Mathematical Structures in Feynman Integrals

Report of Contributions

https://indico.kit.edu/e/mathematicalstructures

Contribution ID: **1** Type: **not specified**

Welcome

Monday, February 13, 2023 2:00 PM (10 minutes)

Presenters: PICLUM, Jan (University of Siegen); BARAKAT, Mohamed (University of Siegen); BRÜSER, Robin (University Siegen); HUBER, Tobias (Siegen U)

Session Classification: Session I

Contribution ID: **2** Type: **not specified**

Wilhelm Killing – Life and mathematical achievements

Monday, February 13, 2023 2:10 PM (50 minutes)

February 11, 2023 marks the 100th anniversary of the death of Wilhelm Killing. Far from all scientific centers, Killing formulated a research program that is still relevant today and has significantly influenced mathematical research for a century, namely the structure theory of Lie algebras. The 100th anniversary of Killing's death is an appropriate opportunity to honour Killing's merits which he has earned in tireless work for research and teaching, with a short portrait, albeit very incomplete.

Presenter: HEIN, Wolfgang

Session Classification: Session I

Contribution ID: **3** Type: **not specified**

2D fishnet integrals and Calabi-Yau geometries

Monday, February 13, 2023 3:00 PM (1 hour)

We argue that Yangian-invariant l-loop fishnet integrals in 2 dimensions are closely related to a family of Calabi-Yau l-folds.
This allows us to reduce the computation of these integrals to the computation of the Calabi-Yau periods. The periods are solutions of the Picard-Fuchs differential equations, which in turn are determined by the Yangian symmetry. Finally, we show that the values of these fishnet integrals admit a natural interpretation as the quantum volume of the Calabi-Yau.

Presenter: DUHR, Claude

Session Classification: Session I

Contribution ID: **4** Type: **not specified**

Epsilon-factorised differential equations for non-trivial geometries

Monday, February 13, 2023 4:30 PM (1 hour)

The method of differential equations is a popular method to compute Feynman integrals. It is particular powerful if the differential equation can be cast into an epsilon-factorised form. In this talk I will discuss how this can be achieved for Feynman integrals which are related to non-trivial geometries like Calabi-Yau manifolds.

Presenter: WEINZIERL, Stefan

Session Classification: Session II

Contribution ID: **5** Type: **not specified**

Constraining the Analytic Properties of Feynman Integrals

Monday, February 13, 2023 5:30 PM (30 minutes)

The analytic properties of Feynman integrals are heavily constrained by basic physical principles such as causality and locality; however, the specific implications of these principles remain only partially known. In this talk, I will describe two methods for deriving concrete constraints on the analytic structure of Feynman integrals. The first of these methods leverages information about the asymptotic expansion of these integrals around singular points, while the second takes into account homological information about the spaces of momenta that describe on-shell scattering processes. I will illustrate these methods in examples involve generic masses.

Presenter: MCLEOD, Andrew

Session Classification: Session II

Contribution ID: **6** Type: **not specified**

Interpolation, Rational Reconstruction and Modular Algorithms

Tuesday, February 14, 2023 9:00 AM (1 hour)

I will give a survey about interpolation, evaluation, and
reconstruction methods for multivariate polynomial and rational problems with applications in linear algebra and Groebner basis. This will include both asymptotically fast as well as practical algorithms.

Presenter: FIEKER, Claus

Session Classification: Session III

Contribution ID: **7** Type: **not specified**

The ice cone family and iterated integrals for Calabi-Yau varieties

Tuesday, February 14, 2023 10:00 AM (30 minutes)

In this talk I will explain how one can compute ice cone integrals in two dimensions for arbitrary loop order and which mathematical structures show up in this procedure. Using a leading singularity analysis we can find two copies of the banana graph which are related to period integrals on Calabi-Yau varieties. This observation allows us to express also the ice cone through iterated Calabi-Yau period integrals. Furthermore, I will explain how the usage of the canonical variable q can help to understand these iterated integrals better.

Presenter: NEGA, Christoph

Session Classification: Session III

Contribution ID: **8** Type: **not specified**

Recent progress in the reconstruction of multi-loop amplitudes

Tuesday, February 14, 2023 11:15 AM (30 minutes)

I will discuss recent progress in techniques for generating and reconstructing multi-loop scattering amplitudes. I will focus on novel developments in physical projectors, reduction to master integrals and partial fraction decomposition, as well as their combination with finite fields and rational reconstruction methods.

Presenter: PERARO, Tiziano

Session Classification: Session IV

Contribution ID: **9** Type: **not specified**

Computing intersection numbers with a rational algorithm

Tuesday, February 14, 2023 11:45 AM (30 minutes)

Intersection theory allows to exploit the vector space structure obeyed by Feynman integrals, turning the decomposition to master integrals into the calculation of the projection of a vector into a basis, via scalar products called intersection numbers. In this talk I will discuss how the calculation of intersection numbers can be achieved via a purely rational algorithm, through the systematic use of polynomial series expansions, and its implementation over the finite fields using the FiniteFlow program.

Presenter: FONTANA, Gaia

Session Classification: Session IV

Contribution ID: **10** Type: **not specified**

A tale of two packages, pfd-parallel and NeatIBP, for the multi-loop computations

Tuesday, February 14, 2023 2:30 PM (30 minutes)

In this talk, we are glad to introduce two packages, pfd-parallel and NeatIBP, based on computational algebraic geometry, aiming at cutting-edge computations for multi-loop Feynman integrals. The former one, powered by Singular and the massive parallelization framework GPI/space, implements multivariate partial fraction algorithms for simplifying analytic coefficients in multi-loop computations. The latter one which implements the syzygy/Module intersection IBP method in a parallelized way, can generate much shorter IBP systems comparing with those from Laporta algorithm. Several nontrivial examples would be presented with the two packages.

Presenter: ZHANG, Yang

Session Classification: Session V

Contribution ID: **11** Type: **not specified**

Massively parallel computer algebra and its application to the computation of Feynman integrals

Tuesday, February 14, 2023 3:00 PM (1 hour)

In this talk, I will first discuss the Singular/GPI-Space project which aims to bring together computer algebra systems and the workflow management system GPI-Space to perform massive parallel computations. I will illustrate how GPI-Space leverages a coordination language based on the idea of Petri nets to efficiently model algorithms. Focusing on specific use cases, I will demonstrate how our approach can be applied, with a particular emphasis on algorithmic methods for Feynman integrals in high-energy physics.

Presenter: BÖHM, Janko

Session Classification: Session V

Contribution ID: **12** Type: **not specified**

Integration-By-parts-Reduction using syzygies

Tuesday, February 14, 2023 4:30 PM (30 minutes)

Integration By parts Reduction to master integrals is an essential step in most modern multi-loop scattering amplitude calculations. In recent years, IBP reduction based on syzygy methods has become increasingly popular, allowing the calculation of many challenging processes. We will discuss our approach to calculate these syzygies using linear algebra and finite field based methods, as well as a few example calculations where these were successfully employed.

Presenter: AGARWAL, Bakul (KIT)

Session Classification: Session VI

Contribution ID: **13** Type: **not specified**

Elliptic substructure of 2-loop kites and 3-loop tadpoles

Wednesday, February 15, 2023 9:00 AM (1 hour)

The generic 2-loop kite integral has 5 internal masses. Its completion by a sixth propagator gives a 3-loop tadpole whose substructure involves 12 elliptic curves. I shall show how to compute all such kites and their tadpoles, with 200 digit precision achieved in seconds, thanks to the procedure of the arithmetic-geometric mean for complete elliptic integrals of the third kind. The number theory of 3-loop tadpoles poses challenges for packages such as HyperInt.

Presenter: BROADHURST, David

Session Classification: Session VII

Contribution ID: **14** Type: **not specified**

Computing motives: an approach to irrationality proofs for periods

Wednesday, February 15, 2023 10:00 AM (1 hour)

Following Francis Brown, I will explain why certain questions in number theory, like irrationality of zeta values, make it desirable to be able to compute some mixed Tate motives of moduli spaces. Then I will explain an algorithm devised by Clément Dupont in his Ph.D. thesis for that purpose, and its implementation in the package MotivesForBiarrangements, which uses CAP (Categories, Algorithms, and Programming), a GAP project developed in Siegen.

Presenter: JUTEAU, Daniel

Session Classification: Session VII

Contribution ID: **15** Type: **not specified**

Amplitudes, Ansätze and Algebraic Geometry

Wednesday, February 15, 2023 11:30 AM (30 minutes)

Multi-particle scattering amplitudes beyond tree level are complicated functions of many variables. In practical computations, amplitudes are organized as a linear combination of transcendental functions with rational functions as coefficients. The computation of these rational functions in cutting-edge cases poses a significant challenge. In the modern approach, these rational functions are computed by using numerical evaluations to constrain an Ansatz. An important problem is thus to construct Ansätze for the rational functions that contain very few free parameters.

In this talk, we discuss an approach to constructing compact Ansätze for rational functions in amplitudes by making use of their non-trivial behavior in singular limits. This behavior imposes powerful constraints on the analytic structure of the rational functions, which we then use to construct compact partial fractions Ansätze. Understanding and implementing these constraints is a non-trivial problem that we solve using tools from (computational) algebraic geometry. Specifically, we interpret the constraints as statements of membership of the numerator of the rational function to certain "ideals" of polynomials, as prescribed by the "Zariski-Nagata" theorem. We will discuss the computational bottlenecks which arise in this framework, as well as potential directions for breaking them.

Presenter: PAGE, Ben

Session Classification: Session VIII

Contribution ID: **16** Type: **not specified**

The Function Space of N=4 Scattering Amplitudes in the Regge Limit to all Orders

Wednesday, February 15, 2023 12:00 PM (30 minutes)

I will review a recent mathematical technique to efficiently compute scattering amplitudes in the forward scattering limit of N=4 SYM and explain how it can be used to fully determine the function space of scattering amplitudes in this limit to all orders in perturbation theory.

Presenter: MARZUCCA, Robin

Session Classification: Session VIII

Contribution ID: **17** Type: **not specified**

One-loop hexagon integral to higher orders in the dimensional regulator

Wednesday, February 15, 2023 2:30 PM (30 minutes)

The state–of–the–art in current two–loop QCD amplitude calculations is at five-particle scattering. In contrast, very little is known at present about two–loop six–particle scattering processes. Computing two–loop six–particle processes requires knowledge of the corresponding one–loop amplitudes to higher orders in the dimensional regulator. In this talk, I will show the analytic results for the one–loop hexagon integral to higher
orders in dimensional regulator obtained via differential equations. I will discuss the function alphabet for general D–dimensional external states, function space up to weight
two and one–fold integral representation up to weight four for all integrals in the integral basis. Finally, I will discuss the difference between the conventional dimensional regularization and the four–dimensional helicity scheme at the level of the master integrals. With this, the one–loop integral basis is ready for two–loop amplitude applications.

Presenter: MATIJAŠIĆ, Antonela

Session Classification: Session IX

Contribution ID: **18** Type: **not specified**

Computer Algebra Methods for Feynman Integrals

Wednesday, February 15, 2023 3:00 PM (1 hour)

We report on a collection of computer algebra algorithms and their implementations that have been used in recent calculations of massive and massless 3-loop Feynman integrals. Among them we will focus on symbolic summation and integration methods, (partial) linear differential and difference equation solvers and the large moment method. A specific feature of all the techniques is that the representation of the closed form output can be given within the class of iterative integrals and sums defined over general letters in terms hyperexponential functions and hypergeometric products.

Presenter: SCHNEIDER, Carsten

Session Classification: Session IX

Contribution ID: **19** Type: **not specified**

Mathematical Structures in Massive Operator Matrix Elements

Wednesday, February 15, 2023 4:30 PM (1 hour)

In this talk we will elaborate, among others, on root-valued alphabets and elliptic structures in Massive Operator Matrix Elements, together with concrete phenomenological results at O(a_s^3).

Presenter: BLÜMLEIN, Johannes

Session Classification: Session X

Contribution ID: **20** Type: **not specified**

Antipodal (Self)-Duality in Planar N=4 Super-Yang-Mills Theory

Thursday, February 16, 2023 9:00 AM (1 hour)

In planar N=4 super-Yang-Mills theory, some scattering amplitudes and form factors that evaluate to multiple polylogarithms can be bootstrapped, or constructed without knowing the precise Feynman integrals, in some cases through eight loops. This allows a window into high orders of perturbation theory for certain scattering processes. Remarkably, the 6-gluon amplitude and the 3-gluon form factor of the chiral stress tensor multiplet are related by a mysterious "antipodal"duality. This duality incorporates the antipode map of the Hopf algebra for multiple polylogarithms, which reverses all the entries in the symbol, thus exchanging the role of branch cuts and derivatives. It has been checked now through eight loops. Moreover, it is a consequence of an antipodal self-duality (ASD) of the 4-gluon form factor. Although ASD has only been checked through two loops, it reduces to the previous duality in (multi)-collinear limits, suggesting that it too holds to all orders. The underlying physical reason for the duality is still a mystery, and it remains unclear whether any version of it holds in more general theories or processes.

Presenter: DIXON, Lance

Session Classification: Session XI

Contribution ID: **21** Type: **not specified**

Motivic Galois theory for Feynman integrals via twisted cohomology

Thursday, February 16, 2023 10:00 AM (1 hour)

I will report on ongoing joint work with Francis Brown, Javier Fresán, and Matija Tapušković, in which we prove that dimensionally regularized Feynman integrals are closed under the action of the motivic Galois group, termwise in the epsilon expansion. This fits into a larger framework of motivic Galois theory for algebraic Mellin transforms, where the main protagonists are formal versions of « twisted » cohomology groups for algebraic varieties.

Presenter: DUPONT, Clément

Session Classification: Session XI

Contribution ID: **22** Type: **not specified**

Progress on 3-loop Feynman integrals for 4-point 1-mass processes

Thursday, February 16, 2023 11:30 AM (30 minutes)

Presenter: SYRRAKOS, Nikolaos

Session Classification: Session XII

Contribution ID: **23** Type: **not specified**

Intersection Numbers from Electromagnetism to Quantum Field Theory

Thursday, February 16, 2023 12:00 PM (1 hour)

Elaborating on the recent developments concerning Feynman integrals' calculus, I shall introduce Intersection Theory for twisted de Rham co-homology, and mainly discuss the impact of Intersection Numbers on the Vector-Space structure of special functions appearing in Mathematics and Physics.

Presenter: MASTROLIA, Pierpaolo

Session Classification: Session XII

Contribution ID: **24** Type: **not specified**

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Presenter: MATIJAŠIĆ, Antonela

Session Classification: Session XII

Contribution ID: **25** Type: **not specified**

Faster modular IBP solving via Ratracer and tricks

Tuesday, February 14, 2023 5:00 PM (30 minutes)

I'd like to present Rational Tracer, a solver of systems of Integration-By-Parts relations (and more) based on modular arithmetic methods, and explain how using new programming ideas we can achieve lower evaluation times and higher flexibility for practical IBP solving.

Presenter: MAGERYA, Vitaly

Session Classification: Session VI