

# Higher-Order Electroweak Contributions to Indirect $CP$ Violation

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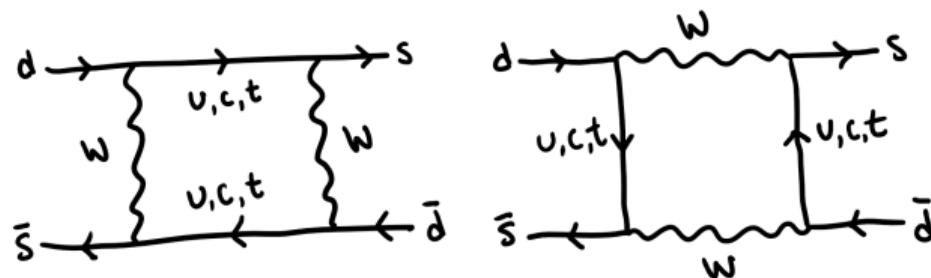
Based on 2108.00017 with J. Brod and S. Kvedaraitė  
and 2207.07669 with J. Brod, S. Kvedaraitė, and A. Youssef

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# $CP$ Violation in Neutral Kaons

- ▶  $CP$  violation first discovered in decays of kaons (1964)
- ▶  $K^0 - \bar{K}^0$  mixing can lead to (indirect)  $CP$ -violation  $\rightarrow$  parameterized by  $\epsilon_K$
- ▶ Sensitive probe of new physics, input for global CKM fit



# Diagonalizing the Hamiltonian

- ▶ Time evolution of  $K^0 - \bar{K}^0$  system described by

$$i \frac{d}{dt} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix} = \left( M - \frac{i}{2}\Gamma \right) \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

- ▶ Diagonalized by linear combinations

$$|K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle, \quad |K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

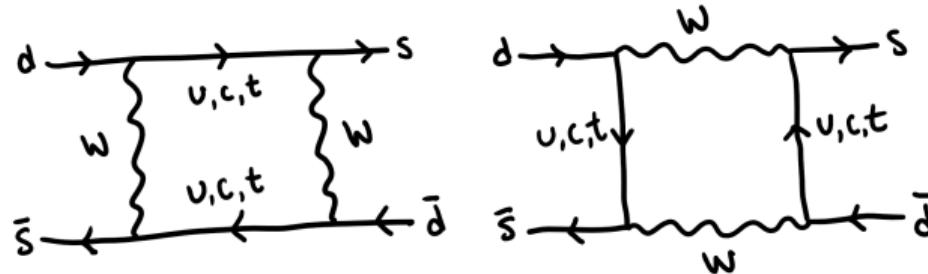
- ▶ If  $|p/q| = 1$ ,  $K_L$  and  $K_S$  CP eigenstates  $\Rightarrow K_L$  (CP odd)  $\rightarrow \pi\pi$  (CP even) forbidden

## Definition of $\epsilon_K$

- Indirect  $CP$  violation suppressed by GIM  $\rightarrow$  sensitive probe of high energies

$$\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle}$$

- Experimentally<sup>1</sup>:  $|\epsilon_K|_{\text{ex}} = 2.228 \pm 0.011 \times 10^{-3}$



<sup>1</sup>PDG 2022.

# Why Electroweak?

- ▶ Re-parameterize Hamiltonian using CKM unitarity

$$\mathcal{H}_{n_f=3}^{|\Delta S|=2} = \frac{G_F^2 M_W^2}{4\pi^2} [\lambda_c^2 C_{S2}''^{cc}(\mu) + \lambda_t^2 C_{S2}''^{tt}(\mu) + \lambda_c \lambda_t C_{S2}''^{ct}(\mu)] Q_{S2}'' + h.c. + \dots$$

$$\mathcal{H}_{n_f=3}^{|\Delta S|=2} = \frac{G_F^2 M_W^2}{4\pi^2} [\lambda_u^2 C_{S2}''^{uu}(\mu) + \lambda_t^2 C_{S2}''^{tt}(\mu) + \lambda_u \lambda_t C_{S2}''^{ut}(\mu)] Q_{S2}'' + h.c. + \dots$$

- ▶ Drastic reduction of perturbative theory errors<sup>23</sup>

$$|\epsilon_K|_{\text{th}} = 1.81(16)_{\text{pert}}(5)_{\text{non-pert}}(23)_{\text{param}} \times 10^{-3} \quad \rightarrow \quad |\epsilon_K|_{\text{th}} = 2.16(6)_{\text{pert}}(7)_{\text{non-pert}}(15)_{\text{param}} \times 10^{-3}$$

- ▶ Perturbative errors  $\lesssim$  expected e/w effects  $\rightarrow$  relevant!

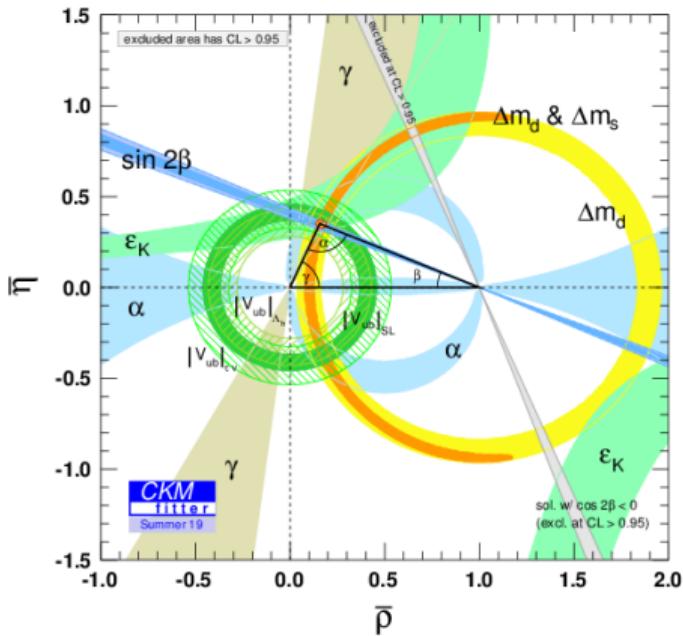
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<sup>2</sup>Brod, Gorbahn, Stamou, Phys.Rev.Lett. 125 (2020) 17, 171803, 1911.06822 [hep-ph].

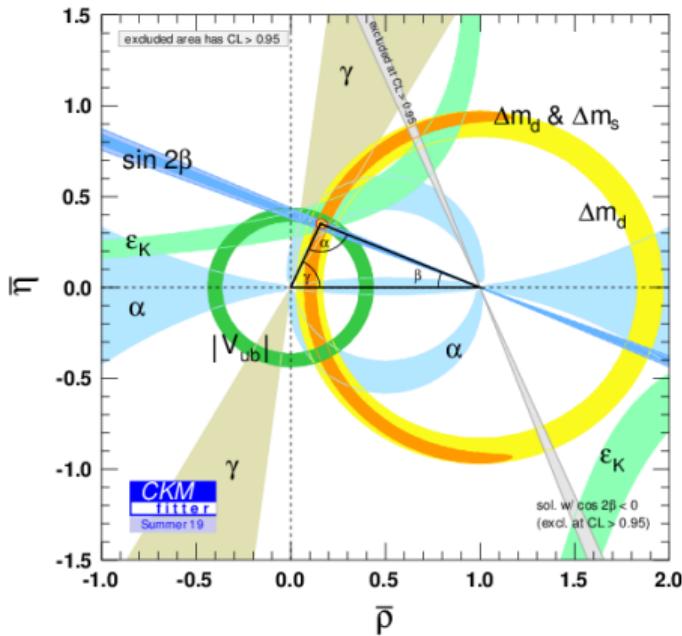
<sup>3</sup>Brod, Gorbahn, Phys.Rev.Lett. 108 (2012) 121801, 1108.2036 [hep-th].

# Reduction of Perturbative Error

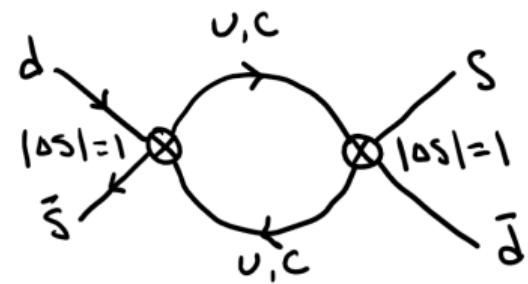
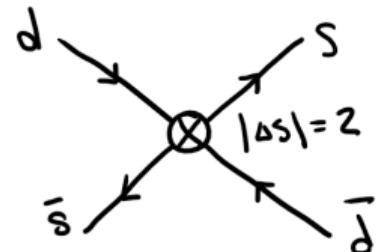
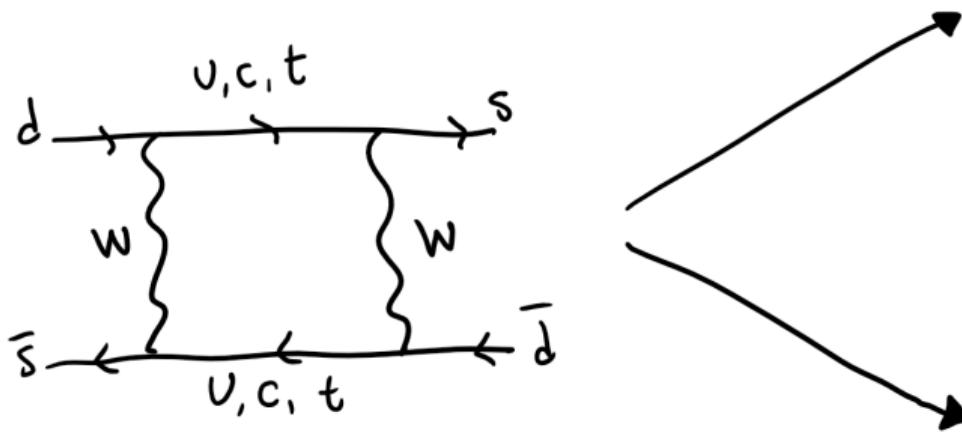
## Before



## After

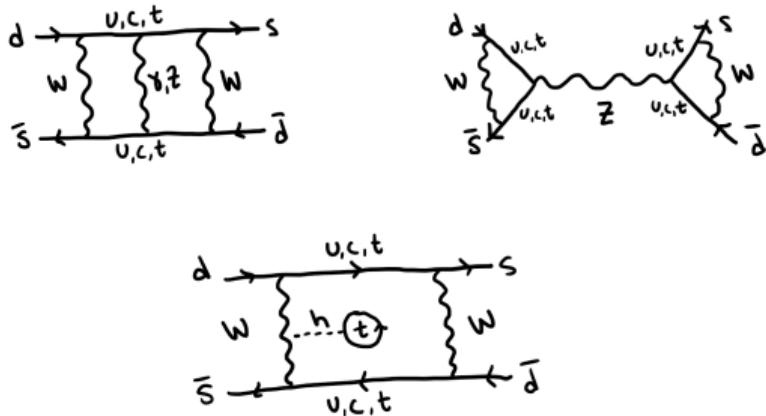


# Different Contributions in EFT



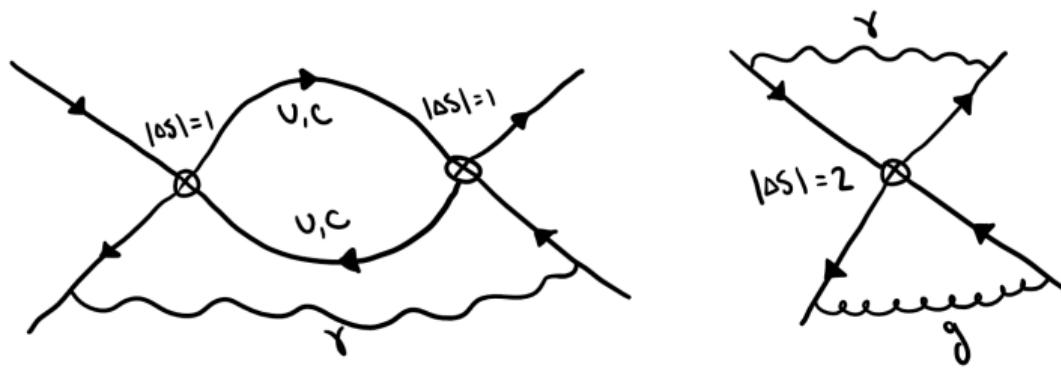
# Top Quark Contribution: E/W Matching

- ▶ Calculate all  $O(30,000)$  two-loop Feynman diagrams ( $p_{\text{ext}} = 0$ )
- ▶ Diagrams generated by qgraf and calculated using self-written FORM routines
- ▶ Separately calculated and verified agreement



## Charm-Top Contribution: Exercise in EFTs

- ▶ Generated via mixing of  $2 \times |\Delta S| = 1 \rightarrow |\Delta S| = 2$  below E/W scale
- ▶ Threshold corrections when decoupling at quark mass scales,  $\mu_b$  and  $\mu_c$



# E/W Contribution to $\epsilon_K$

- ▶ Previous value with QCD-only<sup>4</sup>

$$|\epsilon_K|_{\text{th}} = 2.16(6)_{\text{pert}}(7)_{\text{non-pert}}(15)_{\text{param}} \times 10^{-3}$$

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- ▶ QCD + E/W Top-quark contribution

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<sup>5</sup>Ciuchini et. al, 2111.05153

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# EFT Scheme Dependence

- ▶ Any physical observable must be independent of renormalization scheme!!!
- ▶ Both top and charm contributions are scheme-dependent → cancel with  $\langle Q_{S2} \rangle$  (lattice)
- ▶ Scheme dependence is numerically tiny ( $\lesssim$  order magnitude of E/W shifts) and effect neglected to approximate  $\epsilon_K$
- ▶ Fully consistent determination only possible with lattice QED determination

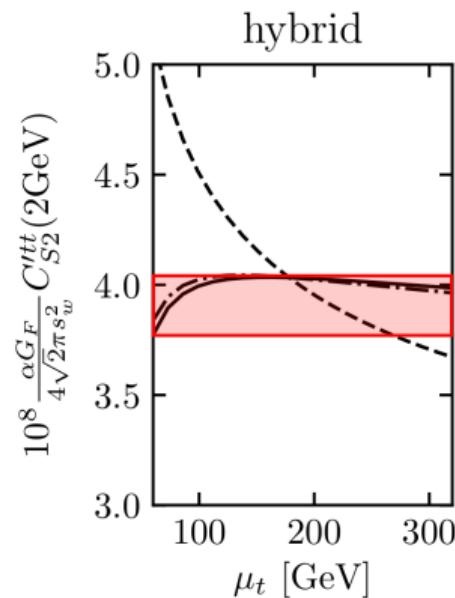
## Conclusions

- ▶ Calculated NLO E/W corrections to  $\epsilon_K$
- ▶ Analogous top calculation:  $B^0 - \bar{B}^0$  system<sup>6</sup> (First independent re-calculation, reproduced numerics)
- ▶ Seeing many  $\mathcal{O}(1\%)$  shifts: important given reduced error
- ▶ Future three-loop QCD top contributions and possible updated lattice calculations will reduce theory uncertainty of  $\epsilon_K$

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<sup>6</sup>Gambino, Kwiatkowski, Pott, 9810400

# Perturbative Theory Error



- ▶ Higher order effects “leak in” (e.g. solving RGEs)
- ▶ Residual scale variation give estimation of size of these effects → Perturbative error