NLO QCD Corrections to inclusive $b \to c\ell\bar{\nu}$ decay spectrum up to $1/m_Q^3$

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based on

T. Mannel, D. Moreno and A. A. Pivovarov, hep-ph/2112.03875

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Context

Testing the flavour sector of the SM is one of the main current activities in particle physics

- Some recent data show persistent tensions with the SM predictions (B-anomalies).
- Could be interpreted as first signals for BSM effects.

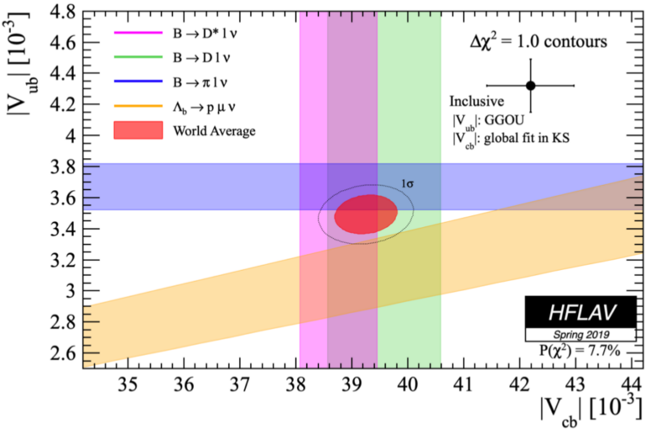
Precision quark-flavour physics may become an important tool to establish the presence of BSM effects. This requires

(a) **Precise measurements**: ongoing BelleII, LHCb experiments

(b) **Precise theoretical calculations**: In the context of B-physics and the HQE means to push for higher orders in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$.

Context

Persistent B anomaly: tension between $|V_{cb}|^{\text{in./ex.}}$



Context

Persistent B anomaly: tension between $|V_{cb}|^{\text{in./ex.}}$.

$$\text{key for precise } |V_{cb}|^{\text{in.}}_{\text{th.unc.}< \, 2\%} \quad \Rightarrow \quad \left.\frac{d\Gamma^{in.}(B \to X_c \ell\bar{\nu})}{d(p_\ell + p_\nu)^2}\right|_{\text{HQE}}$$

Allows precise extraction of a reduced number of NP hadronic matrix elements from data. [M. Fael, T. Mannel and K. Keri Vos, JHEP **02** (2019), 177]

- **New measurements**: in the form of moments of the spectrum.
  [R. van Tonder *et al.* [Belle], PRD **104** (2021), 112011]

- **New theoretical precision (our work)**: we compute

  $$\left.\frac{d\Gamma^{in.}(B \to X_c \ell\bar{\nu})}{dq^2}\right|_{\text{HQE}} \qquad \text{up to } \mathcal{O}\left(\alpha_s/m_b^3\right) , \quad q = p_\ell + p_\nu$$

  with massive final-state quark m_c, analytically.
  [T. Mannel, D. Moreno and A. A. Pivovarov, PRD **105** (2022), 054033]

We expect a further improvement in the precision of $|V_{cb}|^{\text{in.}}$.

HQE for inclusive semileptonic decays

The $\Gamma(B \to X_c \ell \bar{\nu}_\ell)$ can be obtained from

$$\begin{aligned} \Gamma(B \to X_c \ell \bar{\nu}_\ell) &\sim \mathrm{Im}\, \langle B | i \int dx\, T \left\{ \mathcal{L}_{\mathrm{eff}}(x) \mathcal{L}_{\mathrm{eff}}(0) \right\} | B \rangle \\ \mathcal{L}_{\mathrm{eff}}^{b \to c\ell\bar{\nu}_\ell} &= 2\sqrt{2} G_F V_{cb} (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_L \gamma^\mu \ell_L) + \mathrm{h.c.} \end{aligned}$$

Since $m_b \gg \Lambda_{\mathrm{QCD}}$ one can set up an expansion in $\Lambda_{\mathrm{QCD}}/m_b$ (HQE) by using local operators in HQET

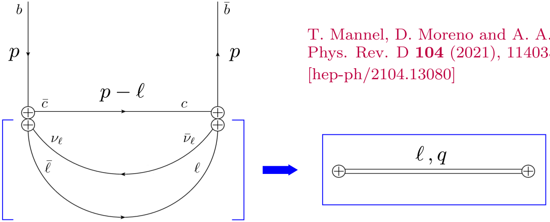
$$\Gamma(B \to X_c \ell \bar{\nu}_\ell) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[C_0 \left(1 - \frac{\mu_\pi^2}{2m_b^2} \right) + C_{\mu_G} \left(\frac{\mu_G^2}{2m_b^2} - \frac{\rho_{LS}^3}{2m_b^3} \right) - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} \right],$$

perturbative and non-perturbative contributions are factorized in:

- **Wilson coefficients**: $C_i(\rho = m_c^2/m_b^2)$ have a perturbative expansion in $\alpha_s(m_b)$, obtained by matching to QCD.
- **Forward ME of HQET operators**: called hadronic parameters μ_π^2, μ_G^2, ρ_{LS}^3 and $\rho_D^3 \sim \langle B | \bar{h}_v [D_{\perp\,\mu}, [D_\perp^\mu, v \cdot D]] h_v | B \rangle$.

HQE for inclusive semileptonic decays

The total width can be written as an integral differential in q^2 by using a dispersion representation for the (massless) lepton-neutrino loop



T. Mannel, D. Moreno and A. A. Pivovarov, Phys. Rev. D **104** (2021), 114035 [hep-ph/2104.13080]

$$\int \frac{d^D k}{(2\pi)^D} \frac{-\mathrm{Tr}(\Gamma^\sigma(\not{k}+\not{\ell})\Gamma^\rho \not{k})}{k^2(k+\ell)^2} = \frac{i}{24\pi^2}\int_0^\infty d(q^2) \underbrace{\frac{1}{\ell^2-q^2+i\eta}(\ell^2 g^{\rho\sigma}-\ell^\rho\ell^\sigma)}_{\text{transverse "effective massive propagator" with mass } q} \;.$$

HQE for inclusive semileptonic decays

The HQE of the decay spectra is writen as follows

$$\frac{d\Gamma(B \to X_c \ell \bar{\nu}_\ell)}{dr} = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[\mathcal{C}_0 \left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) + \mathcal{C}_{\mu_G}\left(\frac{\mu_G^2}{2m_b^2} - \frac{\rho_{LS}^3}{2m_b^3}\right) - \mathcal{C}_{\rho_D}\frac{\rho_D^3}{2m_b^3}\right]$$

where $r = q^2/m_b^2$. The $\mathcal{C}_i(r,\rho)$ are related to the $C_i(\rho)$ by

$$C_i(\rho) \quad = \quad \int_0^{(1-\sqrt{\rho})^2} dr\, \mathcal{C}_i(r,\rho)\,.$$

Experimentalists measure moments of the spectra with low cuts (low q^2 difficult to detect), which we compute in the theory side

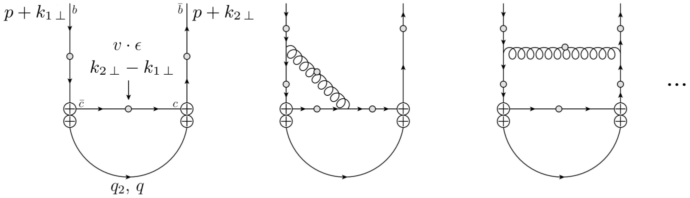
$$M_n(\rho, r_{\rm cut}) \quad = \quad \int_{r_{\rm cut}}^{(1-\sqrt{\rho})^2} dr\, r^n \frac{d\Gamma(r,\rho)}{dr}\,, \qquad \langle q^{2n}\rangle \equiv m_b^{2n}\frac{M_n}{M_0}$$

We compute $\mathcal{C}_i$ and moments analytically up to $\mathcal{O}(\alpha_s/m_b^3)$.

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$

At α_s/m_b^3 we only need to determine the coefficient of ρ_D (Darwin term)

- **Take the amplitude** of quark to quark-gluon scattering with kin. conf.



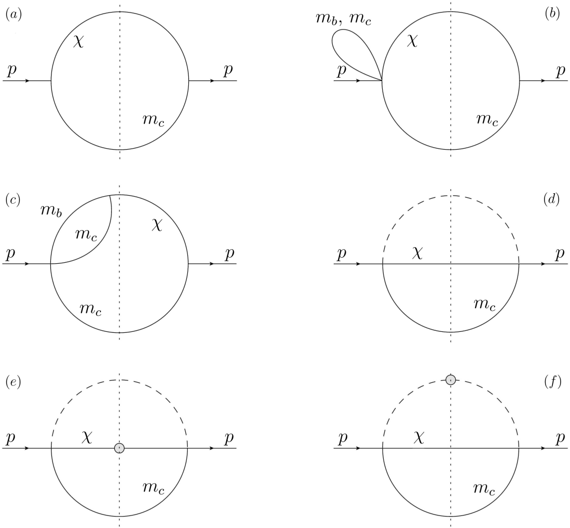
with $p^2 = m_b^2$ and $k_\perp^\mu = k^\mu - v^\mu(v \cdot k)$.

- **Expand** to quadratic order in the small momenta $k_{1\perp}$, $k_{2\perp}$.

- **Project** to the Darwin operator: pick up $k_{1\perp}^{(\alpha} k_{2\perp}^{\beta)}$ structure.

Be careful! We must disantangle contributions to dim. 6 operators $\bar{h}_v(v \cdot D)D_\perp^2 h_v$, ..., that contribute to higher orders after using the EOM.

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$



$$
\begin{aligned}
x_- &= \frac{1}{2}(1 - r + \rho - A) \\
x_+ &= \frac{1}{2}(1 - r + \rho + A) \\
A &= \sqrt{(1 - (\sqrt{r} - \sqrt{\rho})^2)(1 - (\sqrt{r} + \sqrt{\rho})^2)}
\end{aligned}
$$

T. Mannel, D. Moreno and A. A. Pivovarov, PRD 104 (2021), 114035 [hep-ph/2104.13080]

IBP

dim. reg. $D = 4 - 2\epsilon$

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$

Other important remarks:

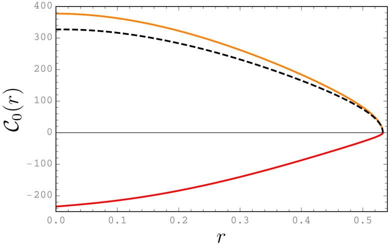
- Renormalization can be performed at differential level ($\epsilon \to 0$ finite).
- Cancellation of poles is delicated and provides a solid check:
  - (a) Requires to consider the mixing under renormalization between HQET operators of different dimension, like [Bauer and Manohar, PRD **57**, 337 (1998)]

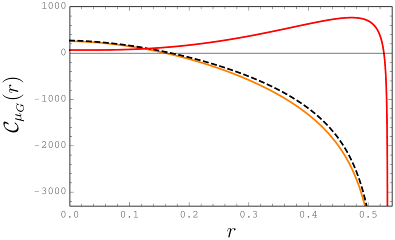
$$\mathcal{O}_\pi^B = \mathcal{O}_\pi^R + \gamma_{\pi D}\frac{\alpha_s}{\pi}\frac{1}{m_b}\mathcal{O}_D$$

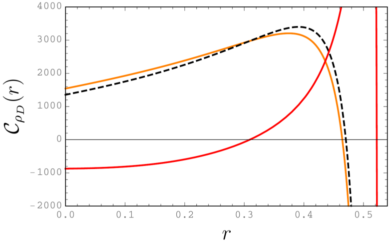
  - (b) γ_{iD} obtained from the combined insertion of operators of the HQE and operators of the HQET Lagrangian.
- $\mathcal{C}_{\rho_D}(\epsilon = 0)$ finite, but integration over r is IR singular at $r_{\max}$ (ϵ dep. must be restored in the IR singular terms).

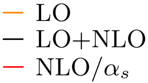
$$C_{\rho_D}^{\rm IR} \sim \int_0^{r_{\max}} dr\,\frac{1}{(r_{\max}-r)^{3/2}} \to \int_0^{r_{\max}} dr\,\frac{1}{(r_{\max}-r)^{3/2+\epsilon}}$$

Numerical analysis





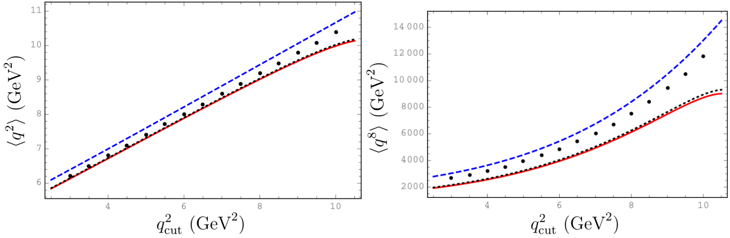


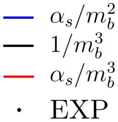


Parameter	Numerical value
$\mu = m_b$	4.8 GeV
$\rho = m_c^2/m_b^2$	0.073
$\alpha_s(m_b)$	0.215
$r_{\max} = (1-\sqrt{\rho})^2$	0.5326

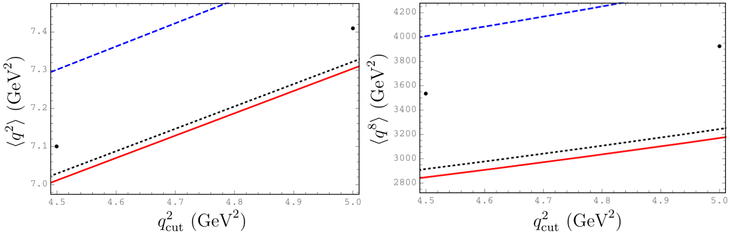
Numerical analysis

Experimentally, one measures moments of the spectrum. Low q^2 is difficult to detect and exp. use cuts while integrating up to the available q^2.





R. Van Tonder *et al.* [Belle] [hep-ex/2109.01685]



Parameter	Numerical value
μ_π^2	0.4 GeV^2
μ_G^2	0.35 GeV^2
ρ_D^3	0.2 GeV^3
ρ_{LS}	-0.15 GeV^3
q^2_{max}	12.27 GeV^2

D. Benson, *et al.* NPB 665, 367 (2003) [hep-ph/0302262]

Final remarks

- We have computed $d\Gamma^{\text{in.}}(B \to X_c \ell\bar{\nu})/dr$ and $M_n(r_{\text{cut}})$ up to $\mathcal{O}(\alpha_s/m_b^3)$ with massive final state quark, analytically.
- We correct C_{ρ_D} at NLO, previously obtained by direct use of 3-loop Feynman integrals.
- Current knowledge of the HQE for $B \to X_c \ell\bar{\nu}$ decay distributions: $(\alpha_s^2,\, \alpha_s/m_b^3,\, 1/m_b^5)$.
- Moments to $\mathcal{O}(\alpha_s^3)$ recently computed
  [M. Fael, K. Schönwald and M. Steinhauser, hep-ph/2205.03410]
- The corrections we have computed are $(\sim 1\%)$, and we expect a small but visible impact on $|V_{cb}|$.
- Overall, this will allow to increase the precision of $|V_{cb}|$ by using $M_n(r_{\text{cut}})$, where a first analysis have given $|V_{cb}| = (41.69 \pm 0.63) \cdot 10^{-3}$
  [F. Bernlochner, M. Fael, K. Olschewsky, E. Persson, R. van Tonder, K. K. Vos and M. Welsch, hep-ph/2205.10274]

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