Karlsruher Institut für Technologie

NRQCD matching coefficients

Young Scientists Meeting 2022

Manuel Egner | Karlsruhe, June 9, 2022

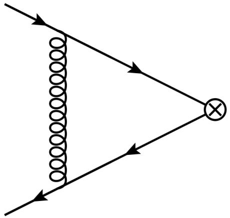
INSTITUTE FOR THEORETICAL PARTICLE PHYSICS

based on:
M. Egner, M. Fael, J. Piclum, K. Schönwald, M. Steinhauser: arXiv:2105.09332
M. Egner, M. Fael, F. Lange, K. Schönwald, M. Steinhauser: arXiv:2203.11231

Outline

QCD and non-relativistic QCD (NRQCD)

- NRQCD: effective theory to describe systems of heavy quark pairs with small relative velocity
  [Beneke, Kiyo, Schuller (2013)] [Pineda (2012)]
- examples: top-quark pair production, quarkonium (bottomonium and charmonium)
- small relative velocity v: the relevant scales, mass $\propto m_q$, momentum $\propto m_q v$ and energy $\propto m_q v^2$ are well separated.



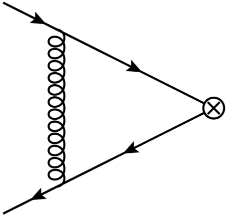
- consider different loop momentum regions according to these scales:[Beneke, Smirnov (1998)]

$$\begin{aligned} \text{hard}: &\quad k_0 \propto m_q \quad k_i \propto m_q \\ \text{soft}: &\quad k_0 \propto m_q v \quad k_i \propto m_q v \\ \text{potential}: &\quad k_0 \propto m_q v^2 \quad k_i \propto m_q v \\ \text{ultrasoft}: &\quad k_0 \propto m_q v^2 \quad k_i \propto m_q v^2 \end{aligned}$$

- NRQCD: integrate out hard region

Matching QCD and NRQCD

- consider operators in full and effective theory
- vector current operator in QCD: $j_v^\mu = \bar{\Psi}\gamma^\mu\Psi$
- vector current in NRQCD: $\tilde{j}_v^k = \phi^\dagger\sigma^k\chi$
  expand j_v^μ in relative velocity $v = |\vec{p}|/m_q$



- matching the full theory to the effective theory:
  require renormalized vertex functions with two external on-shell quarks to be equal up to corrections in $1/m_q$:

$$Z_2\Gamma_v = c_v\tilde{Z}_2\tilde{Z}_v^{-1}\tilde{\Gamma}_v + \mathcal{O}\left(\frac{1}{m_q}\right)$$

- matching coefficient c_v
- Γ_v, $\tilde{\Gamma}_v$: vertex functions in QCD and NRQCD with renormalized m_q and α_s.

Matching QCD and NRQCD

- NRQCD: hard loop momenta are integrated out

$$\Gamma_{QCD} = C\Gamma_{NRQCD}$$

$$(1 + \Gamma_{\mathrm{hard}} + \Gamma_{\mathrm{soft}} + \dots) = (1 + c + \dots)(1 + \Gamma_{\mathrm{soft}} + \dots)$$

$$\to 1 + \Gamma_{\mathrm{hard}} = 1 + c + \dots$$

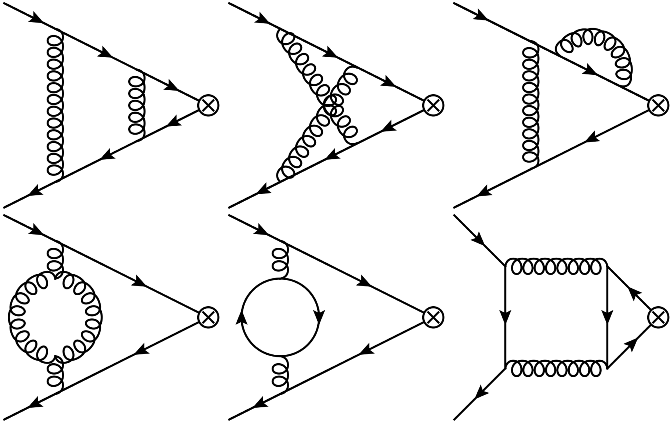
- similar matching equations for axialvector, scalar and pseudoscalar current

$$Z_x Z_2 \Gamma_x = c_x \tilde{Z}_2 \tilde{Z}_x^{-1} \tilde{\Gamma}_x + \mathcal{O}\left(\frac{1}{m_q}\right)$$

with $x \in \{v, a, s, p\}$

Contributing diagrams to c_v at NNLO

- external heavy quarks, mass m_q
- external quarks are on-shell $\left(\frac{q}{2}\right)^2 = m_q^2$
- c_v at NNLO with one mass scale is known [Czarnecki, Melnikov (1998)], [Beneke, Signer, Smirnov (1998)]
- internal quarks, mass $m_2 \neq m_q$



Non-Singlet diagrams

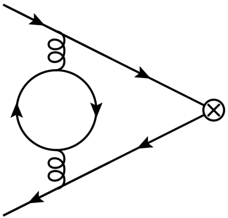
- make use of projector to handle the tensor structure of the amplitude [Kniehl, Onishchenko, Piclum, Steinhauser (2006)]

$$P_\mu^{(v)} = \frac{1}{8\,(d-1)\,m_q^2}\left(\frac{\not{q}}{2}+m_q\right)\gamma_\mu\left(-\frac{\not{q}}{2}+m_q\right)$$
$$\Longrightarrow\ \Gamma_v = \mathrm{Tr}\left[P_\mu^{(v)}\Gamma^\mu\right]$$

- after applying projector: scalar integrals
- reduce to master integrals by using integration by parts relations (IBPs) [Chetyrkin,Tkachov (1981)] with `LiteRed` [Lee (2012)]

- 4 master integrals
- calculate integrals using differential equations
  define $x = m_2/m_q$, $\vec{I} = (I_1, I_2, I_3, I_4)^T$
- differential equation

$$\frac{\mathrm{d}\vec{I}}{\mathrm{d}x} = A(x,\epsilon)\cdot\vec{I}$$



Non-Singlet diagrams - master integrals

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- transformation $\vec{I} = T \cdot \vec{J}$ to ϵ-form [Henn (2013)] with CANONICA [Meyer (2017)]

$$\Longrightarrow \quad \frac{\mathrm{d}\vec{J}}{\mathrm{d}x} = \epsilon \, A'(x) \cdot \vec{J}$$

- solve differential equation order by order in ϵ inserting the Laurent series of $\vec{J}$
- solution of the differential equations contains iterated integrals and integration constants $\rightarrow$ Harmonic Polylogarithms with alphabet

$$f_0(x) = \frac{1}{x}, \quad f_1(x) = \frac{1}{1-x}, \quad f_{-1}(x) = \frac{1}{1+x}$$

- fix integration constants with boundary conditions at $x = 0$, $x = 1$ for which the master integrals are known.

Results

- matching coefficient

$$c_v^{(2)}\Big|_{m_2} = n_m C_F T_F \Bigg[\frac{71}{72} + \frac{3\pi^2}{32x} - \frac{11\pi^2 x}{48} + \frac{35x^2}{24} - \frac{17}{32}\pi^2 x^3 + \frac{2\pi^2 x^4}{9} + \frac{4}{3}x^4 H_0^2$$
$$+ \left(\frac{23}{24} + \frac{19x^2}{24} + \left(\frac{3}{16x} - \frac{11x}{24} - \frac{17x^3}{16} + \frac{4x^4}{3}\right) H_1\right) H_0$$
$$+ \left(-\frac{3}{16x} + \frac{11x}{24} + \frac{17x^3}{16} - \frac{4x^4}{3}\right) H_{0,1}$$
$$+ \left(\frac{3}{16x} - \frac{11x}{24} - \frac{17x^3}{16} - \frac{4x^4}{3}\right) H_{-1,0} + \frac{2}{3}\log\left(\frac{\mu^2}{m_2^2}\right) + \mathcal{O}(\epsilon)\Bigg]$$

with Harmonic Polylogarithms $H_{\vec{a}} = H(a_1, \ldots, a_n; x)$.

- results up including $\mathcal{O}(\epsilon)$.
- numerical value for $m_q = m_b = 5.1\ \mathrm{GeV}$ and $m_2 = m_c = 1.65\ \mathrm{GeV}$

$$c_v^{(2)} = -44.72 + 0.17 n_h + 0.41 n_l + 1.75 n_m + \log\left(\frac{\mu^2}{m_b^2}\right)\left(-20.13 + 0.44\,(n_l + n_m)\right)$$

Scalar, pseudoscalar and axialvector current

- scalar current:

$$\bar{\Psi}\Psi \leftrightarrow -\frac{1}{m_q}\phi^\dagger \vec{p}\cdot\vec{\sigma}\chi$$

- pseudoscalar current:

$$\bar{\Psi}\gamma_5\Psi \leftrightarrow -\mathrm{i}\phi^\dagger\chi$$

- axial-vector current:

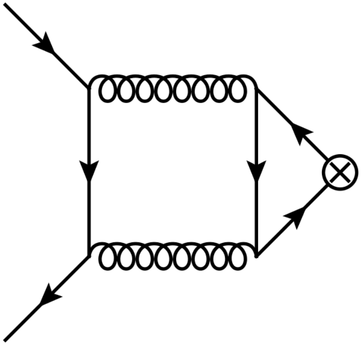
$$\bar{\Psi}\gamma^\mu\gamma_5\Psi \leftrightarrow \frac{1}{2m_q}\phi^\dagger\left[\sigma^k, \vec{p}\cdot\vec{\sigma}\right]\chi$$

- calculation similiar to vector current

$$Z_2 Z_x \Gamma_x = c_x \tilde{Z}_2 \tilde{Z}_x^{-1} \tilde{\Gamma}_x + \mathcal{O}\left(\frac{1}{m_q}\right),$$

  with $x = \{s, p, a\}$
- different projectors
- Non-Singlet contributions: same master integrals as in the vector current case
- matching coefficient consists of Singlet and Non-Singlet part: $c_x^{(2)} = c_{x,\mathrm{non-sing}}^{(2)} + c_{x,\mathrm{sing}}^{(2)}$

Singlet diagrams



- external current does not couple to the external quarks directly
- don't contribute to c_v, but to c_s, c_p, c_a
- external quarks with mass m_q, on-shell $\left(\frac{q}{2}\right)^2 = m_q^2$
- light quark with mass m_2 in the fermion loop
- use Larin scheme for γ_5 in the calculation of Singlet diagrams [Larin (1993)]

Singlet diagrams

- 12 master integrals
- calculate master integrals by using differential equation

$$\frac{\mathrm{d}\vec{I}}{\mathrm{d}x} = A(x,\epsilon)\cdot\vec{I}$$

- $x = m_2/m_q$, perform transformation $x = \frac{2t}{1+t^2} \rightarrow \epsilon$-form with CANONICA [Meyer (2017)]

$$\frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = \epsilon\, A'(t)\cdot\vec{J}$$

- solve differential equation in ϵ-form in terms of Cyclotomic Harmonic Polylogarithms with alphabet

$$f_0(t) = \frac{1}{t}, \quad f_1(t) = \frac{1}{1-t}, \quad f_{-1}(t) = \frac{1}{1+t}, \quad f_{(4,1)}(t) = \frac{t}{1+t^2}$$

  can be handled with `HarmonicSums` [Ablinger (2010)]
- fix constants at boundaries $x = t = 0$ and $x = t = 1$ [Piclum (2007)]

Results

$$
\begin{aligned}
c^{(2)}_{p,\mathrm{sing}}\Big|_{m_2} =& n_m C_F T_F \Bigg[\pi^2\left(\frac{7t^3}{3\left(1+t^2\right)^3}+\frac{2tH_{\{4,1\}}}{1+t^2}\right)-\frac{4t^3H_0}{\left(1+t^2\right)^2}+\frac{4t^3H_0^2}{\left(1+t^2\right)^3}+\frac{16t^3H_0H_1}{\left(1+t^2\right)^3}\\
&+\log(2)\left(-\frac{2t}{1+t^2}+\frac{16t^3H_0}{\left(1+t^2\right)^3}+\frac{16t^3H_1}{\left(1+t^2\right)^3}-\frac{16t^3H_{-1}}{\left(1+t^2\right)^3}\right)+\left(\frac{4t}{1+t^2}-\frac{8tH_0^2}{1+t^2}\right)H_{\{4,1\}}\\
&-\frac{16t^3H_{0,1}}{\left(1+t^2\right)^3}+\left(-\frac{32t^3}{\left(1+t^2\right)^3}+\frac{24tH_0}{1+t^2}\right)H_{0,\{4,1\}}-\frac{32t^3H_{1,\{4,1\}}}{\left(1+t^2\right)^3}-\frac{16t^3H_{-1,0}}{\left(1+t^2\right)^3}\\
&+\frac{32t^3H_{-1,\{4,1\}}}{\left(1+t^2\right)^3}-\frac{24tH_{0,0,\{4,1\}}}{1+t^2}+\frac{8t^3\log^2(2)}{\left(1+t^2\right)^3}-\frac{3t\zeta(3)}{1+t^2}\\
&+i\pi\left\{-\frac{(-1+t)t(1+t)}{\left(1+t^2\right)^2}-\frac{4t^3H_0}{\left(1+t^2\right)^3}-\frac{8tH_0H_{\{4,1\}}}{1+t^2}+\frac{12tH_{0,\{4,1\}}}{1+t^2}\right\}+\mathcal{O}\left(\epsilon\right)\Bigg]
\end{aligned}
$$

- matching coefficients up to and including $\mathcal{O}(\epsilon)$
- singlet contributions including one mass scale are known [Kniehl, Onishchenko, Piclum, Steinhauser (2006)]
  cross check: limits $t \to 1$ and $t \to 0$ agree

Non-Singlet matching coefficient at N^3LO

- use the results for the massive form factors $\rightarrow$ see talk of Fabian!
- the matching coefficients are obtained with the expansion at threshold around $x = \sqrt{4 - s/m^2} = 0$
- scaling of loop momenta at threshold and integrals:

- hard (h): $k_0 \sim m,\ k_i \sim m$
- soft (s): $k_0 \sim x \cdot m,\ k_i \sim x \cdot m$
- potential (p): $k_0 \sim x^2 \cdot m,\ k_i \sim x \cdot m$
- ultrasoft (u): $k_0 \sim x^2 \cdot m,\ k_i \sim x^2 \cdot m$

- $I_a \sim x^{-0\epsilon} \cdot$ Taylor expansion $(h-h-h)$
- $I_b \sim x^{-2\epsilon} \cdot$ Taylor expansion $(h-h-p), (h-h-s)$
- $I_c \sim x^{-4\epsilon} \cdot$ Taylor expansion $(h-h-u), (h-p-p), \ldots$
- $I_d \sim x^{-6\epsilon} \cdot$ Taylor expansion $(h-p-u), (h-s-u), \ldots$

- insert this ansatz into system of differential equations $\rightarrow$ linear equations for expansion coefficients
- reduce system with `Kira` [Klappert, Lange, Maierhöfer, Usovitsch (2020)] and `FireFly` [Klappert, Lange (2020)]
- sum contributions from all regions, match to results from the form factor calculation
- insert the solutions into ansatz for $(h-h-h)$ $\rightarrow$ master integrals in the hard expansion

Results

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- significant improvement to previous results for Non-Singlet c_v [Marquard, Piclum, Seidel, Steinhauser (2014)] where the master integrals are calculated numerically with `FIESTA`[Smirnov (2016)]

$$c_v^{(3)} = C_F^3 c_{FFF} + C_F C_A^2 c_{FFA} + C_F C_A^2 c_{FAA} + \text{fermionic and singlet contributions}$$

$$\begin{aligned} c^v_{FFF} &= 36.55(0.53) & \rightarrow & \quad 36.49486246 \\ c^v_{FFA} &= -188.10(0.83) & \rightarrow & \quad -188.0778417 \\ c^v_{FAA} &= -97.81(0.38) & \rightarrow & \quad -97.73497327 \end{aligned}$$

$\rightarrow$ reproduce old results with much better precision

- results for Non-Singlet contributions for all four currents
- results for the anomalous dimension of the currents and renormalization constants $\tilde{Z}_x$ in NRQCD are obtained,
  precision is high enough to reconstruct the analytic expression with the PSLQ algorithm [Ferguson, Bailey, Arno (1999)]

$\Upsilon(1S)$ decay

Decay width [Beneke, Kiyo, Schuller (2007)]

$$\Gamma = \frac{4\pi\alpha^2}{9m_b^2}\left[|\Psi_1(0)|^2 c_v \left(c_v - \frac{E_1}{m_b}\left(c_v + \frac{d_v}{3}\right) + \ldots\right)\right]$$

with

$$\left|\Psi_1^{\mathrm{LO}}(0)\right|^2 = \frac{8m_b^3\alpha_s^3}{27\pi}, \qquad E_1^{\mathrm{LO}} = -\frac{4m_b\alpha_s^2}{9}$$

calculation up to NNNLO including m_c effects up to NNLO,
perturbative expansion in α_s of:

- vector current matching coefficient c_v (NNNLO) [Marquard, Piclum, Seidel, Steinhauser (2014)]
- derivative current matching coefficient d_v (NLO) [Luke, Savage (1998)]
- wave function at the origin $\Psi_{n=1}(0)$ (NNNLO) [Beneke, Kiyo, Schuller (2007)]
- bound-state energy levels $E_{n=1}$ (NLO) [Pineda, Ynduráin (1997)]

can be found in QQbar_threshold [Beneke, Kiyo, Maier, Piclum (2016)]

$\Upsilon(1S)$ decay

- consider hard charm quark
  $\rightarrow$ charm integrated out when matching QCD to NRQCD
  $\rightarrow$ expanding in $\alpha_s^{(n_l=3)}(\mu)$
  $\rightarrow$ m_c contributions to c_v at NNLO

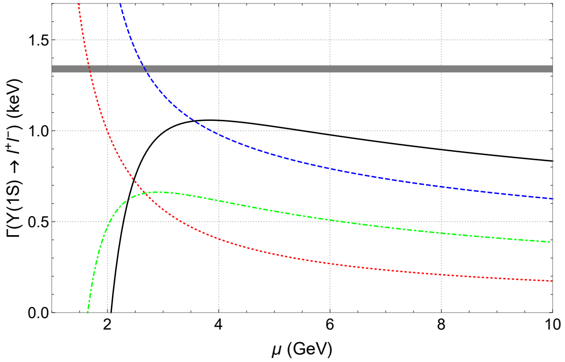
numerical values:

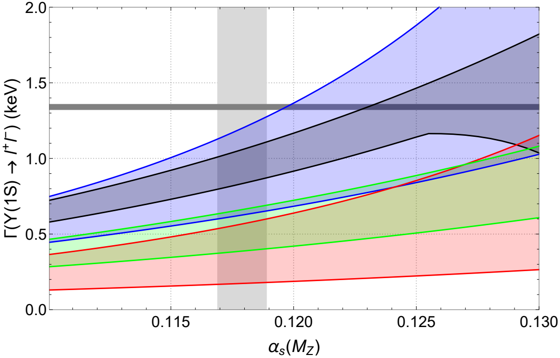
- renormalization scale $\mu = 3.5\,\mathrm{GeV}$
- use RunDec 3 [Herren, Steinhauser (2017)] for masses and strong coupling constant
- use the pole mass of the bottom-quark

Decay width

$$\Gamma\left(\Upsilon(1S) \rightarrow l^+ l^-\right)\Big|_{\mathrm{pole}} = \frac{2^5 (\alpha_s^{(n_l=3)})^3 \alpha^2 m_b}{3^5}\left(1 + 0.374 + (0.916 + 0.020_{c_v}) - 0.032\right)$$
$$= 1.041 + 0.009_{c_v}\ \mathrm{keV}$$
$$= \left[1.051 \pm 0.047(\alpha_s)^{+0.007}_{-0.217}(\mu)\right]\ \mathrm{keV}$$

$\Upsilon(1S)$ decay





$\Gamma\left(\Upsilon(1S)\right)$ depending on μ and $\alpha_s(M_Z)$ up to LO, NLO, NNLO, NNNLO

$$\Gamma\left(\Upsilon(1S)\to l^+l^-\right)\Big|_{\text{pole}} = \left[1.051 \pm 0.047(\alpha_s)^{+0.007}_{-0.217}(\mu)\right]\ \text{keV}$$

Comparing results

- calculation with massless charm quark [Beneke, Kiyo, Marquard, Penin, Piclum, Seidel, Steinhauser (2014)]

$$\Gamma\Big|_{\mathrm{pole},m_c=0} = \left[1.04 \pm 0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)\right]\ \mathrm{keV}$$

- calculation with massive charm quark

$$\Gamma\Big|_{\mathrm{pole}} = \left[1.051 \pm 0.047(\alpha_s)^{+0.007}_{-0.217}(\mu)\right]\ \mathrm{keV}$$

- experimental value [Beringer et al. (2012)]

$$\Gamma\left(\Upsilon(1S) \to l^+ l^-\right) = 1.340(18)\ \mathrm{keV}$$

$\rightarrow$ finite charm mass leads to small shift of the decay width, but cannot explain the difference between theoretical and experimental value

- possible explanation: big non-perturbative contributions to the wave function

$$\Gamma = \frac{4\pi\alpha^2}{9m_b^2}\left[|\Psi_1(0)|^2 c_v \left(c_v - \frac{E_1}{m_b}\left(c_v + \frac{d_v}{3}\right) + \ldots\right)\right]$$

Conclusion

NNLO:

- calculation of two mass scale contributions to c_v, c_s, c_p, c_a, both Non-Singlet and Singlet
- correction leads to a small shift of $\Gamma\left(\Upsilon(1S) \to l^+ l^-\right)$ towards the experimental value

N^{3}LO:

- numerical results for Non-Singlet contributions to c_v, c_s, c_p, c_a with high precision (10 digits)
- calculation of the current renormalization constants $\tilde{Z}_x$ in NRQCD at N^{3}LO

Thank you for your attention!

Results Non-Singlet

- compare the limits $x \to 1$ and $x \to 0$ to already known n_l and n_h contributions [Czarnecki, Melnikov (1998)], [Beneke, Signer, Smirnov (1998)]
- limit $m_2 \to m_q$ reproduces the contribution of the bottom quark loop

$$c_v^{(2)}\Big|_{m_2} \xrightarrow{x \to 1} c_v^{(2)}\Big|_{m_q}$$

- limit $m_2 \to 0$ produces infinite term $\implies$ Coulomb singularity which is regulated by the mass m_2

$$c_v^{(2)}\Big|_{m_2} \xrightarrow{x \to 0} C_F T_F \left(\frac{3\pi^2}{32x} + \frac{11}{18} + \frac{2}{3} \log\left(\frac{\mu^2}{m_q^2}\right) + \mathcal{O}(x) \right)$$

$$= C_F T_F \frac{3\pi^2}{32x} + c_v^{(2)}\Big|_{m=0}$$

DEQ in ϵ-form

insert ansatz

$$\vec{J} = \frac{\vec{j}_{-2}}{\epsilon^2} + \frac{\vec{j}_{-1}}{\epsilon} + \vec{j}_0 + \vec{j}_1\epsilon + \vec{j}_2\epsilon^2 + \mathcal{O}\left(\epsilon^3\right).$$

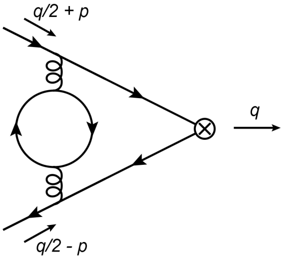
into differential equation in ϵ-form $\mathrm{d}\vec{J}/dx = \epsilon A'(x) \cdot \vec{J}$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\vec{j}_{-2} &= 0, \\ \frac{\mathrm{d}}{\mathrm{d}x}\vec{j}_{-1} &= A'(x) \cdot \vec{j}_{-2}, \\ \frac{\mathrm{d}}{\mathrm{d}x}\vec{j}_0 &= A'(x) \cdot \vec{j}_{-1}, \\ &\dots \end{aligned} \quad \rightarrow \quad \begin{aligned} \vec{j}_{-2} &= \vec{c}_{-2}, \\ \vec{j}_{-1} &= \vec{c}_{-1} + \int^x \mathrm{d}x' A'(x') \cdot \vec{c}_{-2}, \\ \vec{j}_0 &= \vec{c}_0 + \int^x \mathrm{d}x' A'(x') \cdot \vec{c}_{-1} \\ &\quad + \int^x \mathrm{d}x' \tilde{A}(x') \cdot \int^{x'} \mathrm{d}x'' A'(x'') \cdot \vec{c}_{-2}, \\ &\dots \end{aligned}$$

Scalar current

- scalar current:

$$\bar{\Psi}\Psi \quad \leftrightarrow \quad -\frac{1}{m_q}\phi^\dagger \vec{p}\cdot\vec{\sigma}\chi$$



- operator in effective theory $\propto p$
  $\rightarrow$ incoming momenta $q_1 = \frac{q}{2} + p$, $q_2 = \frac{q}{2} - p$, relative momentum p
  $\rightarrow$ expand up to first order in p

  - for example

$$\frac{1}{\left(-\frac{\not{q}}{2} + \not{k} + \not{p} + m_q\right)} \approx \frac{1}{\left(-\frac{\not{q}}{2} + \not{k} + m_q\right)}\left(1 - \frac{\not{p}}{\left(-\frac{\not{q}}{2} + \not{k} + m_q\right)} + \dots\right)$$

  - projector [Kniehl, Onishchenko, Piclum, Steinhauser (2006)]

$$P^{(s)} = \frac{1}{8m_b^2}\left(\left(-\frac{\not{q}}{2} + m_q\right)\mathbb{1}\left(-\frac{\not{q}}{2} + m_q\right) + \left(-\frac{\not{q}}{2} + m_q\right)\frac{m_q}{p^2}\not{p}\left(\frac{\not{q}}{2} + m_q\right)\right)$$

Pseudoscalar current

- pseudoscalar current:

$$\bar{\Psi}\gamma_5\Psi \;\leftrightarrow\; -\mathrm{i}\phi^\dagger\chi$$

- non-singlets: use anticommuting γ_5
- singlets: treat γ_5 in d dimensions according to [Larin (1993)] :

$$\gamma_5 \to \frac{i\epsilon^{\mu\nu\rho\sigma}}{4!}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$$
$$\gamma^\mu\gamma_5 \to \frac{i\epsilon^{\mu\nu\rho\sigma}}{3!}\gamma_\nu\gamma_\rho\gamma_\sigma$$

  - multiply projector
  - strip off ϵ-tensors from the amplitude
  - multiplied ϵ-tensors can be expressed in terms of metric tensors
  - take trace over d-dimensional γ-matrices
- projector [Kniehl, Onishchenko, Piclum, Steinhauser (2006)]

$$p^{(p)} = \frac{1}{8m_q^2}\left(-\frac{\not{q}}{2}+m_q\right)\gamma_5\left(\frac{\not{q}}{2}+m_q\right)$$

Axialvector current

- axial-vector current:

$$\bar{\Psi}\gamma^{\mu}\gamma_5\Psi \;\leftrightarrow\; \frac{1}{2m_q}\phi^{\dagger}\left[\sigma^k, \vec{p}\cdot\vec{\sigma}\right]\chi$$

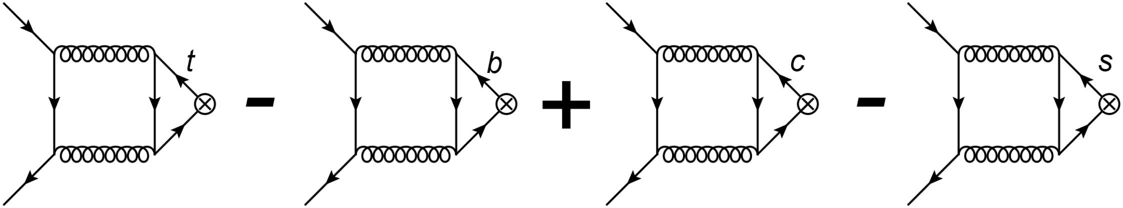
- ensure that anomaly-like contributions cancel for the singlet diagrams $\rightarrow$ introduce current containing upper and lower component of quark doublet, e.g.

$$j_a^{\mu} = \bar{t}\gamma^{\mu}\gamma_5 t - \bar{b}\gamma^{\mu}\gamma_5 b$$

in this case:

$$j_a^{\mu} = \bar{t}\gamma^{\mu}\gamma_5 t - \bar{b}\gamma^{\mu}\gamma_5 b + \bar{c}\gamma^{\mu}\gamma_5 c - \bar{s}\gamma^{\mu}\gamma_5 s$$

including m_t, m_b, m_c and massless strange quark



- treat γ_5 and expansion in p as described above
- projector [Kniehl, Onishchenko, Piclum, Steinhauser (2006)]

$$P_\mu^{(a,k)} = -\frac{1}{8m_q^2}\left(\frac{1}{d-1}\left(-\frac{\not{q}}{2}+m_q\right)\gamma_\mu\gamma_5\left(-\frac{\not{q}}{2}+m_q\right) - \frac{1}{d-2}\left(-\frac{\not{q}}{2}+m_q\right)\frac{m_q}{p^2}\left((d-3)\,p_\mu+\gamma_\mu\not{p}\right)\gamma_5\left(\frac{\not{q}}{2}+m_q\right)\right)$$

$\Upsilon(1S)$ decay

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two scenarios

- scenario A: hard charm quark
  charm integrated out when matching QCD to NRQCD
  $\to$ expanding in $\alpha_s^{(n_l=3)}(\mu)$
  m_c contributions:
  - c_v, starting at NNLO

- scenario B: soft charm quark
  charm is not integrated out when matching QCD to NRQCD
  $\to$ expanding in $\alpha_s^{(n_l=4)}(\mu)$
  m_c contributions:
  - wave function, starting at NLO
  - binding energy, starting at NNNLO

numerical values:

- renormalization scale $\mu = 3.5\ \mathrm{GeV}$
- use `RunDec` 3 [Herren, Steinhauser (2017)] for masses and strong coupling constant

$\Upsilon(1S)$ decay - scenario B

- no decoupling of the charm quark: $\alpha_s^{(n_l=4)}$
- calculation up to NNNLO, including charm mass effects up to NNLO
  $\rightarrow$ charm mass effect on wave function

Decay width

$$\begin{aligned}\Gamma\left(\Upsilon(1S)\rightarrow l^+l^-\right)\Big|_{\text{pole},B} =& \frac{2^5\alpha^2 m_b(\alpha_s^{(n_l=4)})^3}{3^5}\left(1+(0.259+0.0137_{m_c})+(0.869+0.039_{m_c})-0.178\right)\\ =&1.011+0.039_{m_c}\ \text{keV}\\ =&\left[1.050\pm 0.045(\alpha_s)^{+0.024}_{-0.155}(\mu)\right]\ \text{keV}\end{aligned}$$

Comparing scenario A and B

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- calculation with massless charm quark [Beneke, Kiyo, Marquard, Penin, Piclum, Seidel, Steinhauser (2014)]

$$\Gamma\Big|_{\mathrm{pole},m_c=0} = \left[1.04 \pm 0.04(\alpha_s)^{+0.02}_{-0.15}(\mu)\right]\ \mathrm{keV}$$

- both scenarios lead to same prediction

$$\Gamma\Big|_{\mathrm{pole},A} = \left[1.051 \pm 0.047(\alpha_s)^{+0.007}_{-0.217}(\mu)\right]\ \mathrm{keV}$$
$$\Gamma\Big|_{\mathrm{pole},B} = \left[1.050 \pm 0.045(\alpha_s)^{+0.024}_{-0.155}(\mu)\right]\ \mathrm{keV}$$

- experimental value [Beringer et al. (2012)]

$$\Gamma\left(\Upsilon(1S) \to l^+ l^-\right) = 1.340(18)\ \mathrm{keV}$$

$\Upsilon(1S)$ with m_b^{PS}

- consider the bottom mass in the potential-subtracted mass scheme
- scenario A: m_b^{PS} related to pole mass by [Beneke (1998)]

$$m_{b,\mathrm{pole}} = m_b^{\mathrm{PS}}(\mu_f) - \Sigma_{i=0}^{\infty}\delta m_i^{\mathrm{PS}}$$

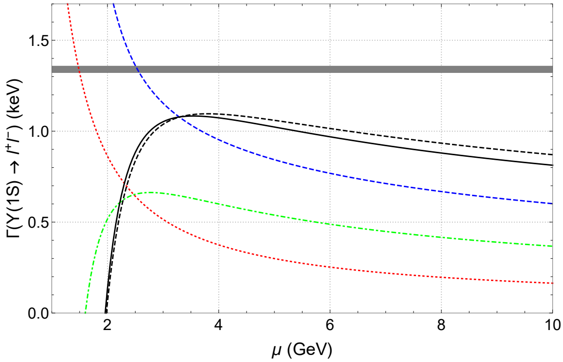
  with δm_i^{PS} originating from Coulomb potential
- scenario B: m_c corrections to δm_i^{PS} [Beneke, Maier, Piclum, Rauh (2014)]
- two different masses, set $\mu_f = 2\mathrm{GeV}$: $m_b^{\mathrm{PS}}|_A = 4.520\mathrm{GeV}$, $m_b^{\mathrm{PS}}|_B = 4.484\mathrm{GeV}$

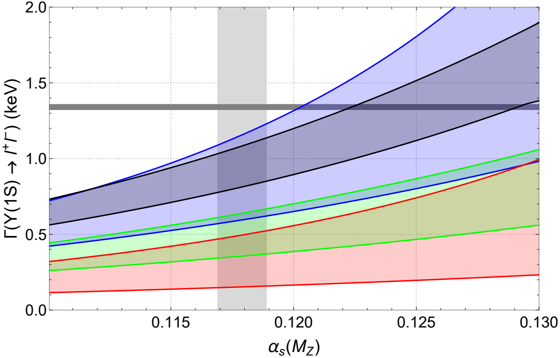
$\Upsilon(1S)$ with m_b^{PS}

$$\Gamma\left(\Upsilon(1S)\to l^+l^-\right)\big|_{\mathrm{PS},A} = \frac{2^5\alpha^2 m_b(\alpha_s^{(n_l=3)})^3}{3^5}\left(1+0.485+(1.001+0.017_{c_v})+0.125\right)$$
$$=1.076+0.007_{c_v}\ \mathrm{keV}$$
$$=\left[1.083\pm 0.053(\alpha_s)^{+0.001}_{-0.270}(\mu)\right]\ \mathrm{keV}$$

$$\Gamma\left(\Upsilon(1S)\to l^+l^-\right)\big|_{\mathrm{PS},B} = \frac{2^5\alpha^2 m_b(\alpha_s^{(n_l=4)})^3}{3^5}\left(1+(0.374+0.042_{m_c})+(0.939+0.048_{m_c})-0.029\right)$$
$$=1.050+0.041_{m_c}\ \mathrm{keV}$$
$$=\left[1.091\pm 0.052(\alpha_s)^{+0.006}_{-0.218}(\mu)\right]\ \mathrm{keV}$$

$\Upsilon(1S)$ with $m_b^{\rm PS}$





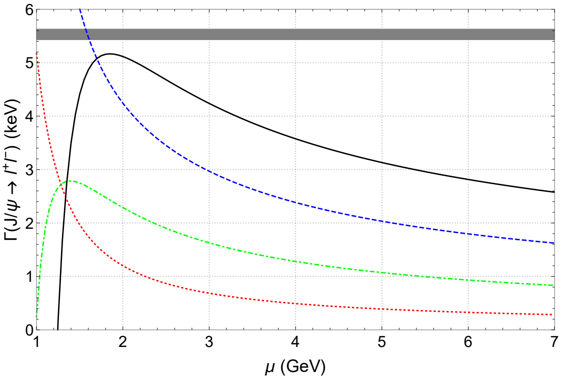
$\Gamma\left(\Upsilon(1S)\right)$ with the PS-mass depending on μ and $\alpha_s(M_Z)$ up to LO, NLO, NNLO, NNNLO

$$\Gamma\left(\Upsilon(1S) \to l^+l^-\right)\big|_{\mathrm{PS},A} = \left[1.083 \pm 0.053(\alpha_s)^{+0.001}_{-0.270}(\mu)\right]\ \mathrm{keV}$$

J/Ψ decay

- apply same formalism to J/Ψ
- three light quarks, hard charm quark
- numerical values:
  - $m_{c,pole} = 1.65\mathrm{GeV}$
  - $\mu = 2\mathrm{GeV}$
  - $\alpha_s^{(3)}(2\mathrm{GeV}) = 0.2943$
- experimental value [Zyla et al. (2020)]

$$\Gamma\left(J/\Psi \to l^+ l^-\right) = 5.53 \pm 0.10\mathrm{keV}$$



Decay width

$$\Gamma = \frac{2^7 \alpha^2 m_b (\alpha_s^{(3)})^3}{3^5} (1 + 0.875 + 1.596 + 0.654)$$
$$= \left[5.08 \pm 0.35(\alpha_s)^{+0.03}_{-2.25}(\mu)\right] \ \mathrm{keV}$$