Constraining the Analytic Properties of Feynman Integrals

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Mathematical Structures in Feynman Integrals
Siegen, February 13, 2023

arXiv:2109.09744 [hep-th], arXiv:2211.07633 [hep-th], and ongoing work
with H. Hannesdottir, M. Schwartz, and C. Vergu

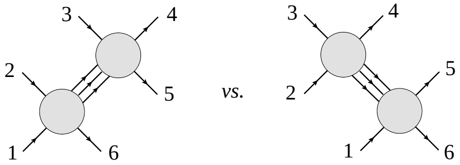
Motivation

Feynman integrals are **highly constrained by basic physical principles**, but the concrete implications of these principles are not yet fully understood

$\Rightarrow$ What are the full implications of principles like locality and causality for the analytic structure of Feynman integrals and scattering amplitudes?

Feynman integrals have also been **empirically observed** to exhibit intriguing analytic properties

$\Rightarrow$ The **sequential discontinuities** of Feynman integrals often obey generalized versions of the **Steinmann relations** [Drummond, Foster, Gürdoğan (2017)] [Caron-Huot, Dixon, von Hippel, AJM, Papathanasiou (2018)]



[Steinmann (1960)] (see also Dixon's talk)

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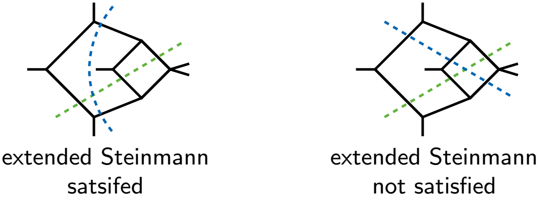

(even in the nonplanar sector) [Abreu, Ita, Page, Tschernow (2021)]

Constraints from Landau Analysis

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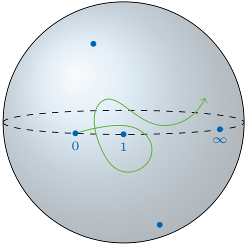
We bring to this question a well-developed understanding of the types of **iterated integrals** that are known to appear in Feynman integrals

- The first class of iterated integrals that naturally arise are **multiple polylogarithms**



Multiple Polylogarithms

- Multiple polylogarithms come equipped with a motivic coaction, which can be used to systematically expose their analytic structure
- In particular, the **symbol** of a polylogarithmic Feynman integral $\mathcal{I}(p)$ transparently encodes its salient analytic properties:

$$\mathcal{S}\big(\mathcal{I}(p)\big) = \sum \overbrace{a_1(p)}^{\text{first discontinuity}} \otimes \overbrace{a_2(p)}^{\text{second discontinuity}} \otimes \overbrace{a_3(p)}^{\cdots} \otimes \cdots \otimes \underbrace{a_{w-2}(p)}_{\cdots} \otimes \underbrace{a_{w-1}(p)}_{\text{second derivative}} \otimes \underbrace{a_w(p)}_{\text{first derivative}}$$

Constraints from Landau Analysis

Motivated by the two ways of understanding the information encoded in the symbol, we pursue two general strategies for constraining the analytic structure of Feynman integrals:

Constrain their **derivatives** by studying their behavior when expanded near branch points

[Hannesdottir, AJM, Schwartz, Vergu (2021)]

asymptotic analysis

Constrain their allowed **sequences of discontinuities** by studying where singularities—and therefore branch points—can appear in these integrals

[Pham (1967)] [Hannesdottir, AJM, Schwartz, Vergu (2022)]

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homological analysis

In the remainder of the talk, we'll see how these strategies work in examples involving generic masses

Constraining Derivatives

Landau Analysis Review

- The locations where Feynman integrals can become singular and develop branch cuts are described by solutions to the **Landau Equations** [Landau (1959)]

$$\alpha_e(q_e^2 - m_e^2) = 0 \qquad \sum_{e \in \text{loop}} \alpha_e q_e^\mu = 0$$

- Near a branch points that is approached as some kinematic variable $\varphi \to 0$, the **leading non-analytic behavior** of a Feynman integral is expected to take the form

$$\mathcal{I}(p, \varphi \to 0) \sim C(p)\, \varphi^\gamma \log^\nu \varphi + \ldots$$

All-Mass Example

Consider the class of Feynman integrals **with generic masses in D dimensions**

- Near a branch point that corresponds to an **ℓ-loop diagram with E nonzero Feynman parameters**, these integrals are expected to behave as [Landau (1959)]

$$\mathcal{I}(p, \varphi \to 0) \sim \begin{cases} C(p)\varphi^{\gamma} \log \varphi & \text{if } \gamma \in \mathbb{Z}, \gamma \geq 0 \\ C(p)\varphi^{\gamma} & \text{otherwise} \end{cases} \qquad \gamma = \frac{\ell D - E - 1}{2}$$

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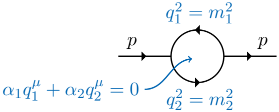
For example, two-particle thresholds and pseudothresholds are associated with the bubble Landau diagram

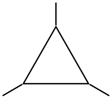


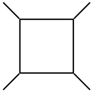
$$\Rightarrow \quad p^2 = (m_1 \pm m_2)^2$$

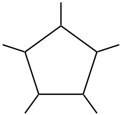
$$\gamma = (D-3)/2$$

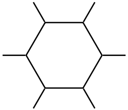
All-Mass Example

- The branch cuts that develop near the **two-particle thresholds** of all-mass Feynman integrals in different dimensions thus behave as

$D = 3$  $\sim \log \varphi$

$D = 4$  $\sim \varphi^{\frac{1}{2}}$

$D = 5$  $\sim \varphi \log \varphi$

$D = 6$  $\sim \varphi^{\frac{3}{2}}$

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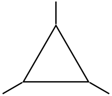
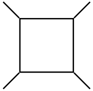
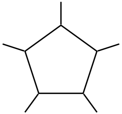
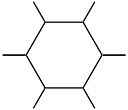
$D = 3$  $\sim \log \varphi$

$D = 4$  $\sim \varphi^{\frac{1}{2}}$

$D = 5$  $\sim \varphi \log \varphi$

$D = 6$  $\sim \varphi^{\frac{3}{2}}$

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If we can predict the leading-order behavior of Feynman integrals near a given branch point, what constraints does this put on the symbol of this integral?

Constraining Derivatives

Study the order at which non-analytic behavior appears when polylogarithms are expanded around the branch points in their symbol

$$\lim_{\varphi \to 0} (a_1 \otimes \cdots \otimes \varphi \otimes \cdots \otimes a_n) \sim \varphi^p \log^q \varphi$$

Approximate the value of Feynman integrals near their branch points

$$\mathcal{I}(\varphi \to 0) \sim \varphi^\gamma \log^\nu \varphi$$

Compare these expansions to put new constraints on the positions of branch points in the symbols of Feynman integrals

Logarithmic Singularities of Symbols

- For example, we can study the contribution coming from a **generic polylogarithm** that involves a symbol term in which a single letter becomes singular as $\varphi \to 0$:

$$a_1(p) \otimes \cdots \otimes a_{m-1}(p) \otimes \varphi \otimes a_{m+1}(p) \otimes \cdots \otimes a_n(p)$$

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- Writing this contribution as an iterated integral over a generic integration contour that ends on the $\varphi = 0$ surface, we find a **leading non-analytic contribution**

$$\sim \varphi^{n-m} \log \varphi \ + \ \ldots$$

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Non-analytic contributions are **power-suppressed by the number of letters after φ**:

$$a_1 \otimes \cdots \otimes a_{m-1} \otimes \varphi \otimes \underbrace{a_{m+1} \otimes \cdots \otimes a_n}_{n-m}$$

New Constraints on Symbol Letters

We conclude that any generic polylogarithmic integral with leading behavior

$$\mathcal{I}(p, \varphi \to 0) \ \sim \ \varphi^{\gamma} \log \varphi$$

(i) cannot involve symbol letters that vanish as $\varphi \to 0$ in the last γ entries:

$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = \sum a_1 \otimes \cdots \otimes a_{n-\gamma} \otimes \underbrace{a_{n-\gamma+1} \otimes \cdots \otimes a_n}_{\substack{\text{no logarithmic branch} \\ \text{points at } \varphi = 0}}$$

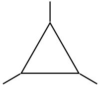
(ii) must have at least one term in which a logarithmic branch point at $\varphi = 0$ appears in the $n - \gamma$ entry (and nowhere else):

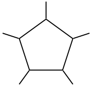
$$\mathcal{S}\big(\mathcal{I}(p,\varphi)\big) = a_1 \otimes \cdots \otimes a_{n-\gamma-1} \otimes \varphi \otimes a_{n-\gamma+1} \otimes \cdots \otimes a_n \ + \ \ldots$$

[Hannesdottir, AJM, Schwartz, Vergu (2021)]

All-Mass Example

We recall that the logarithmic branch cuts in odd-dimensional all-mass Feynman integrals were suppressed by $\varphi^{\frac{D-3}{2}}$ near two-particle thresholds:

$D = 3$  $\sim \log \varphi$

$D = 5$  $\sim \varphi \log \varphi$

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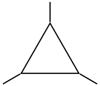
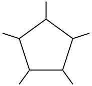
$D = 3$  $\sim \log \varphi$ $\Rightarrow$ φ appears in **last entry** of the symbol

$D = 5$  $\sim \varphi \log \varphi$ $\Rightarrow$ φ appears in **second-to-last entry** of the symbol

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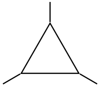
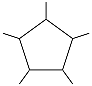
$D = 3$  $\sim \log\varphi$ $\Rightarrow$ φ appears in **last entry** of the symbol

$D = 5$  $\sim \varphi\log\varphi$ $\Rightarrow$ φ appears in **second-to-last entry** of the symbol

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- The one-loop n-gon symbols in n dimensions are known at one loop for all n
  [Schläfli (1860)] [Aomoto (1977)] [Davydychev, Delbourgo (1998)]
- Our analysis correctly predicts the position of **all logarithmic branch points** that appear in these one-loop symbols

Singularities of Symbols

We can similarly analyze symbol terms in which algebraic branch points at $\varphi \to 0$ occur in the symbol, as well as terms in which multiple branch points occur:

$$a_1 \otimes a_2 \otimes \cdots \otimes a_m \otimes \cdots \otimes a_{n-1} \otimes a_n$$

Location of Branch Points	**Leading Non-Analytic Behavior**
$a_m = \varphi$	$\sim \varphi^{n-m} \log \varphi$
$a_{m-r+1} = \cdots = a_m = \varphi$	$\sim \varphi^{n-m} \log^r \varphi$
$a_m = \frac{b+\sqrt{\varphi}}{b-\sqrt{\varphi}}$	$\sim \varphi^{n-m+\frac{1}{2}}$
$a_{m-r+1} = \cdots = a_m = \frac{b+\sqrt{\varphi}}{b-\sqrt{\varphi}}$	$\sim \varphi^{n-m+\frac{1}{2}}$

This provides us with a dictionary between the leading behavior of Feynman integrals near their branch points and where these branch points can appear in generic symbols

Constraining Discontinuities

Sequential Discontinuities

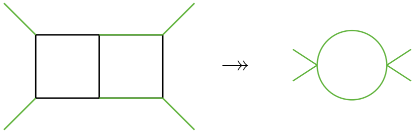
Having learned about the **locations in the symbol** at which specific branch points can appear, we now explore the possible **sequences of discontinuities** that can appear

Sequential Discontinuities

Having learned about the **locations in the symbol** at which specific branch points can appear, we now explore the possible **sequences of discontinuities** that can appear

- To do so, we first recall that each solution to the Landau equations comes with an associated on-shell graph, or **Landau diagram**

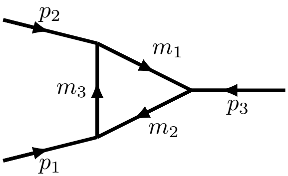
$$\alpha_e(q_e^2 - m_e^2) = 0 \quad \Rightarrow \quad \begin{Bmatrix} q_e^2 = m_e^2 \\ q_e^2 \neq m_e^2 \end{Bmatrix}$$



All-Mass Example

For example, the all-mass triangle integral in three dimensions is given by

$\mathcal{I} =$



$$= \frac{\pi^2}{4\sqrt{D}} \Bigg[\log\left(-\frac{y_{12} + y_{23}\, y_{13} + i\sqrt{D}}{y_{12} + y_{23}\, y_{13} - i\sqrt{D}} \right) + \log\left(-\frac{y_{23} + y_{13}\, y_{12} + i\sqrt{D}}{y_{23} + y_{13}\, y_{12} - i\sqrt{D}} \right) + \log\left(-\frac{y_{13} + y_{12}\, y_{23} + i\sqrt{D}}{y_{13} + y_{12}\, y_{23} - i\sqrt{D}} \right) + i\pi \Bigg]$$

in the region where $y_{12} > 1$, $y_{13} < -1$, $y_{23} > 1$, and $D < 0$, where

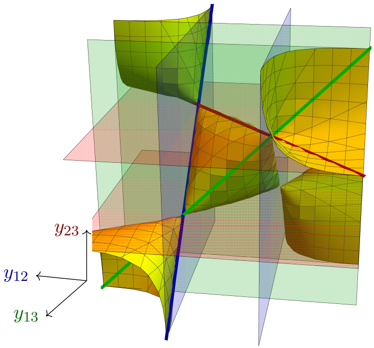
$$y_{ij} = \frac{(p_i + p_j)^2 - m_i^2 - m_j^2}{2m_i m_j} \qquad , \qquad D = 1 - y_{12}^2 - y_{23}^2 - y_{13}^2 - 2y_{12}\, y_{23}\, y_{13} \,.$$

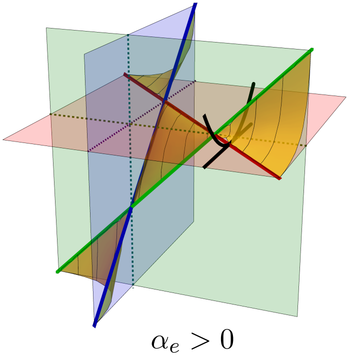
In this integral:

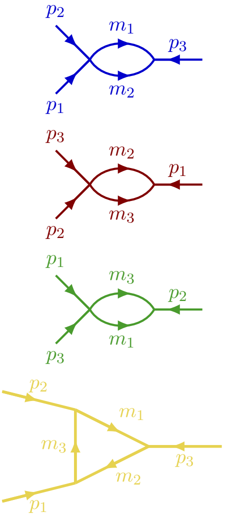
- the **triangle Landau diagram** encodes to the algebraic branch point at $D = 0$
- the three **bubble Landau diagrams** encode the logarithmic branch points at $y_{ij} = \pm 1$

All-Mass Example

We can derive constraints on the discontinuities of Feynman integrals by understanding how these singularity surfaces intersect [Pham (1967)]

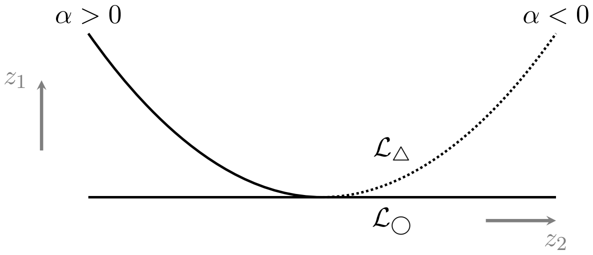






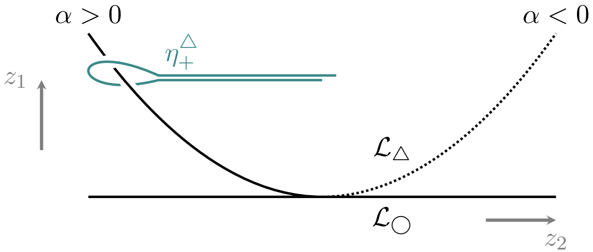
- We are generally most interested in the **α-positive** parts of these singularity surfaces, as it is only these singularities that will be encountered on the physical sheet

From Homology to Discontinuity Relations



From Homology to Discontinuity Relations

$$\mathrm{Disc}_{\triangle} = \mathbb{1} - \eta_{+}^{\triangle}$$



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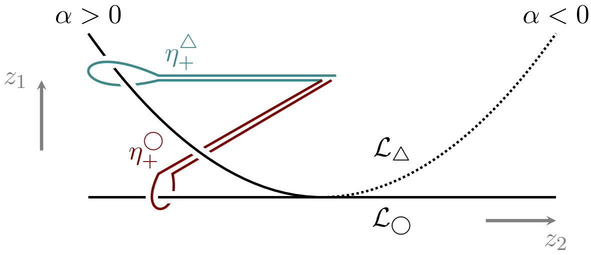

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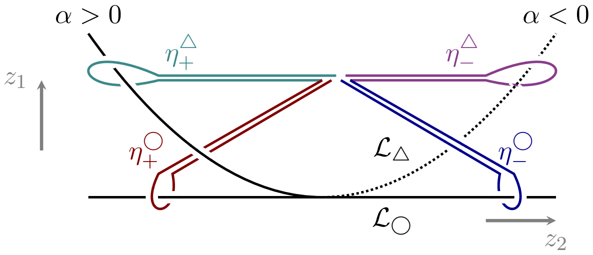

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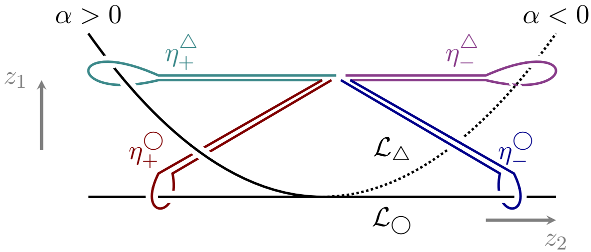

- ◦ Due to the tangential intersection, nontrivial relations exist between different compositions of these paths
- ◦ This implies new relations between the discontinuities of $\mathcal{I}$

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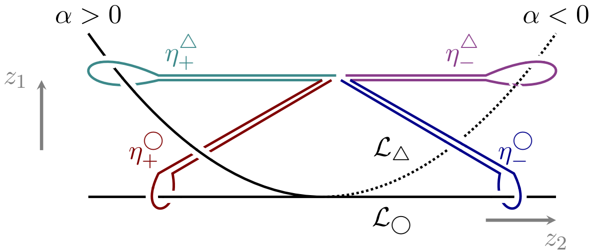

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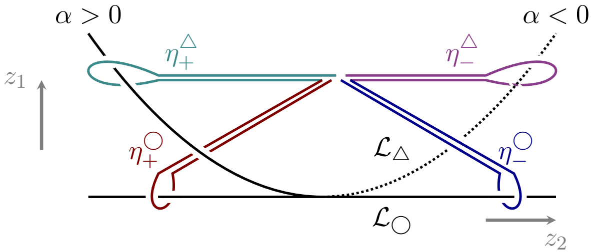

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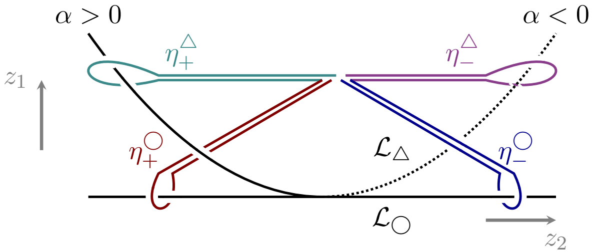

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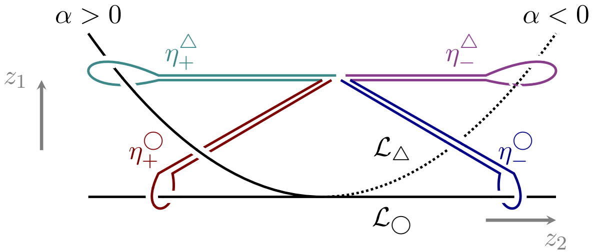
$$\begin{aligned}
\mathrm{Disc}_{\triangle} \circ \mathrm{Disc}_{\bigcirc}\, \mathcal{I} &= (\mathbb{1} - \eta_{+}^{\triangle}) \circ (\mathbb{1} - \eta_{+}^{\bigcirc})\, \mathcal{I} \\
&= (\mathbb{1} - \eta_{+}^{\triangle} - \eta_{+}^{\bigcirc} + \eta_{+}^{\bigcirc} \circ \eta_{-}^{\triangle})\, \mathcal{I} \\
&= (\mathbb{1} - \eta_{+}^{\triangle} - \eta_{+}^{\bigcirc} + \eta_{+}^{\bigcirc})\, \mathcal{I}
\end{aligned}$$

From Homology to Discontinuity Relations

$$\text{Disc}_{\triangle} = \mathbb{1} - \eta_+^{\triangle}$$

$$\text{Disc}_{\bigcirc} = \mathbb{1} - \eta_+^{\bigcirc}$$

$$\eta_+^{\triangle} \circ \eta_+^{\bigcirc} = \eta_+^{\bigcirc} \circ \eta_-^{\triangle}$$



- Due to the tangential intersection, nontrivial relations exist between different compositions of these paths
- This implies new relations between the discontinuities of $\mathcal{I}$

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&= \text{Disc}_{\triangle}\, \mathcal{I}
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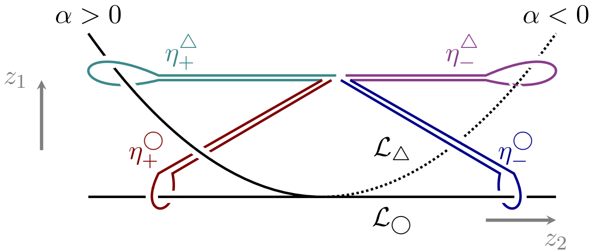

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$$\mathrm{Disc}_{\triangle} \circ \mathrm{Disc}_{\bigcirc} \, \mathcal{I} = \mathrm{Disc}_{\triangle} \, \mathcal{I}$$

- It can be checked that this identity is indeed satisfied by the triangle integral

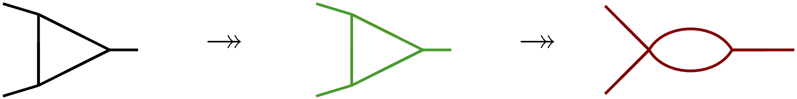
Hierarchical Discontinuities

This proof generalizes to any pair of codimension-one singularities in the physical region, as long as a sequence of graph contractions $G_{\mathcal{I}} \twoheadrightarrow G_{\mathcal{L}_1} \twoheadrightarrow G_{\mathcal{L}_2}$ exists:

$$\mathrm{Disc}_{\mathcal{L}_1} \circ \mathrm{Disc}_{\mathcal{L}_2} \, \mathcal{I} = \mathrm{Disc}_{\mathcal{L}_1} \, \mathcal{I}$$

(we must also require that these singularities involve at least one nonzero α per loop)

- In the triangle integral example, this sequence of contractions was given by



where the first graph is associated with the original Feynman integral, and the others represent Landau diagrams

[Pham (1967)] [Hannesdottir, AJM, Schwartz, Vergu (2022)]

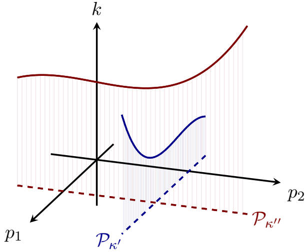
Generalized Steinmann Relations

One can similarly prove a **generalized version of the Steinmann relations**:

Whenever two Landau diagrams $G_{\mathcal{L}_1}$ and $G_{\mathcal{L}_2}$ are not related by contraction, we have

$$\mathrm{Disc}_{\mathcal{L}_1} \circ \mathrm{Disc}_{\mathcal{L}_2}\, \mathcal{I} = 0$$

if the corresponding solutions to the Landau equations cannot be simultaneously satisfied



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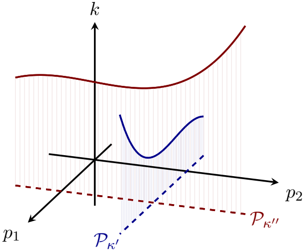

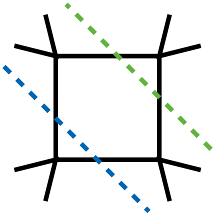

$= 0$

[Pham (1967)] [Hannesdottir, AJM, Schwartz, Vergu (2022)]

Conclusions

In this talk, two general strategies were highlighted for deriving constraints on Feynman integrals:

- The asymptotic behavior of polylogarithmic Feynman integrals near their branch points can be used to put **constraints on the locations of these branch points in iterated integrals**
- The manner in which different singularity surfaces intersect in Feynman integrals can be used to derive **constraints on their allowed sequences of discontinuities**

While rigorous results have currently only been worked out for all-mass integrals, we expect progress can also be made in cases involving degenerate or vanishing masses using the same strategies

- Note that these all-mass results already apply whenever a Feynman integral can be **contracted to all-mass Landau diagrams**

In addition to teaching us about the mathematical structure of perturbative quantum field theory, these results provide constraints that will prove useful in the future for **bootstrap methods**

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Thanks!