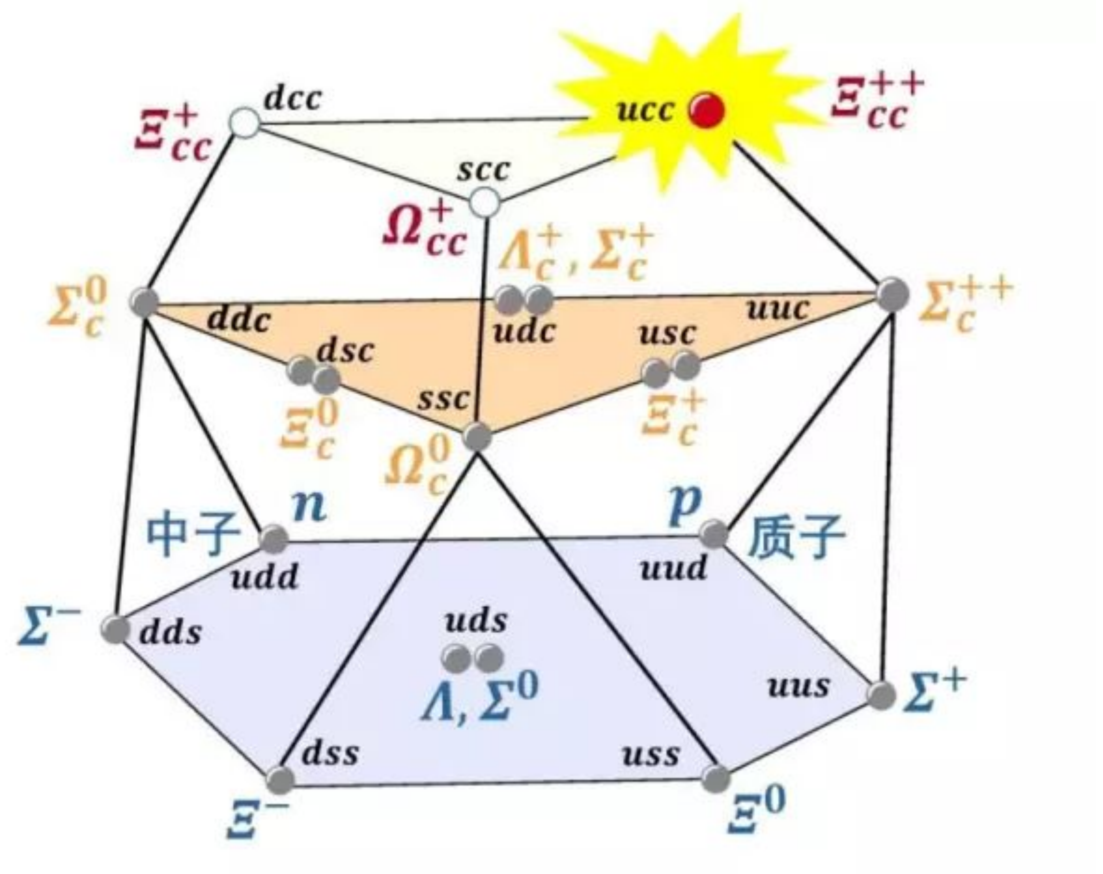
Exp: LHCb, PRL122, 211803 (2019)

Topological diagrammatic approach successfully predicted the charm CPV !!!

Double-charm baryons

- Double-charm baryons are predicted in SU(4) quark model

  De Rujula, Georgi and Glashow, 1975; Jaffe, J. E. Kiskis, 1976; Ponce, 1979



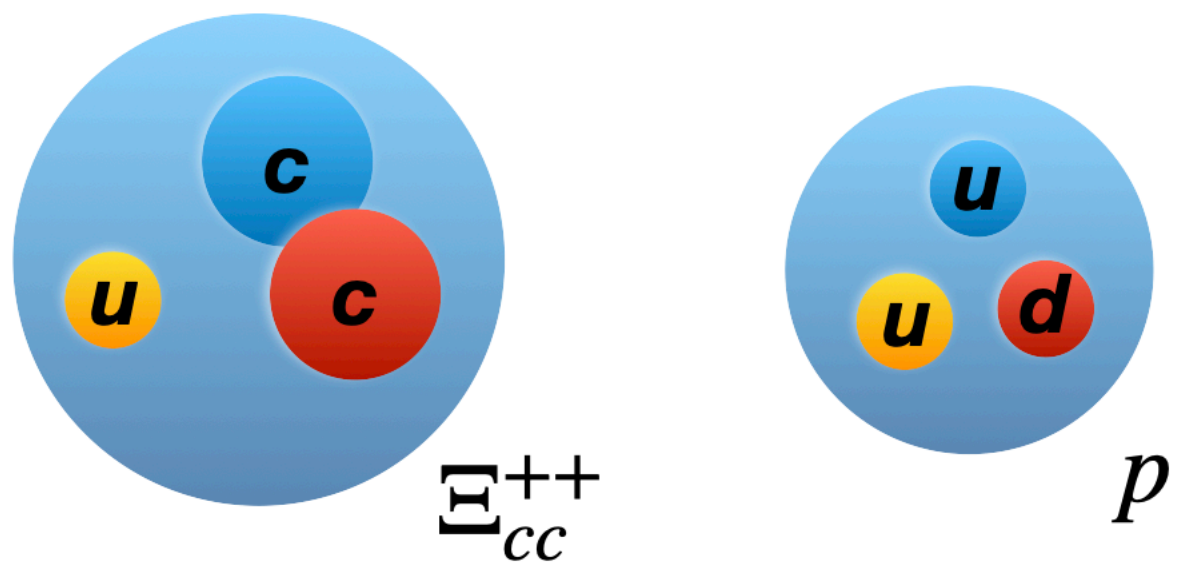
- A heavy 'double-star' system with an attached light 'planet', much different from light baryons. Open a new window for QCD properties

- Two problems in the exp searches: Production and Decay

- Production problem was solved at the beginning of LHC running

- Decay properties are the key problem in the LHCb searches of doubly charmed baryons.

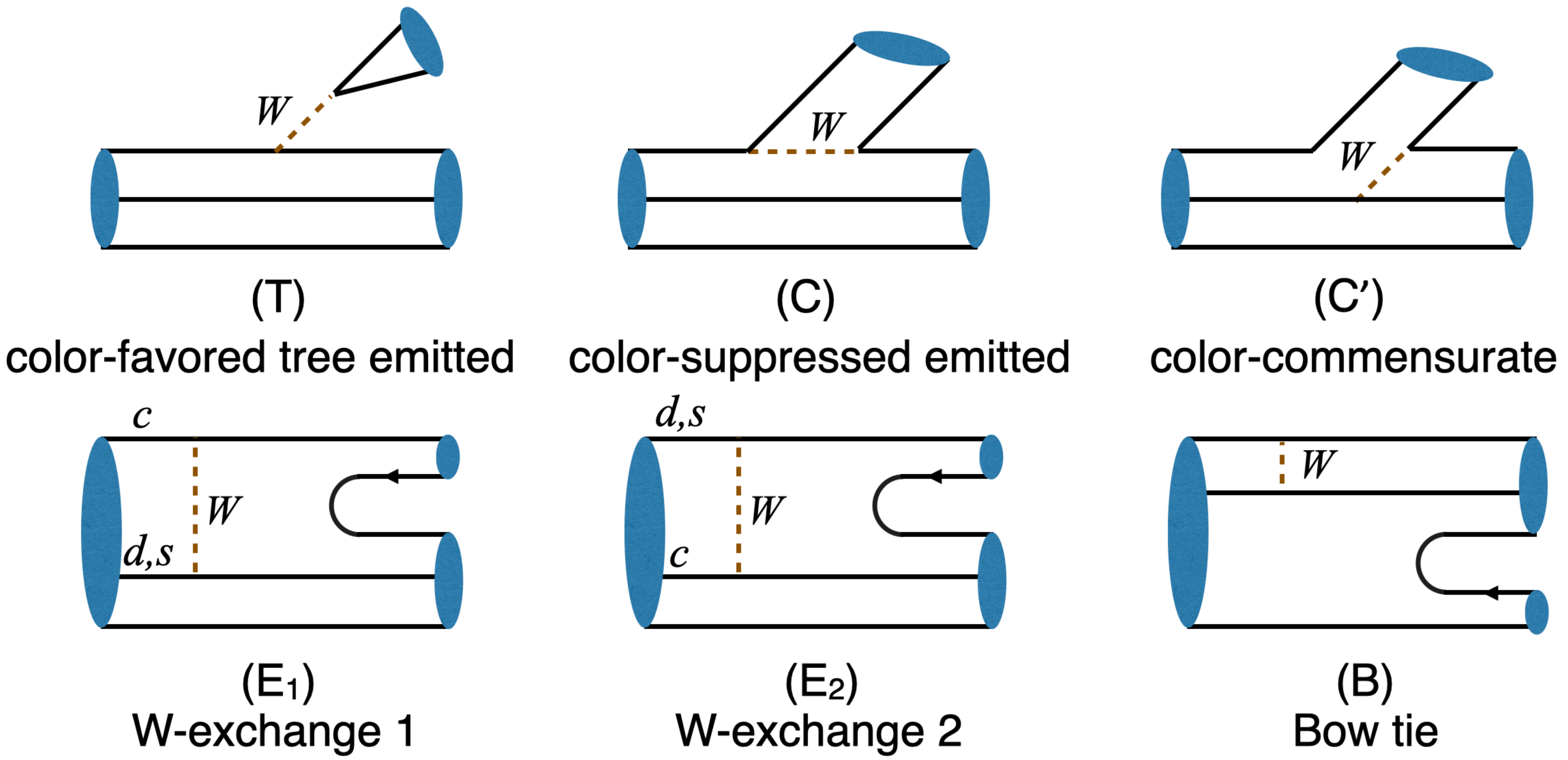
  FSY, Sci.China.PMA, 2020



- Statistics requires: largest branching ratios and easily detected

Branching Fractions: topological diagrams

- Topological diagrammatic approach is suitable for hadronic charm decays
- Include non-perturbative contributions
- It works well in D decays
- It needs experimental data to extract the topological amplitudes
- No experimental data available in double-charm baryons



Chau,’86; Chau,Cheng,’87; Bhattacharya, Rosner, ’08; Cheng, Chiang,’10; Li, Lu, FSY, ‘12

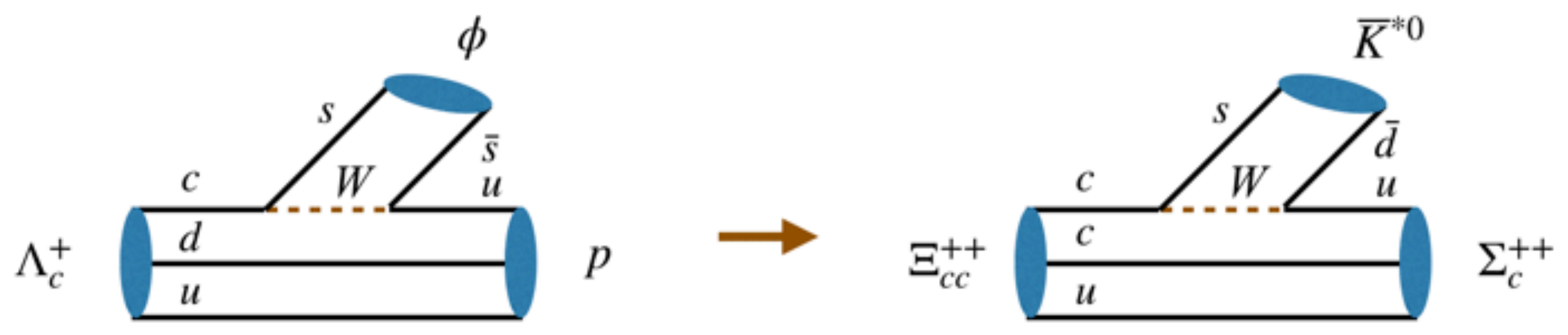
Branching Fractions: topological diagrams

- Hierarchy of topological diagrams in heavy quark expansion

SCET: |C/T|~|C'/T|~|E/T|~$O(\Lambda_{QCD}/m_Q)$ Leibovich, Ligeti, Stewart, Wise, 2004

charm decay: |C/T|~|C'/T|~|E/T|~$O(\Lambda_{QCD}/m_c)$~1

- BESIII measurements on Λ_c^+ decays are important BESIII, 2016



$Br(\Lambda_c^+ \to p\phi)/|V_{us}|^2 = 2\,\%$ → $Br(\Xi_{cc}^{++} \to \Sigma_c^{++}\bar{K}^{*0}) = O(\%)$

↓

$\Xi_{cc}^{++} \to \Lambda_c^+\pi^+K^-\pi^+$

$\Lambda_c^+ \to$

Modes	Representation	$\mathcal{B}_{\rm exp}$
$p\overline{K}^0$	$\lambda_{sd}(C+E)$	$(3.04\pm0.17)\%$
$\Lambda^0\pi^+$	$\lambda_{sd}(T-C'+B-E)/\sqrt{2}$	$(1.24\pm0.08)\%$
$\Delta^{++}K^-$	$\lambda_{sd}E$	$(1.18\pm0.27)\%$

So far dynamics is almost ready

Non-leptonic decays of Ξ_{cc}^{++}

Modes	Br(first)($\times 10^{-3}$)	Br(final)($\times 10^{-3}$)	Representation
$p(D^+/D^0\pi^+)$ 😀	8.	0.2	Ccm Vcd Vud
$p(D_s^+/D^0K^+)$	0.4	0.01	Ccm Vcd Vus
$(pK^-\pi^+/\Sigma^+)(D^+/D^0\pi^+)$ 😀	80.	2.	Ccm Vcs Vud
$(pK^-\pi^+/\Sigma^+)(D_s^+/D^0K^+)$	3.	0.1	Ccm Vcs Vus
$(\Lambda_c^+\pi^+)(\pi^+\pi^-)$	3.	0.2	-(Ct Vcd Vud)
$(\Lambda_c^+\pi^+)(K^+\pi^-)$	0.2	0.008	Ct Vcd Vus
$(\Lambda_c^+\pi^+)(K^-\pi^+)$ 😀	50.	3.	Ct Vcs Vud
$(\Lambda_c^+\pi^+)(K^+K^-)$	2.	0.08	Ct Vcs Vus
$\Lambda_c^+\pi^+$ 😀	30.	1.	Ccb Vcd Vud + T Vcd Vud
$\Lambda_c^+K^+$	1.	0.06	Ccb Vcd Vus + T Vcd Vus
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)\pi^+$ 😀	400.	20.	Ccb Vcs Vud + T Vcs Vud
$(\Lambda_c^+\pi^+K^-/\Xi_c^+)K^+$	20.	0.9	Ccb Vcs Vus + T Vcs Vus

- discovery channels: Br=O($10^{-3\sim-4}$)

$$\boxed{\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+}$$

Talk at LHCb China group in 2016.12 and LHCb charm group in 2017.03

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Discovery potentials of doubly charmed baryons *

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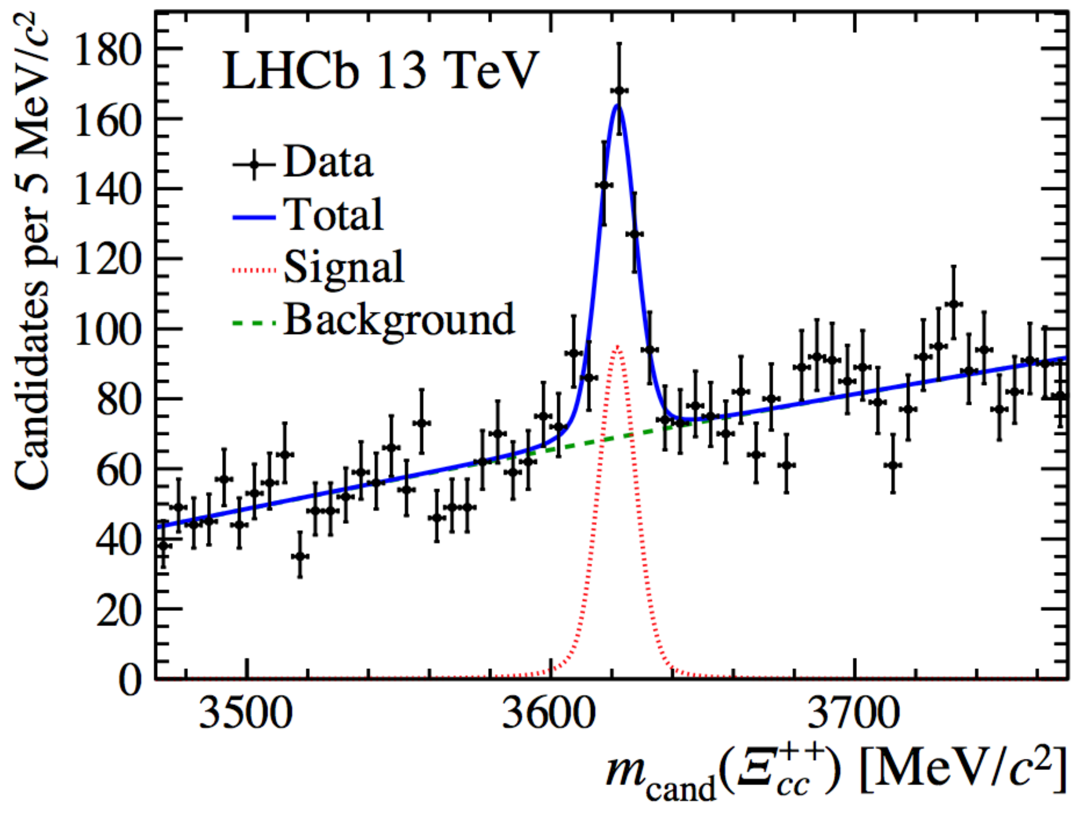
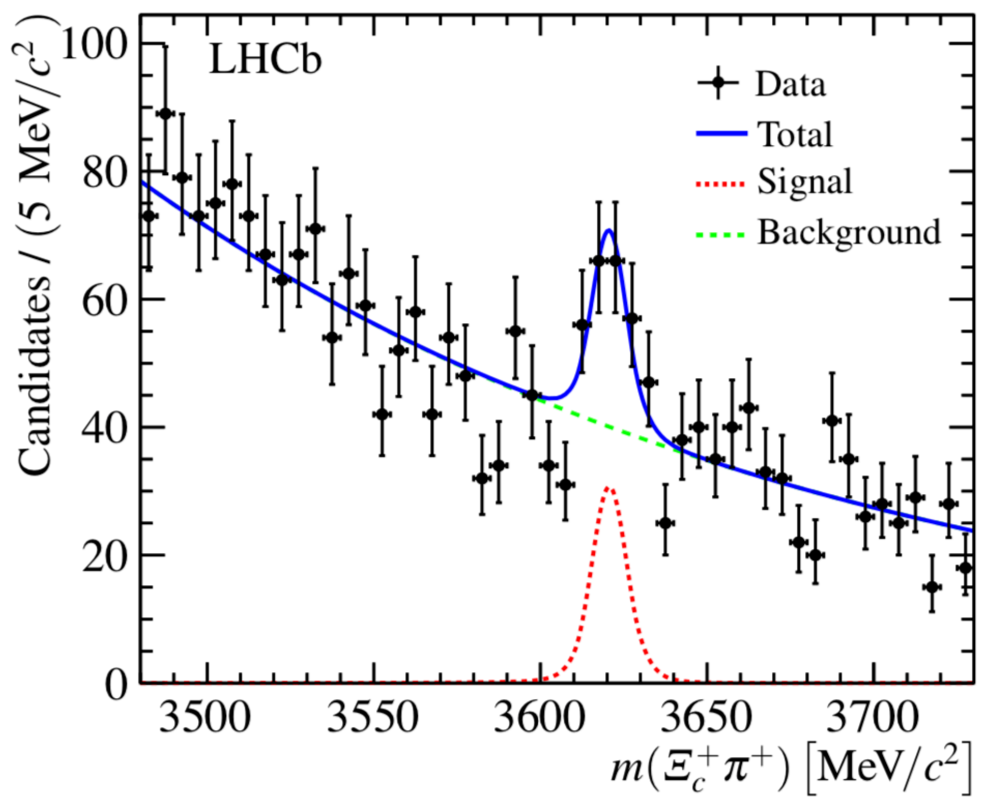
$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_c^+ \pi^+$ are the most favorable decay modes

LHCb 2017



$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

LHCb 2018



$$\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$$

Topological diagrammatic approach successfully predicted the discovery channels of double-charm baryon !!!

What's more?

- Topological diagrammatic approach is powerful:
  successfully predict the charm CPV and Ξ_{cc}^{++} discovery channels. So far so good.
- Currently it is a phenomenological approach, but what is its mathematical foundation?
- Further studies: Deep understanding on the topological approach
  - What is the complete set of topological diagrams?
  - Are they all independent with each other?
  - Can the SU(3) breaking effects be systematically studied?

Topological diagrams = SU(3) irreducible representations