

# Dark Matter in $S_3$ -Symmetric Three-Higgs Doublet Model With Spontaneous CP Violation

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**Anton Kunčinas**

Centro de Física Teórica de Partículas – CFTP and Dept de Física Instituto Superior Técnico – IST,  
Universidade de Lisboa, Portugal

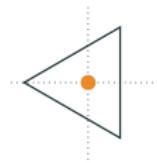
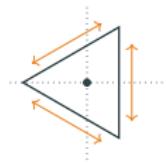
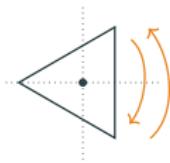
In collaboration with: O. M. Øgreid, P. Osland, M. N. Rebelo  
Based on [2108.07026] and [2204.05684]

**DISCRETE 2022**

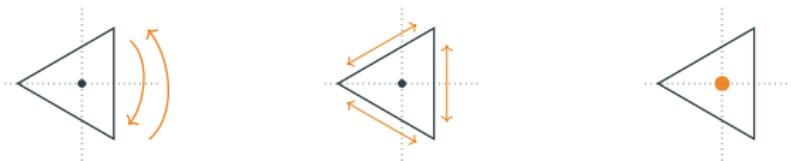
November 8, 2022



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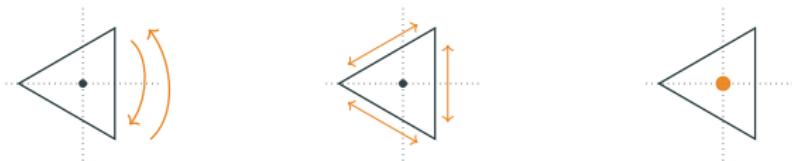


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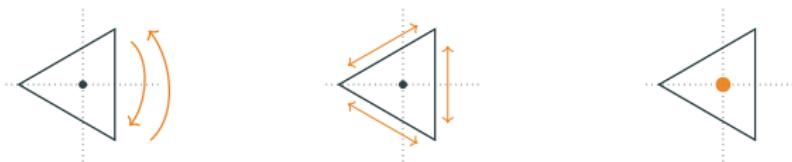
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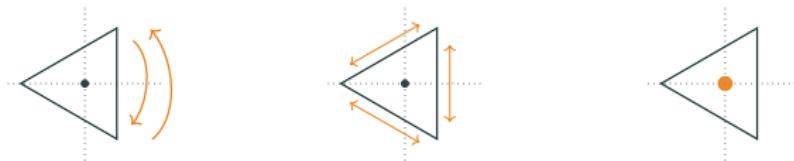


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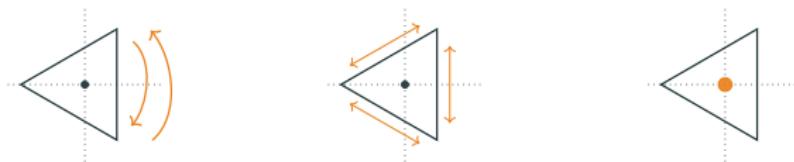
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Possible DM candidates: 3 exact  $V(S_3)$  and 8 softly broken  $V(S_3)$  [App].

## Comparing Two Models

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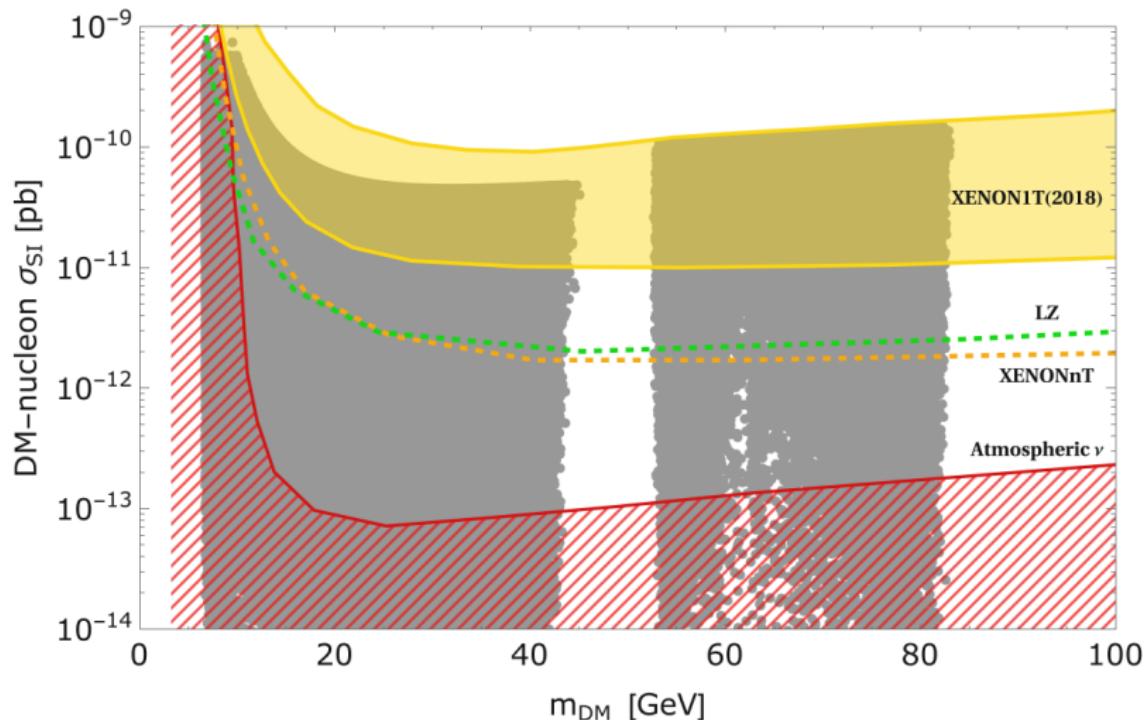
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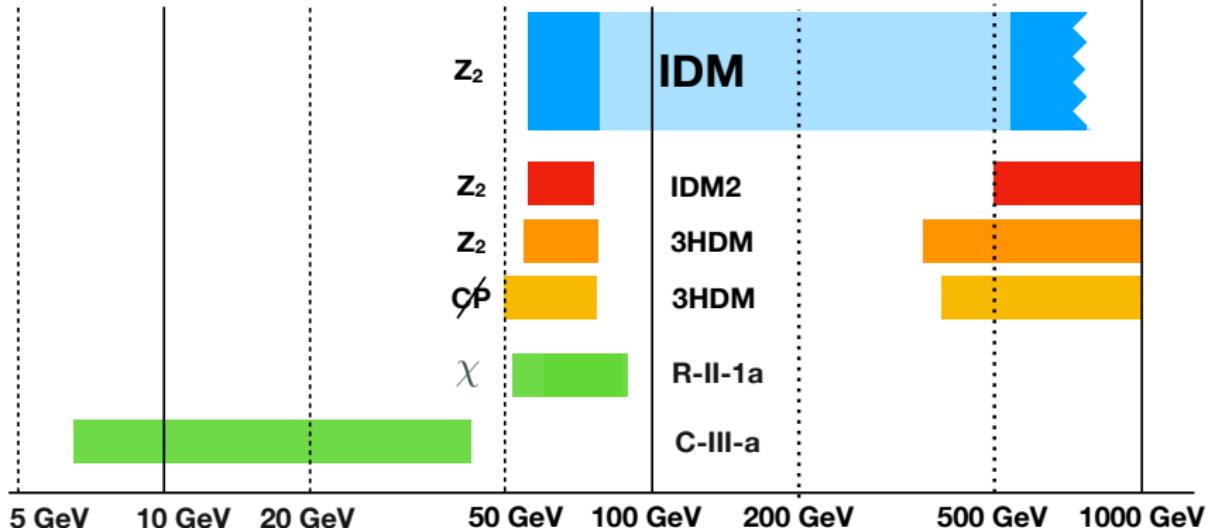
Theoretical and experimental constraints evaluated:

- Cut 1: perturbativity, stability, unitarity checks, LEP constraints;
- Cut 2:  $h \rightarrow \{VV, FF\}$ ,  $S$  and  $T$ ,  $\bar{B} \rightarrow X(s)\gamma$ ;
- Cut 3:  $h \rightarrow \{\text{invisible}, \gamma\gamma\}$ ,  $\Omega_{\text{CDM}} h^2$ , direct searches;

# Results: Direct Detection of Dark Matter



## SCALAR DM MASS RANGES

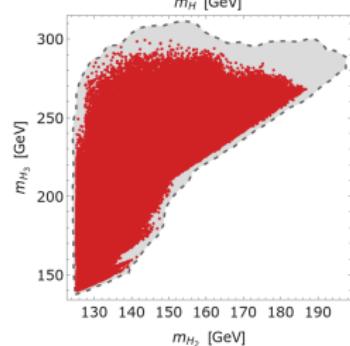
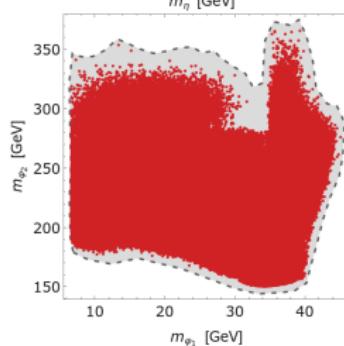
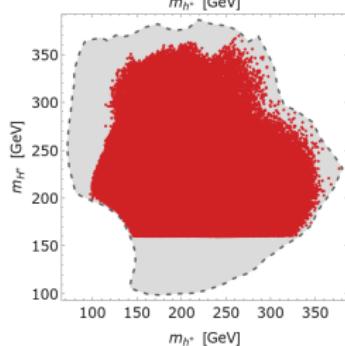
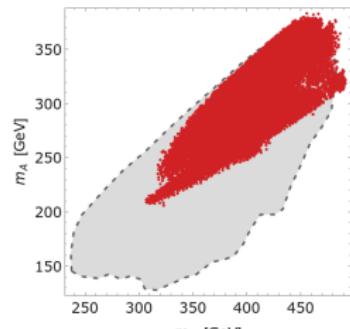
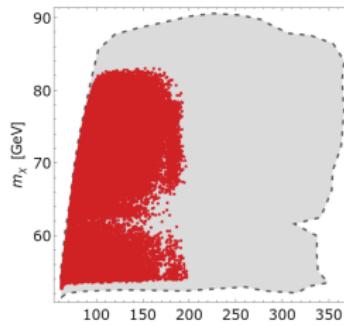
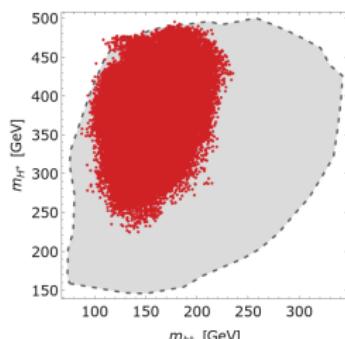


IDM: (A. Belyaev, G. Cacciapaglia, I. P. Ivanov, F. Rojas-Abatte, M. Thomas),  
(J. Kalinowski, W. Kotlarski, T. Robens, D. Sokolowska, A. F. Zarnecki);

IDM2 (one inert doublet): (M. Merchant, M. Sher);

3HDM and ~~CP~~-3HDM (two inert doublets): (A. Cordero-Cid, J. Hernández-Sánchez, V. Keus, S. F. King, S. Moretti,  
D. Rojas, D. Sokolowska)

# HiggsTools (HiggsBounds and HiggsSignals)



Cut 3

HiggsTools

# Indirect Dark Matter Detection

Generalised NFW profile with  $\rho = 0.3 \text{ GeV/cm}^3$ .

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C-III-a:  $\text{Br}(\varphi_1\varphi_1 \rightarrow b\bar{b}) \in \{0.8; 0.92\}$ ,  
 $\text{Br}(\varphi_1\varphi_1 \rightarrow \tau^-\tau^+) \in \{0.04; 0.18\}$ .

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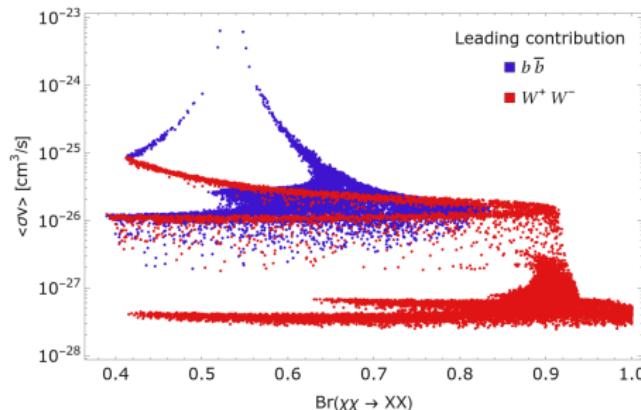
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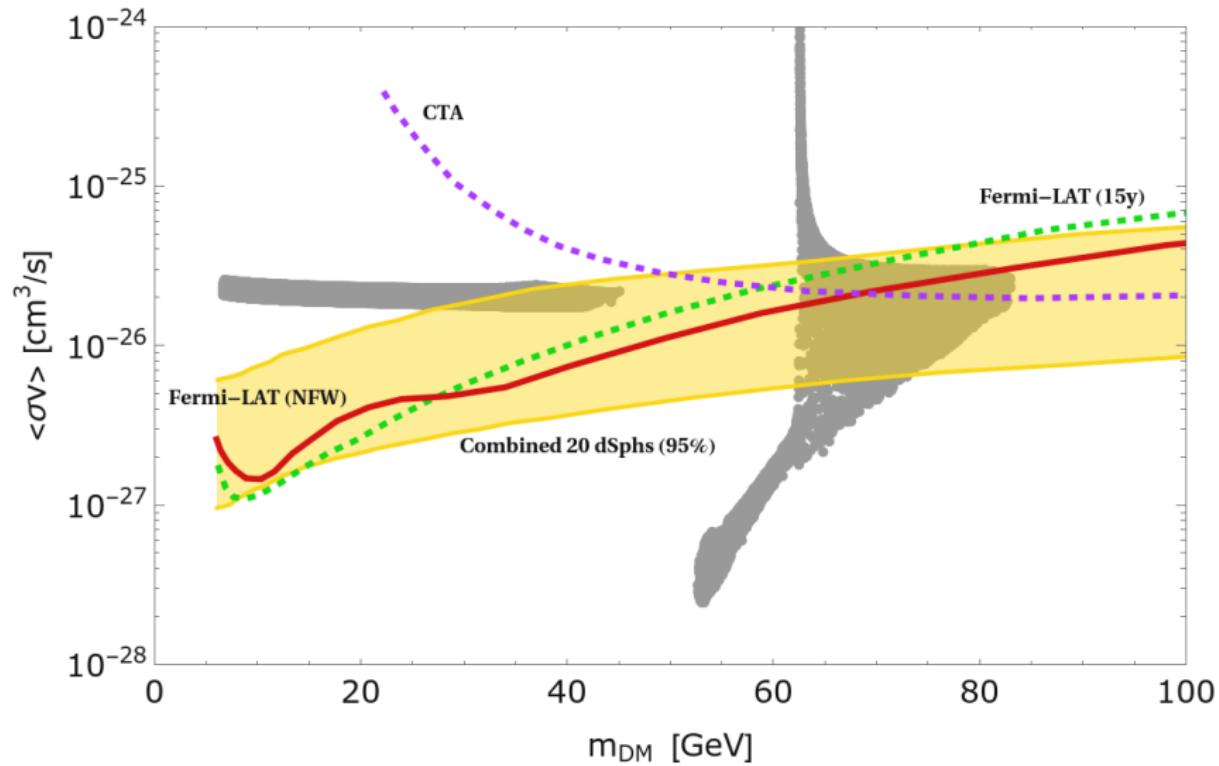
$$\text{Br}(\chi\chi \rightarrow b\bar{b}) \in \{0.38; 0.84\},$$
$$\text{Br}(\chi\chi \rightarrow \{gg, \tau^-\tau^+, W^-W^+\}) \in \{0.08; 0.45\},$$

R-II-1a: or

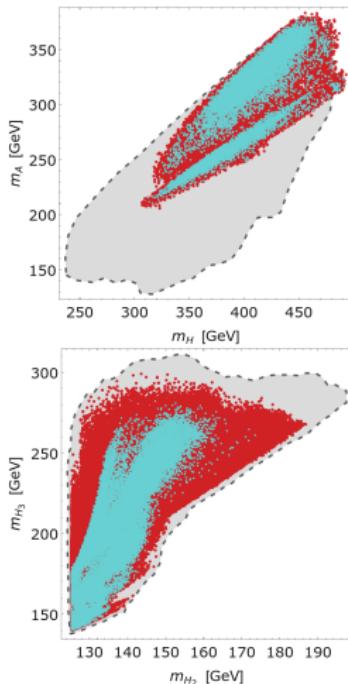
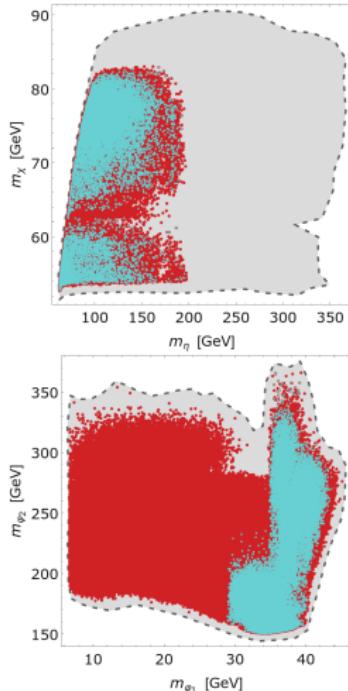
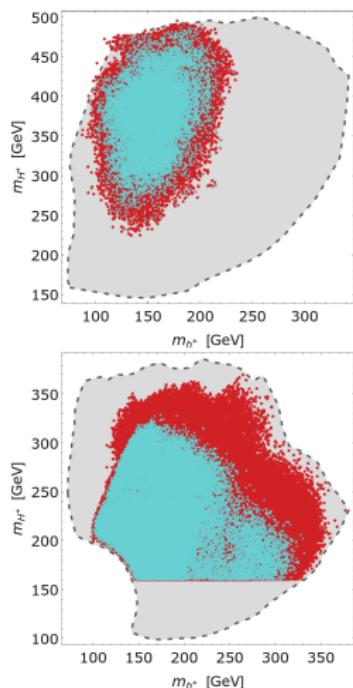
$$\text{Br}(\chi\chi \rightarrow W^-W^+) \in \{0.38; 0.99\},$$
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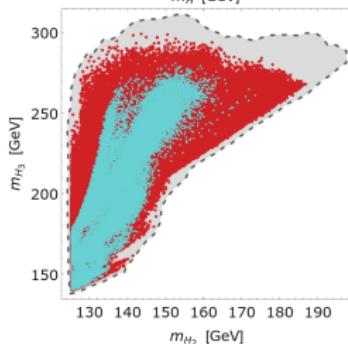
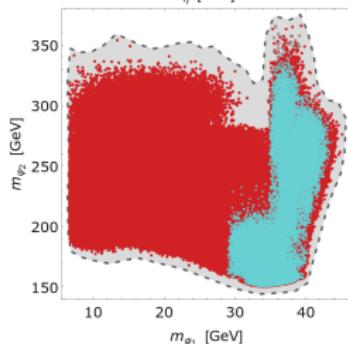
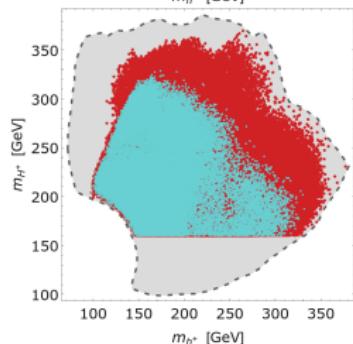
# Indirect Dark Matter Detection



# HiggsTools (HiggsBounds and HiggsSignals)



R-II-1a



C-III-a

Cut 3

HiggsTools

Indirect DM detection,  
XENONnT + LZ,  
 $\text{Br}(h \rightarrow \text{inv.}) \leq 0.10$ .

# Probing $Hf\bar{f}$ Coupling CP Properties

$$\mathcal{L} \supset -\frac{m_f}{v} \overline{\psi_f} (\kappa_f + i\gamma_5 \tilde{\kappa}_f) \psi_f.$$

ATLAS:  $\alpha = \arctan(\tilde{\kappa}_f/\kappa_f)$ , with  $|\alpha| \in \{0; 43^\circ\}$ ,

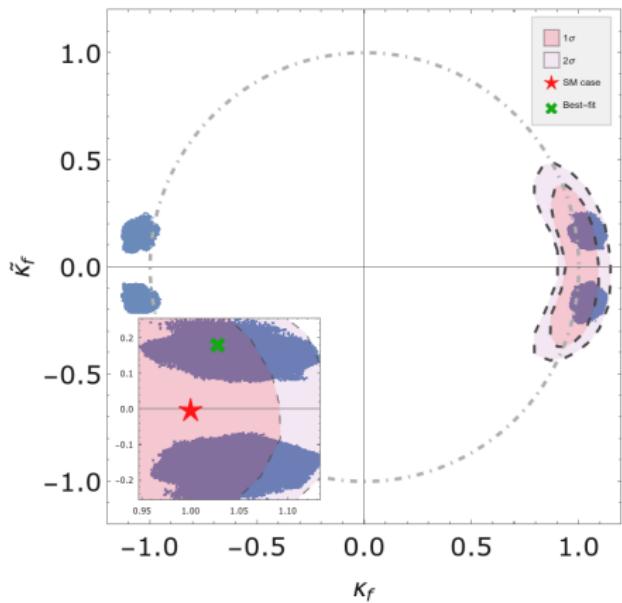
CMS:  $f_{CP} = \frac{|\tilde{\kappa}_f|^2}{|\kappa_f|^2 + |\tilde{\kappa}_f|^2} \times \text{sign}(\tilde{\kappa}_f/\kappa_f)$ , with  $f_{CP} = 0.00 \pm 0.33$ .

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C-III-a:  $|\alpha| \in \{3.34^\circ; 14.91^\circ\}$ ,  
 $f_{CP} \in \{-0.07; 0.06\}$ .

## What Else?

$$\Gamma^{\text{tot}} = \begin{cases} \text{C-III-a: } \mathcal{O}(\text{GeV}), \\ \text{R-II-1a: } \in \{10^{-12}; 100\} \text{ GeV.} \end{cases}$$

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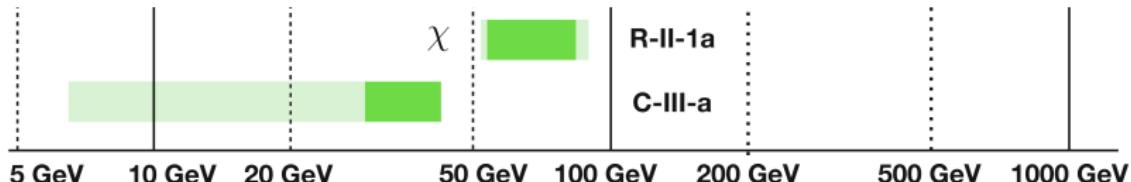
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Plans: measure  $\cancel{CP}$  via EDMs for C-III-a.

# Conclusions

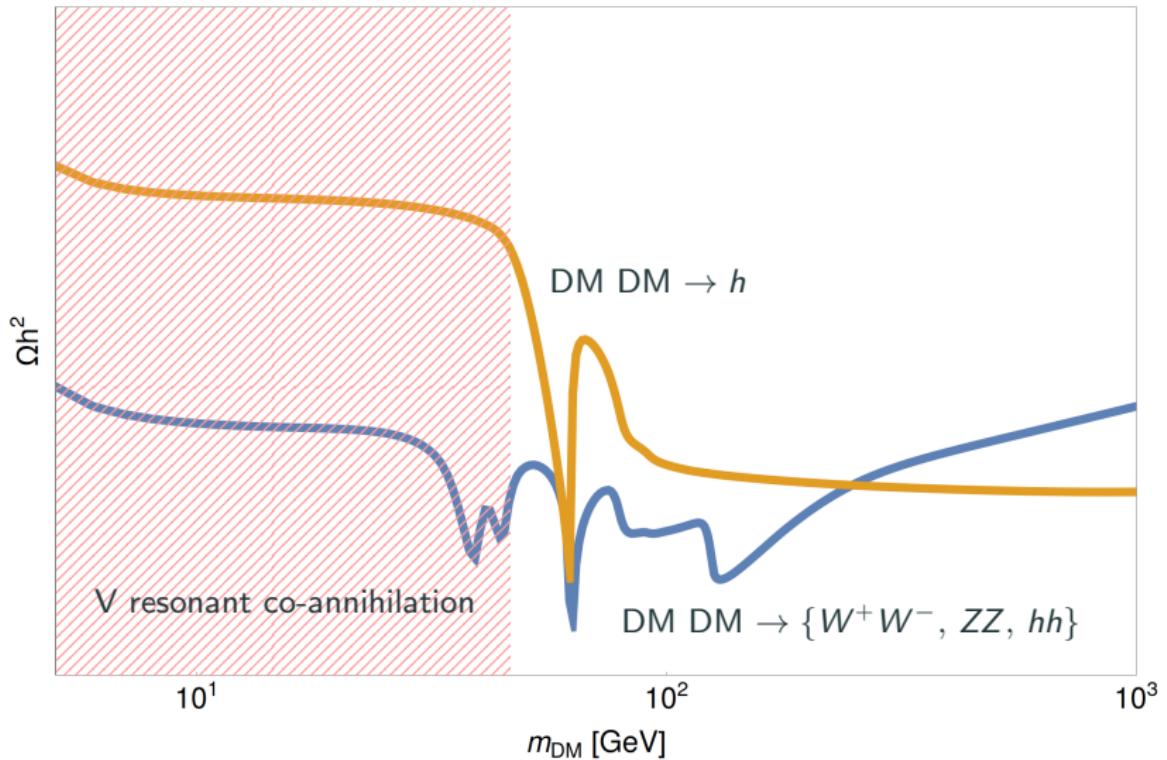
- Multi-Higgs-doublet models are phenomenologically rich and can accommodate a dark matter candidate;
- Viable dark matter regions: R-II-1a {52; 83} GeV, C-III-a {29; 44} GeV;



Work supported by the Fundação para a Ciência e a Tecnologia (FCT, Portugal) PhD fellowship with reference UI/BD/150735/2020 as well as through the FCT projects CERN/FIS-PAR/0002/2021, UIDB/00777/2020, UIDP/00777/2020, CERN/FIS-PAR/0008/2019, PTDC/FIS-PAR/29436/2017.



## Appendix: Inert Doublet Model Relic Density



## Appendix: Yukawa Interactions

Whenever  $w_S \neq 0$  we can construct a trivial Yukawa sector:

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} (y_{ij}^u) w_S^*, \quad \mathcal{M}_d = \dots$$

Fermions can transform non-trivially under  $S_3$ :

$$\mathbf{2} : (Q_1 \ Q_2)^T, (u_{1R} \ u_{2R})^T, (d_{1R} \ d_{2R})^T \quad \text{and} \quad \mathbf{1} : Q_3, u_{3R}, d_{3R},$$

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_1^* \\ y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & y_4^u w_2^* \\ y_5^u w_1^* & y_5^u w_2^* & y_3^u w_S^* \end{pmatrix}, \quad \mathcal{M}_d = \dots$$

$$\mathbf{2} : Q_3, \mathbf{1}' u_{3R} : \quad \mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_2^* \\ y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & -y_4^u w_1^* \\ y_5^u w_2^* & -y_5^u w_1^* & y_3^u w_S^* \end{pmatrix},$$

$$\mathbf{1} : Q_3, \mathbf{1}' : u_{3R} : \quad \mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_2^* \\ y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & -y_4^u w_1^* \\ y_5^u w_1^* & y_5^u w_2^* & 0 \end{pmatrix}.$$

## Appendix: Continuous Symmetries

Massless state:

$$\begin{aligned}\mathcal{V}(Uh) &= \mathcal{V}(h), \\ \langle 0 | (Uh) | 0 \rangle &\neq \langle 0 | h | 0 \rangle.\end{aligned}$$

Results of [2001.01994]:

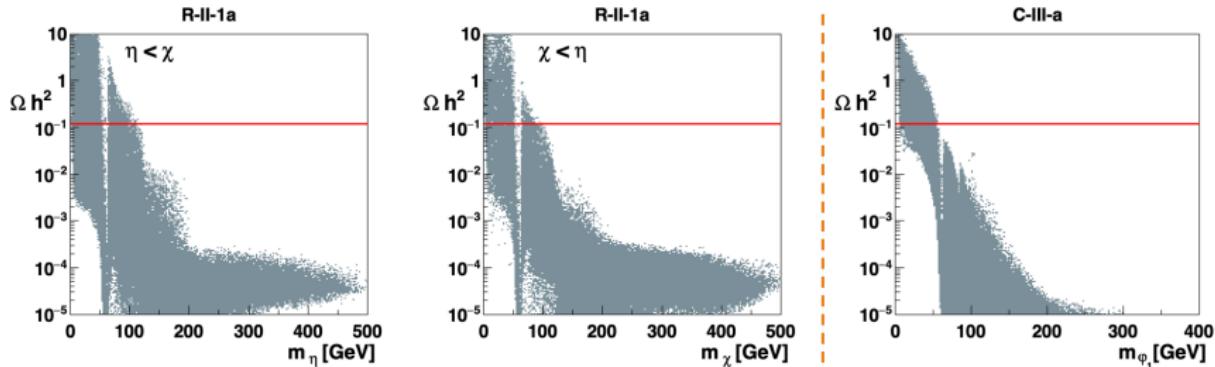
Constraints	Continuous symmetries	# of massless states
$[\lambda_4 = 0]$	$O(2)$	1
$\cdots + [\lambda_7 = 0]$	$O(2) \otimes U(1)_{h_S}$	2
$\cdots + [\lambda_2 + \lambda_3 = 0]$	$SU(2)$ $[ O(2) \otimes U(1)_{h_1} \otimes U(1)_{h_2} \otimes U(1)_{h_S} ]$	3

## Appendix: Dark Matter in $S_3$ -Symmetric Three-Higgs-Doublet Models

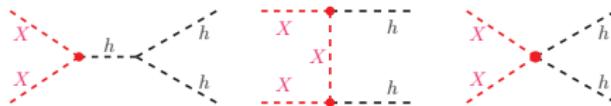
Vacuum	vevs	$\lambda_4$	symmetry	# massless states	fermions under $S_3$
R-I-1	(0, 0, $w_S$ )	✓	$S_3, h_1 \rightarrow -h_1$	none	trivial
R-I-2a	( $w$ , 0, 0)	✓	$S_2$	none	non-trivial
R-I-2b,2c	( $w, \pm\sqrt{3}w, 0$ )	✓	$S_2$	none	non-trivial
R-II-1a	(0, $w_2$ , $w_S$ )	✓	$S_2, h_1 \rightarrow -h_1$	none	trivial
R-II-2	(0, $w$ , 0)	0	$h_1 \rightarrow -h_1, h_S \rightarrow -h_S$	1	non-trivial
R-II-3	( $w_1, w_2, 0$ )	0	$h_S \rightarrow -h_S$	1	non-trivial
R-III-s	( $w_1, 0, w_S$ )	0	$h_2 \rightarrow -h_2$	1	trivial
C-I-a	( $\hat{w}_1, \pm i\hat{w}_1, 0$ )	✓	cyclic $\mathbb{Z}_3$	none	non-trivial
C-III-a	(0, $\hat{w}_2 e^{i\sigma_2}$ , $\hat{w}_S$ )	✓	$S_2, h_1 \rightarrow -h_1$	none	trivial
C-III-b	( $\pm i\hat{w}_1, 0, \hat{w}_S$ )	0	$h_2 \rightarrow -h_2$	1	trivial
C-III-c	( $\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0$ )	0	$h_S \rightarrow -h_S$	2	non-trivial
C-IV-a	( $\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S$ )	0	$h_2 \rightarrow -h_2$	2	trivial

Possible DM candidates: 3 (exact  $S_3$ ) and 8 (softly broken  $S_3$ ) solutions.

## Appendix: Relic Density

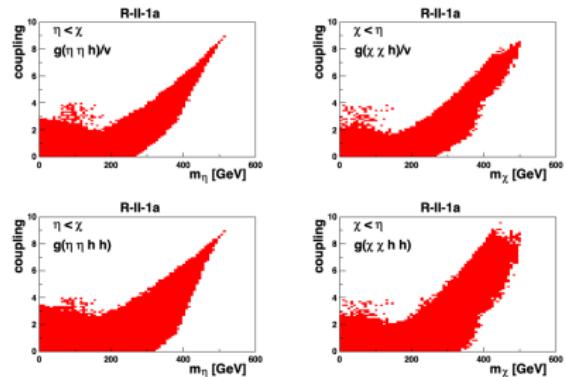


Scans performed using micrOMEGAs 5.3.35.



Trilinear and quartic couplings are not tuneable!

$$\frac{g(XXh)}{v} \Big|_{SM} = g(XXhh) \Big|_{SM} = \frac{1}{v^2} [m_h^2 + 2m_X^2].$$



## Appendix: SU(2) Doublets in Terms of the Mass Eigenstates

R-II-1a:

$$h_1 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}(\eta + i\chi) \end{pmatrix},$$

$$h_2 = \begin{pmatrix} \sin \beta G^+ - \cos \beta H^+ \\ \frac{1}{\sqrt{2}}(\sin \beta v + \cos \alpha h - \sin \alpha H + i(\sin \beta G^0 - \cos \beta A)) \end{pmatrix},$$

$$h_S = \begin{pmatrix} \cos \beta G^+ + \sin \beta H^+ \\ \frac{1}{\sqrt{2}}(\cos \beta v + \sin \alpha h + \cos \alpha H + i(\cos \beta G^0 + \sin \beta A)) \end{pmatrix}.$$


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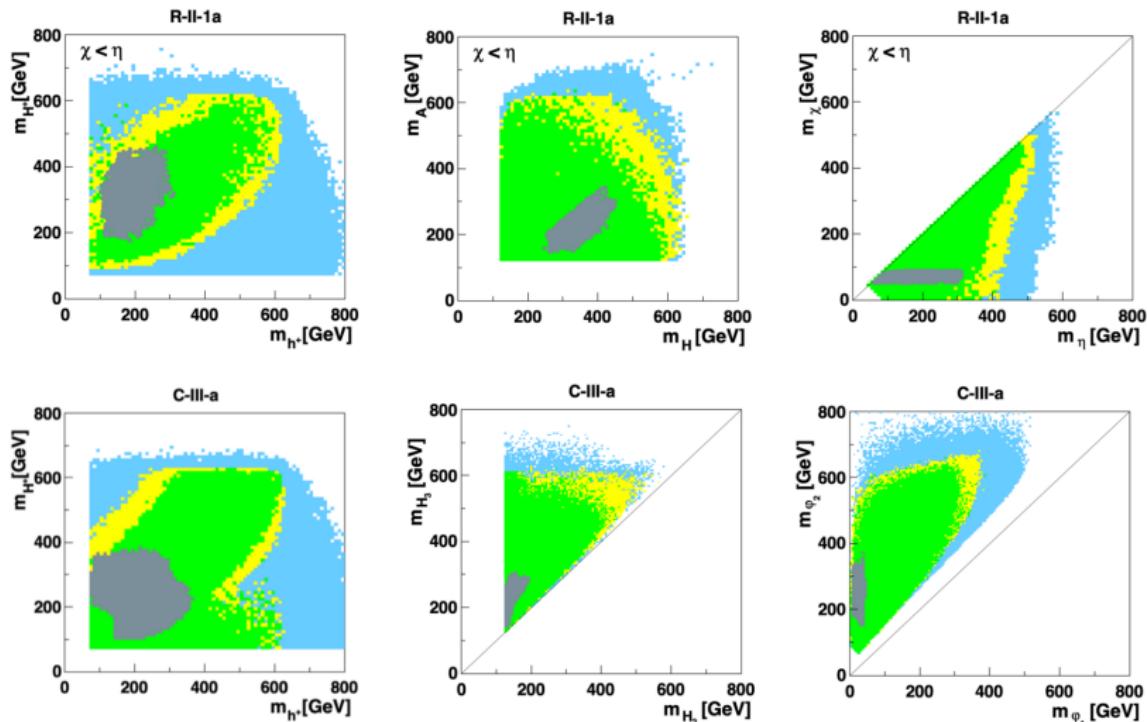
C-III-a:

$$h_1 = e^{i\gamma} \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2) \end{pmatrix},$$

$$h_2 = e^{i\sigma} \begin{pmatrix} \sin \beta G^+ - \cos \beta H^+ \\ \frac{1}{\sqrt{2}}(\sin \beta v + i \sin \beta G^0 + \sum_{i=1}^3 [\sin \beta \mathcal{R}_{i1}^0 - \cos \beta (\mathcal{R}_{i2}^0 + i\mathcal{R}_{i3}^0)] H_i) \end{pmatrix},$$

$$h_S = \begin{pmatrix} \cos \beta G^+ + \sin \beta H^+ \\ \frac{1}{\sqrt{2}}(\cos \beta v + i \cos \beta G^0 + \sum_{i=1}^3 [\cos \beta \mathcal{R}_{i1}^0 + \sin \beta (\mathcal{R}_{i2}^0 + i\mathcal{R}_{i3}^0)] H_i) \end{pmatrix}.$$

## Appendix: Scalar Masses



Cut 1

Cut 2:  $3 - \sigma$

Cut 2:  $2 - \sigma$

Cut 3