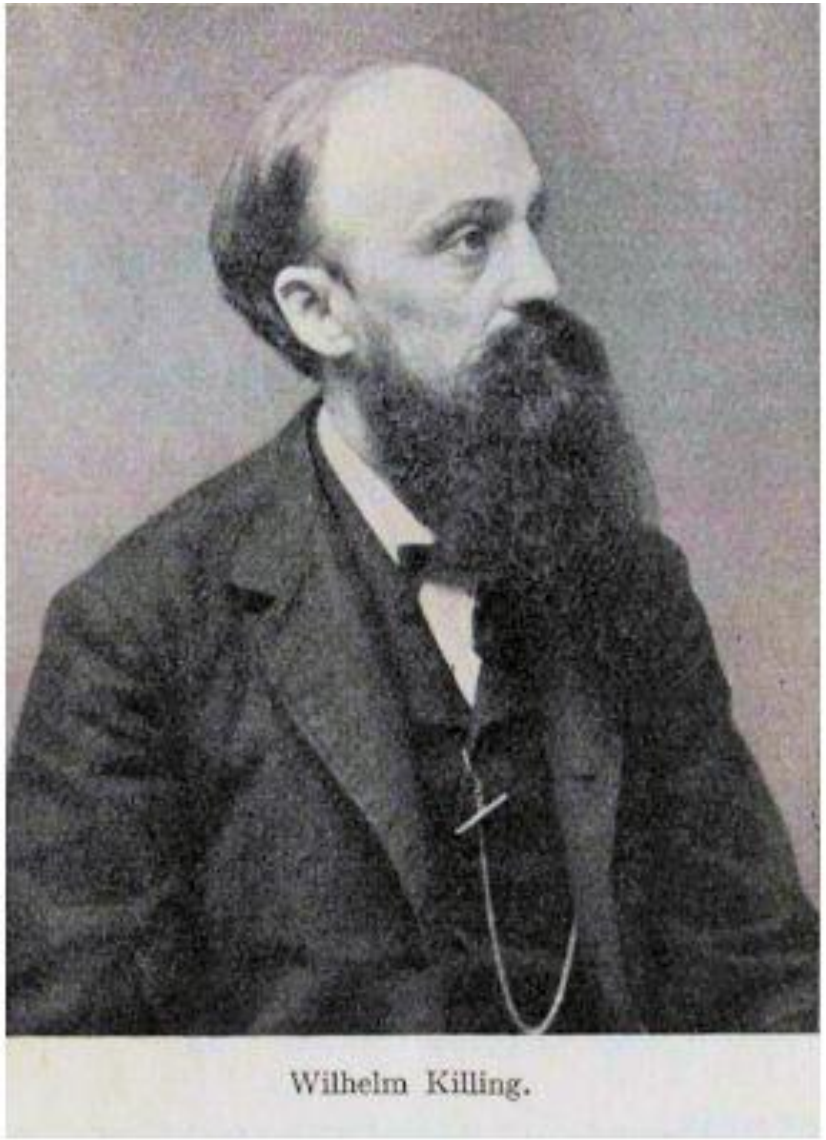


Wilhelm Killing (1847-1923) -

Life and mathematical achievements

Father

Franz Joseph Killing (1812 – 1898)

Court Clerk (Gerichtssekretär) in Burbach

Father

Franz Joseph Killing (1812 – 1898)

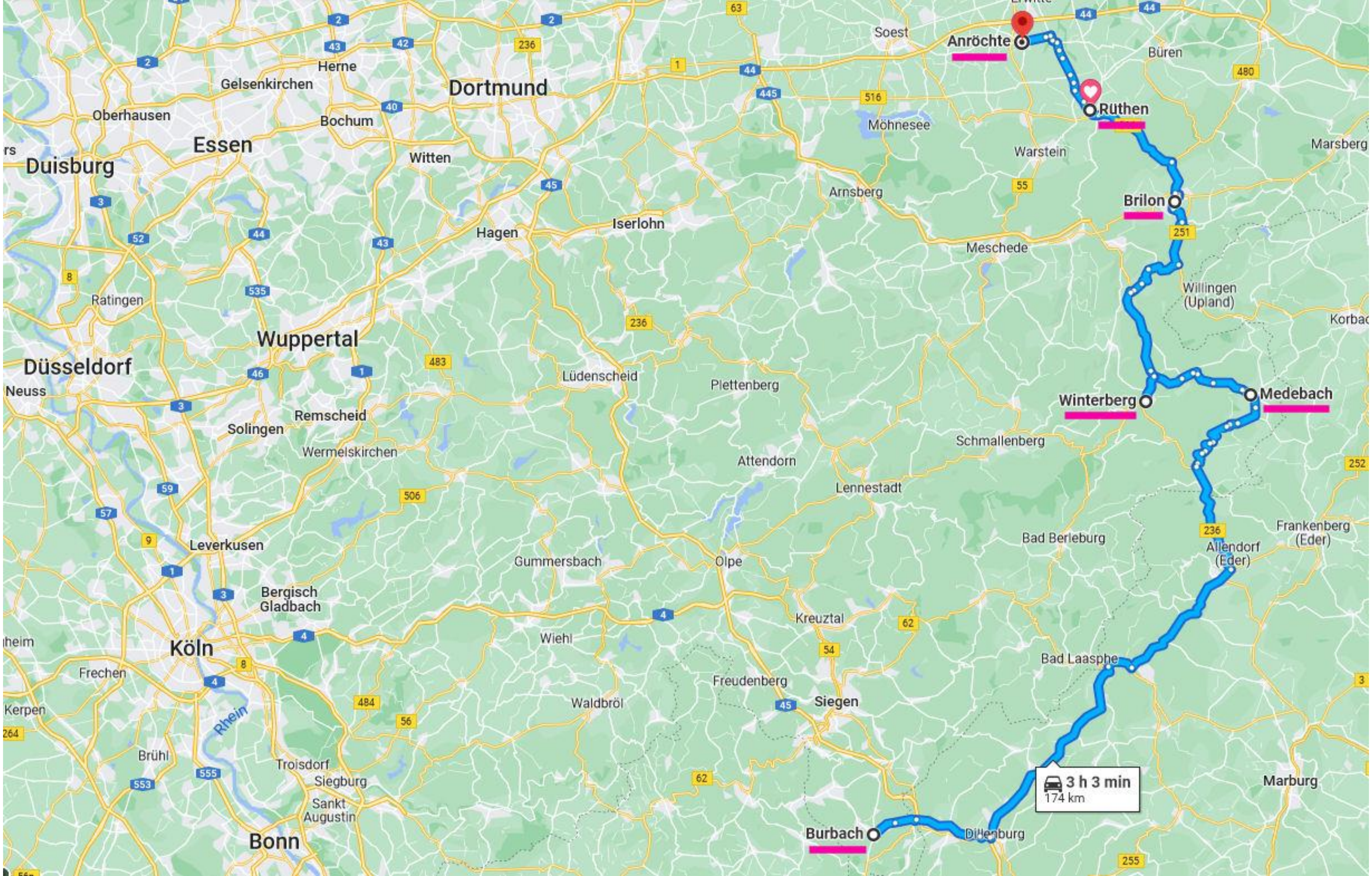
Court Clerk (Gerichtssekretär) in Burbach

Mother

Anna Catharina Kortenbach (1815 – 1883),

Daughter of the pharmacist

Wilhelm Kortenbach in Burbach


Anröchte
Rüthen
Brilon
Winterberg
Medebach
Burbach
3 h 3 min
174 km
Soest
Büren
Möhnesee
Warstein
Marsberg
Arnsberg
Meschede
Willingen (Upland)
Korbach
Schmallenberg
Attendorn
Lennestadt
Bad Berleburg
Frankenberg (Eder)
Allendorf (Eder)
Kreuztal
Bad Laasphe
Freudenberg
Siegen
Dillenburg
Marburg
Dortmund
Herne
Gelsenkirchen
Bochum
Oberhausen
Essen
Duisburg
Witten
Hagen
Iserlohn
Ratingen
Wuppertal
Düsseldorf
Neuss
Lüdenscheid
Plettenberg
Remscheid
Solingen
Wermelskirchen
Leverkusen
Gummersbach
Olpe
Bergisch Gladbach
Köln
Wiehl
Frechen
Kerpen
Waldbröl
Brühl
Rhein
Troisdorf
Siegburg
Sankt Augustin
Bonn

1860 – 1864 pupil at the gymnasium Brilon

graduated with „excellent grades“,

„ganz vorzüglichen Noten“

1865 – 1867 study at the

„Königlich Theologische und Philosophische Akademie“

in Münster

1865 – 1867 study at the

„Königlich Theologische und Philosophische Akademie“

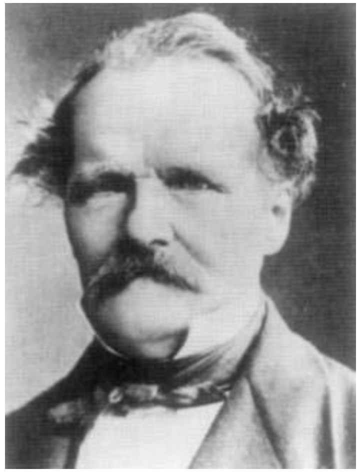
in Münster

„The students showed no interest in science itself,

they wished (with very few exceptions) to study only

what was needed for the examinations.“

1867–1872 study Universität Berlin



E. E. Kummer
1810-93

H. v. Helmholtz
1821-94

Karl Weierstraß
1815-97

1872 Promotion bei Weierstraß

Gymnasium teacher in Berlin 1873 - 1878

K. marries Anna Commer, daughter of a musicologist

Four sons and three daughters

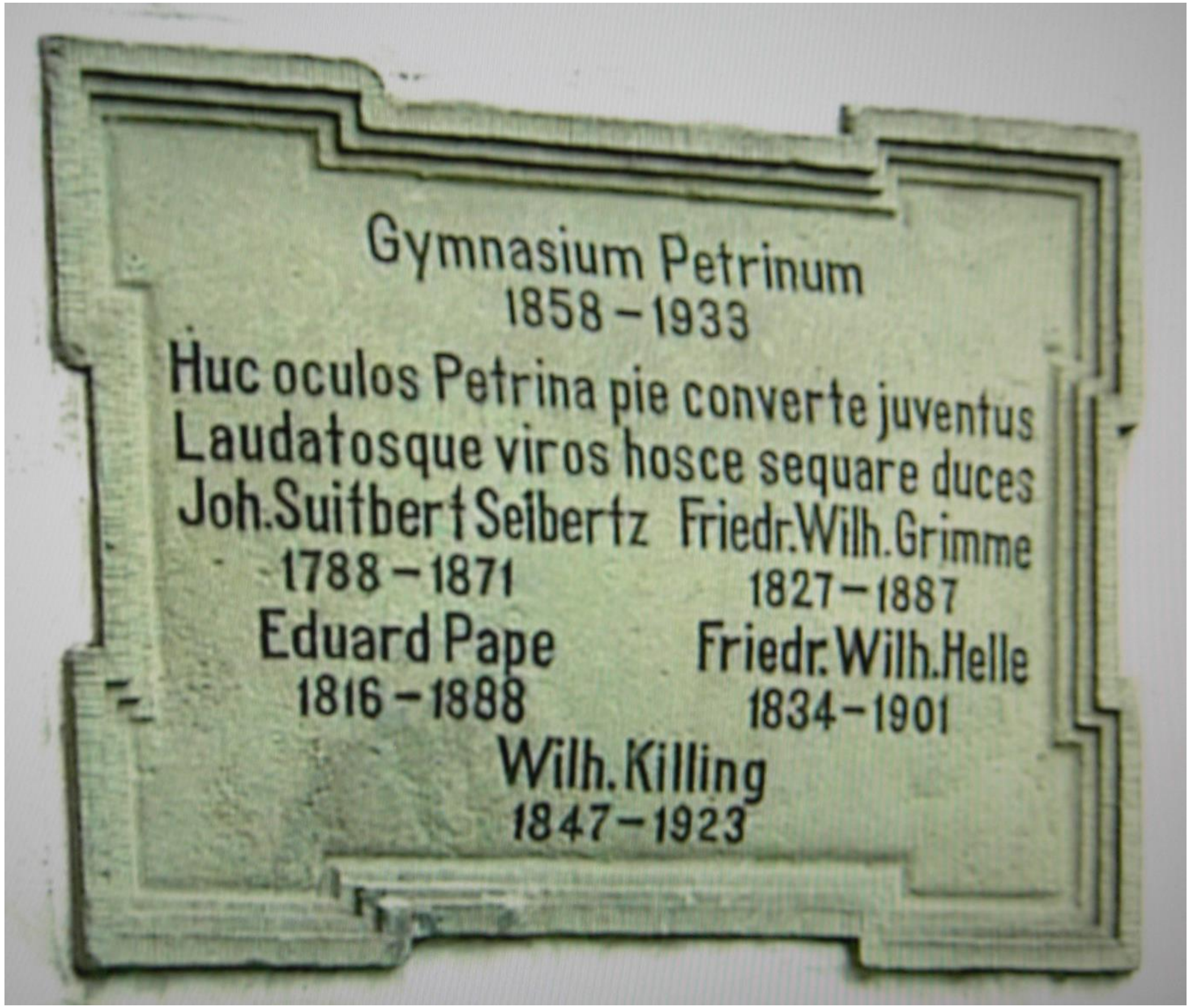
- Two sons died as infants
- The third son died shortly before completing his
  habilitation in history of music
- The fourth son died in a military camp 1918

Gymnasium teacher in Brilon 1878 – 1882

„Hierher, petrinische Jugend, die Augen voll Achtung nun wende, und eifere nach diesen hochgepriesenen Männern“

“Here, Petrine youth, turn the eyes full of respect, and strive after these highly praised men”

Commemorative plaque at the Nikolaikirche in Brilon, the former gymnasium church.



Gymnasium Teacher in Brilon 1878 – 1882

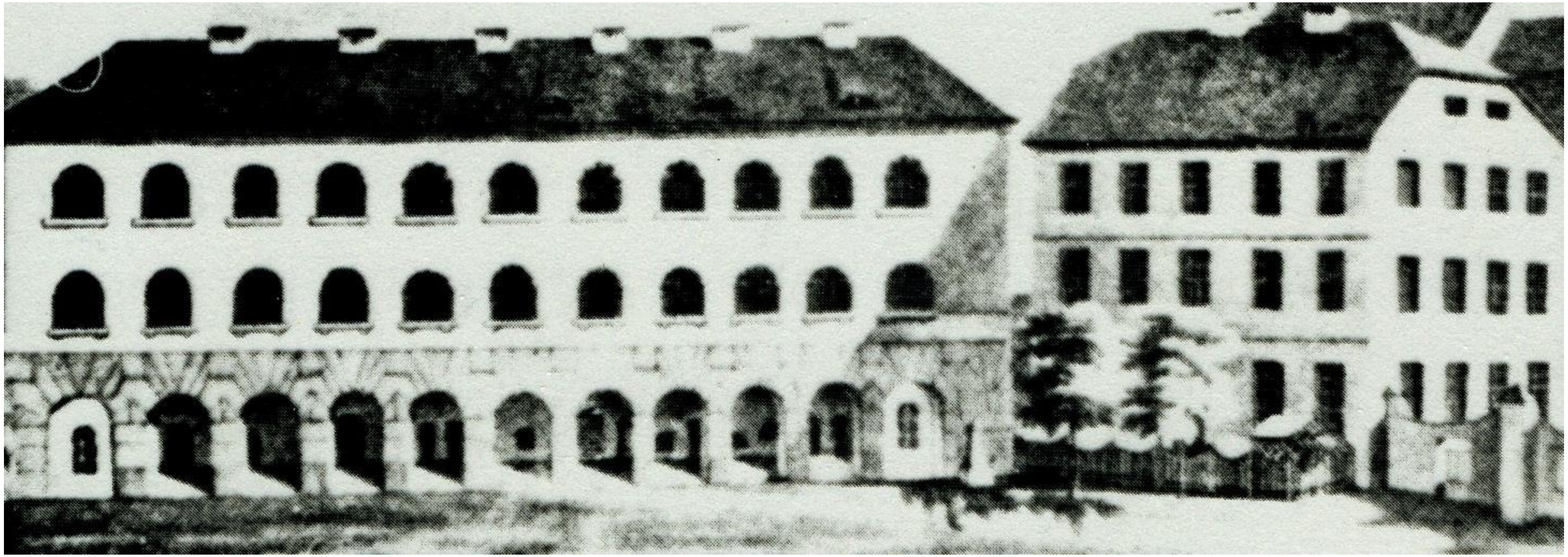
1880 *Grundbegriffe und Grundsätze der Geometrie*

Fundamental concepts and fundamental propositions

Programmschrift Gymnasium Brilon

1882 - 1892

Professor at the Lyceum Hosianum in Braunsberg/Braniewo



Lyceum Hosianum um 1835

Memorial tablet on the wall of the Lyceum

“In memory of the famous mathematicians Karl Weierstrass (1815-1897) and Wilhelm Killing (1847-1923), who taught here at the Catholic Gymnasium and Lyceum Hosianum.”

PTM and DMV, July 2008

Lyceum Hosianum 2008

1884 Erweiterung des Raumbegriffs

Programmschrift Lyceum Hosianum

1884 Erweiterung des Raumbegriffs

Programmschrift Lyceum Hosianum

Analytic form of an infinitesimal motion:

$x = (x_1 , \dots , x_n) \longmapsto (x_1 + dx_1 , \dots , x_n + dx_n),\ x_i$ reell

$dx_i = u_i(x)dt,$ $u_i(x)$ continous, differentiable

$x \longmapsto x + u(x)dt,$ $u(x) = (u_1(x), \dots , u_n(x))$

The inf. motions of a space form form a real vectorspace.

Let the motion u send x to

$$y = x + u(x)dt$$

and the motion v send y to

$$z = y + v(y)ds$$

$$= x + u(x)dt + v(x + u(x)dt)ds.$$

If $v_j(x + u(x)dt)$ is expanded in a Taylor series about x and higher order terms are neglected, then

$$z = x + (uv)(x)ds,$$

where the function uv is defined by

$$(uv)_j ds = u_j dt + v_j ds + \sum u_i \frac{\partial v_j}{\partial x_i}\, dt ds.$$

If the order of the application of the two motions is reversed, similar considerations show that

$$(vu)_j ds = u_j dt + v_j ds + \sum v_i \frac{\partial u_j}{\partial x_i}\, dt ds \text{ , } \quad \text{hence}$$

$$(uv - vu)_j = [u,v]_j = \sum \left(u_i \frac{\partial v_j}{\partial x_i} - v_i \frac{\partial u_j}{\partial x_i} \right) dt.$$

Killing states, that $x \longmapsto x + [u,v]dt$

is also an inf. motion of the same space form.

Killing proves:

$[u, v] + [v, u] = 0$ and the ***„Jacobian“***

$[u,[v,w]] + [v,[w,u]] + [w,[u,v]] = 0,$ i.e.

The inf. motions of a space form form a Lie algebra.

in der Arbeit behindert wurde. [illegible] dass die Untersuchungen, zu denen ich nun einmal gekommen bin, gar wenig für mich passen. Auf dem nun einmal betretenen Gebiete glaube ich nur mit saurer Arbeit etwas, und dann nur etwas Weniges, leisten zu können. Daher fehlt mir die Liebe, die Hingebung an die Arbeit. Mehrmals habe ich andern Gebieten mich zugewandt. Aber schliesslich wird hoffentlich der Gedanke überwiegen, dass, nachdem ich die betr. Untersuchungen begonnen habe, ich sie auch zu Ende führen muss.

„… The investigations I have come to are hardly suitable for me. In the field I have entered, I think I can only achieve something with sour work, and then only a little. That is why I lack love, devotion to work. Several times I have turned to other fields. But in the end, I hope, the thought will prevail that, having begun the investigations, I must complete them”.

Felix Klein
1849-1925

Friedrich Engel
1861-1941

Sophus Lie
1842-1899

Die Zusammensetzung der stetigen endlichen Transformationsgruppen I, II, III, IV,

Math. Ann. 31 (1888), 33 (1888), 34 (1889), 36 (1890)

Adjoint representation of the Lie algebra L

$$\mathrm{ad}: L \mapsto \mathrm{Hom}(L,L)$$

$$\mathrm{ad}: X \mapsto \mathrm{ad}X, \quad \mathrm{ad}X(Y) = [X,Y]$$

1894 Élie Cartan‘s doctoral thesis

„Sur la structure des groupes de transformations finis et continus“

Cartan writes in the introduction:

„Le présent travail a pour but d‘exposer et de compléter en certains point les recherches de M. Killing, en y introduisant toute la rigueur desirable.“

1892 – 1919 Professor at the Royal Academie/WWU Münster

Rektor 1897/98

1892 – 1919 Professor at the Royal Academie/WWU Münster

Rektor 1897/98

Einführung in die Grundlagen der Geometrie,
1. Bd. 1893, 357 S.,   2. Bd. 1898, 361 S.

Lehrbuch der analytischen Geometrie in homogenen Koordinaten
1. Teil 1900, 220 S.,   2. Teil 1900, 361 S.

Handbuch des mathematischen Unterrichts,
1. Bd. 1910, 456 S.,   2. Bd. 1913, 472 S.

1900 Lobachevsky-Prize

of the Kazan Physico-mathematical Society

(1897 Lie, 1903 Hilbert)

1900 Lobachevsky-Prize

„*Such recognition of my work, the shortcomings of which I myself recognise only too clearly, I would not have thought possible so far.* [...] *The wish of my youth that my life should not be entirely unfruitful for mathematics may well be regarded as fulfilled.*“

Together with his wife
Anna (+1928), sons
Joseph (+1910) and
Max (+1918)
and daughters
Maria (+1969) and
Anka (+1945),
W. Killing rests in the
family grave in the
Central cemetery in
Münster.

Coleman: *„Throughout his life Killing evinced a high sense of duty and a deep concern for anyone in physical or spiritual need.*

St Francis of Assisi was his model, so at the age of 39 he, together with his wife, entered the Third Order of the Franciscans.

His students loved and admired Killing because he gave himself unsparingly of time and energy to them, never being satisfied for them to become narrow specialists.“

„This makes Killing look almost a mathematical saint, but this probably goes too far.“

In a more familiar notation
(and more in the spririt of Sophus Lie)

If $\quad U = \sum u_i \frac{\partial}{\partial x_i} \quad$ and $\quad V = \sum v_i \frac{\partial}{\partial x_i}$

then $\quad UV - VU = \sum [u, v]_i \frac{\partial}{\partial x_i}$

Anders als bei Killing bilden bei Lie die „endlichen Transformationen“ den Ausgangspunkt seiner Untersuchungen, genauer: Systeme von endlichen Transformationen, die bei Komposition eine Gruppe bilden.

Es handelt sich dabei Transformationen von reellen Veränderlichen $x_1, \ldots, x_n$

$$x_i' = f_i(x_1, \ldots, x_n; t_1, \ldots, t_m), \quad i = 1, \ldots, n$$

die in einer Umgebung des Punktes $(t_1^0, \ldots, t_m^0)$, für den die zugehörige Transformation die Identität ist, stetig von den Parametern $t_1, \ldots, t_m$ abhängen.

Lie entwickelt um den Punkt $(t_1^0, \ldots, t_m^0)$ in eine Taylorreihe

$$f_i(x_1, \ldots, x_n; t_1^0 + t_1, \ldots, t_m^0 + t_m) = x_i + \sum_{j=1}^{m} t_j \xi_{ij}(x_1, \ldots, x_n) + \cdots$$

In Killings Terminologie ist

$$\xi_{ij} = u_i^{(j)} \quad (j\text{-te Komponentenfunktion von } u_i).$$

Lie führt als **„Symbole infinitesimaler Transformationen“** die Differentialoperatoren

$$X_i(f) = \sum_{j=1}^{n} \xi_{ij} \frac{\partial f}{\partial x_i} \quad (1 \le i \le m)$$

ein und erhält so die „Lie-Algebra“ der Gruppe.

Nun werden die Differentialoperatoren

$$X_j(f) = \sum_{i=1}^{n} \xi_{ij}(x_1, \ldots, x_n) \frac{\partial f}{\partial x_i} \quad (j = 1, \ldots, m)$$

als „Symbole infinitesimaler Transformationen“ eingeführt. Unter Berücksichtigung der Terme zweiter Ordnung in der Taylorentwicklung folgt aus den Gruppeneigenschaften (bei Killing aus der Geometrie)

$$(X_k X_l) := \sum_{i,j=1}^{n} \left(\xi_{jk} \frac{\partial \xi_{il}}{\partial x_j} - \xi_{jl} \frac{\partial \xi_{ik}}{\partial x_j} \right) = \sum_{j=1}^{m} c_{klj} X_j,$$

Beispiel:

$$f_1(x,y;t) = \cos t \cdot x - \sin t \cdot y$$
$$f_2(x,y;t) = \sin t \cdot x + \cos t \cdot y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix} = \exp tX = E + tX + \frac{t^2}{2} X^2 + \dots$$

mit

$$X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Vernachlässigung der Terme höherer als 1. Ordnung gibt

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} t,$$

Für kleines t also eine infinitesimale Transformation der obigen Form, wenn

$$u_1(x_1, x_2) = -x_2 \ ,$$

$$u_2(x_1, x_2) = x_1 \ .$$

Jägerstr. 27 in Burbach
Most likely the Birthplace of
Wilhelm Carl Joseph Killing

John Coleman:
“The Greatest Mathematical Paper of all Time”
in Math. Intelligencer 11.3 (1989)

His conclusion: „Euclid's Elements and Newton's Principia are more important than Z.v.G.II. But if you can name one paper in the past 200 years of equal significance to the paper which was sent off diffidently to Felix Klein on 2 February 1888 from an isolated outpost of Bismarck's empire, please inform the Editor of the Mathematical Intelligencer.“