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On the topology and geometry of higher graph manifold

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Our understanding of 3–manifolds has illuminated two distinct classes of importance; hyperbolic manifolds and graphmanifolds. These are by now considered the basic blocks featured in the geometrisation program of Thurston, famously consolidated by Perelman.

From one perspective graph manifolds are exactly the manifolds that {\it collapse}, in the sense that they admit a family of smooth metrics whose volumes tend to zero while their sectional curvatures remain bounded. Historically the term {\it graph manifold} was introduced by Waldhausen in the 1960's. It highlighted the fact that the fundamental group can be described as a graph of groups and that the manifolds were built up from fundamental pieces that are (heuristically) described in terms of circle bundles over 2-orbifolds.

Recently Frigerio, Lafont and Sisto proposed a family of generalised graph manifolds; products of k–tori with hyperbolic (n-k)–manifolds with truncated cusps are glued along their common n–toral boundaries. They explored multiple topological aspects of this family and raised some questions. For example, in their definition k is allowed to equal zero, so that one subfamily is made up of hyperbolic manifolds glued along truncated cusps. They asked if the minimal volume of such manifolds is achieved by the sum of the hyperbolic volumes of the pieces.

Together with Chris Connell we answered this question positively. In so doing we realized that a natural family we termed higher graph manifolds could be defined; bundles of infranilpotent manifolds over negatively curved bases are glued along boundaries (when possible). This family further extends the one proposed by Frigerio, Lafont and Sisto. We first characterize the higher graph manifolds that admit volume collapse, by explicitly constructing sequences of metrics of bounded curvature whose volume collapses (this builds on earlier work by Fukaya). Various results about the simplicial volume and volume entropy of this family are calculated. Then we exploit the graph structure of the fundamental group to show that these manifolds obey the coarse Baum-Connes conjecture, have finite asymptotic dimension and do not admit metrics of positive scalar curvature. Finally we use several of the produced results to prove that when the infranilmanifold fibre has positive dimension the Yamabe invariant vanishes.

Further, in joint work with Noé Bárcenas and Daniel Juan Pineda we proved that the Borel conjecture also holds for higher graph manifolds.

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