The non-leptonic $\bar{B}^0_{d(s)} \to D^+_{(s)} K^-(\pi^-)$ decays from Light-Cone Sum Rule (LCSR)

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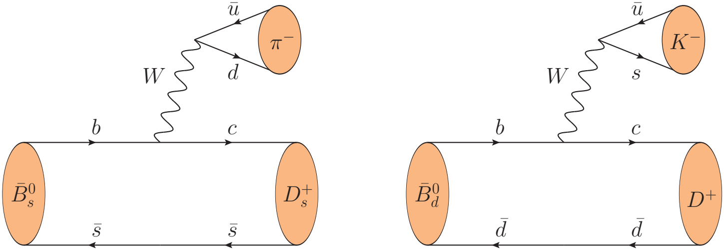
In collaboration with Maria Laura Piscopo

Introduction

The non-leptonic decays $\bar{B}^0_{d(s)} \to D^+_{(s)} K^-(\pi^-)$



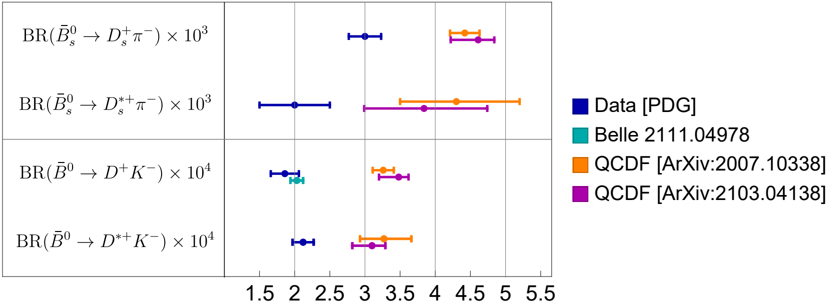
- Tree-level decays induced by $b \to c\bar{u}q$ transitions $q = d, s$
- No penguin and annihilation topologies
- "Golden" modes for the QCD factorisation (QCDF) approach

[Beneke, Buchalla, Neubert, Sachrajda (1999-2001)]

The non-leptonic decays $\bar{B}^0_{d(s)} \to D^+_{(s)} K^-(\pi^-)$

[Bordone, Gubernari, Huber, Jung, van Dyk, arXiv: 2007.10338]

[Cai, Deng, Li, Yang, arXiv: 2103.04138]



- Tension between QCDF predictions and data $2 - 5.5\,\sigma$!

[see talk by Nico Gubernari]

Reasons for discrepancies ?

- Problems with QCDF? Form factors?
  Power corrections (soft-gluon effects)?

- QED corrections (incl. ultra-soft photon emission)?
  [Beneke, Böer, Finauri, Vos, arXiv: 2107.03819]

- Rescattering effects? [Endo, Iguro, Mishima, arXiv: 2109.10811]

- New physics? [Cai, Deng, Li, Yang, arXiv: 2103.04138]
  [Fleischer, Malami, arXiv: 2109.04950]
  [Fleischer, Malami, arXiv: 2110.04240]
  [Iguro, Kitahara, arXiv: 2008.01086]

  ▷ Interplay with collider constraints [Bordone, Grejio, Marzocca, arXiv: 2103.10332]

Non-leptonic decays $\bar{B}^0 \to D^+ P^-$ *from LCSR*

Non-leptonic decays $\bar{B}^0 \to D^+ P^-$

- The Effective Hamiltonian

$q = d, s$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \left(C_1 O_1^q + C_2 O_2^q \right) + \text{h.c.}$$

- The effective $\Delta B = 1$ four-quark operators

$$O_1^q = \left(\bar{c}^i \Gamma^\mu b^i \right) \left(\bar{q}^j \Gamma_\mu u^j \right) \qquad O_2^q = \left(\bar{c}^i \Gamma^\mu b^j \right) \left(\bar{q}^j \Gamma_\mu u^i \right)$$

$\Gamma_\mu = \gamma_\mu (1 - \gamma_5)$

- Use the relation

$$O_2^q = 2\, O_T^q + \frac{1}{N_c} O_1^q$$

$$O_T^q = \left(\bar{c}^i t_{ij}^a \Gamma^\mu b^j \right) \left(\bar{q}^n t_{nm}^a \Gamma_\mu u^m \right).$$

The amplitude

- The amplitude of the $\bar{B}^0_{q'} \to D^+_{q'} P^-$ process $\quad q' = d, s$

$$\mathcal{A}(\bar{B}^0_{q'} \to D^+_{q'} P^-) = -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq} \left[C_1 \langle D^+_q P | O^q_1 | \bar{B}^0_q \rangle + C_2 \langle D^+_q P | O^q_2 | \bar{B}^0_q \rangle \right]$$

$$= -\frac{G_F}{\sqrt{2}} V_{cb} V^*_{uq} \Bigg[\underbrace{\left(C_1 + \frac{C_2}{3} \right) \langle O^q_1 \rangle}_{\equiv \mathcal{A}_{\rm F} + \mathcal{O}(\alpha_s^2)} + \underbrace{2\, C_2\, \langle O^q_T \rangle}_{\equiv \mathcal{A}_{\rm NF} + \mathcal{O}(\alpha_s)} \Bigg]$$

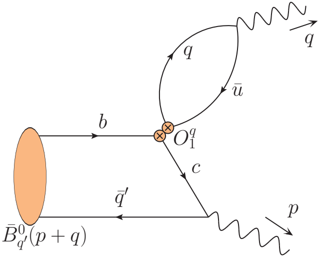
- $R_{\rm NF} \equiv \dfrac{\mathcal{A}_{\rm NF}}{\mathcal{A}_{\rm F}}$ $\quad \langle O^q_X \rangle \equiv \langle D^+_{q'} P | O^q_X | \bar{B}^0_{q'} \rangle$
- Computation at leading order in α_s
- Estimate $\langle O^q_1 \rangle$, $\langle O^q_T \rangle$ and $R_{\rm NF}$ using the LCSR method

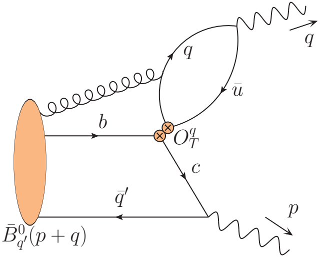
[Balitsky, Braun, Kolesnichenko (1989)]

[Khodjamirian (2000)]

[Khodjamirian, Mannel, Pivovarov, Wang (2010)]

The correlation functions





- Starting point of the LCSR – correlation functions

$$F^{\mu}_{1,T}(p,q) = i^2 \int d^4x \int d^4y \, e^{ip\cdot x} e^{iq\cdot y} \langle 0| \mathrm{T}\{ j^5_D(x), O_{1,T}(0), j^{\mu}_P(y)\} \, |B(p+q)\rangle$$

$$j^5_D(x) = i m_c \, \bar{q}' \gamma_5 c \qquad j^{\mu}_P(y) = \bar{u} \, \gamma^{\mu} \gamma^5 q$$

Light-cone OPE for the correlation functions

- For $p^2 \ll m_c^2$, $q^2 \ll 0 \quad \Rightarrow \quad x^2 \sim y^2 \sim 0$ (light-cone dominance)
- Use the light-cone expansion of the quark propagator up to $G_{\alpha\beta}$
- In the coordinate representation [AR (2017)]

$$
\begin{aligned}
S(x,0)\Big|_{x^2\sim 0} &= -\frac{im^2}{4\pi^2}\left[\frac{K_1(m\sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not{x}}{-x^2}K_2(m\sqrt{-x^2})\right] \\
&- \frac{ig_s}{16\pi^2}\int_0^1 du\Bigg[mK_0(m\sqrt{-x^2})(G(ux)\cdot\sigma) \\
&+ \frac{im}{\sqrt{-x^2}}K_1(m\sqrt{-x^2})\left[\bar{u}\not{x}(G(ux)\cdot\sigma) + u(G(ux)\cdot\sigma)\not{x}\right]\Bigg]
\end{aligned}
$$

$K_n(z)$ – the modified Bessel functions of second kind

- For massless quark one can just take the limit $m \to 0$

Light-cone OPE for the correlation functions

- Use the two- and three-particle B-meson LCDAs up to twist-4

[Braun, Ji, Manashov (2017)]

$$\langle 0|\bar{q}(z)h_v(0)|\bar{B}(v)\rangle\Big|_{z^2\sim 0} \sim \int_0^\infty d\omega\, e^{-i\omega(v\cdot z)} f\big(\underbrace{\{\phi_+, \phi_-, g_+\}(\omega)}_{\equiv\phi(\omega) \quad \text{2p. LCDAs}}\big)$$

$$\langle 0|\bar{q}(z_1)g_s G_{\mu\nu}(z_2)h_v(0)|\bar{B}(v)\rangle\Big|_{z_1^2\sim z_2^2\sim 0} \sim \int_0^\infty d\omega_1 \int_0^\infty d\omega_2\, e^{-i\omega_1(v\cdot z_1)} e^{-i\omega_2(v\cdot z_2)}$$

$$\times f_{\mu\nu}\big(\underbrace{\{\psi_A, \psi_V, X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, W, Z\}(\omega_1,\omega_2)}_{\equiv\psi(\omega_1,\omega_2) \quad \text{3p. LCDAs}}\big)$$

- $\{\psi_A, \psi_V, X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, W, Z\} \quad \Rightarrow \quad \{\phi_3, \phi_4, \psi_4, \tilde{\psi}_4\}$

(with definite twist)

The light-cone OPE results

- Lorentz decomposition of the correlation functions

$$F^{\mu}_{1,T}(p,q) = F_{1,T}(p^2,q^2)\, q^{\mu} + \tilde{F}_{1,T}(p^2,q^2)\, p^{\mu}$$

- One needs only $F_{1,T}(p^2,q^2)$
- The light-cone OPE result can be presented as

$$F_1(p^2,q^2)^{\mathrm{OPE}} = \int_0^{\infty} d\omega_1 \sum_n \sum_{\phi} \frac{g(q^2)\, B^n_{\phi}(\omega_1)\, \phi(\omega_1)}{(m_c^2-\tilde{p}^2)^n}$$

$$F_T(p^2,q^2)^{\mathrm{OPE}} = \int_0^{1} du \int_0^{\infty} d\omega_1 \int_0^{\infty} d\omega_2 \sum_n \sum_{\psi} \frac{C^n_{\psi}(\omega_1,\omega_2,u)\, \psi(\omega_1,\omega_2)}{-\tilde{q}^2\,(m_c^2-\tilde{p}^2)^n}$$

- $\tilde{p}^{\mu} = p^{\mu} - \omega_1 v^{\mu}, \quad \tilde{q}^{\mu} = q^{\mu} - u\omega_2 v^{\mu}$

The light-cone OPE results

- Change of variables: $\omega_1 \to f_1(s_1), \quad \omega_2 \to f_2(s_2)$
- Bringing OPE to the quasi-dispersion form

$$F_1(p^2, q^2)^{\rm OPE} = \int\limits_{m_c^2}^{\infty} ds_1 \sum_n \sum_\phi \frac{g(q^2)\, \bar{B}^n_\phi(s_1)\, \bar{\phi}(s_1)}{(s_1 - p^2)^n}$$

$$F_T(p^2, q^2)^{\rm OPE} = \int\limits_0^1 du \int\limits_{m_c^2}^{\infty} ds_1 \int\limits_0^{\infty} ds_2 \sum_n \sum_\psi \frac{\bar{C}^n_\psi(s_1, s_2, u)\, \bar{\psi}(s_1, s_2)}{(s_2 - q^2)\,(s_1 - p^2)^n}$$

Hadronic dispersion relations

- Start again from the correlation functions

$$F^{\mu}_{1,T}(p,q) = i^2 \int d^4x \int d^4y\, e^{ip\cdot x} e^{iq\cdot y} \langle 0|\mathrm{T}\{j^5_D(x), O_{1,T}(0), j^{\mu}_P(y)\}\,|B(p+q)\rangle$$

- Write down the double hadronic dispersion relation in two steps, first in q^2-channel, then in p^2

$$F_{1,T}(q^2,p^2) = \frac{1}{\pi^2} \int\limits_{m_\pi^2}^{\infty} ds' \int\limits_{m_D^2}^{\infty} ds \frac{\mathrm{Im}_{p^2}\mathrm{Im}_{q^2} F_{1,T}(q^2=s', p^2=s)}{(s'-q^2)(s-p^2)}$$

- Isolate the ground P- and D-meson states in q^2 and p^2 channels
- Use the following matrix elements

$$\langle 0|\, j^{\mu}_P(0)|P(k)\rangle = i\, f_P k^{\mu} \qquad \langle 0|\, j^5_D(0)|D(k)\rangle = m_D^2 f_D$$

Hadronic dispersion relations

- Assume quark-hadron duality

$$\rho_h^P(s') = \frac{1}{\pi}\mathrm{Im}_{q^2} F^{\mathrm{OPE}}(q^2 = s')\Theta(s' - s_0^P)$$
$$\rho_h^D(s) = \frac{1}{\pi}\mathrm{Im}_{p^2} F^{\mathrm{OPE}}(p^2 = s)\Theta(s - s_0^D)$$

- The double Borel Transform: $q^2 \to M'^2$, $p^2 \to M^2$
- Results for $\langle O_1 \rangle$, $\langle O_T \rangle$ from SR

$$\langle O_{1,T} \rangle = \frac{-i}{\pi^2 m_D^2 f_D f_P} \int_0^{s_0^P} ds' \int_{m_c^2}^{s_0^D} ds\, e^{(m_P^2 - s')/M'^2} e^{(m_D^2 - s)/M^2} \underbrace{\rho_{1,T}^{\mathrm{OPE}}(s, s')}_{\mathrm{Im}_{s'}\mathrm{Im}_s F^{\mathrm{OPE}}(s,s')}$$

Very preliminary
numerical results

Very preliminary numerical results

- Consider the $\bar{B}_s^0 \to D_s^+ \pi^-$ decay
- Use the exponential model for B-meson LCDAs

[Braun, Ji, Manashov (2017)]

- Very preliminary estimates

$$\frac{\mathcal{A}_{\rm NF}}{\mathcal{A}_{\rm F}} = [0.1, 2.1]\%$$

$$|\mathcal{A}_{\rm F}| = [1.0, 3.2]\,\mathrm{GeV}^3$$

$$|\mathcal{A}_{\rm NF}| = [0.001, 0.057]\,\mathrm{GeV}^3$$

$$\mathcal{A}_{\rm F} = (C_1 + C_2/3)\langle O_1\rangle$$

$$\mathcal{A}_{\rm NF} = 2C_2\langle O_T\rangle$$

$$\mathrm{BR}(\bar{B}_s^0 \to D_s^+ \pi^-) = [0.8, 8.7] \times 10^{-3}$$

- Large uncertainties due to the scale variation (LO) and parameters $\lambda_{B_s}, \lambda_E^2, \lambda_H^2$

Other previous estimates

- [Block, Shifman (1993)]: estimate of soft-gluon effects in $\bar{B}_d^0 \to D^+\pi^-$ decays using the two-point QCD SR
  - ▷ The correlator $\mathcal{A}^\alpha = 2\int d^4x \langle D|\mathrm{T}\{O_T(x), j_\pi^\alpha(0)\}|\bar{B}\rangle e^{iq\cdot x}$
  - ▷ Short-distance OPE and saturation by the pion
  - ▷ An estimate $\dfrac{\mathcal{A}_{\mathrm{NF}}}{\mathcal{A}_{\mathrm{F}}} \sim 0.13$
- [Halperin (1994)]: estimate of soft-gluon effects in $\bar{B}^0 \to D^0\pi^0$ using the two-point QCD SR
- [Cui, Li (2004)]: estimate of soft-gluon effects in $\bar{B}^0 \to D^0\pi^0$ using LCSR with the π-meson LCDAs

Conclusion

Conclusion

- There are tensions between the QCDF predictions and data on several non-leptonic B-meson decays
- Possible reasons for discrepancy are discussed at this workshop
- LCSR as an alternative method to estimate the soft-gluon effects in non-leptonic B-meson decays and, though with large uncertainties, their branching fractions
- Very preliminary analysis reveals that $\mathcal{A}_{\rm NF}/\mathcal{A}_{\rm F} \sim 1\%$, still small, but one order of magnitude larger than in the previous work

  [Bordone, Gubernari, Huber, Jung, van Dyk, arXiv: 2007.10338]
- Note: the work still in progress! More insights follow