CONSISTENT KINETIC MIXING

Based on arXiv:2207.00023 with Martin Bauer
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Patrick Foldenauer
IFT UAM-CSIC Madrid

CSIC
Consejo Superior de Investigaciones Científicas

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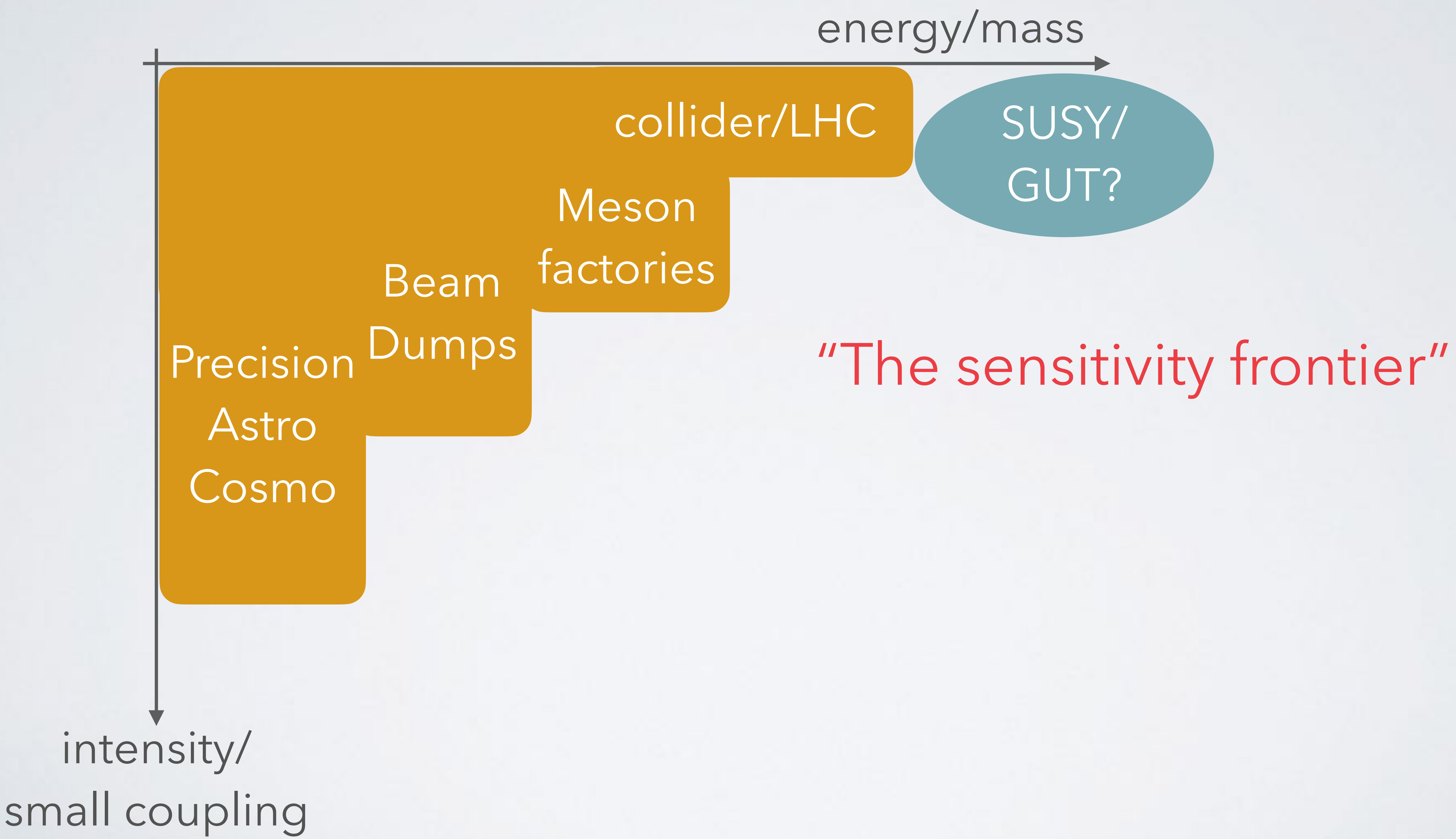
ift Instituto de Física Teórica UAM-CSIC

WHERE TO LOOK FOR BSM

- Many UV theories predict heavy new states with sizeable couplings (e.g. SUSY, GUTs, String Models, …)



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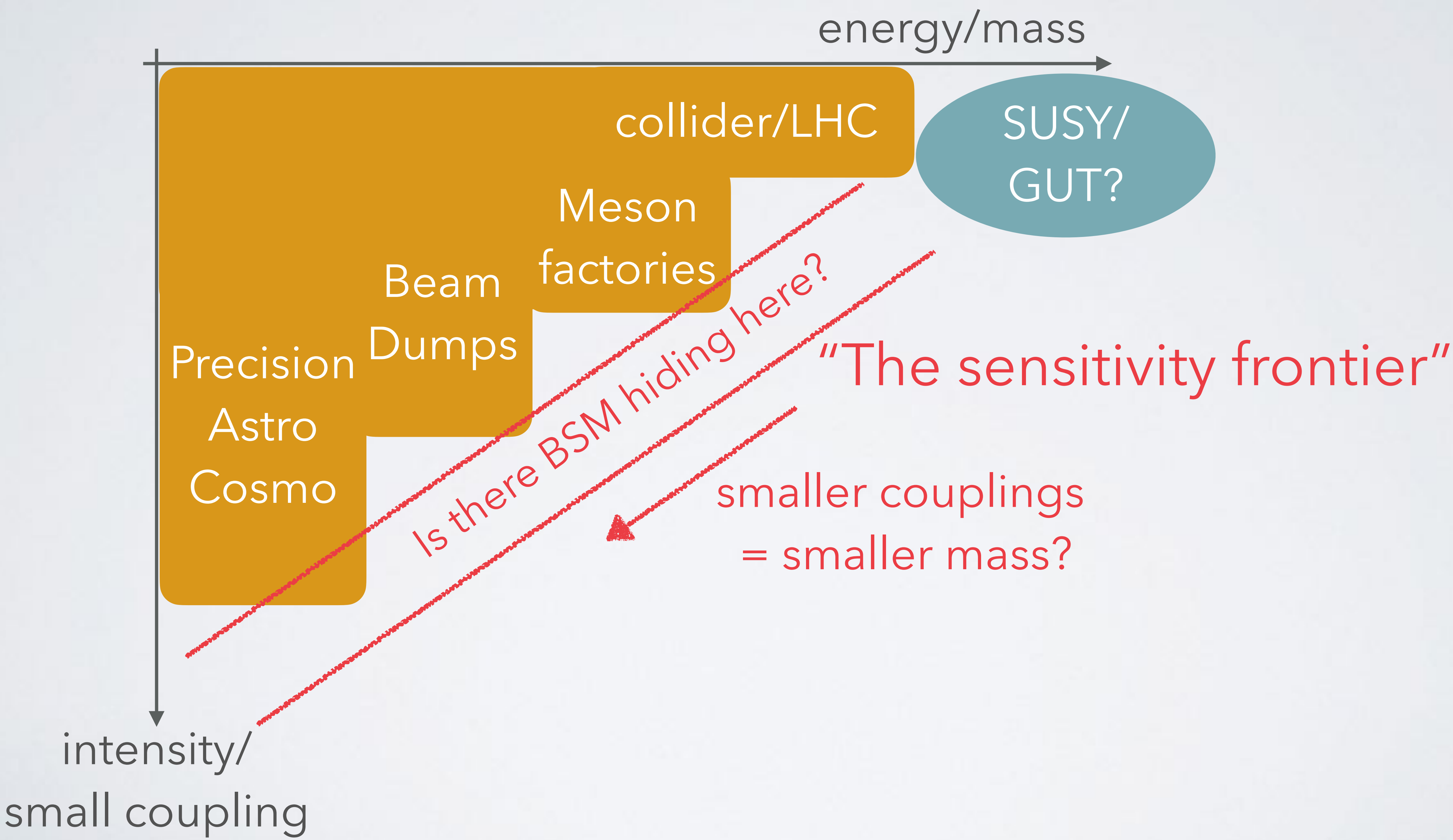

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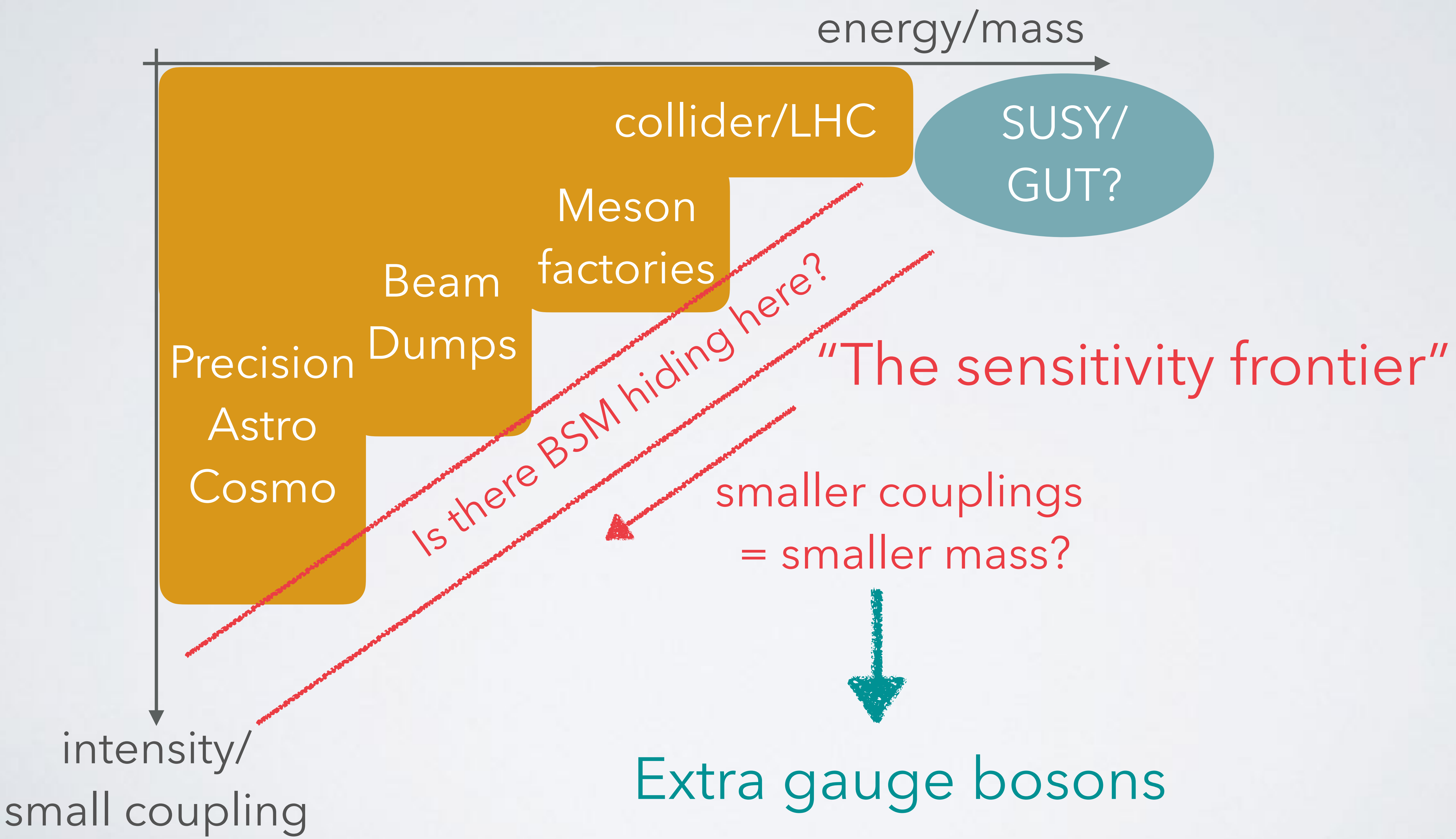


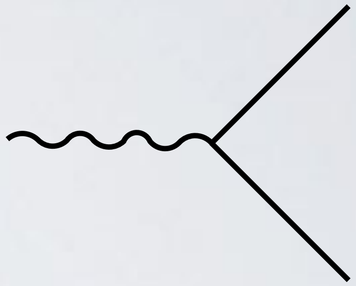
HIDDEN PHOTONS

$$\mathcal{L} \supset -\frac{\epsilon_A}{2} F_{\mu\nu} X^{\mu\nu}$$

[Okun '82; Holdom '86]

- For light mediators $M_X \ll M_Z$ kinetic terms can be diagonalised by simple field redefinition:

$$A^\mu \to A^\mu - \epsilon_A X^\mu \quad \rightarrow \quad e A_\mu J^\mu_{\rm EM} - \epsilon_A\, e X_\mu J^\mu_{\rm EM}$$



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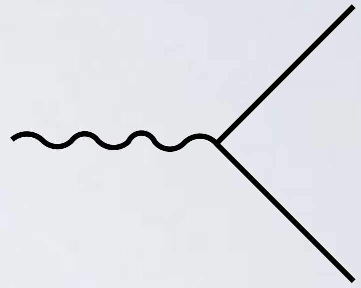

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- If $U(1)_X$ is broken by VEV f of scalar, mass is related to coupling:

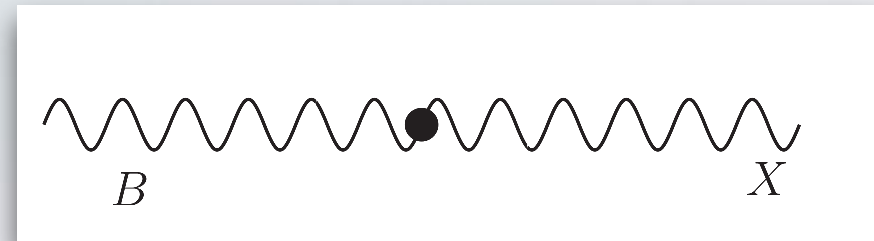
$$\mathcal{L} = (D_\mu S)^\dagger D^\mu S \supset g_x^2 f^2 X_\mu X^\mu$$

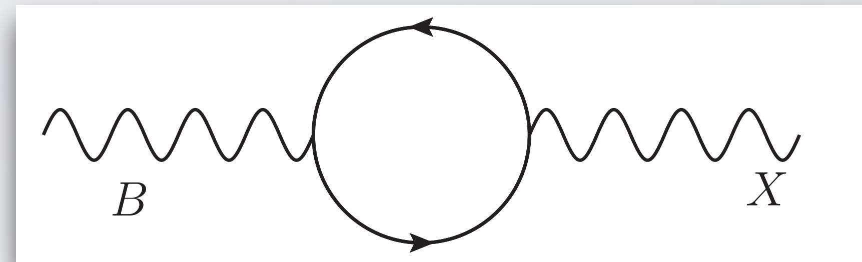
$$\Rightarrow \quad M_X = g_x f$$

smaller couplings
⇒ smaller mass!

KINETIC MIXING —``COMMON LORE"

- How does kinetic mixing with photon arise? Cannot be fundamental!





- Kinetic mixing requires **either tree-level mixing** between the new $U(1)_X$ and the SM hypercharge $U(1)_B$ **or** is induced at the **loop-level** if there are **fields charged under both** $U(1)_B$ **and** $U(1)_X$

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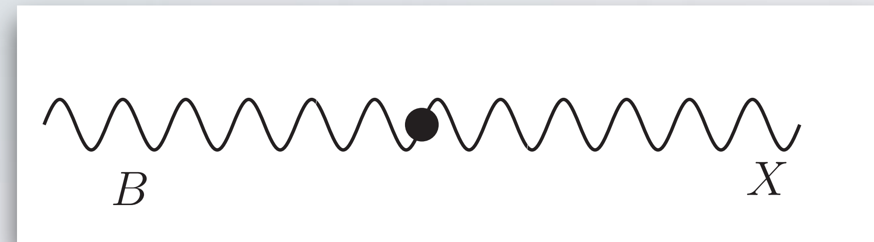

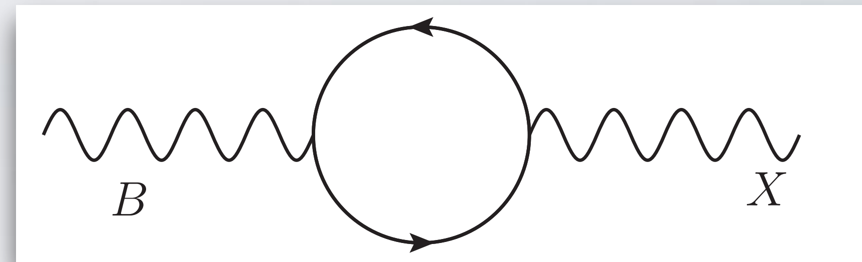

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$$\mathcal{L} \supset -\frac{\epsilon_B}{2} B_{\mu\nu} X^{\mu\nu}$$

$$B_\mu = c_w A_\mu - s_w Z_\mu$$

$$\mathcal{L} \supset -c_w \frac{\epsilon_B}{2} F_{\mu\nu} X^{\mu\nu} + s_w \frac{\epsilon_B}{2} Z_{\mu\nu} X^{\mu\nu}$$

$$\Rightarrow \epsilon_A = c_w \epsilon_B$$

KINETIC MIXING — THE FULL PICTURE

- There is a dim-6 operator that induces mixing with the weak bosons (in theories with $SU(2)_L$ **multiplets charged under** $U(1)_X$ generated at loop level)

$$\mathcal{O}_{WX} = \frac{c_{WX}}{\Lambda^2}\, H^\dagger \sigma^i H\, W^i_{\mu\nu} X^{\mu\nu}$$

New physics scale

- This **operator induces kinetic mixing** after EWSB

$$\mathcal{O}_{WX} \supset -\frac{\epsilon_W}{2}\, W^3_{\mu\nu} X^{\mu\nu}$$

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$$W^3_\mu = s_w A_\mu + c_w Z_\mu$$

$$\mathcal{O}_{WX} \supset -s_w \frac{\epsilon_W}{2} F_{\mu\nu} X^{\mu\nu} - c_w \frac{\epsilon_W}{2} Z_{\mu\nu} X^{\mu\nu}$$

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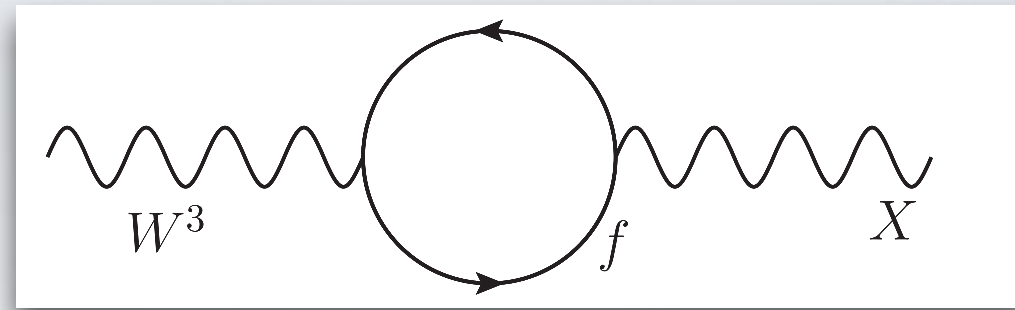
LOOP GENERATION OF ϵ_W



- The operator $\mathcal{O}_{WX}$ captures the $SU(2)_L$ contributions to kinetic mixing of the X boson with W^3. After EWSB one can identify

$$\Pi^{\mu\nu}_{WX} = \Pi_{WX}\,[g^{\mu\nu} p_1 \cdot p_2 - p_1^{\mu} p_2^{\nu}] + \Delta_{WX}\, g^{\mu\nu}$$

ϵ_W

- The kinetic mixing due to $U(1)_X$ charged $SU(2)_L$ multiplets reads

$$\Pi_{WX} = -\frac{g\, g_x}{8\pi^2} \sum_f T_3^f \,(v_X^f + a_X^f) \int_0^1 dx\, x(1-x)\, \log\left(\frac{\mu^2}{m_f^2 - x(1-x) q^2}\right)$$

Sum over $SU(2)_L$ dofs

$SU(2)_L$ charge

$U(1)_X$ charge $(v_X^f + a_X^f) = 2\, Q_L^f$

loop function

WHY IS THIS IMPORTANT?

- Beyond kinetic mixing $U(1)_X$ can be coupled to SM by gauge interactions

$$\mathcal{L}_{\rm int} = -g_x\, J^\mu_X X_\mu$$

$SU(2)_L$ multiplets!

$$J^\mu_X = \sum_\psi \bar\psi\, Q_\psi\, \gamma^\mu \psi \qquad \text{with } \psi = Q, L, u, d, \ell, \nu$$

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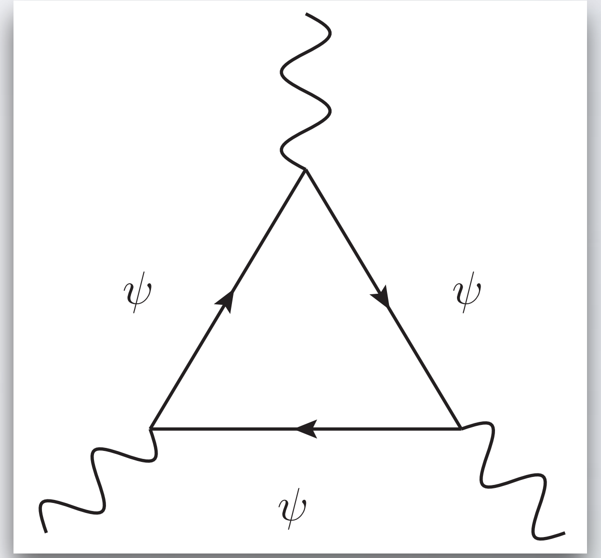
- **Phenomenologically viable** anomaly-free combinations are:

$B - L$	$L_\mu - L_e$	$L_e - L_\tau$	$L_\mu - L_\tau$
charging quarks & leptons	charging 1st & 2nd generation leptons	charging 1st & 3rd generation leptons	charging 2nd & 3rd generation leptons

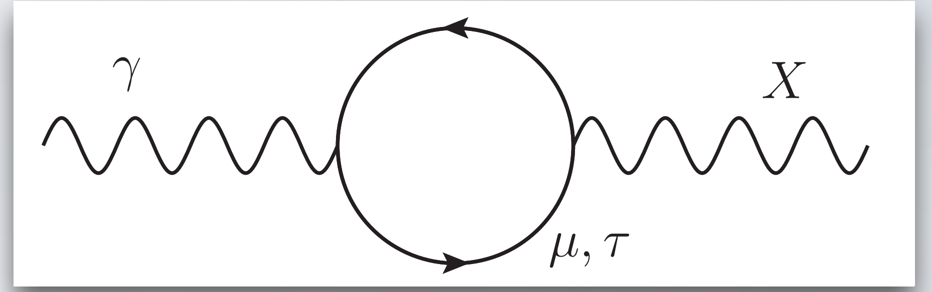


- **All of these models charge SM $SU(2)_L$ multiplets, and thus necessarily induce $\mathcal{O}_{WX}$ at the renormalizable level ($\Lambda = v$)!**

MATCHING EXAMPLE: $U(1)_{L_\mu - L_\tau}$

- In the IR the loop mixing with the photon is computed to

$$\epsilon_A = \frac{e g_{\mu\tau}}{6\pi^2} \log\left(\frac{m_\mu}{m_\tau}\right)$$



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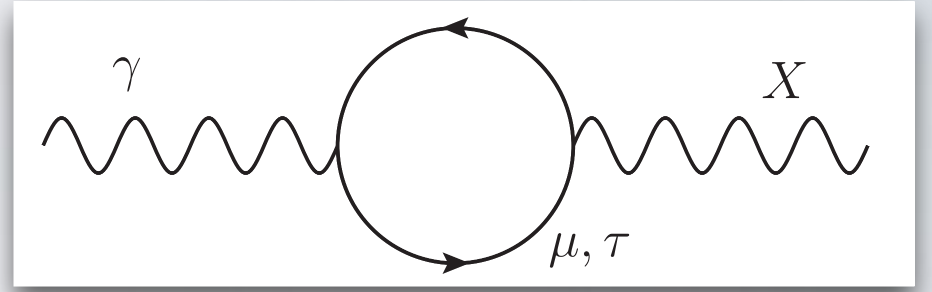
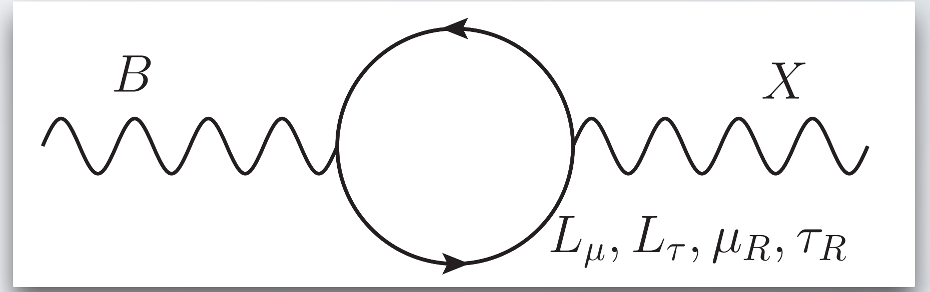


- The naive UV computation yields

$$\epsilon_B = \frac{g' g_{\mu\tau}}{24\pi^2} \left[3 \log\left(\frac{m_\mu}{m_\tau}\right) + \log\left(\frac{m_{\nu_\mu}}{m_{\nu_\tau}}\right)\right]$$



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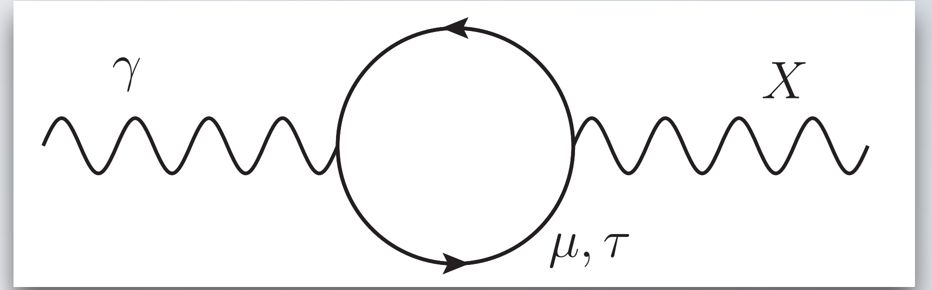

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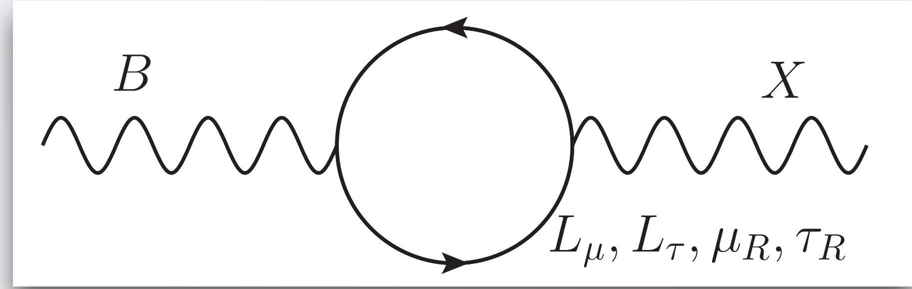
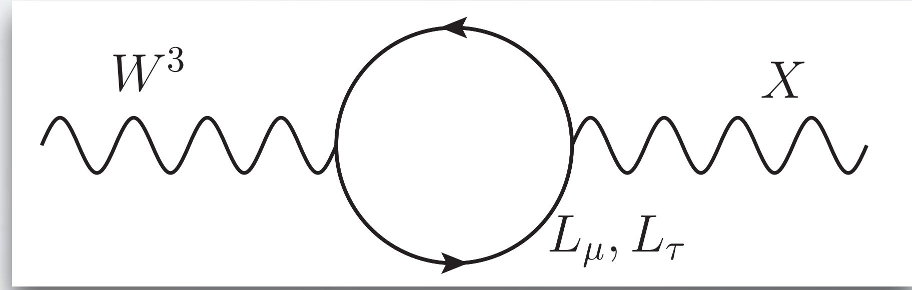


$$\Rightarrow \quad \epsilon_A \neq c_w\, \epsilon_B \quad ✗$$

- We are missing the mixing with the $SU(2)_L$

$$\epsilon_W = \frac{g g_{\mu\tau}}{24\pi^2}\left[\log\left(\frac{m_\mu}{m_\tau}\right) - \log\left(\frac{m_{\nu_\mu}}{m_{\nu_\tau}}\right)\right]$$



$$\Rightarrow \quad \epsilon_A = c_w\, \epsilon_B + s_w\, \epsilon_W \quad ✓$$

HIGGS LOW-ENERGY THEOREM

One-loop corrections: vacuum polarization

- Starting from the low-energy Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}, Z_{\mu\nu}, X_{\mu\nu}\right)\left[\begin{pmatrix} 1 & 0 & \epsilon_A \\ 0 & 1 & \epsilon_Z \\ \epsilon_A & \epsilon_Z & 1 \end{pmatrix} + \mathbf{\Pi}\right]\begin{pmatrix} F^{\mu\nu} \\ Z^{\mu\nu} \\ X^{\mu\nu} \end{pmatrix}$$

we can derive the Higgs decay amplitudes via the **low-energy theorem:**

$$\lim_{p_h \to 0} \mathcal{M}(h \to V_i V_j) \to \frac{\partial}{\partial v}\mathcal{M}(V_i \to V_j) = \partial_v [G^T \, \mathbf{\Pi} \, G]_{ij}$$

[Ellis, Gaillard, Nanopoulos '76]

[Shifman, Vainshtein, Voloshin, Zakharov '79]

vacuum polarizations in canonical normalisation

$$\mathbf{\Pi} = \begin{pmatrix} \Pi_{\gamma\gamma} & \Pi_{\gamma Z} & \Pi_{\gamma X} \\ \Pi_{\gamma Z} & \Pi_{ZZ} & \Pi_{ZX} \\ \Pi_{\gamma X} & \Pi_{ZX} & \Pi_{XX} \end{pmatrix}$$

$$G = \begin{pmatrix} 1 & 0 & -\frac{\epsilon_A}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \\ 0 & 1 & -\frac{\epsilon_Z}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \\ 0 & 0 & \frac{1}{\sqrt{1-\epsilon_A^2-\epsilon_Z^2}} \end{pmatrix}$$

$$G^T \, \mathbf{\Pi} \, G = \mathbf{\Pi} - \begin{pmatrix} 0 & 0 & \epsilon_A \Pi_{\gamma\gamma} + \epsilon_Z \Pi_{\gamma Z} \\ \cdot & 0 & \epsilon_A \Pi_{\gamma Z} + \epsilon_Z \Pi_{ZZ} \\ \cdot & \cdot & 2\epsilon_A \Pi_{\gamma X} + 2\epsilon_Z \Pi_{ZX} \end{pmatrix}$$

HIGGS LOW-ENERGY THEOREM

- In the SM we find then for mixing with a new $U(1)_X$ in the IR

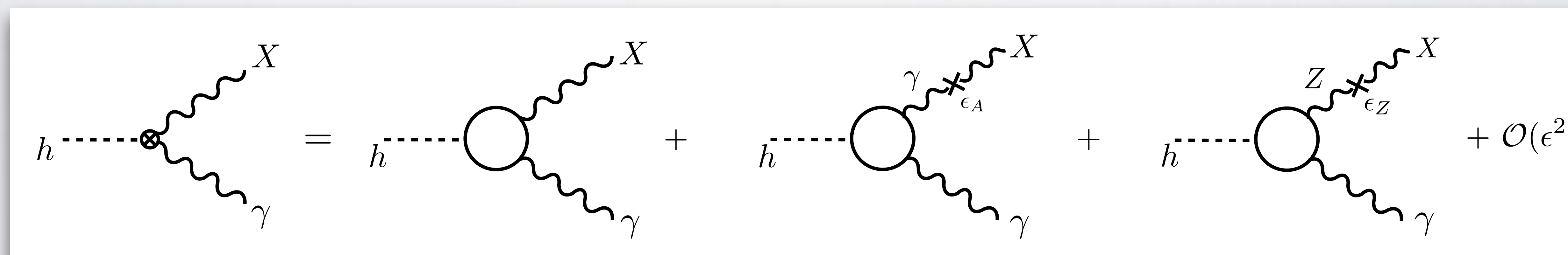
$$\partial_v \Pi_{\gamma X}(0) = \sum_f N_c^f \frac{e\, g_x}{12\,\pi^2\, v} Q_f\, v_X^f$$

$$\partial_v \Pi_{ZX}(0) = \sum_f N_c^f \frac{e\, g_x}{24\,\pi^2\, v} \frac{T_3^f - 2\, s_w^2\, Q_f}{s_w c_w} v_X^f$$

$$\partial_v \Pi_{XX}(0) = \sum_f N_c^f \frac{g_x^2}{24\,\pi^2\, v} v_X^{f\,2}$$

The sums run over all fermions with $m_f \gg m_h$ (i.e. the top quark)

- Allows us to derive **universal branching ratios** for models gauging B



$$\mathcal{BR}_{h\to\gamma X} \simeq \left(0.92\, g_x^2 + 6.36 g_x \epsilon_A + 11.01 \epsilon_A^2\right) \cdot 10^{-3}$$

$\Rightarrow \sim 10^{-8}$ (for $g_x \sim 10^{-4}$ and $\epsilon_A \sim 10^{-3}$) **in reach of FCC-hh!**

CONCLUSIONS

- **Hidden Photons** are well motivated particles that could **hide along the *sensitivity frontier*** (i.e. weaker coupling, smaller mass)

- The **commonly quoted matching** $\epsilon_A = c_w \epsilon_B$ is **incomplete!**

- There is a **dim-6 operator inducing** $W^3 X$ **mixing**. In models with $SU(2)_L$ multiplets charged under $U(1)_X$ it is generated at loop level

$$\Rightarrow \quad \epsilon_A = c_w\, \epsilon_B + s_w\, \epsilon_W$$

- The ϵ_W **contribution is essential** in anomaly-free U(1) models to obtain **correct IR mixing!**

- **Higgs low-energy theorems** automatically **incorporate all decay amplitudes** comprehensively **at fixed order** in ϵ

THANK YOU!

BACKUP

HIGGS LOW-ENERGY THEOREM

- Have derived the vacuum polarisation amplitudes for kinetic mixing

$$\partial_v \, \Pi^n_{ij} = \frac{g_i g_j}{48\pi^2} N^n_c (v^n_i \, v^n_j + a^n_i a^n_j) \; \partial_v \log\left(\frac{\det \mathcal{M}^\dagger_n \mathcal{M}_n}{\mu^2} \right)$$

color factor

$U(1)$ charges

mass matrix
of multiplet