Analytic leading two-loop Yukawa corrections to $gg \to HH$

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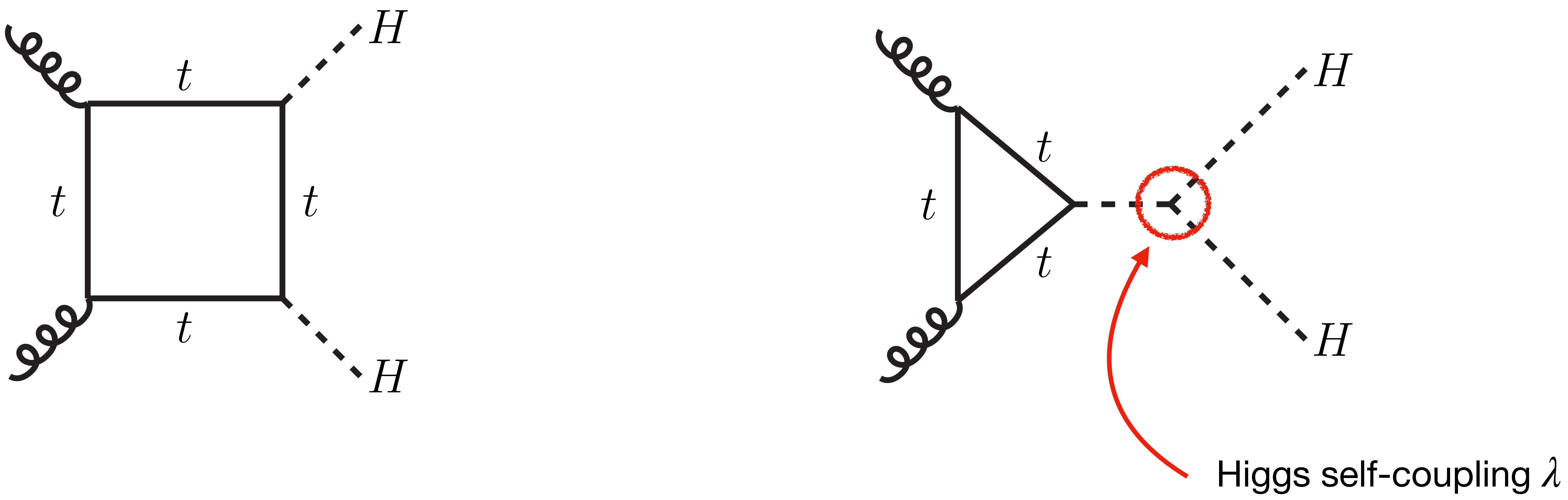
Motivation: Higgs self-coupling

- Probe Higgs self-coupling in pair productions, and compare with the Standard Model value

  $\lambda = m_H^2/(2v^2) \approx 0.13$ in the Higgs potential

$$V(H) \;=\; \frac{1}{2} m_H^2 H^2 + \lambda\, v\, H^3 + \frac{\lambda}{4} H^4$$

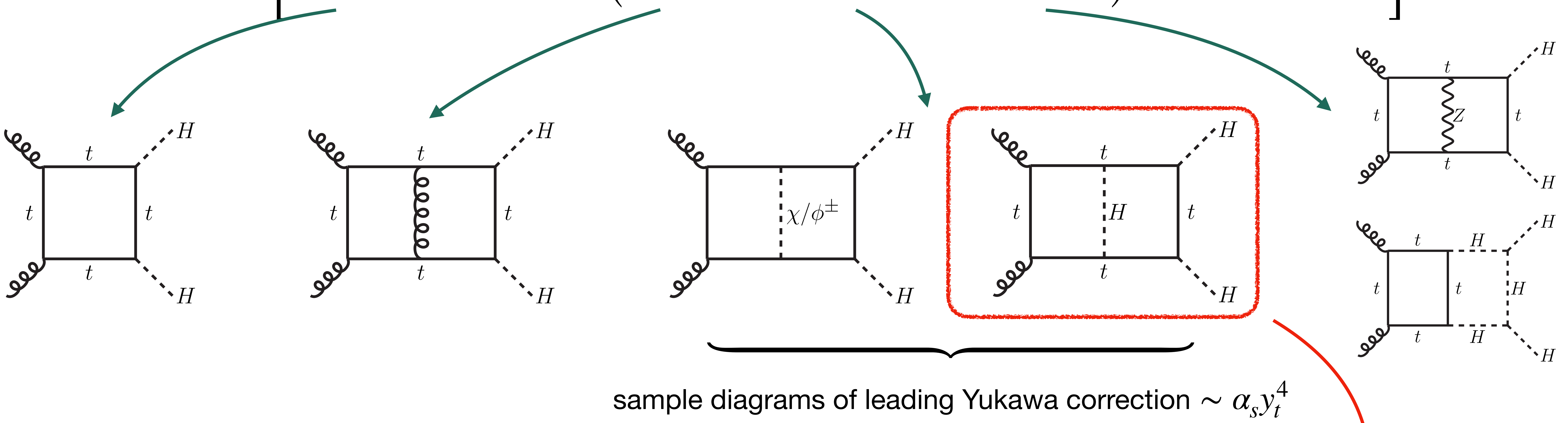
- Gluon-fusion channel dominates Higgs pair production at LHC



Motivation: two-loop Yukawa corrections

- Perturbative expansion of amplitudes in loop-induced gluon-fusion channel

$$\mathscr{A}_{gg\to HH} \approx \alpha_s \left[y_t^2 A_1^{(1)} + y_t\lambda A_1^{(2)} + \left(\alpha_s y_t^2 A_2^{(1)} + y_t^4 A_2^{(2)} + \alpha_{ew}^{(y_t,\lambda,\alpha)} A_2^{(3)} \right) + \mathcal{O}\left(\alpha_s^2, \alpha_s\alpha_{ew}\right) \right]$$



- Two-loop Yukawa (EW) corrections are not small, $y_t^2 = \frac{\alpha\, m_t^2}{2\sin^2\theta_w\, m_W{}^2} \sim \frac{1}{2}\alpha_s$

- **Aim:** analytic high energy expansion to leading Yukawa corrections with internal Higgs in $p_T^H \geq 150\,\mathrm{GeV}$ region

Previous calculations

- **NLO QCD corrections with full m_t-dependence are known**
  - Numerical approach [Borowka, Geiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke, 16'], [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher, 18']
  - Expansions in different regions [Daoson, Dittmaier, Spira, 98'], [Grigo, Hoff, Melnikov, Steinhauser, 13'], [Degrassi, Giardino, Gröber, 16', and Bonciani, 18'], [Gröber, Maier, Raum, 17'], [Davies, Mishima, Steinhauser, Wellmann, 18', 19'], [Xu, Yang, 18', and Wang, Xu, 20']
  - Expansions + numerical approach [Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann, 19'], [Bellafronte, Degrassi, Giardino, Gröber, Vitti, 22']

- **NNLO and N^3LO QCD corrections are available in large-m_t limit / expansion**
  - At NNLO [de Florian, Mazzitelli, 13'], [Grigo, Melnikov, Steinhauser, 14' and Hoff, 15'], [Davies, Herren, Mishima, Steinhauser, 19', 21'], [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, 18']
  - At N^3LO [Spira, 16'], [Gerlach, Herren, Steinhauser, 18'], [Banerjee, Borowka, Dhabi, Gehrmann, Ravindran, 18'], [Chen, Li, Chao, Wang, 19']

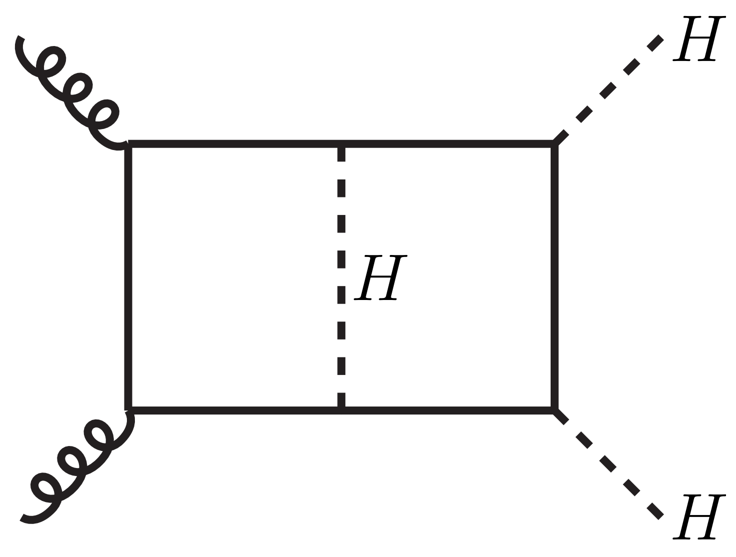
High energy expansion

Why?

- Complete analytic solution is out of reach $\Leftarrow$ double box diagram with 9 massive lines and 2 masses $\{m_t, m_H\}$
- New physics from high energies can have sizeable impacts in large p_T^H region
- High energy expansion + Padé improved approximation yield convergent analytic solution in $p_T^H \geq 150\,\mathrm{GeV}$

  $\Rightarrow$ accurate and fast evaluations for future phenomenology studies

How?



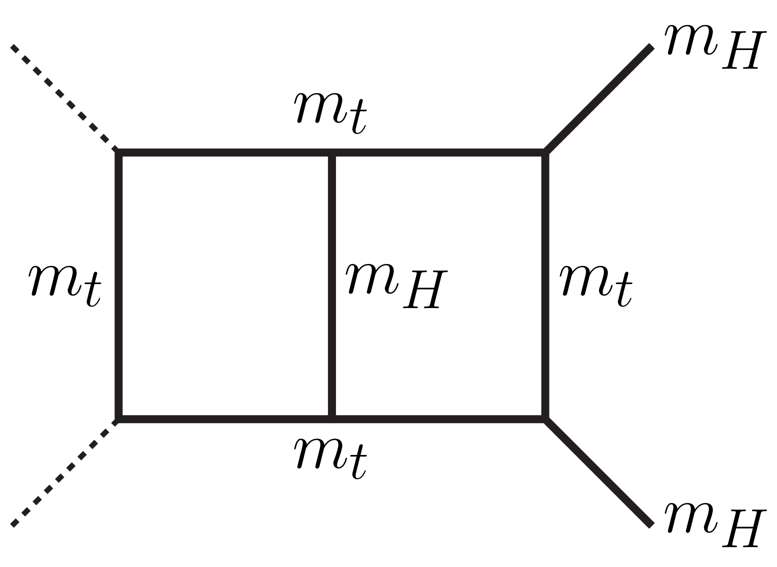
kinematic invariants

$s = (q_1 + q_2)^2$

$t = (q_1 + q_3)^2$

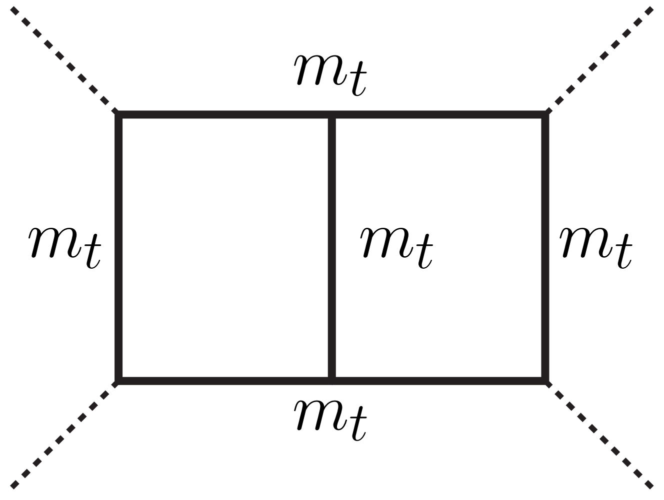
$u = (q_2 + q_3)^2$

scalar master integral (MI)



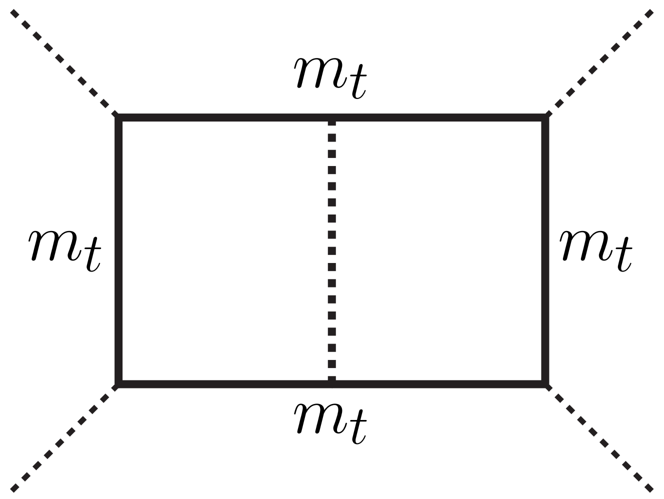
solid line: massive
dashed line: massless

two expansions



$s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2$

- general to full EW corrections
- need entire new family of MIs



$s, t \gg m_t^2 \gg \left(m_H^{\text{int}}\right)^2, \left(m_H^{\text{ext}}\right)^2$

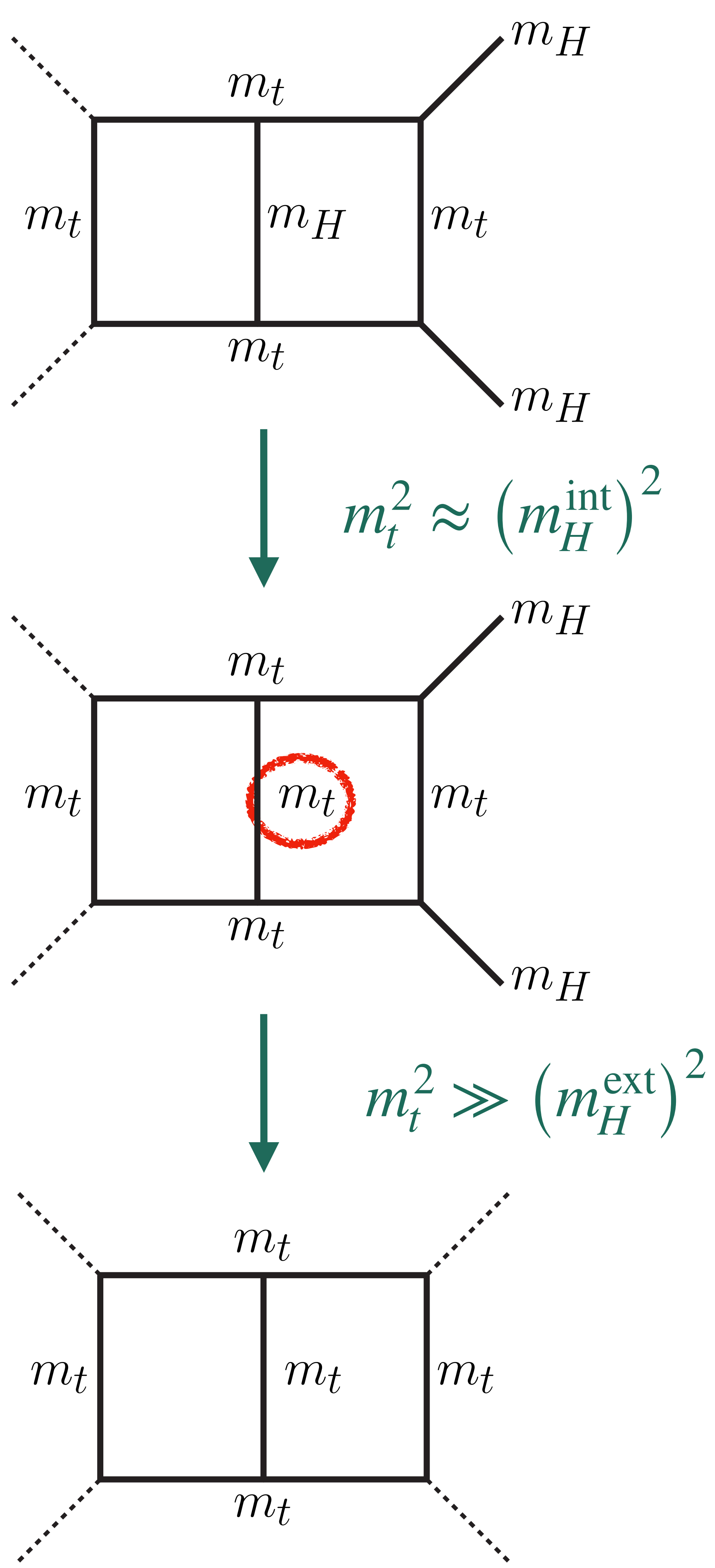
- reduce to known QCD MIs
- non-trivial to full EW corrections

Equal-mass expansion in high energies

Expansion Hierarchies: $s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2$

Simple Taylor expansions:

1. Expand internal lines in $\delta = 1 - \frac{m_H^{(\text{int})}}{m_t} \Rightarrow \frac{1}{p^2 - m_H^2} = \frac{1}{p^2 - m_t^2\left[1 - (2-\delta)\,\delta\right]}$
2. Expand external legs in limit $m_t \gg m_H^{(\text{ext})}$



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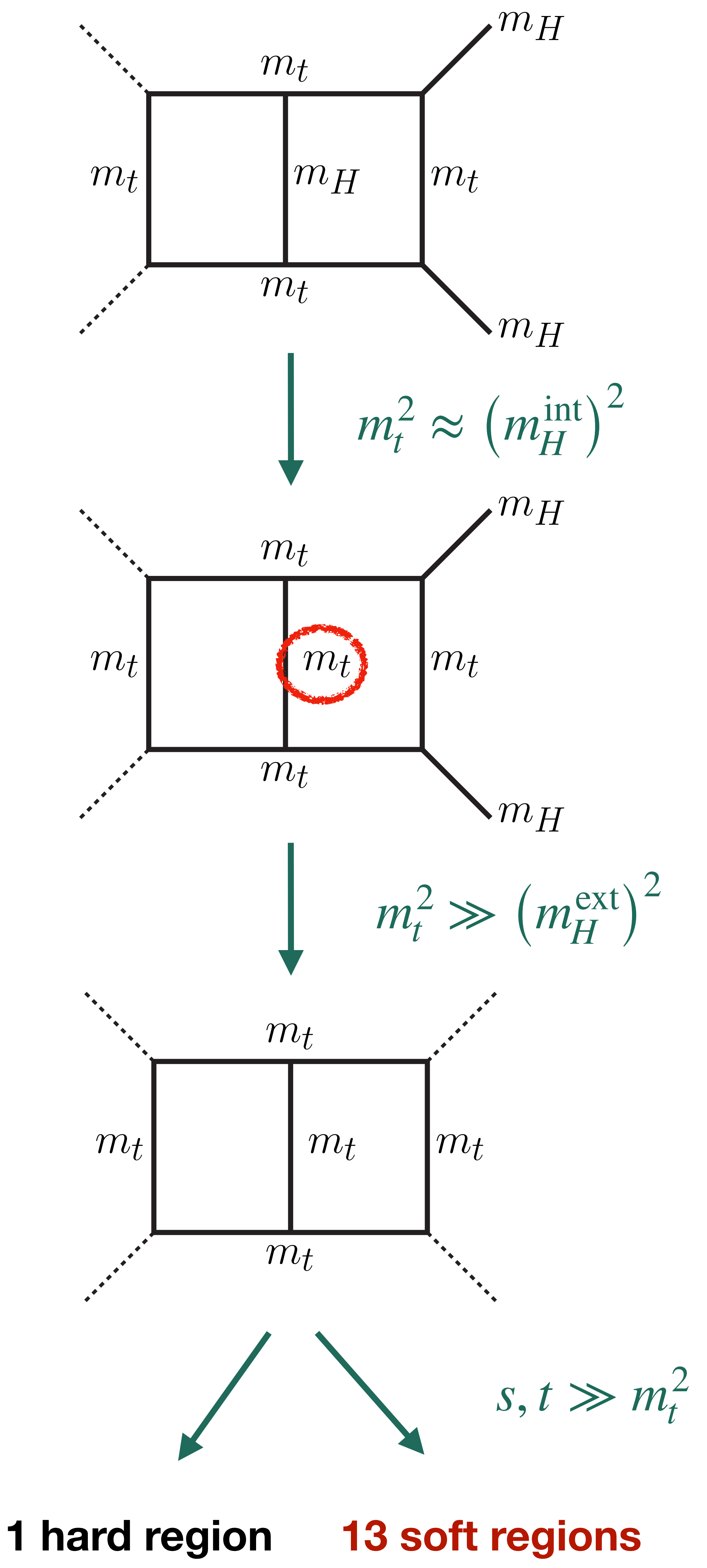
Asymptotic expansion: $s, t \gg m_t^2$

- Perform expansion in alpha-representation with **`asy2.1.m`** [Pak, Janzten, Smirnov[2]]

  $\Rightarrow$ generate 1 hard region and 13 soft regions scalings

$$(\alpha_1, \ldots, \alpha_7) \overset{\chi}{\sim} \underbrace{(0,0,0,0,0,0,0)}_{\textbf{hard}},\ \underbrace{(0,0,0,0,1,1,1)}_{\textbf{soft-1}},\ \ldots,\ \underbrace{(1,1,1,1,1,1,0)}_{\textbf{soft-13}}$$



Higher-order $(m_t^2/s)^n$ corrections

MIs, differential equations, asymptotic expansions and Mellin-Barnes integrals

1. Amplitudes are generated by `qgraf` [Nogueira], `q2e&exp` [Harlander, Seidensticker, Steinhauser], `FORM` [Vermaseren], and integration-by-parts (IBP) reduction yields 140 MIs to $\mathcal{O}(\epsilon^0)$ by `FIRE6 and ImproveMasters.m` [Chukharev, Smirnov^2]
2. Linear system of coupled differential equations for 140 MIs derived with `LiteRed` [Lee]

$$\frac{\partial}{\partial\left(m_t^2\right)}\mathbf{I} = M(s,t,m_t^2,\epsilon)\,\mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{I}_1, \ldots, \mathcal{I}_{140}\right)^T$$

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3. Higher-order $(m_t^2/s)^n$ corrections can be solved by inserting **power-log ansatz** for each MI

$$\mathcal{I}_n = \sum_{i=-2}^{0}\sum_{j=-1}^{40}\sum_{k=0}^{i+4} C_{(n)}^{ijk}(s,t)\,\epsilon^i\left(m_t^2\right)^j \log^k(m_t^2)$$

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challenging & require improved method

4. Boundary conditions are calculated in limit $m_t \to 0$ by Mellin-Barnes method and analytic summations with help of `MB.m` [Czakon], `HarmonicSums.m` [Ablinger], `Sigma.m and EvaluateMultiSums.m` [Schneider]

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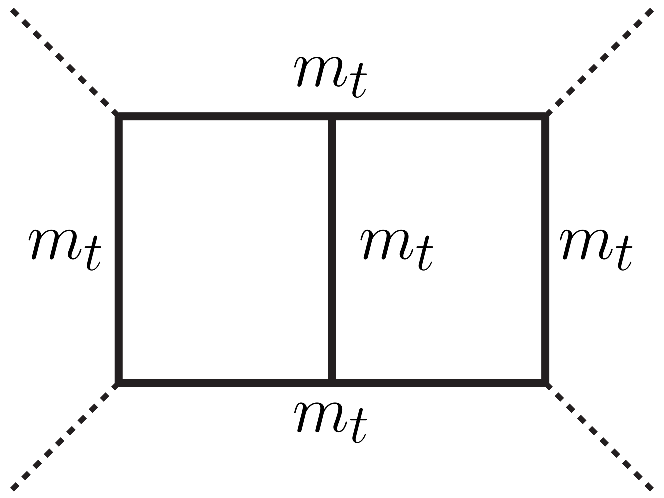
5. Solve all MIs to $\mathcal{O}\left(m_t^{80}\right)$ by **KIRA.m** [Klappert, Lange, Maierhöfer, Usovitsch] and apply Padé approximations to form factors

$$F^N = \lim_{x\to 1}\sum_{i=0}^{N} f_i\,(m_t^2)^i\,x^i \qquad \Rightarrow \qquad F^N = \lim_{x\to 1}\frac{a_0 + a_1x + \ldots + a_nx^n}{1 + b_1x + \ldots + b_mx^m} = \lim_{x\to 1}[n/m](x)$$

Boundary conditions

Complexity of boundary contributions

- Choosing ϵ-finite basis for 140 MIs $\Rightarrow$ 7-line top-sector integrals **(1,1,1,1,1,1,1,0,0), (1,1,1,1,1,1,2,0,0), (1,1,1,1,1,1,1,-1,0), (1,1,1,1,1,1,1,-1,-1)**, where last two indices denote irreducible numerators (scalar products).
- The most complicated boundary is "7+2"-line integral **(1,1,1,1,1,1,1,-1,-1)** at $\mathcal{O}(m_t^0)$:



with two numerators

introduce propagators into alpha representation →

$$\frac{1}{(D_i)^\lambda} = \begin{cases} \dfrac{1}{\Gamma(\lambda)} \displaystyle\int_0^\infty \mathrm{d}\alpha\, \alpha^{\lambda-1}\, e^{-D_i\,\alpha} & \text{for } \lambda > 0 \\ (-1)^{|\lambda|} \dfrac{\partial^{|\lambda|}}{\partial \alpha^{|\lambda|}}\, e^{-D_i\,\alpha}\Big|_{\alpha=0} & \text{for } \lambda < 0 \end{cases}$$

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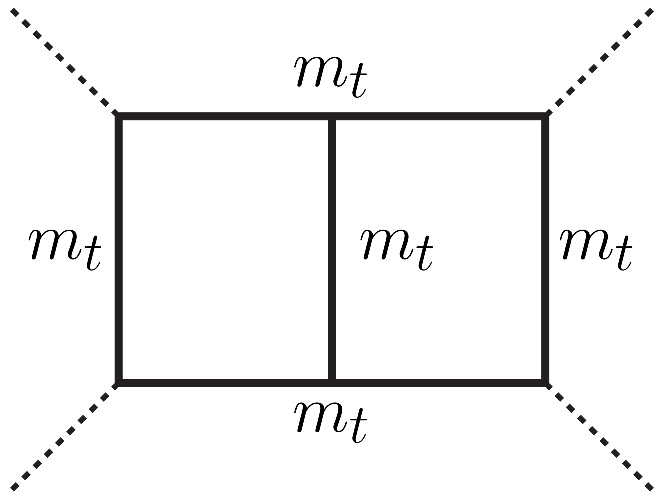

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region-regulators $\{\delta_1, \ldots, \delta_7\}$

$$\mathcal{I}_{7,2} = \int_0^\infty \left(\prod_{i=1}^{7} d\alpha_i \frac{\alpha_i^{\delta_i}}{\Gamma(1+\delta_i)} \right) \left(\frac{\partial}{\partial \alpha_9} \frac{\partial}{\partial \alpha_8} \tilde{\mathcal{U}}^{-d/2} e^{-\tilde{\mathcal{F}}/\tilde{\mathcal{U}}} \right) \Bigg|_{\alpha_8=\alpha_9=0}$$

$$\mathcal{U} = \tilde{\mathcal{U}}\Big|_{\alpha_8=\alpha_9=0} \quad \text{and} \quad \mathcal{F} = \tilde{\mathcal{F}}\Big|_{\alpha_8=\alpha_9=0}$$

$$= \underbrace{\int_0^\infty \left(\prod_{i=1}^{7} d\alpha_i \frac{\alpha_i^{\delta_i}}{\Gamma(1+\delta_i)} \right) \mathcal{U}^{-d/2} e^{-\mathcal{F}/\mathcal{U}}}_{\text{template integrals } \mathbf{T}(\delta_1, \ldots, \delta_7, \epsilon)} \underbrace{\left(\hat{\mathcal{O}}(d^2, \{\alpha_i\}) + \hat{\mathcal{O}}(S^2, \{\alpha_i\}) + \hat{\mathcal{O}}(T^2, \{\alpha_i\}) + \hat{\mathcal{O}}(d\,S, \{\alpha_i\}) + \hat{\mathcal{O}}(d\,T, \{\alpha_i\}) + \hat{\mathcal{O}}(S\,T, \{\alpha_i\}) \right)}_{\text{Shift operators to template integrals}}$$

template integrals $\mathbf{T}(\delta_1, \ldots, \delta_7, \epsilon)$

Shift operators to template integrals (rational functions in $\{\alpha_i\}$)

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1. Apply asymptotic expansion $S, T \gg m_t^2$ in Euclidean region
   $\Rightarrow$ 1 hard region ✅ and 13 soft regions
2. Apply Mellin transformation to 13 soft template integrals $\mathbf{T}_n$
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Dimension of MB rep.	Number of integrals
1-dim	2003
2-dim	515
3-dim	14

Compact expression compared to IBP approach for $\mathcal{I}_{7,2}$ ✓

Boundary conditions

Mellin-Barnes (MB) integrals, residue theorem and non-vanishing arc

1. Derive MB representation for soft-template integrals via parametric integration and Mellin transformation:

$$(x+y)^\lambda = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{\Gamma(-\lambda+z)\Gamma(-z)}{\Gamma(-\lambda)} x^z y^{\lambda-z} \quad \text{with} \quad z \in \mathbb{C}$$

2. Apply shift operators and fix the integration contours as straight lines, e.g. $\{\mathrm{Re}(z_1) = -1/7, \ldots\}$, and perform analytical continuation and expansion in regulators with `MB.m` [Czakon]

3. Apply residue theorem by closing the contour to left or right semi-circle, and sum up residues, e.g.

$$\int_{-1/7-i\infty}^{1/7-i\infty} \frac{dz_1}{2\pi i} f(z_1) = \sum_{k=0}^{\infty} \mathrm{Res}_{z_1=k}\left[f(z_1)\right] - \int_{\mathrm{arc}} \frac{dz_1}{2\pi i} f(z_1) \quad \text{with} \quad f(z_1) = \frac{z_1^8 \Gamma(-z_1)^2 \Gamma(z_1)^2}{(z1+1)^3 (z_1+2)^3}$$

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‼ arc integral can be non-zero ‼

$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

by **`HarmonicSums.m & Sigma.m`** [Ablinger, Schneider]

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‼ arc integral can be non-zero ‼

solve arc contribution by adding auxiliary scale:

$$\int_{\mathrm{arc}} \frac{dz_1}{2\pi i} \xi^{z_1} f(z_1) = -\sum_{k=0}^{\infty} \frac{k^6}{(1+k)^3(2+k)^3} \xi^k \log(\xi) \overset{\xi\to 1}{=} -1$$

Boundary conditions

Mellin-Barnes (MB) integrals, residue theorem and non-vanishing arc

1. Derive MB representation for soft-template integrals via parametric integration and Mellin transformation:

$$(x+y)^\lambda = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} \frac{\Gamma(-\lambda+z)\Gamma(-z)}{\Gamma(-\lambda)} x^z y^{\lambda - z} \quad \text{with} \quad z \in \mathbb{C}$$

2. Apply shift operators and fix the integration contours as straight lines, e.g. $\{\mathrm{Re}(z_1) = -1/7, \ldots\}$, and perform analytical continuation and expansion in regulators with **MB.m** [Czakon]

3. Apply residue theorem by closing the contour to left or right semi-circle, and sum up residues, e.g.

$$\int_{-1/7-i\infty}^{1/7-i\infty} \frac{dz_1}{2\pi i} f(z_1) = \sum_{k=0}^{\infty} \mathrm{Res}_{z_1=k}\left[f(z_1)\right] - \int_{\mathrm{arc}} \frac{dz_1}{2\pi i} f(z_1) \quad \text{with} \quad f(z_1) = \frac{z_1^8 \Gamma(-z_1)^2 \Gamma(z_1)^2}{(z1+1)^3 (z_1+2)^3}$$

$$-18\zeta(3) - \frac{3\pi^2}{2} - \frac{21\pi^4}{10} + 240$$

by **HarmonicSums.m & Sigma.m** [Ablinger, Schneider]

!! arc integral can be non-zero !!

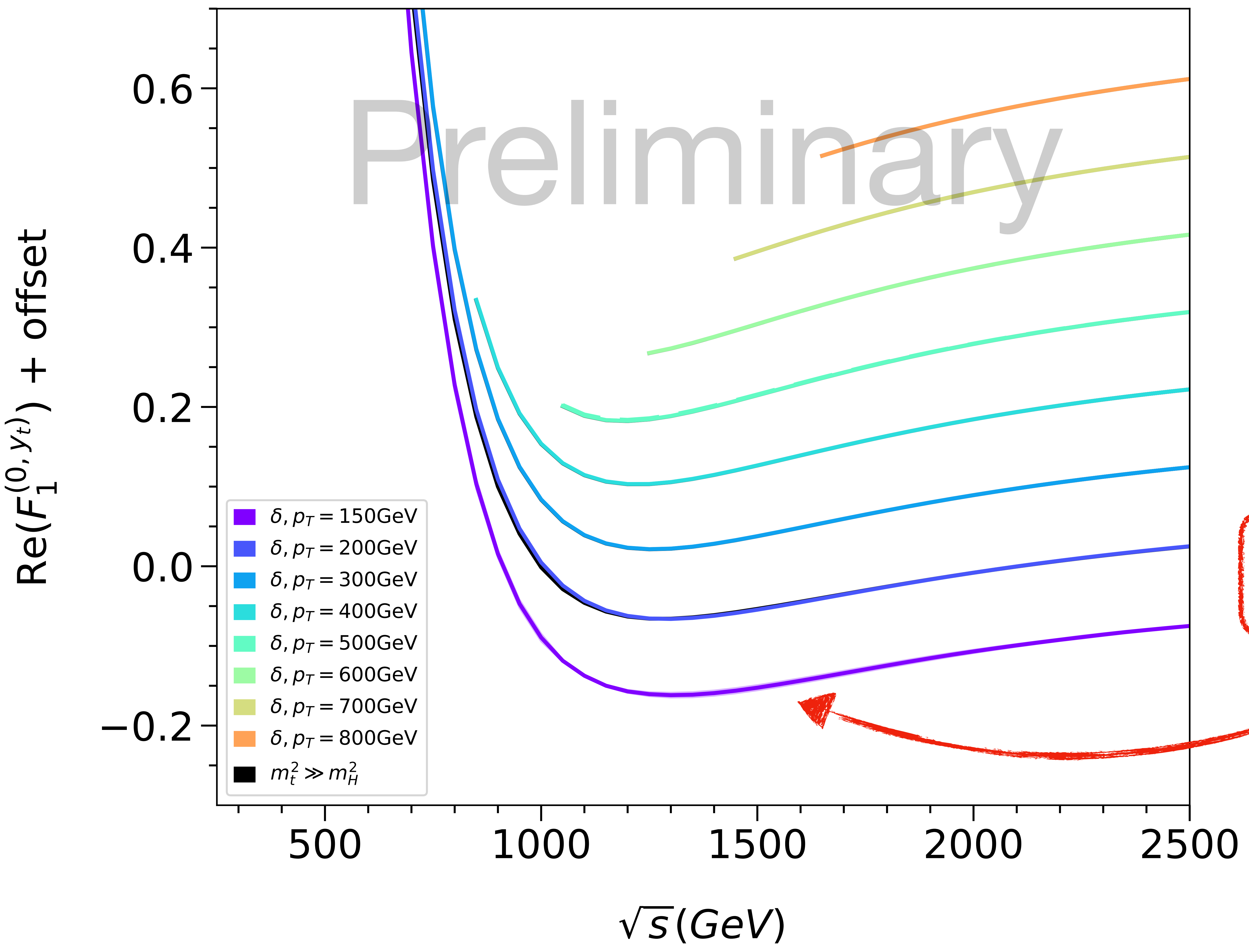
solve arc contribution by adding auxiliary scale:

$$\int_{\mathrm{arc}} \frac{dz_1}{2\pi i} \xi^{z_1} f(z_1) = -\sum_{k=0}^{\infty} \frac{k^6}{(1+k)^3(2+k)^3} \xi^k \log(\xi) \overset{\xi \to 1}{=} -1$$

- We use expansion and ansatz fitting for more complicated 2-dim MB integral with **nested arc contributions**

$$\int \frac{dz_1}{2\pi i} \frac{dz_2}{2\pi i} \left(\frac{t}{s}\right)^{z_1} \hat{\Gamma}(z_1) \hat{\Gamma}(z_2) \hat{\Gamma}(z_1, z_2) \quad \text{with } \hat{\Gamma} \text{ being products of Gamma functions,}$$

Form factors for $gg \to HH$ with various fixed p_T^H



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu} F_{\mathrm{box1}} + T_2^{\mu\nu} F_{\mathrm{box2}}^{\mu\nu}$$

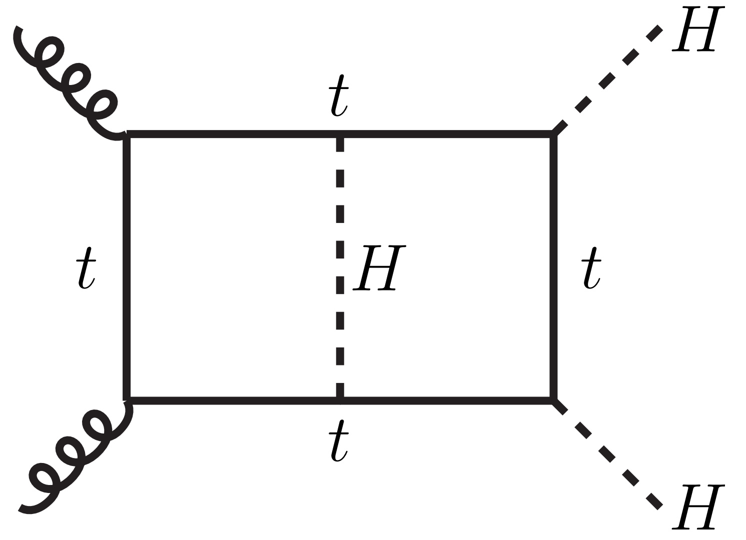
$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

Padé improved high energy expansions converge even at $p_T^H = 150$ GeV

Color lines: Padé improved equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\mathrm{int}}\right)^2$ using MIs from $\mathscr{O}(m_t^{52})$ to $\mathscr{O}(m_t^{60})$

Black lines: Padé improved expansion in $m_t^2 \gg m_H^2$

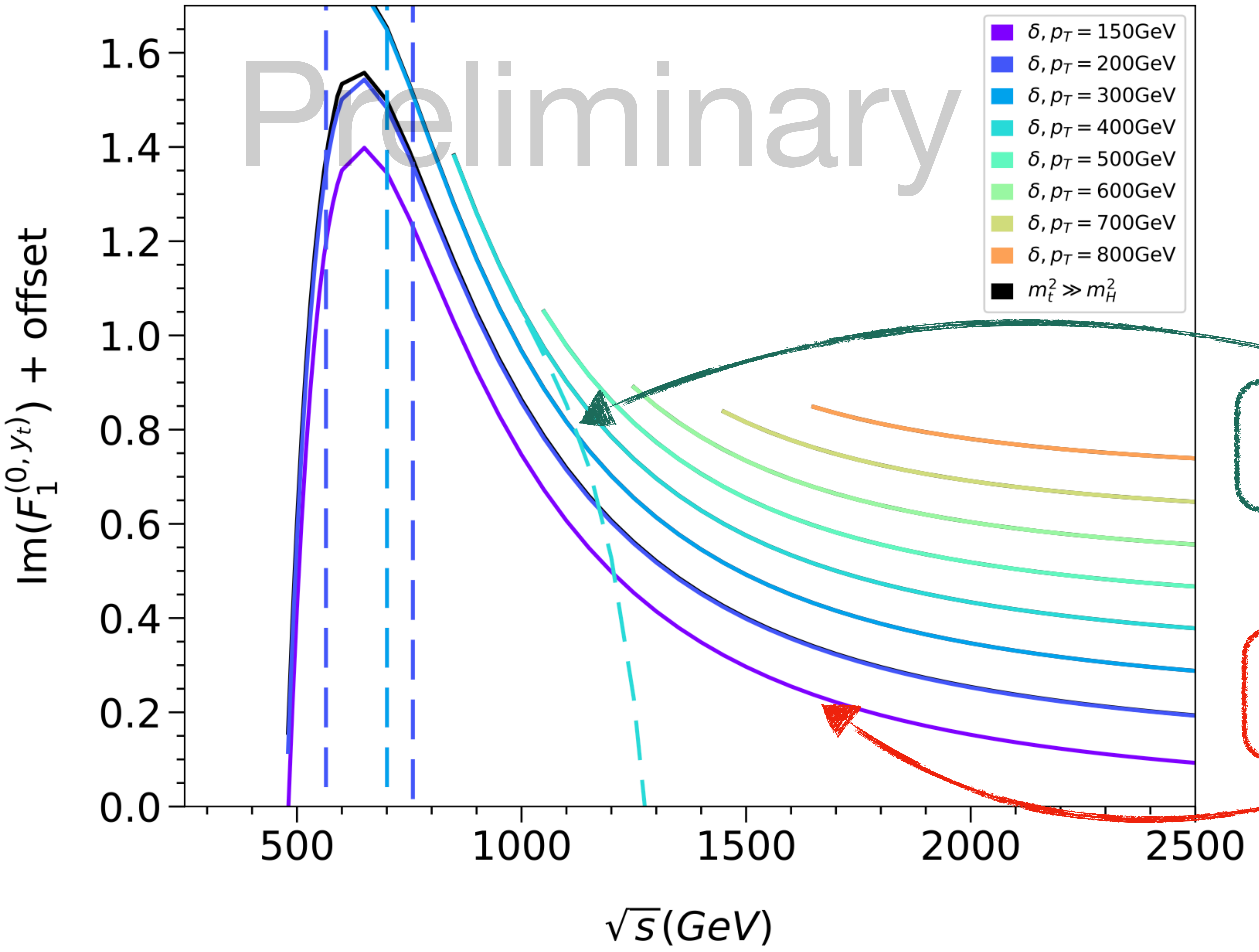
Conclusions and outlook



- We analytically compute leading two-loop Yukawa corrections to Higgs pair production in high energy limit.
- In particular, we propose two-expansion approaches for the technical challenging contribution from the exchange of Higgs boson inside the top-loop.
- We calculate new set of 140 master integrals up to $\mathcal{O}(m_t^{80})$ for fully massive double box families with Mellin-Barnes method and differential equations.
- Padé improved approximations for both expansions show good convergent results for $p_T^H > 150$ GeV region.
- Proof of principle for multiple asymptotic expansion approach to reduce numbers of scales, and make analytical calculations tractable.
- Future work, complete full Yukawa corrections and extend the equal-mass expansion approach to other EW corrections.

Backup Slides

Form factors for $gg \to HH$ with various fixed p_T^H



$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

Naive high energy expansions starts to diverge at $p_T^H = 400$ GeV

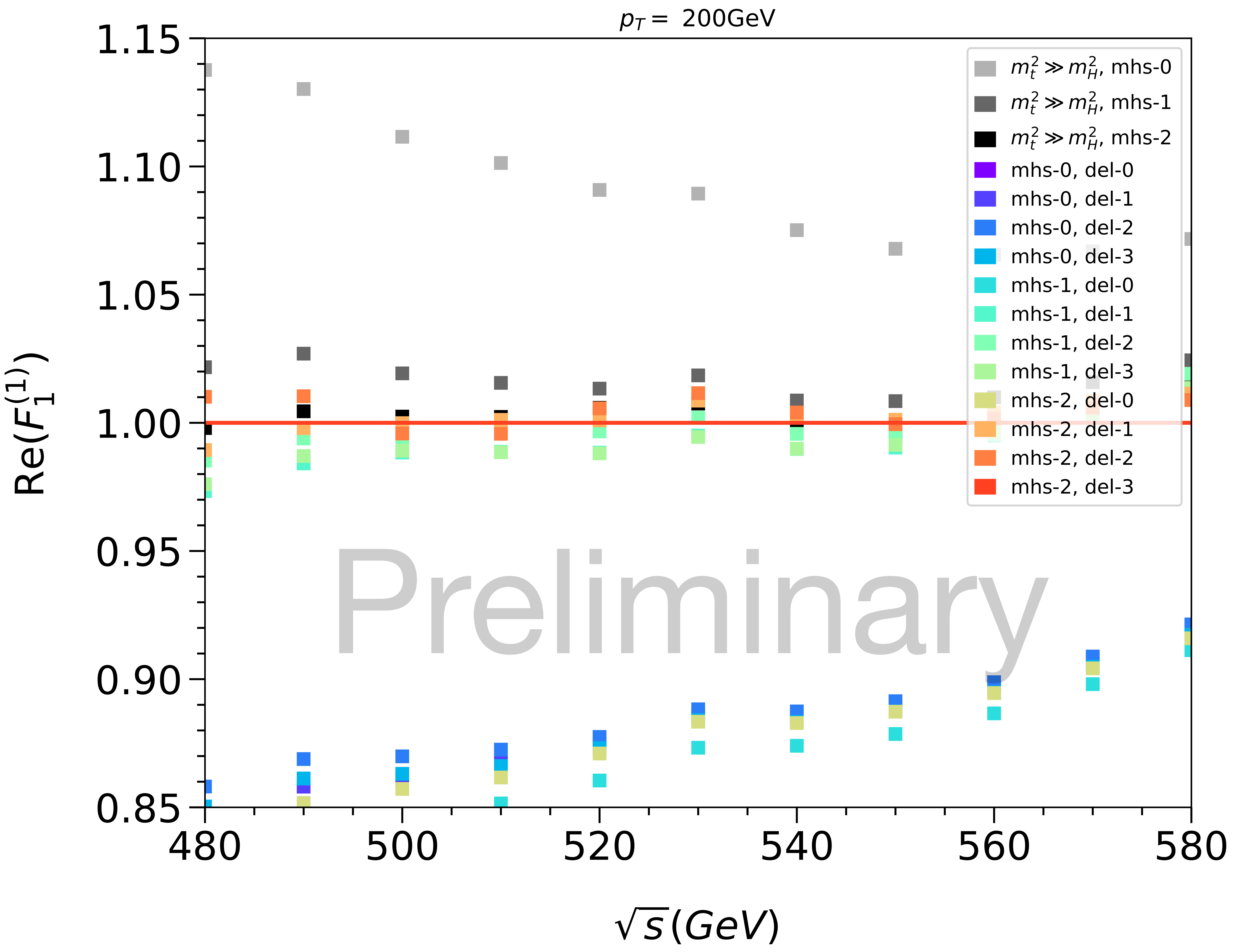
Padé improved high energy expansions converge even at $p_T^H = 150$ GeV

Color lines: Padé improved equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\text{int}}\right)^2$ using MIs from $\mathcal{O}(m_t^{52})$ to $\mathcal{O}(m_t^{60})$.

Dashed color lines: Naive equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\text{int}}\right)^2$ at high energies.

Black lines: Padé improved expansion in $m_t^2 \gg m_H^2$

Convergence of expansions for $gg \to HH$ form factors



Plot shows convergences of different expansion orders by ratios to the benchmark at fixed $p_T^H = 200$ GeV.

The benchmark is expansion at $\mathcal{O}\left(m_{H_{(\mathrm{ext})}}^4, \delta^3, m_t^{32}\right)$.

Color lines: Padé improved δ expansions in $m_t^2 \approx \left(m_H^{\mathrm{int}}\right)^2$ to $\mathcal{O}(m_t^{32})$.

Black-gray lines: Padé improved expansion in $m_t^2 \gg m_H^2$ to $\mathcal{O}(m_t^{32})$.