# The lepton propagator PROPOSAL as electromagnetic interaction model in CORSIKA 8

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# Introduction

- Monte Carlo simulation using (modified) EGS 4
  - Full Monte Carlo simulation of electromagnetic (sub)showers
    - Compton scattering, $e^+e^-$ pair production, photoelectric effect
    - Bhabha scattering, Møller scattering, bremsstrahlung, annihilation
    - Coulomb multiple scattering according to Molière's theory
  - Added photoproduction of muons $\gamma \to \mu\mu$
  - Added photohadronic interaction $\gamma N \to X$
  - Added LPM effect
  - Mortran Code
- Analytical calculations using the Nishimura-Kamata-Greisen (NKG) formulæ
  - Very fast
  - Gives only particle densities at selected points in the shower plane

- Electromagnetic interactions calculated with PROPOSAL (PRopagator with Optimal Precision and Optimized Speed for All Leptons)
- PROPOSAL originally developed for muon (and tau lepton) propagation
  - Developed due to differences between existing muon propagation codes with the aim to exclude as far as possible algorithmic uncertainties, such that only theoretical uncertainties from cross sections remain
  - Full Monte Carlo simulation
    - Ionization ($\mu e$ scattering), bremsstrahlung, pair production ($\mu Z \rightarrow \mu eeZ$), photonuclear interaction ($\mu N \rightarrow \mu X$), decay
    - Rare processes: muon trident production ($\mu Z \rightarrow \mu\mu\mu Z$), weak interaction ($\mu N \rightarrow \nu X$)
    - Multiple parametrizations for the cross sections of the different processes, continuously updated with newer parametrizations
    - Coulomb multiple scattering according to Molière's theory or Highland's approximation to Molière's theory
  - Modular C++14 code (C++17 to build unit tests)

# Extensions to PROPOSAL for EM showers in C8



- Added photon cross sections
- Added parametrizations for bremsstrahlung and photopair production as in EGS4
- Added interface for mean step length
  - Stand-alone PROPOSAL calculates the step-length itself
  - Within C8 mean free path required, calculates step-length outside PROPOSAL with its own random number generator

# Differences regarding electromagnetic interactions

# Differences between EM interactions in EGS and PROPOSAL

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- Cross sections taken over from EGS4 manual to PROPOSAL code
  - electron/positron bremsstrahlung: relativistic parametrization from Koch and Motz, 1959 with empirical correction below 50 MeV from Storm and Israel, 1970
  - Photopair production: analogous parametrization with empirical corrections
  - Compton, Bhabha, and Møller scattering, as well as annihilation
- Processes present in PROPOSAL, but absent in EGS4
  - Triplet production $eZ \to eeeZ$
  - Muon photopair production (following Burkhardt et al., 2002, also in C7)
  - Photohadronic interaction (also in C7)

- Photoelectric effect
  - C7 (via EGS4) uses tables
  - C8 (via PROPOSAL) uses parametrization in closed form based on photoeffect on $K$ shell electrons (Sauter, 1931a, 1931b) and empirical parametrization of ratio between total photoeffect cross section and $K$ shell photoeffect cross section (Hubbell, 1969)
- Photohadronic interactions
  - C7 uses three resonances + continuum for the total cross section, with HDPM as low-$E$ hadronic interaction model from 2 GeV onwards (one or two pions at lower energies) and the chosen high-$E$ hadronic interaction model above 80 GeV with $\pi$ or $\eta$ as initial particle
  - C8 uses an interpolation of measured data for the total cross section with a high-$E$ extrapolation (Caldwell et al., 1979) and hands over a $\rho$ to the high-$E$ hadronic interaction model; currently there is no low-$E$ hadronic interaction model yet.

# Propagation algorithm

# Propagation of particles

- interaction length

$$\lambda_{\text{int}} = \frac{A}{N_A \rho \sigma} \tag{1}$$

- probability to traverse a distance $\lambda$ without interaction

$$P(x) = \frac{1}{\lambda_{\text{int}}} e^{-\lambda/\lambda_{\text{int}}} \tag{2}$$

- sufficient for calculation of attenuation factors

- Simulation of all energy losses impossible due to infrared divergence: Bremsstrahlung cross section diverges $d\sigma/dv \sim 1/v$ for $v \to 0 \Rightarrow$ infinitely many secondary particles, total cross section diverges
- Separate losses into soft and hard losses
  - soft losses: continuous treatment
  - hard losses: stochastic treatment
- Cutoff (relative $v_{\text{cut}}$ or absolute $e_{\text{cut}}$) is an artificial scale; has to be chosen sufficiently small so as not to influence the simulation results

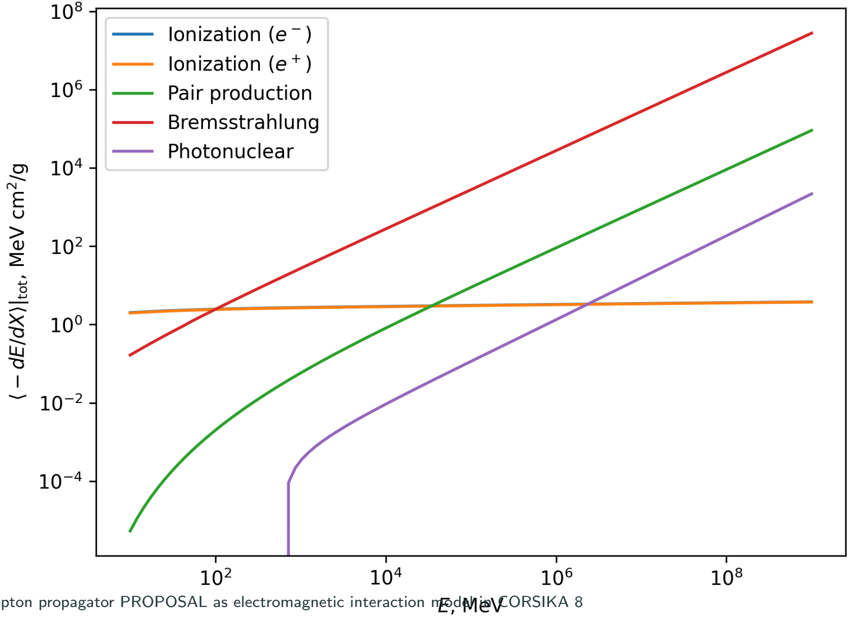
- Probability of stochastic hard loss over a distance $dX$

$$dP(E) = dX \left. \frac{dN}{dx} \right|_{\text{hard}}, \qquad \left. \frac{dN}{dX} \right|_{\text{hard}} = \sum_{\text{processes}} \frac{N_{\text{A}}}{A} \int_{v_{\text{cut}}}^{v_{\text{max}}} \frac{d\sigma}{dv} \tag{3}$$

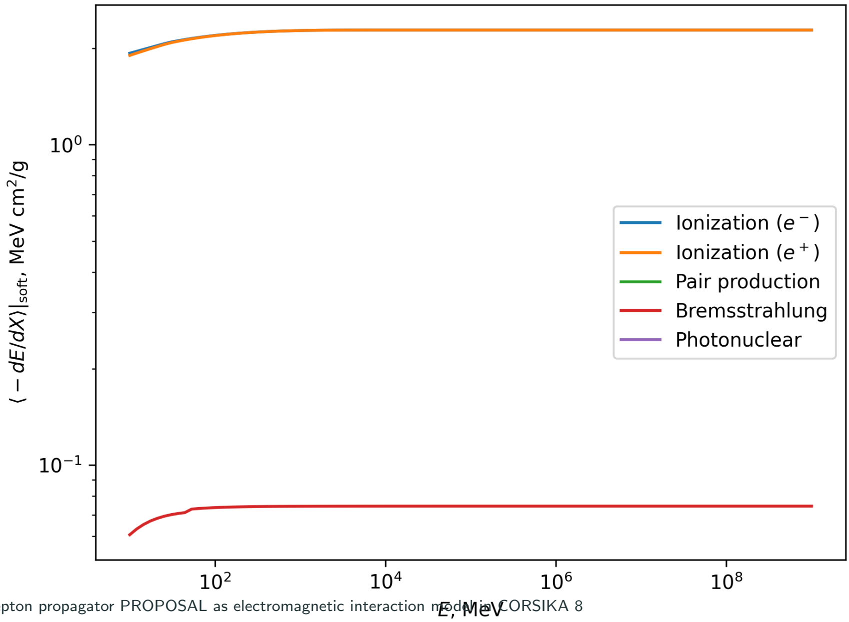
- energy loss excluding hard losses per distance

$$\left. \left\langle -\frac{dE}{dX} \right\rangle \right|_{\text{soft}} = \frac{N_{\text{A}}}{A} \sum_{\text{processes}} \int_{v_{\text{min}}}^{v_{\text{cut}}} v \frac{d\sigma}{dv} dv \tag{4}$$

# Total energy loss of electrons/positrons

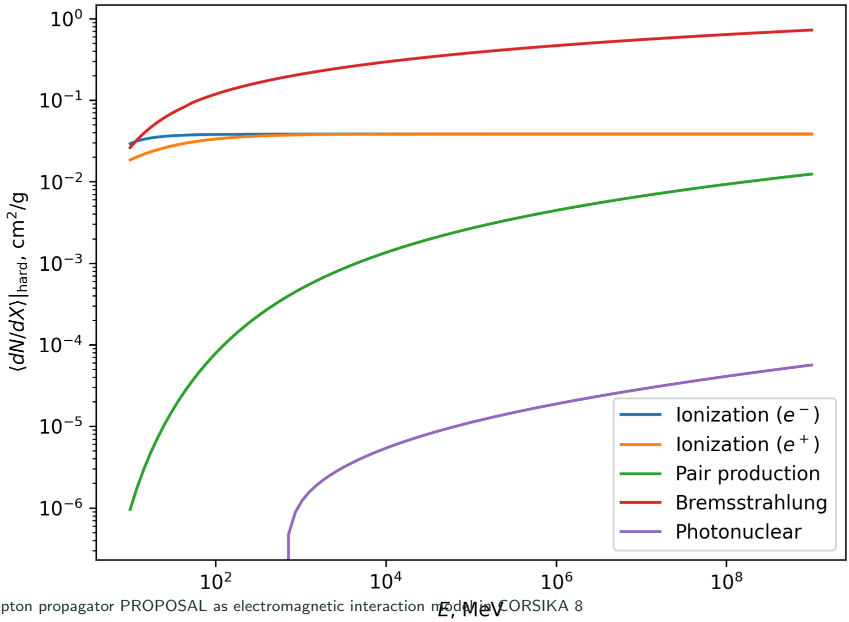


# Soft energy loss of electrons/positrons



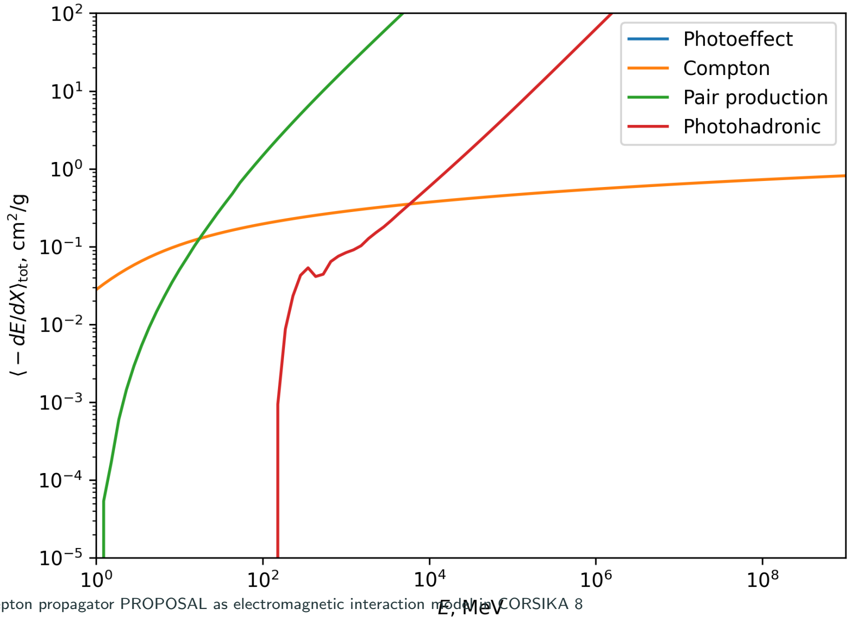
# Hard interaction rate of electrons/positrons



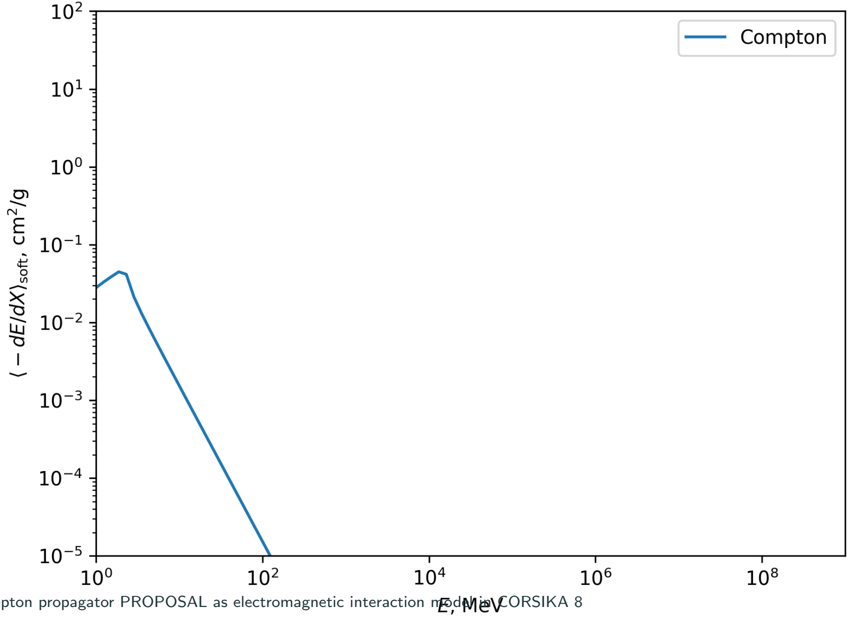


# Total energy loss of photons



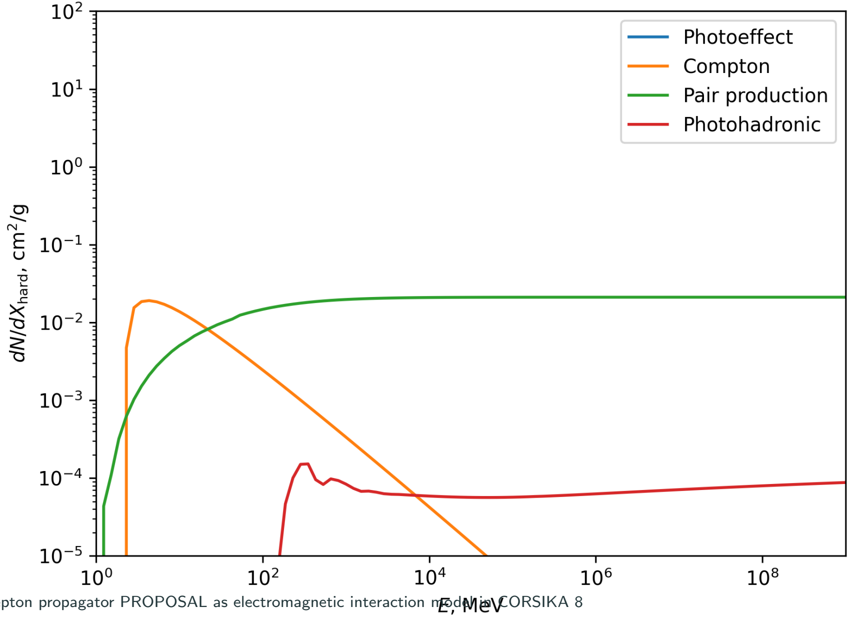


# Soft energy loss of photons



# Hard interaction rate of photons





- $\Rightarrow$ one random number determines the energy $E_f$ and distance $x_f - x_i$ of the next interaction

$$-\ln \xi = \int_{E_i}^{E_f} \frac{\frac{dN}{dx}(E)\Big|_{\text{hard}}}{-\left.\frac{dE}{dx}\right|_{\text{soft}}} dE \qquad (5)$$

- another random number determines which process at which component of the medium and with which relative energy loss $v$ is chosen based on the differential cross-section $d\sigma/dv$

# Transition from random number $\xi$ to process and relative energy loss $v$ for muons in ice

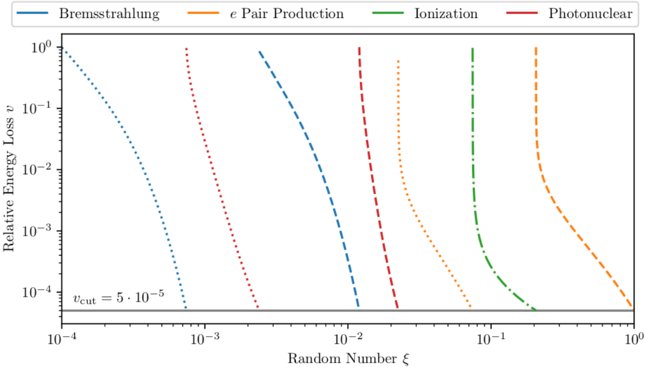



**Figure 5.4:** The stacked and inverted cumulative distribution of a stochastic loss for a muon with an initial energy of $10^5$ MeV in ice using an energy loss cut of 500 MeV. The interaction is split between the probabilities for the different targets; Hydrogen (dotted), Oxygen (dashed), and Ice (dash-dotted).

Soedingrekso, 2021

- Instead of solving the energy integral, inside C8 the interaction length ($\propto (dN/dx|_{\text{hard}})^{-1}$) and an exponentially distributed random number is used to sample the next interaction point
- Continuous losses are taken into account after the next interaction point has been sampled
    - step to next interaction point may be rejected if interaction length has increased due to continuous losses
- To allow for this, an API was written that allows to access the necessary functions, which were previously not public to the end user

- In both cases, the propagation is significantly accelerated by saving results of numerical integration into interpolation tables
- $\langle -dE/dX \rangle|_{\text{soft}}$, $dN/dX|_{\text{hard}}$, mean free path, ratios of cross sections, ...

# Multiple scattering

- Elastic scattering on nuclei between stochastic interactions
- Several parametrizations for multiple scattering available in PROPOSAL
  - Molière scattering theory
  - Highland approximation to Molière's theory
- additional change of direction and position at end of step between stochastic losses
  - In C8, currently only the direction is updated
- important for correct simulation of lateral shower profiles (see following presentation on validation of EM showers)

# Work in progress

# LPM effect

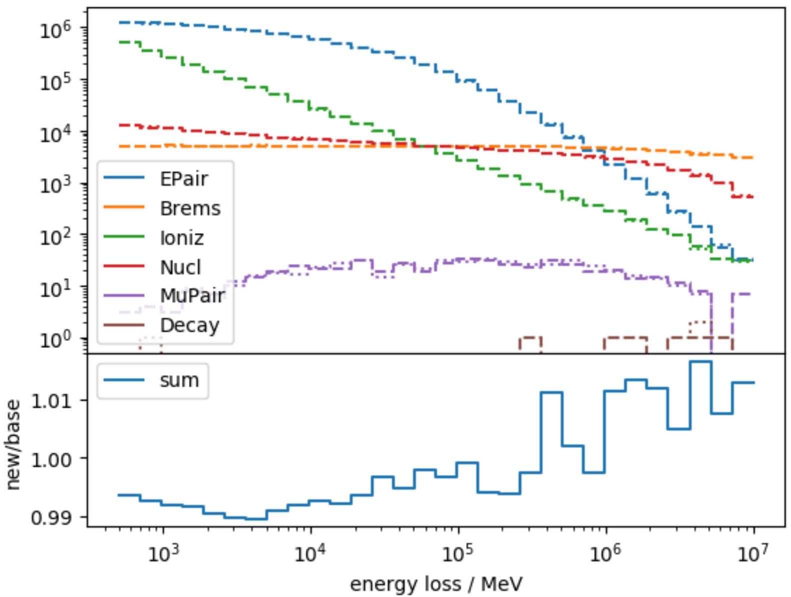
- density dependent effect at high energies
- suppresses small bremsstrahlung losses and symmetric pair production events
- difficulty lies in dependence on density that does not allow inclusion in the interpolation tables for inhomogeneous media
- in C7: Neumann rejection of bremsstrahlung and pair production losses based on additional random number and ratio of LPM and BH cross sections
- first draft implementation for C8 exists, needs more testing

# Conclusion & Outlook

- PROPOSAL is the first (and by now only) electromagnetic interaction model in CORSIKA 8
- Can be used independently as propagation library as well as to propose interactions of leptons and photons inside the C8 framework
- Changes and extensions to PROPOSAL and the creation of the interface to the CORSIKA 8 framework made possible the simulation of full showers including hadronic, muonic, and electromagnetic components
- Modular structure makes addition of new media, parametrizations or particles possible with moderate effort
  - Systematic studies of effect of different parametrizations on propagation are possible

# Backup

# Secondary distribution of 10 TeV muons in ice



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