Gauged Quintessence

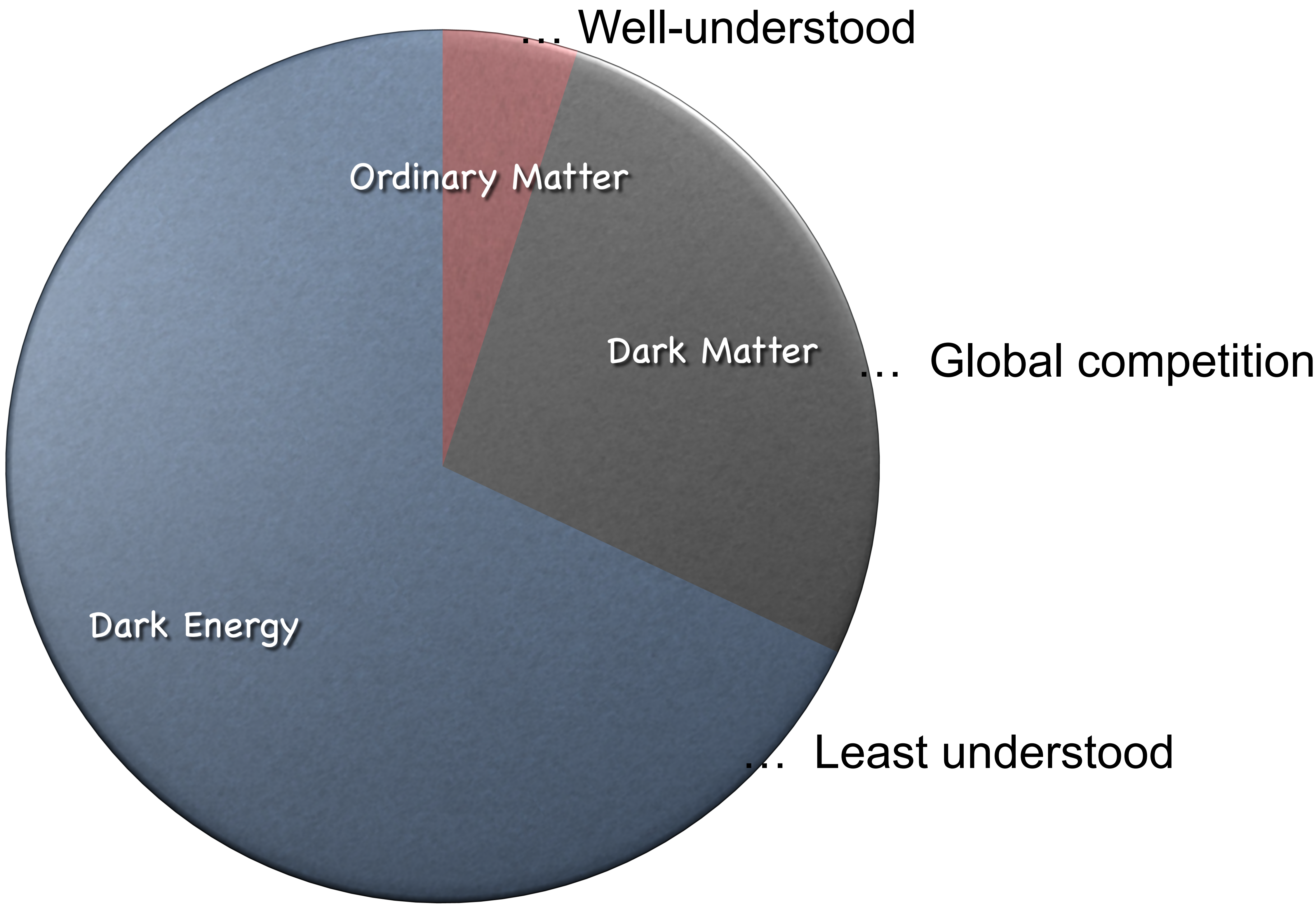
Hye-Sung Lee
(KAIST)

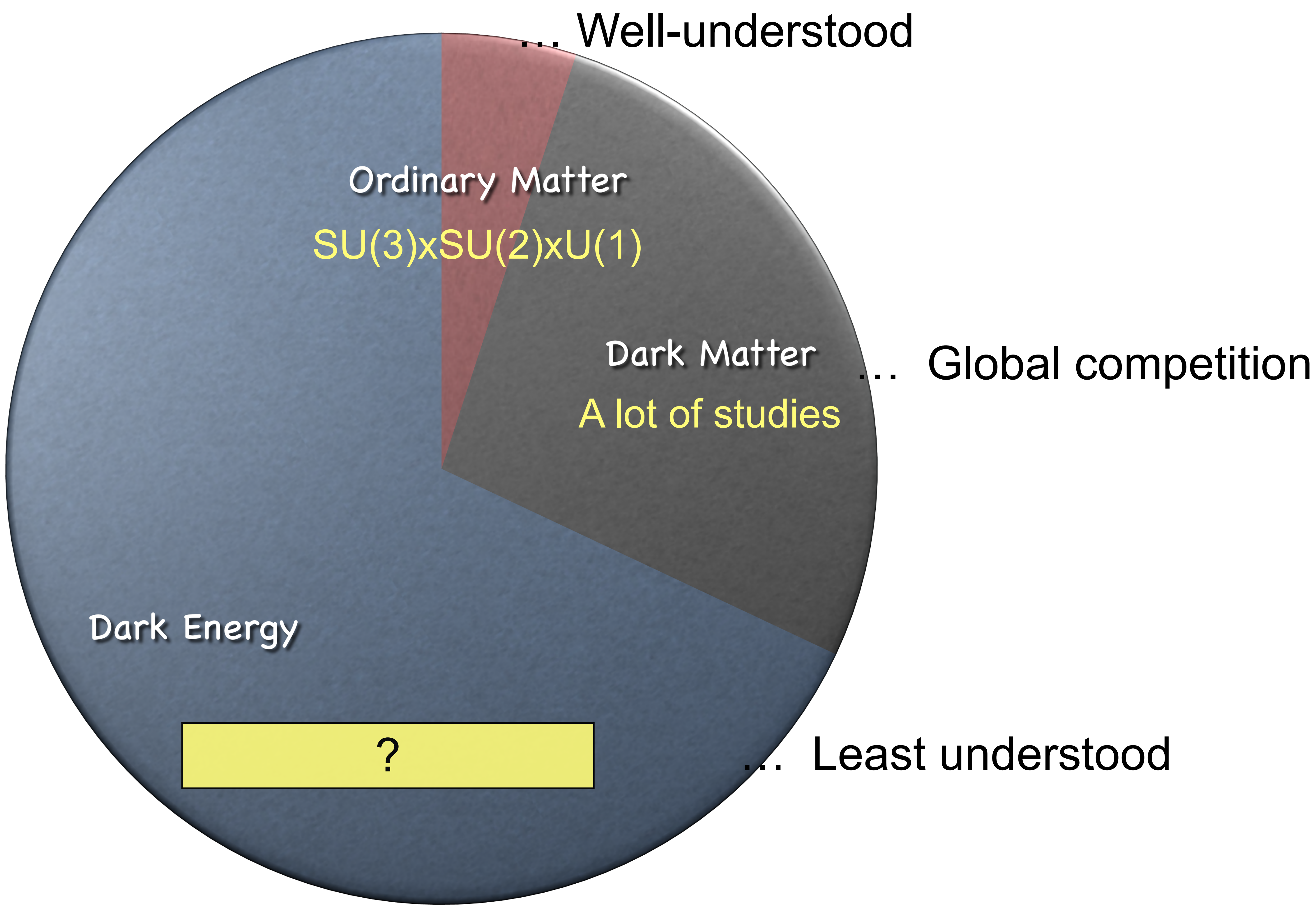
based on JCAP 02, 005 (2023) & JCAP 09, 017 (2023)
with Kunio Kaneta, Jiheon Lee, Jaeok Yi

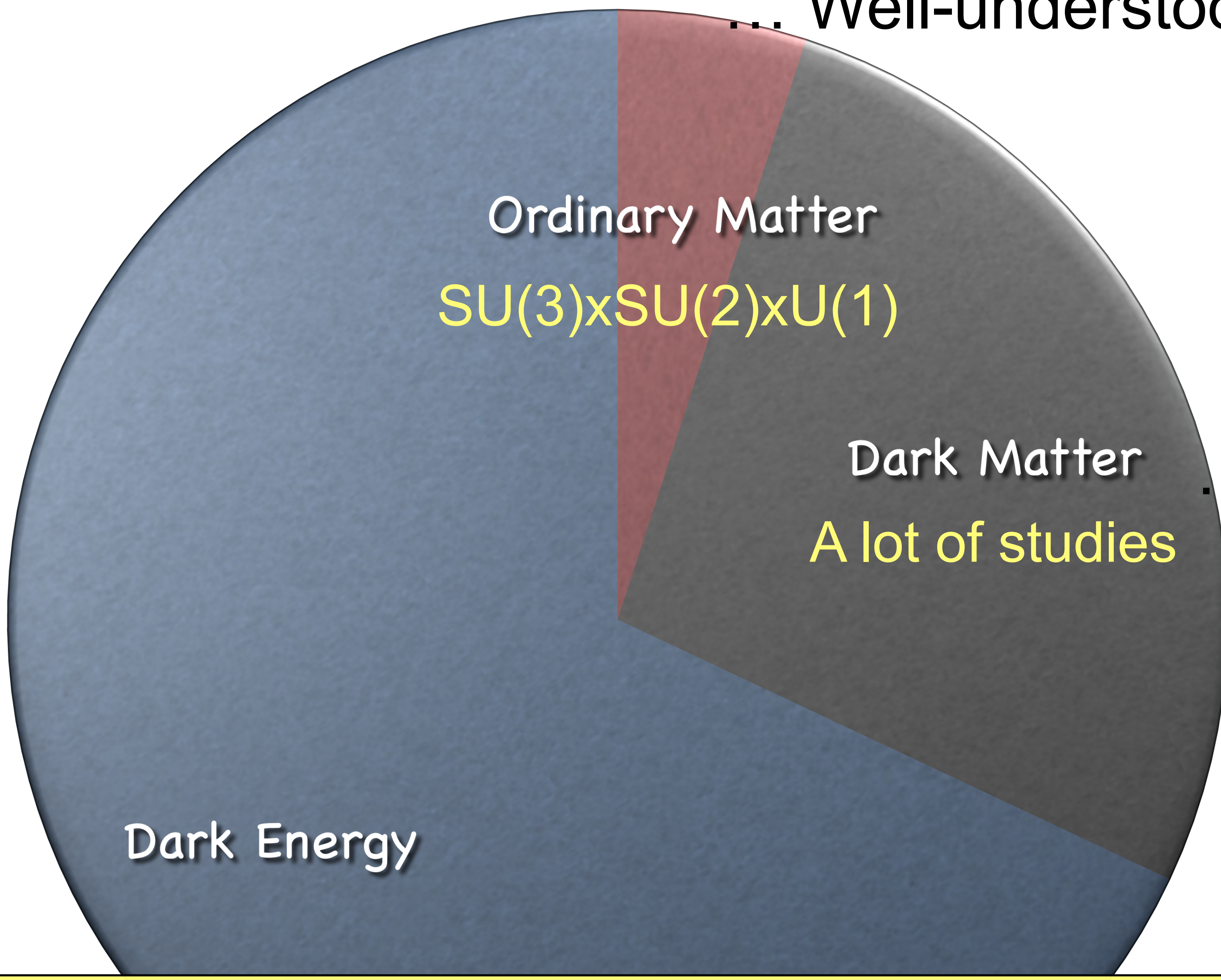
Light Dark World 2023
Karlsruhe Institute of Technology
September 19-21, 2023

The universe is an enormous direct product of representations of symmetry groups.

\- Steven Weinberg -


... Well-understood
Ordinary Matter
Dark Matter
... Global competition
Dark Energy
... Least understood


... Well-understood
Ordinary Matter
SU(3)xSU(2)xU(1)
Dark Matter
... Global competition
A lot of studies
Dark Energy
?
... Least understood



… Well-understood

… Global competition

… Least understood

Quintessence [Ratra, Peebles (1988)]
: first dark energy field model with a singlet scalar

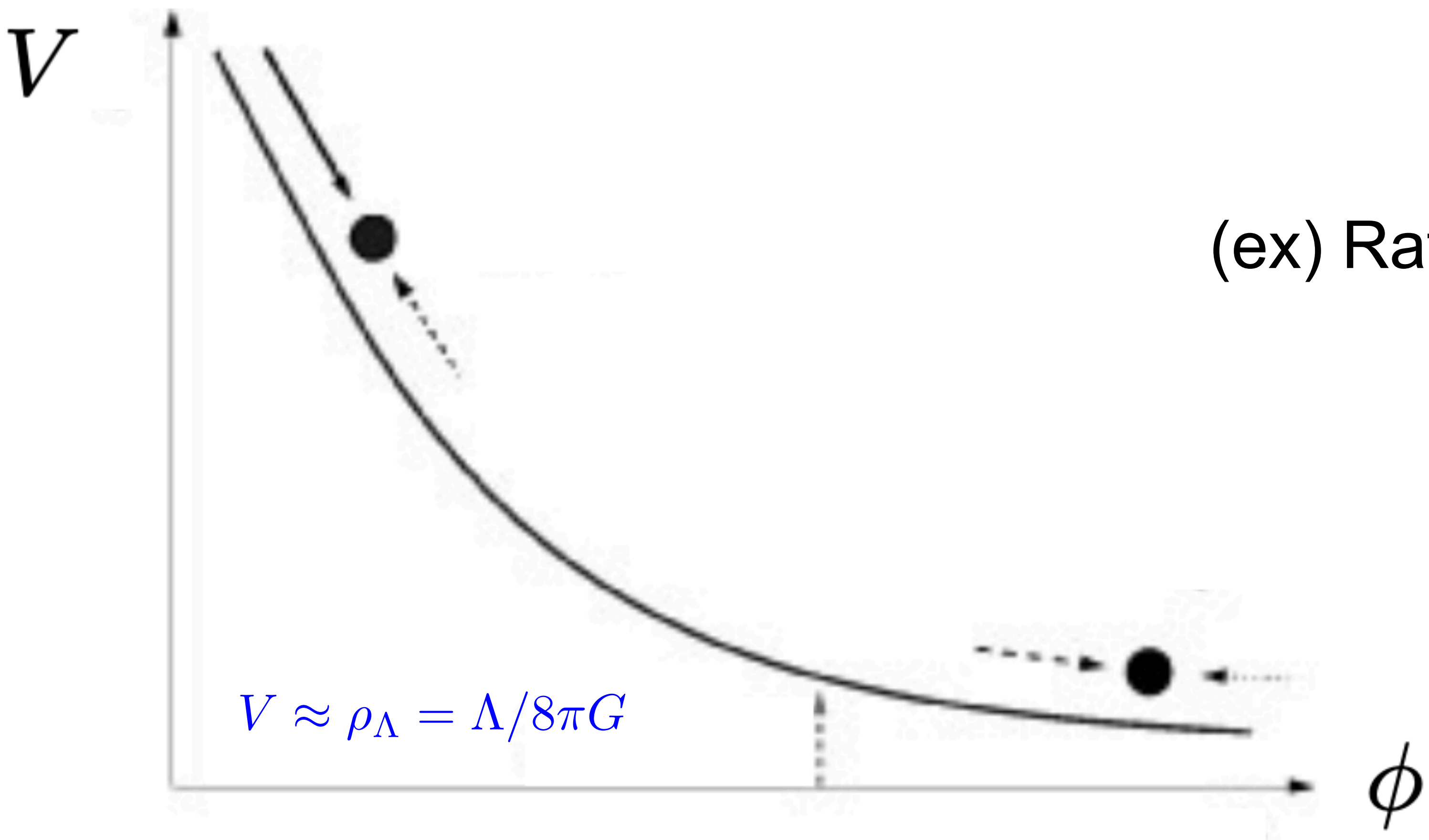
Gauged Quintessence [Kaneta, LEE, Lee, Yi (2023)]
: first gauge symmetry model in
the quintessence scalar field

Outline of this talk

1. Quintessence at a glance
2. Gauged quintessence
3. Misalignment mechanism for the vector boson

Quintessence at a glance

Quintessence



(ex) Ratra-Peebles inverse power potential

$$V(\phi) = \frac{M^{\alpha+4}}{|\phi|^{\alpha}}$$

Quintessence

- Proposed by Ratra and Peebles (1988).
- Dynamic dark energy model with a scalar field (ϕ).
- A scalar rolls down a potential slowly in the late universe.
- Its potential energy is identified as the dark energy.
- Tracking behavior: The ϕ initial value does not really matter. Only the potential determines the present time value of ϕ and its equation of state (addressing the cosmological coincidence problem) [Steinhardt, Wang, Zlatev (1999)].

Quintessence

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{2}m_{Pl}^2 R - \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)\Big]$$

$g_{\mu\nu} = \mathrm{Diag}\{-1, a(t)^2, a(t)^2, a(t)^2\}$

$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}$$

(m_ϕ decreases for a Ratra-Peebles potential.)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

(equation of motion)

$H \equiv \frac{\dot{a}}{a}$ (Hubble parameter)

$$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

(equation of state)

$\rho \propto a^{-3(1+w)}$

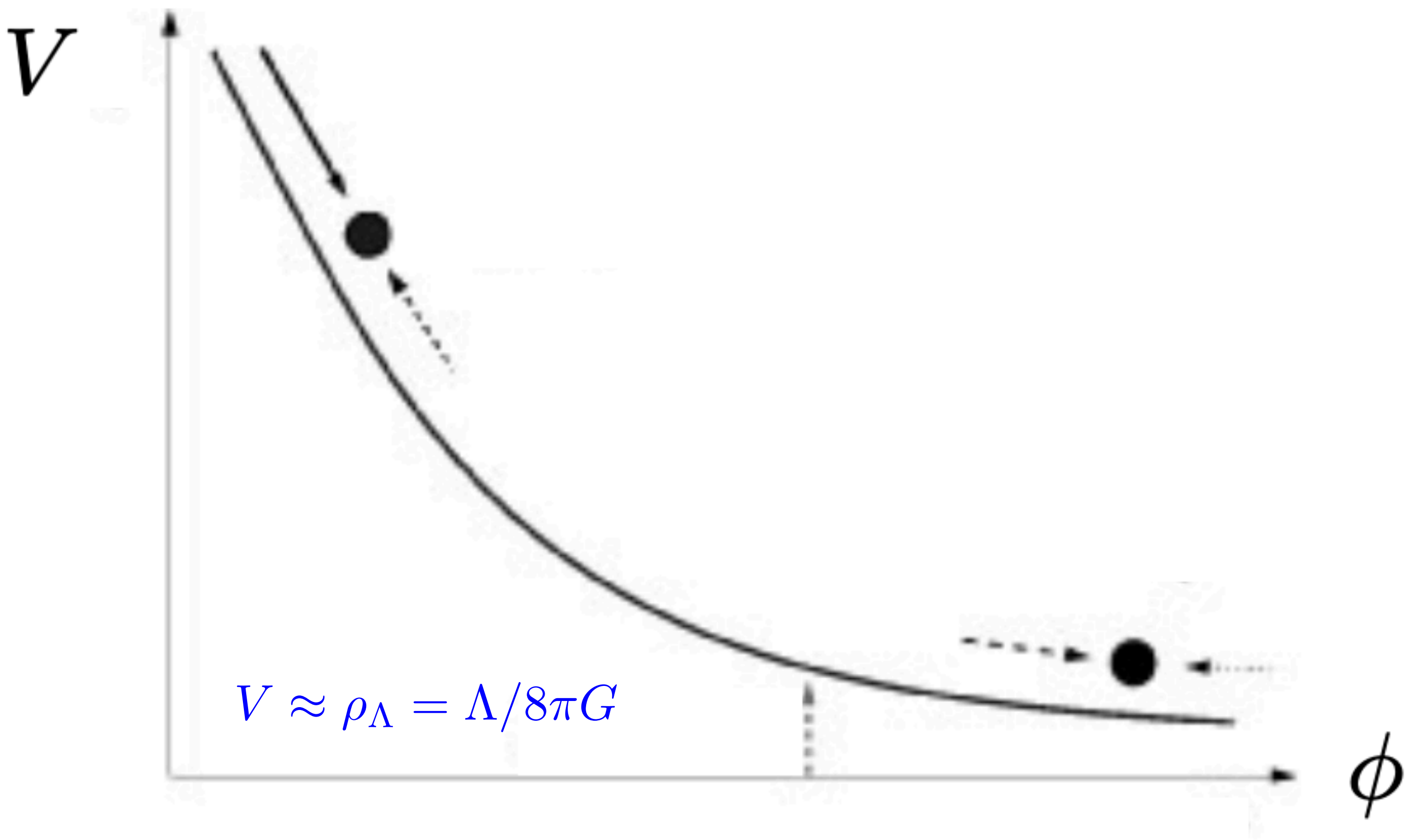
$$= -1 + \frac{\dot{\phi}^2}{V} + \cdots$$

for $\dot{\phi}^2 \ll V(\phi)$ (slow-roll)

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho$

(w < -1/3 for the accelerated expansion. w = -1 for $\mathbf{\Lambda}$)

Quintessence



Conditions for the quintessence dark energy

$V \sim 10^{-123} M_{Pl}^4 \sim 3 \times 10^{-47}\,\mathrm{GeV}^4$ (present dark energy density)

$m_\phi \lesssim H_0 \sim 10^{-42}\,\mathrm{GeV}$ (slow-roll)

Gauged quintessence

Gauged Quintessence

We introduce a dark U(1) gauge symmetry to the quintessence scalar.

$$\Phi = \frac{1}{\sqrt{2}}\phi\, e^{i\eta}$$: complex scalar under the U(1)$_{\text{dark}}$ gauge symmetry

(ϕ : quintessence scalar)

(η : longitudinal component of the dark gauge boson X)

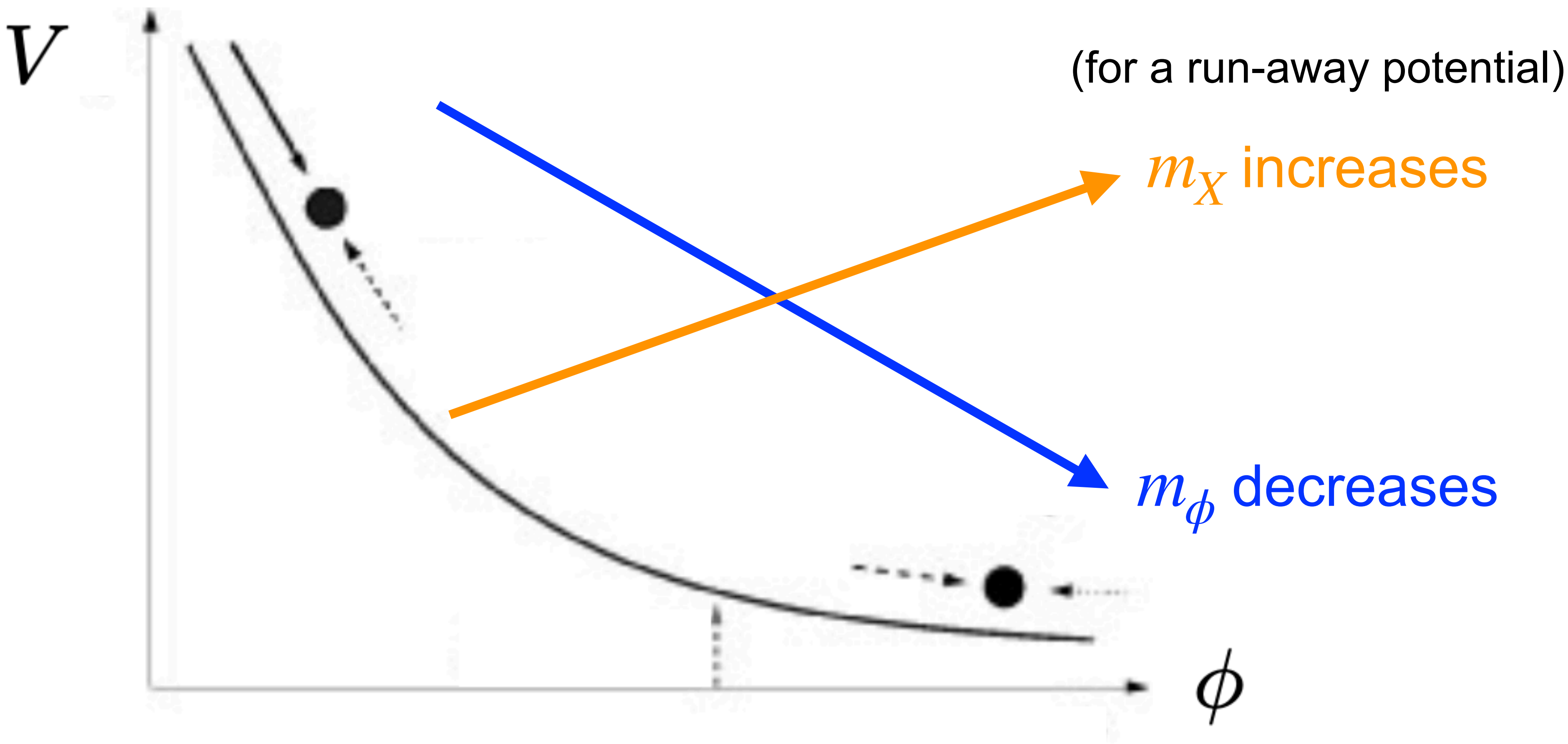
$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}m_{Pl}^2 R - |D_\mu\Phi|^2 - V_0(\Phi) - \frac{1}{4}\mathbb{X}_{\mu\nu}\mathbb{X}^{\mu\nu}\Big] \qquad D_\mu \equiv \partial_\mu + i g_X \mathbb{X}_\mu$$

$$= \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}m_{Pl}^2 R - \frac{1}{2}(\partial_\mu\phi)^2 - V_0(\phi) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \underbrace{\frac{1}{2}g_X^2\phi^2 X_\mu X^\mu}_{V_{\text{gauge}}}\Big]$$

(in unitary gauge : $\eta = 0, \quad X_\mu = \mathbb{X}_\mu + \frac{1}{g_X}\partial_\mu\eta$)

$$m_\phi^2|_0 = \frac{\partial^2 V_0}{\partial\phi^2} + \frac{\partial^2 V_{\text{gauge}}}{\partial\phi^2}\,, \quad m_X^2|_0 = g_X^2\phi^2$$ (tree-level masses)

Masses vary over cosmic evolution



$$V_0(\phi) = \frac{M^{\alpha+4}}{|\phi|^\alpha}$$

As the quintessence ϕ rolls down the potential, both m_ϕ and m_X change over cosmic evolution.

$$m_\phi^2|_0 = \frac{\partial^2 V_0}{\partial \phi^2} + \frac{\partial^2 V_{\rm gauge}}{\partial \phi^2} \,, \quad m_X^2|_0 = g_X^2 \phi^2$$ (tree-level masses)

Gauged Quintessence

Equations of motion for ϕ and X (coupled via V_{gauge})

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_0}{\partial \phi} + g_X^2 X_\mu X^\mu \phi = 0$$

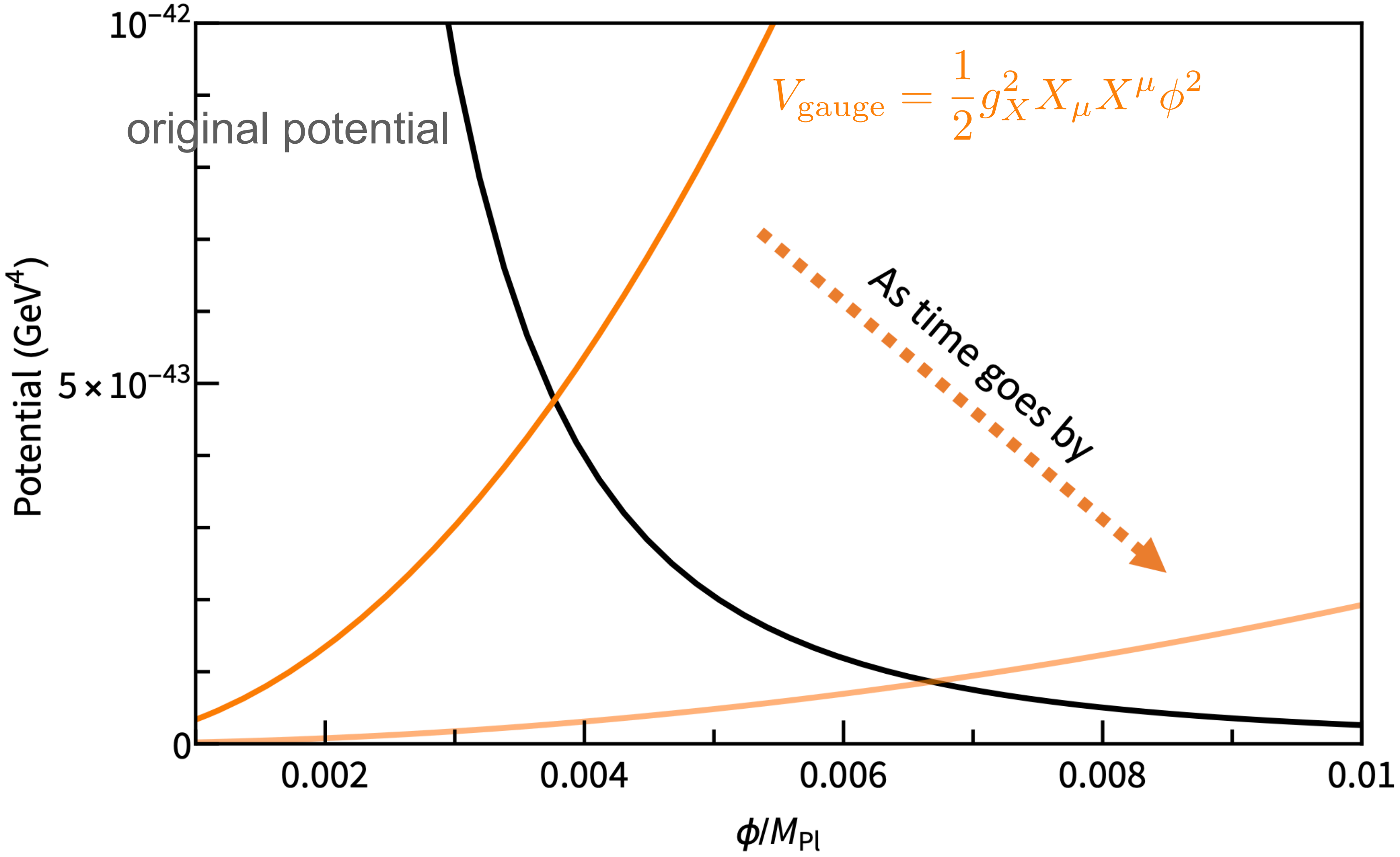
$$\partial_\mu X^{\mu\nu} + 3HX^{0\nu} - g_X^2 \phi^2 X^\nu = 0$$

$$V_{\text{gauge}} = \frac{1}{2} g_X^2 \phi^2 X_\mu X^\mu$$

Energy-momentum tensor

$$\begin{aligned} T_{\mu\nu} =& (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \phi)(\partial^\alpha \phi) - g_{\mu\nu} V_0(\phi) \\ & - \frac{1}{2} g_{\mu\nu} g_X^2 \phi^2 X_\alpha X^\alpha + g_X^2 \phi^2 X_\mu X_\nu + X_{\mu\alpha} X_\nu^\alpha - \frac{g_{\mu\nu}}{4} X_{\alpha\beta} X^{\alpha\beta} \end{aligned}$$

Potential modified by the gauge symmetry



$$V = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2$$

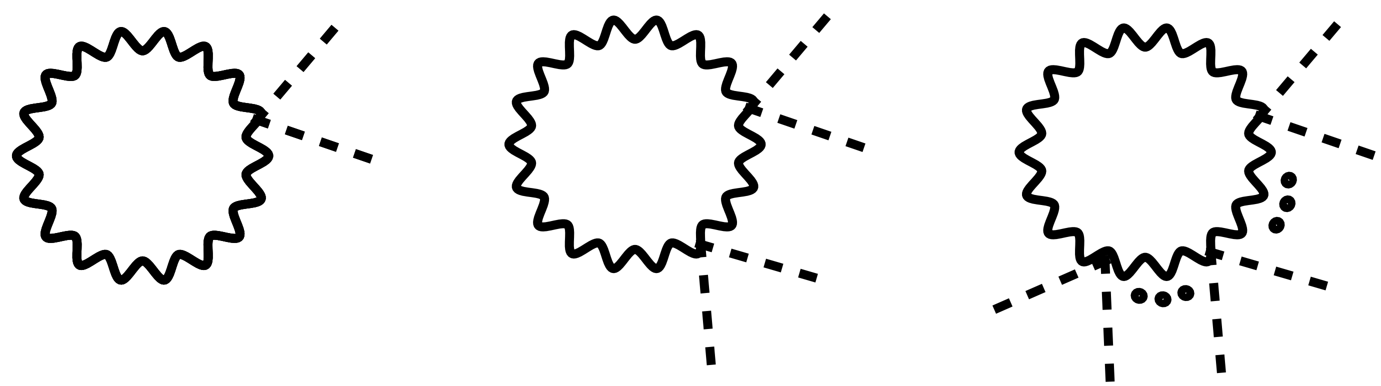
Quantum corrections in the gauged quintessence

1-loop effective potential in the gauged quintessence model

$$V_{\rm eff} = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2 + \frac{\Lambda^2}{32\pi^2} V_0'' + \frac{(V_0'')^2}{64\pi^2}\left(\ln \frac{V_0''}{\Lambda^2} - \frac{3}{2}\right) + \frac{3(m_{\rm X}^2|_0)^2}{64\pi^2}\left(\ln \frac{m_{\rm X}^2|_0}{\Lambda^2} - \frac{5}{6}\right)$$

1-loop correction of the quintessence

Additional 1-loop correction due to the X-boson



Quantum corrections in the gauged quintessence

1-loop effective potential in the gauged quintessence model

$$V_{\rm eff} = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2 + \frac{\Lambda^2}{32\pi^2} V_0'' + \frac{(V_0'')^2}{64\pi^2}\left(\ln\frac{V_0''}{\Lambda^2} - \frac{3}{2}\right) + \frac{3(m_X^2|_0)^2}{64\pi^2}\left(\ln\frac{m_X^2|_0}{\Lambda^2} - \frac{5}{6}\right)$$

$$m_\phi^2 = \frac{\partial^2 V_{\rm eff}}{\partial\phi^2} = V_0'' + g_X^2 X_\mu X^\mu + \frac{\Lambda^2}{32\pi^2} V_0'''' + \frac{V_0'' V_0''''}{32\pi^2}\left(\ln\frac{V_0''}{\Lambda^2} - 1\right) + \frac{9 g_X^2 m_X^2|_0}{16\pi^2}\left(\ln\frac{m_X^2|_0}{\Lambda^2} + \frac{1}{3}\right)$$

$$m_X^2 = g_X^2\left(\phi^2 + \frac{V_0''}{32\pi^2}\ln\frac{V_0''}{\Lambda^2}\right)$$

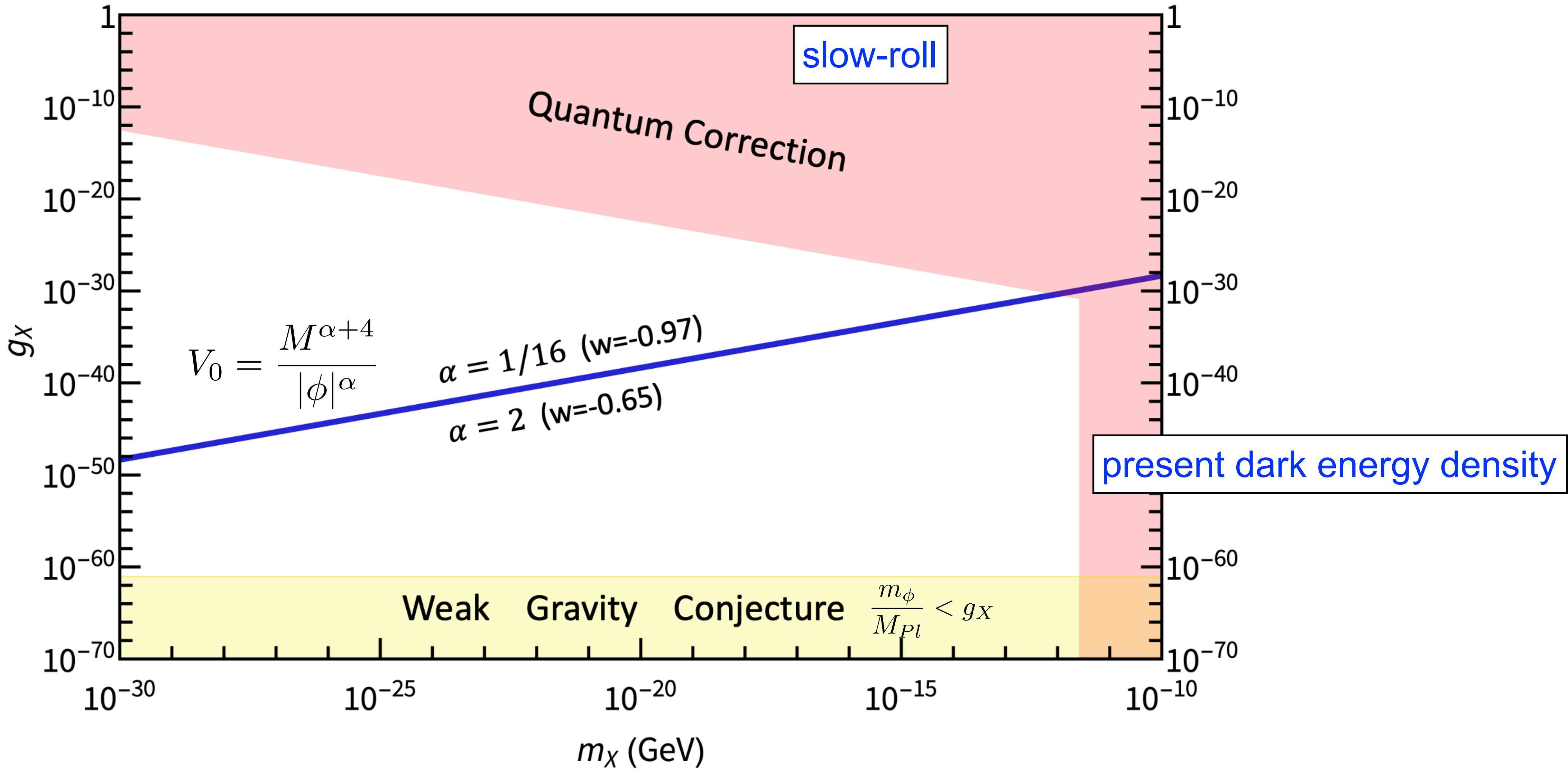
independent of potential V_0

Conditions for the quintessence dark energy

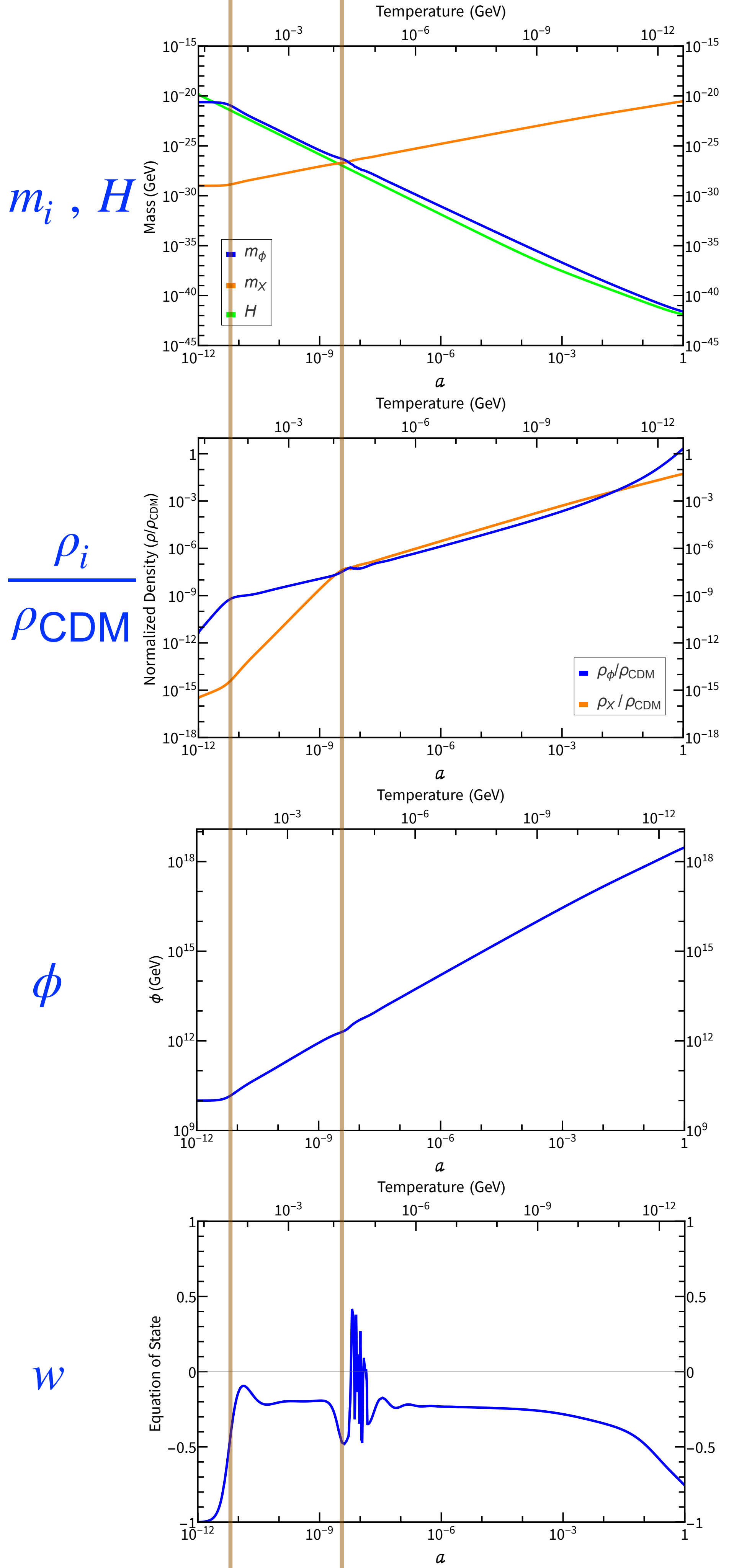
$V \sim 10^{-123} M_{Pl}^4 \sim 3 \times 10^{-47}\,\mathrm{GeV}^4$ (present dark energy density)

$m_\phi \lesssim H_0 \sim 10^{-42}\,\mathrm{GeV}$ (slow-roll)

Potential-independent constraints (at present universe)



(Blue band: Ratra-Peebles potential case with the tracking behavior.)



X may have a sizable relic density, but we assume it is a subdominant DM (less than 10% relic density of the dominant CDM).

The dynamics of ϕ and X change drastically when the hierarchy between m_ϕ, m_X and H change over time.

$(i)\ H > m_\phi,\ m_X$: Both ϕ and X are frozen by Hubble friction.
$(ii)\ m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.
$(iii)\ m_X,\ m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_0}{\partial \phi} + g_X^2 X_\mu X^\mu \phi = 0$$
$$\partial_\mu X^{\mu\nu} + 3H X^{0\nu} - g_X^2 \phi^2 X^\nu = 0$$

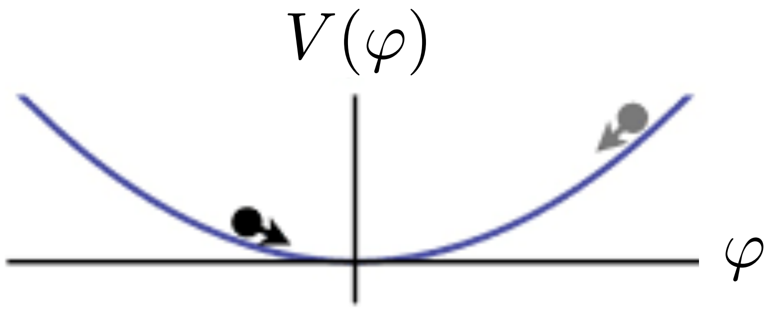
Benchmark parameters: $\alpha = 1$, $M = 2.2 \times 10^{-6}$ GeV, $g_X = 10^{-39}$
$\dot{X} = 0$, $\dot{\phi} = 0$ (at $a = 10^{-12}$)

Misalignment mechanism for the vector boson

Misalignment mechanism for coherent scalar oscillation

[Preskill, Wise, Wilczek (1983)] [Abbott, Sikivie (1983)] [Dine, Fischler (1983)]

Misalignment mechanism is a popular production mechanism of a coherent scalar field (such as QCD axion DM). [See Cem Eröncel's talk]



$$\ddot{\varphi} + 3H\dot{\varphi} + m_\varphi^2 \varphi = 0 \qquad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m_\varphi^2\varphi^2$$

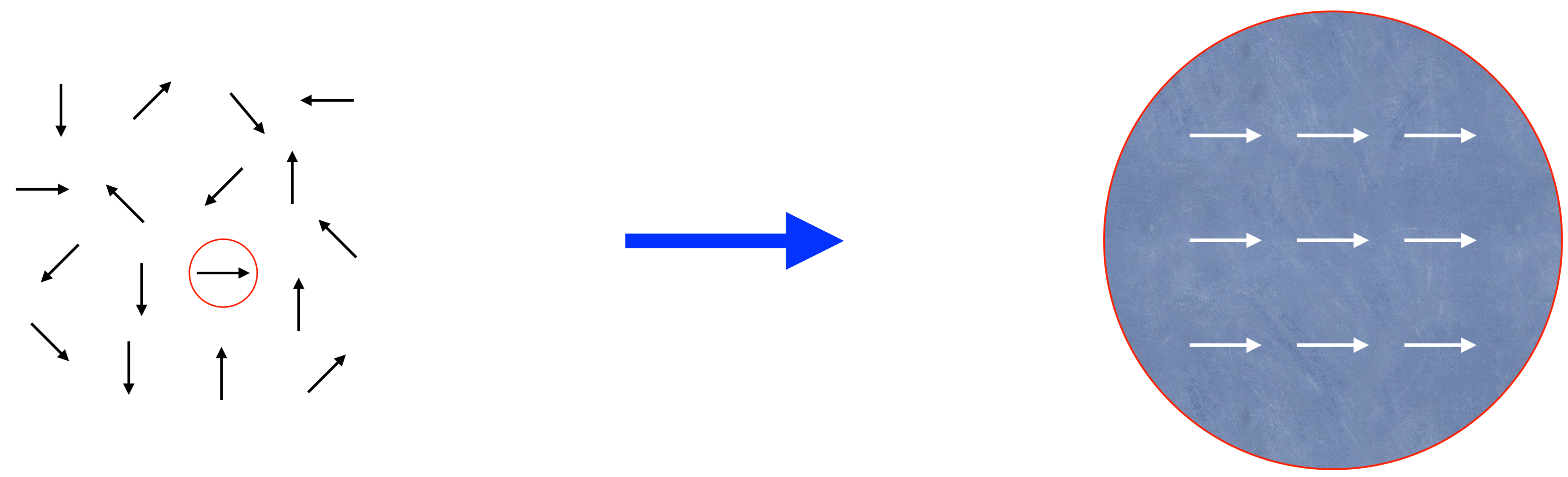
(i) Inflation makes φ spatially homogeneous: $\varphi(t, \vec{x}) = \varphi(t)$.

(ii) Initially, Hubble friction is large ($H > m_\varphi$), which makes φ frozen and ρ_φ constant.

(iii) When Hubble friction decreases sufficiently ($H \lesssim m_\varphi$), a coherent φ oscillation begins around the potential minimum.

(iv) The oscillator has $p_\varphi = 0$, behaving as non-relativistic matter ($\rho_\varphi \propto a^{-3}$); φ is a CDM despite of lightness. (QCD axion DM: $m_a \sim 10^{-6} - 10^{-2}$ eV)

Misalignment mechanism for coherent vector oscillation



$\vec{X} = (0, 0, X)$

vector fields in random direction
(before the inflation)

vector fields in one direction in visible universe
(during/after inflation)

$$\partial_\mu X^{\mu\nu} + 3H X^{0\nu} - m_X^2 X^\nu = 0$$

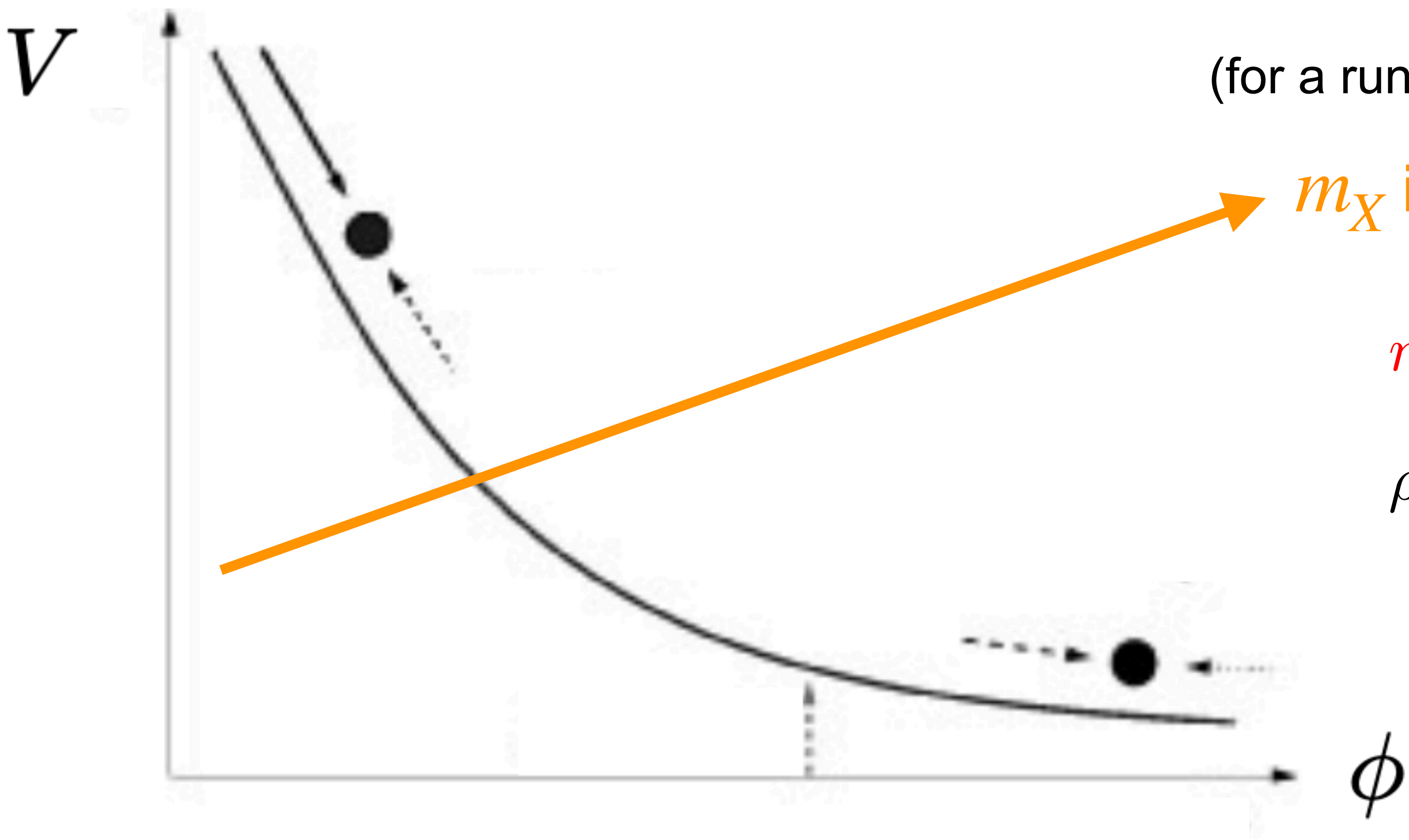
zero mode (spatially homogeneous): $X_\mu(t, \vec{x}) = X_\mu(t) = (X_0(t), \vec{X}(t))$

$$X_0 = 0 \qquad \ddot{X} + H\dot{X} + m_X^2 X = 0 \qquad \rho_X = \frac{1}{2a^2}\left(\dot{X}^2 + m_X^2 X^2\right)$$

Unlike the scalar case, the ρ_X is highly suppressed by the scale factor, and it is hard to retain the ρ_X through the inflation. (Typical inflation e-folding is 60.)

Naive misalignment does not really work for a sizable vector boson production.

Vector misalignment in the gauged quintessence model



(for a run-away potential)

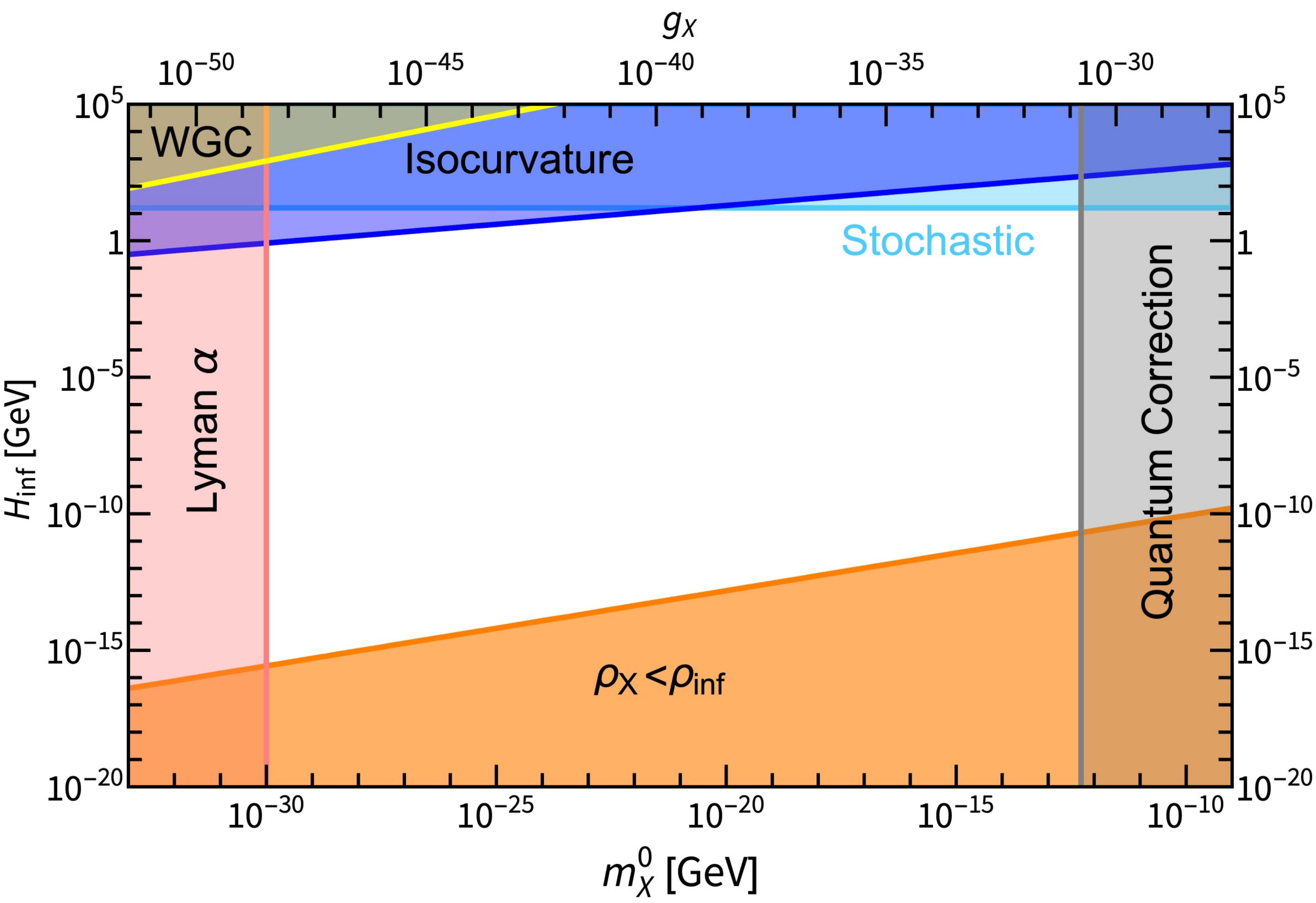
m_X increases

$$m_X = g_X \phi$$

$$\rho_X = \frac{1}{2a^2}\left(\dot{X}^2 + m_X^2 X^2\right)$$

As ϕ may increase by many orders of magnitude, m_X may increase by many orders of magnitude too overcoming the suppression by the scale factor.

Vector misalignment in the gauged quintessence model



Misalignment mechanism with a mass-varying vector boson may work to produce a sizable vector boson energy density.

Concluding remarks

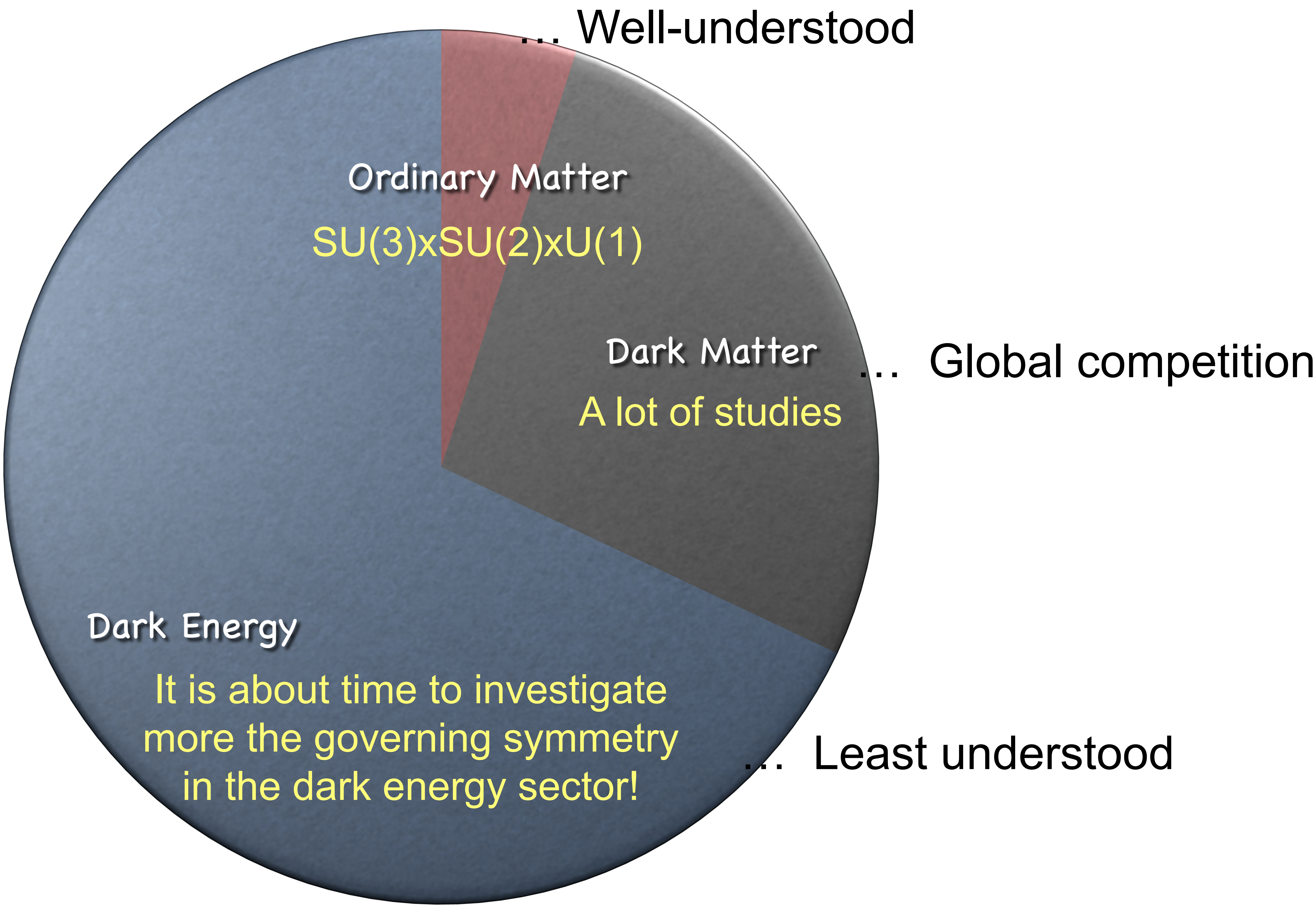
Summary

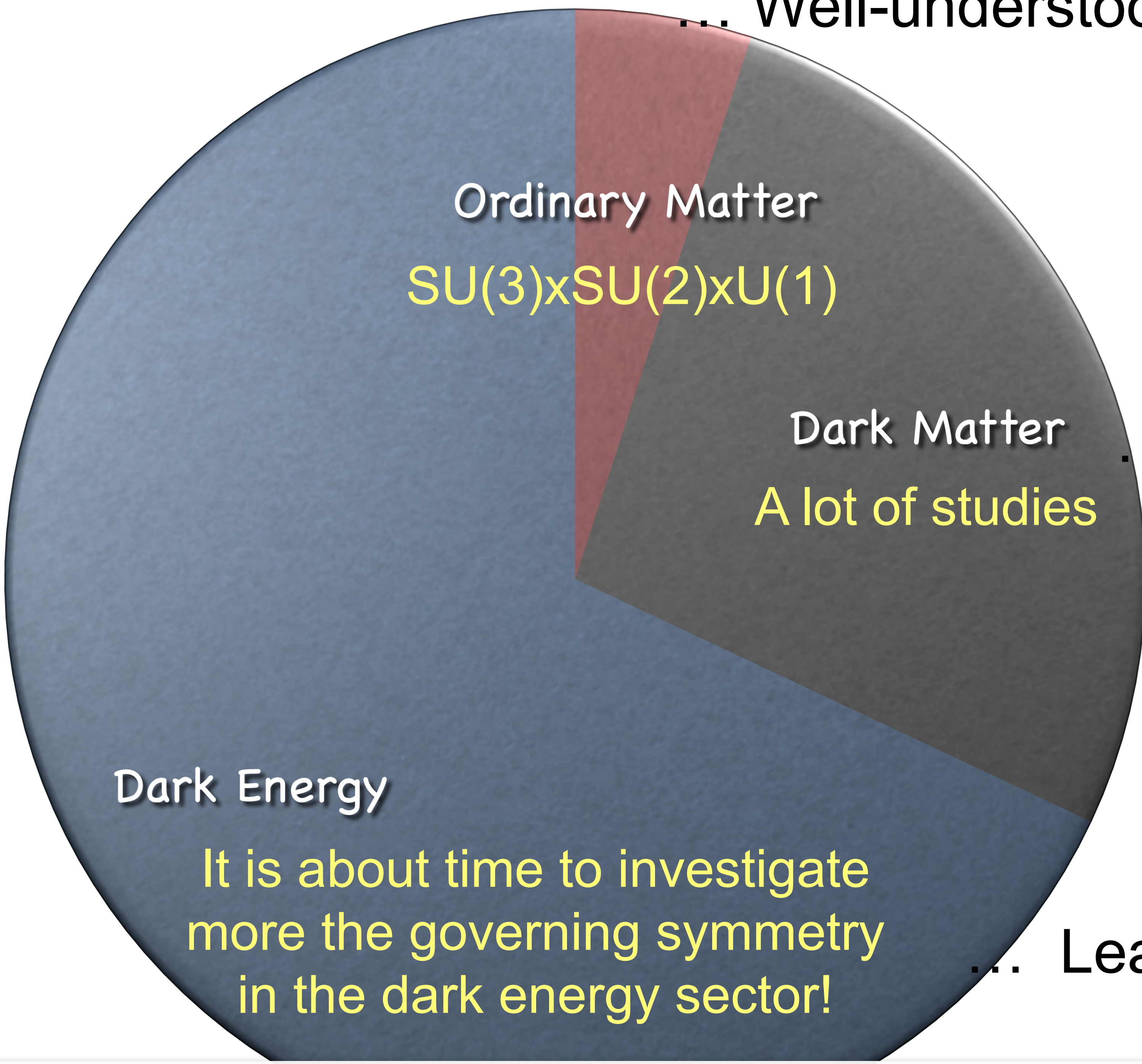
1. We introduced the first gauge symmetry model for a popular quintessence dark energy scalar field.

2. The interaction between the quintessence and the gauge boson ($V_{\text{gauge}} = \frac{1}{2}g_X^2\phi^2 X_\mu X^\mu$) brings many interesting features to the universe evolution.

3. The mass-varying effect of the X gauge boson may overcome the problem of the vector boson misalignment mechanism (scaling factor suppression).

4. Our study may serve as a proof of concept that the dark energy sector can be studied using the gauge principle. (Gauge interaction with the dark energy)


... Well-understood
Ordinary Matter
SU(3)xSU(2)xU(1)
Dark Matter
... Global competition
A lot of studies
Dark Energy
It is about time to investigate
more the governing symmetry
in the dark energy sector!
... Least understood



... Well-understood

... Global competition

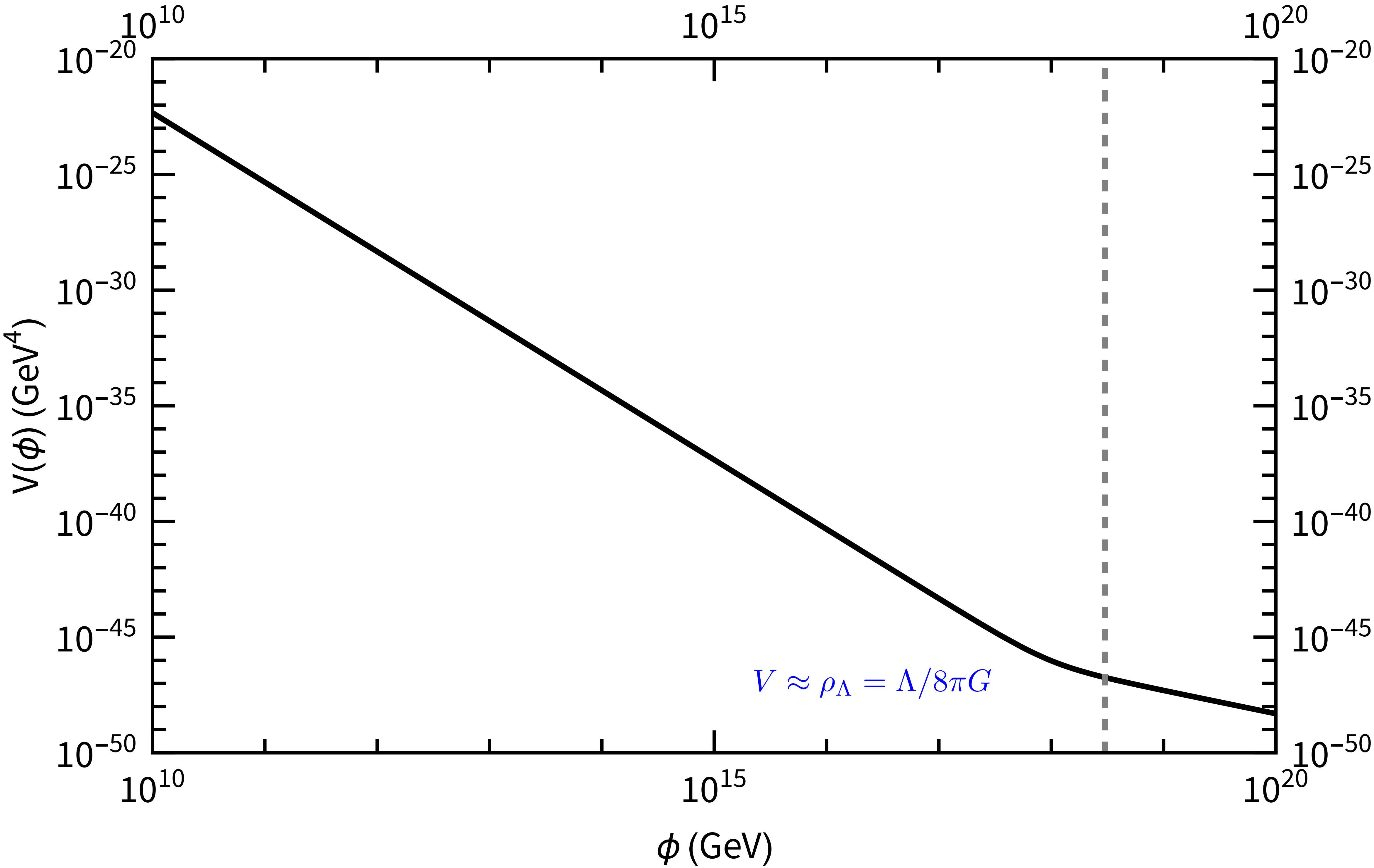
... Least understood

> The universe is an enormous direct product of representations of symmetry groups.
>
> – Steven Weinberg –

Thank you

Back-up

Ratra-Peebles potential (with quantum corrections)



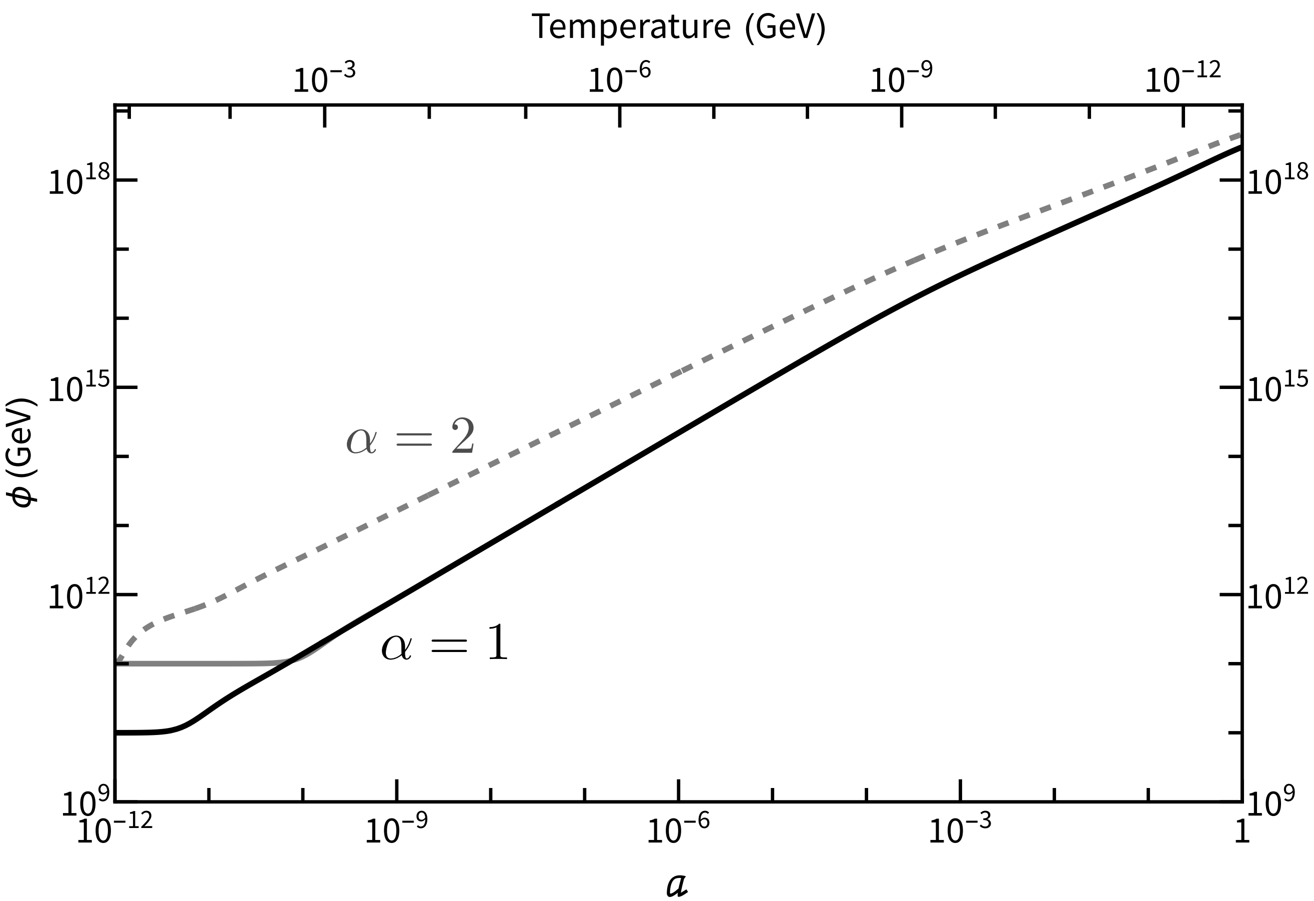
Adiabatic condition

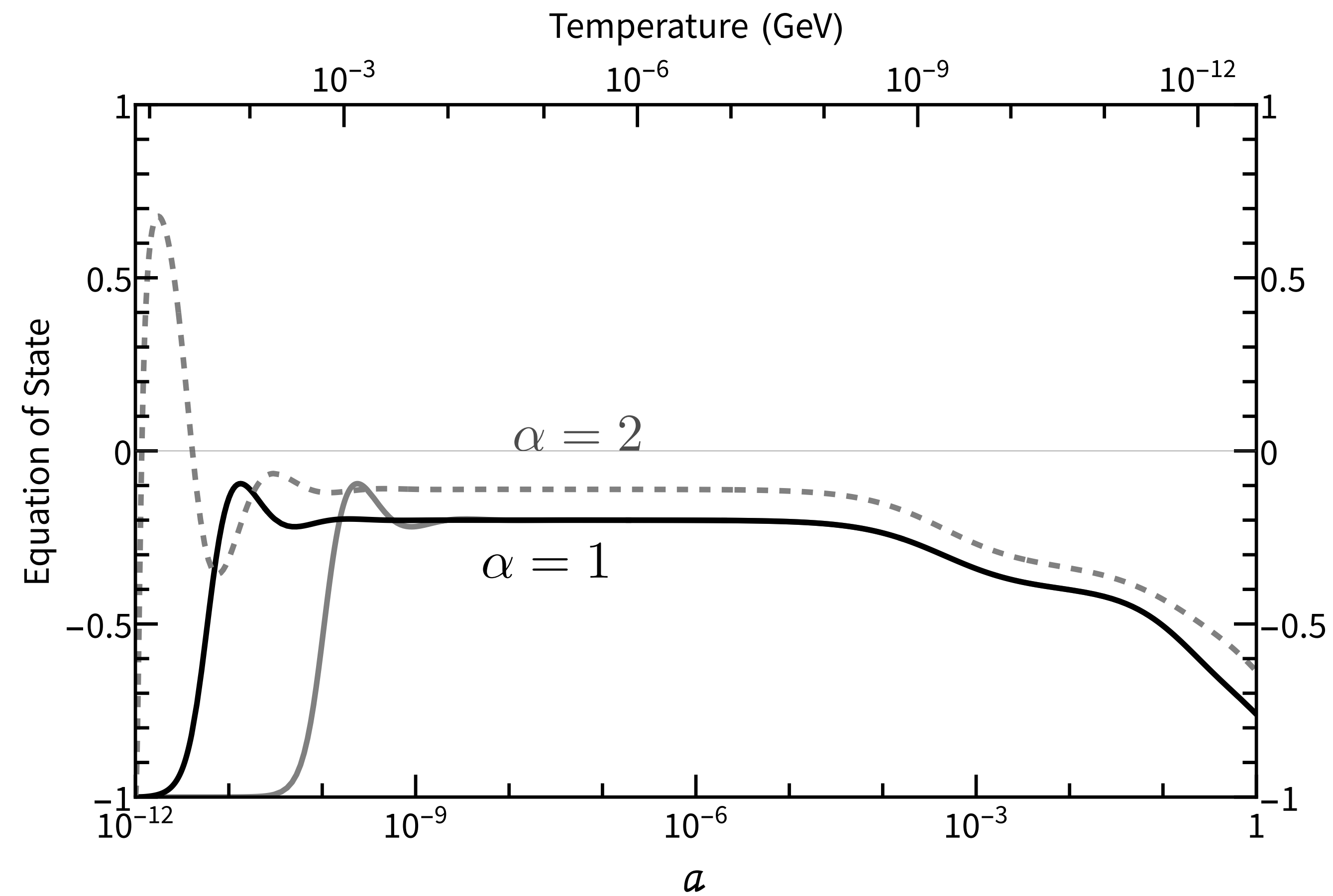
It should be noted that in both cases we will only consider the case the adiabatic condition [77], which can be written as

$$\frac{dm_X}{dt} \ll m_X^2 \,, \tag{4.1}$$

is always satisfied. If this condition is violated, the WKB-like solution for the wave function of X_μ cannot be used. In other words, non-perturbative X_μ production could be non-negligible in such a case. The adiabatic condition may break in some cases[6], which can bring intriguing phenomenology. Furthermore, the violation of the adiabaticity indicates that the fragmentation of a condensate (ϕ and/or X_μ) may take place through gauge or self interactions[7] in a similar manner of other coherent states, such as inflaton [78] and axion-like particle [79]. However, in this paper, we will investigate the simple cases in which this condition is valid.

Quintessence dynamics (without a gauge symmetry)





$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$V(\phi) = \frac{M^{\alpha+4}}{|\phi|^{\alpha}}$$

Balancing between the potential slope and Hubble friction results in the common tracking solution for the quintessence. The present values are not sensitive to its initial values (quintessence tracking behavior).

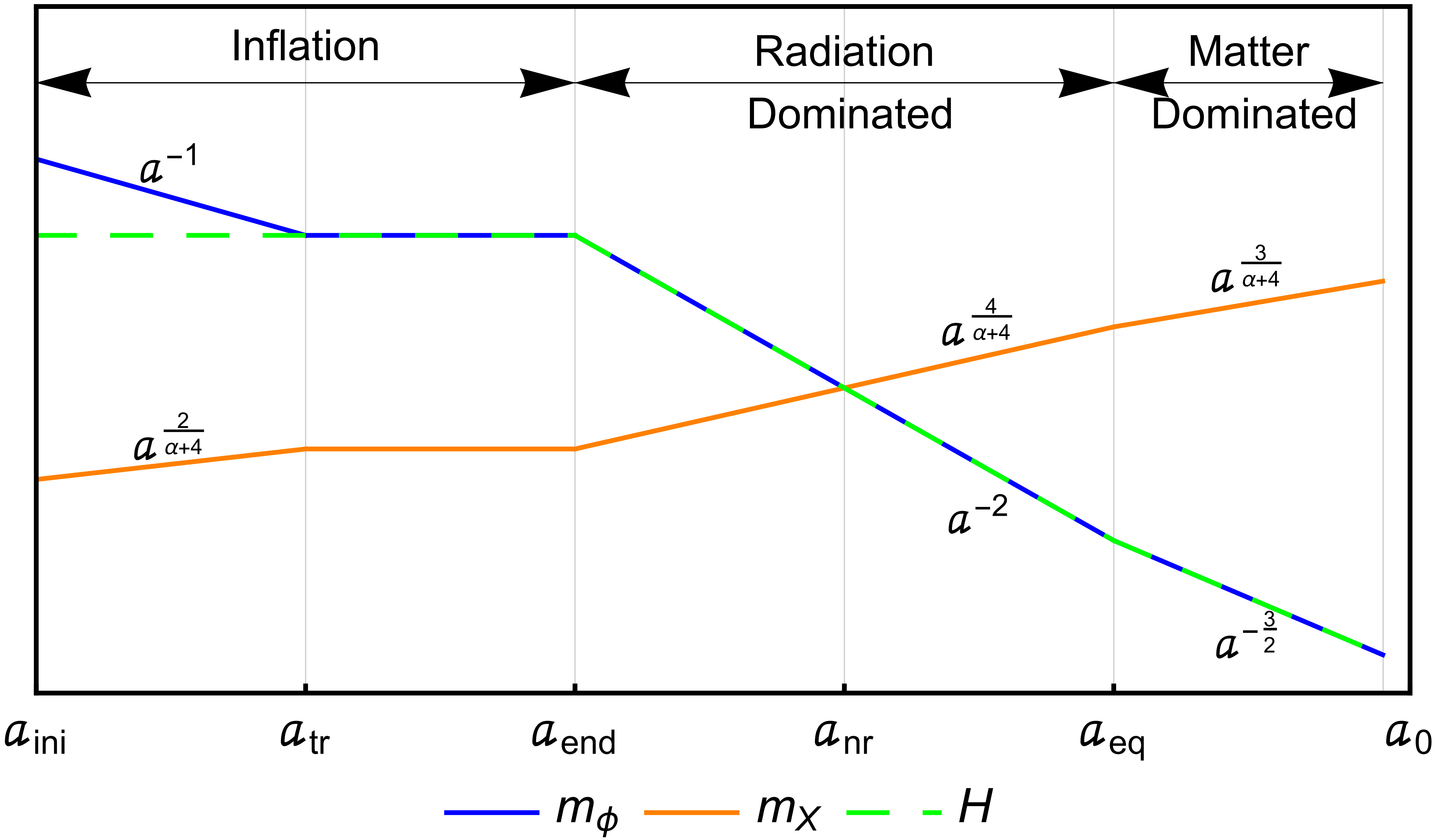
Boltzmann equations for mass varying ϕ and X

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\frac{\dot{m}_X}{m_X}(\rho_X - 3p_X)$$
$$\dot{\rho}_X + 3H(\rho_X + p_X) = \frac{\dot{m}_X}{m_X}(\rho_X - 3p_X)$$

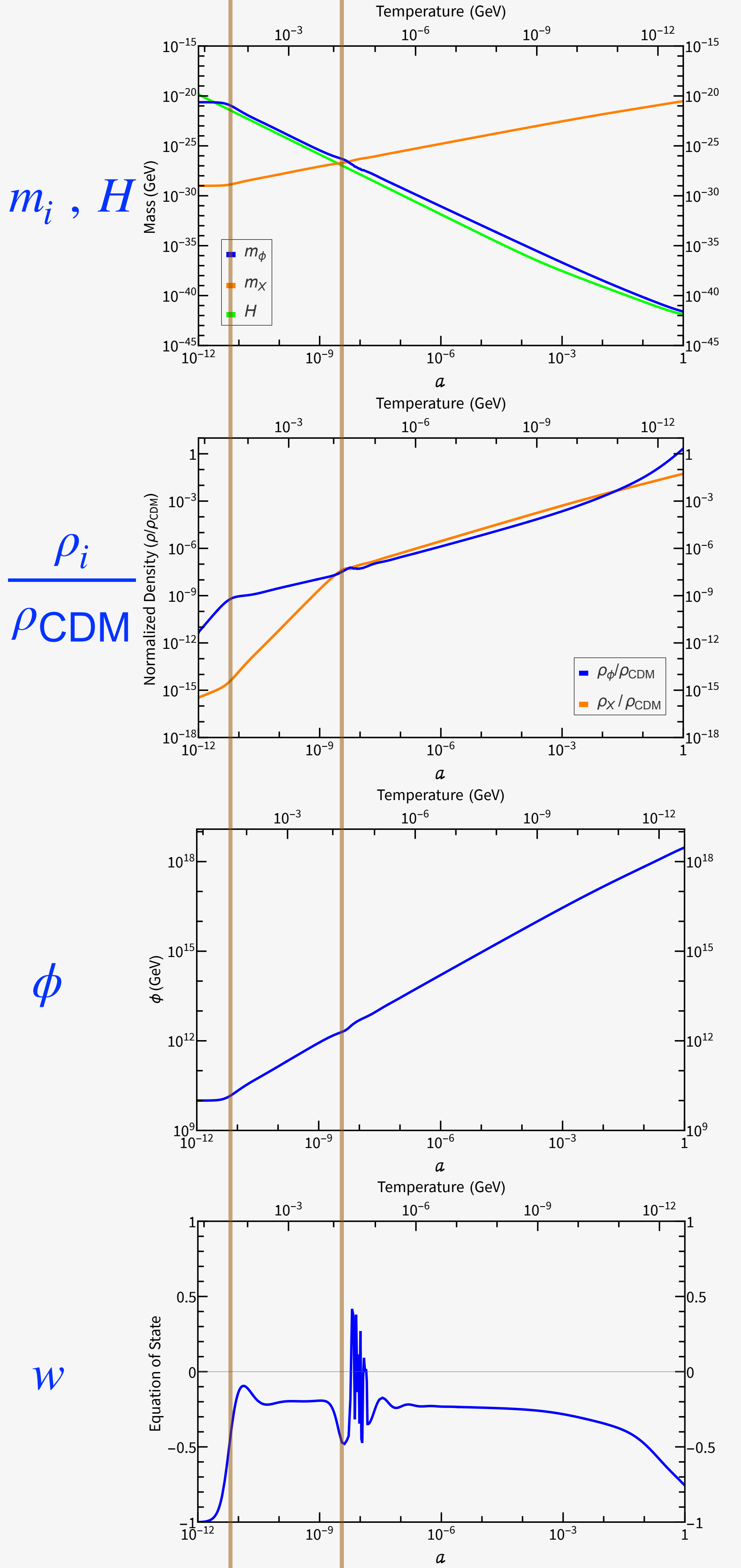
The energy flow between the quintessence scalar and the dark gauge boson is proportional to the $\dot{m}_X$.

$$\dot{m}_X > 0: \ \text{Energy flows from } \phi \text{ to } X$$
$$\dot{m}_X < 0: \ \text{Energy flows from } X \text{ to } \phi$$

Vector misalignment in the gauged quintessence model



Evolution of the universe



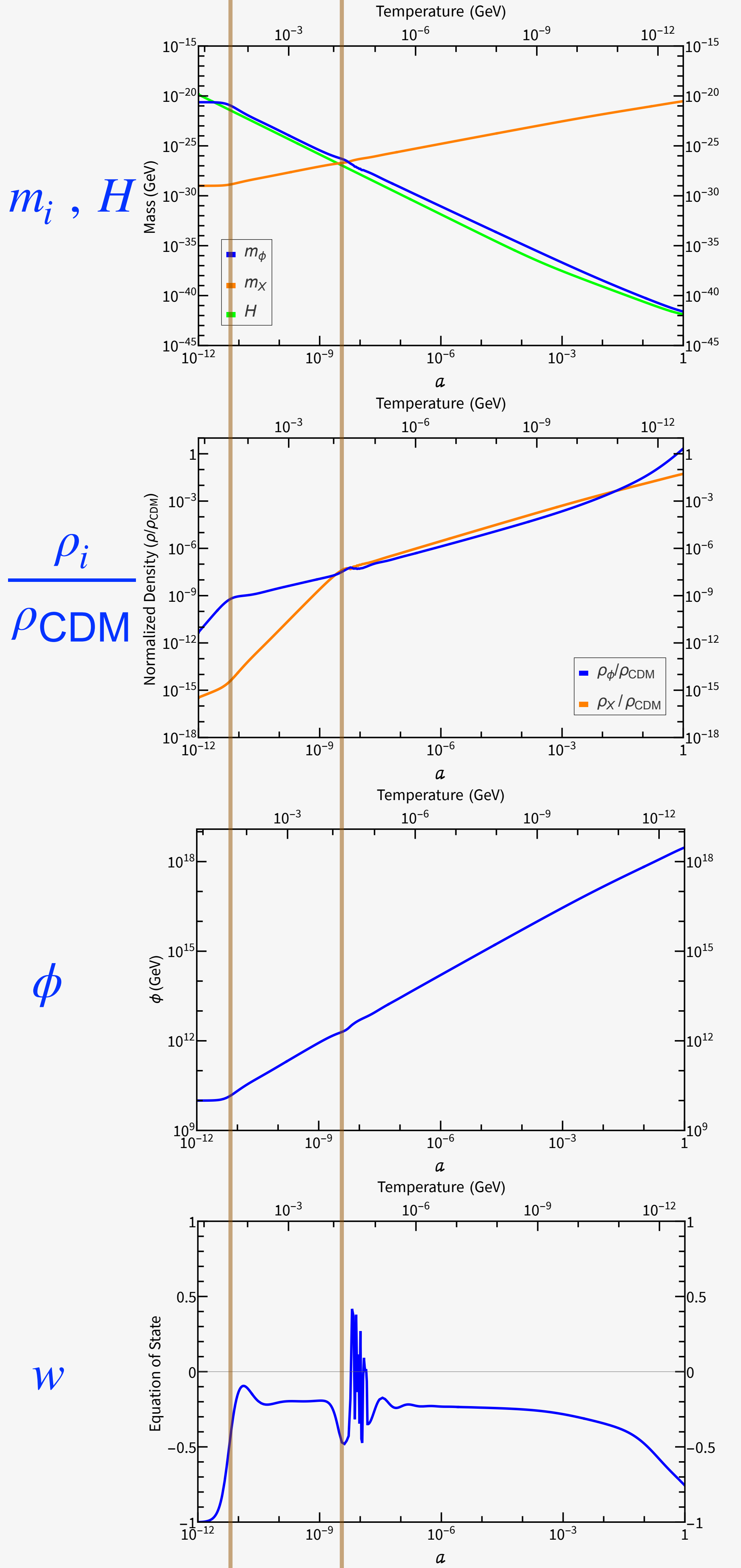
X may have a sizable relic density, but we assume it is a subdominant DM (less than 10% relic density of the dominant CDM).

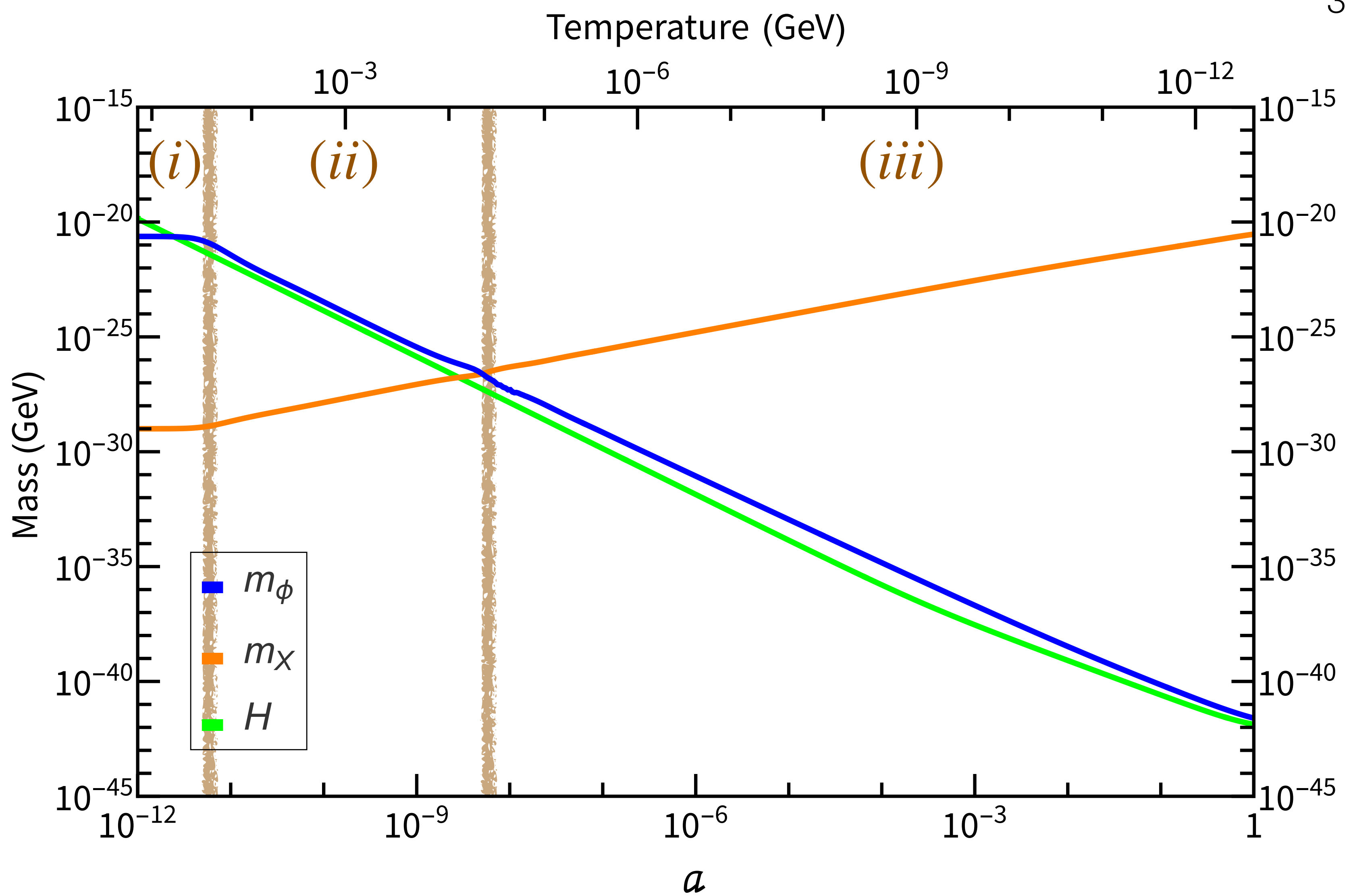
The dynamics of ϕ and X change drastically when the hierarchy between m_ϕ, m_X and H change over time.

(i) $H > m_\phi, m_X$ (ii) $m_\phi > H > m_X$ (iii) $m_X, m_\phi > H$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_0}{\partial \phi} + g_X^2 X_\mu X^\mu \phi = 0$$
$$\partial_\mu X^{\mu\nu} + 3H X^{0\nu} - g_X^2 \phi^2 X^\nu = 0$$

Benchmark parameters: $\alpha = 1$, $M = 2.2 \times 10^{-6}$ GeV, $g_X = 10^{-39}$
$\dot{X} = 0$, $\dot{\phi} = 0$ (at $a = 10^{-12}$)

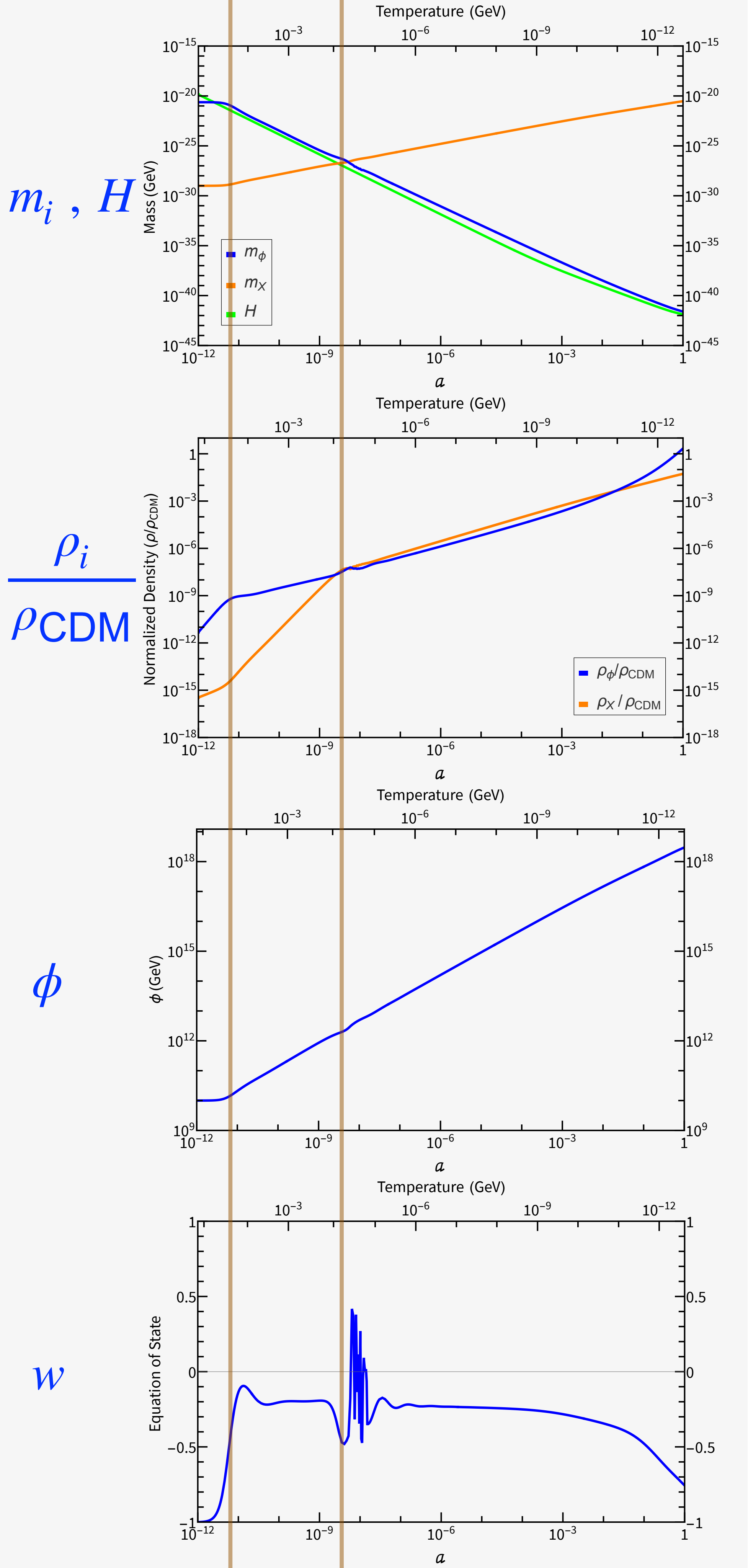


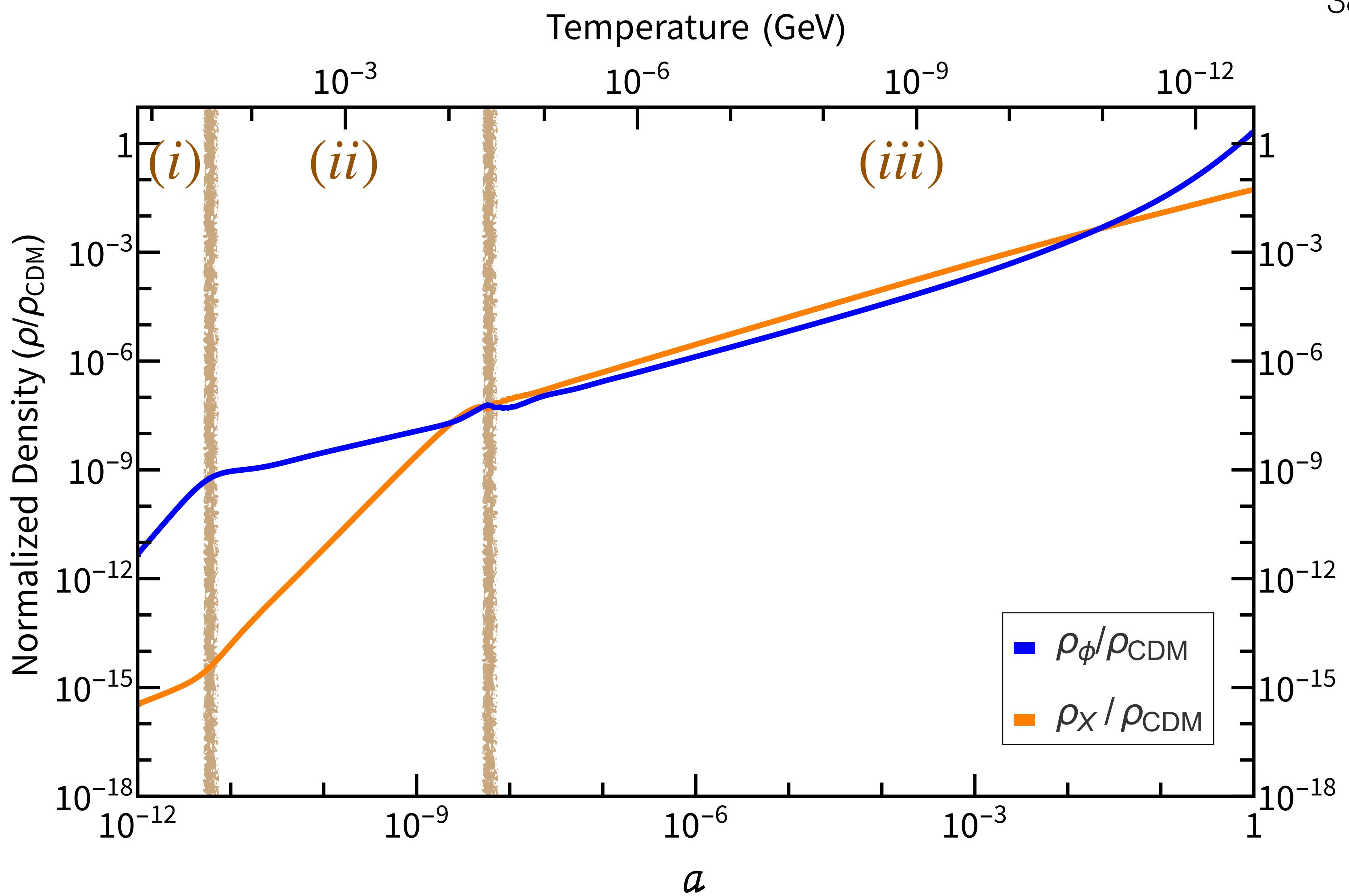


$(i)\ H > m_\phi,\ m_X$: Both ϕ and X are frozen by Hubble friction.

$(ii)\ m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

$(iii)\ m_X,\ m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.





(i) $H > m_\phi, m_X$: Both ϕ and X are frozen by Hubble friction.

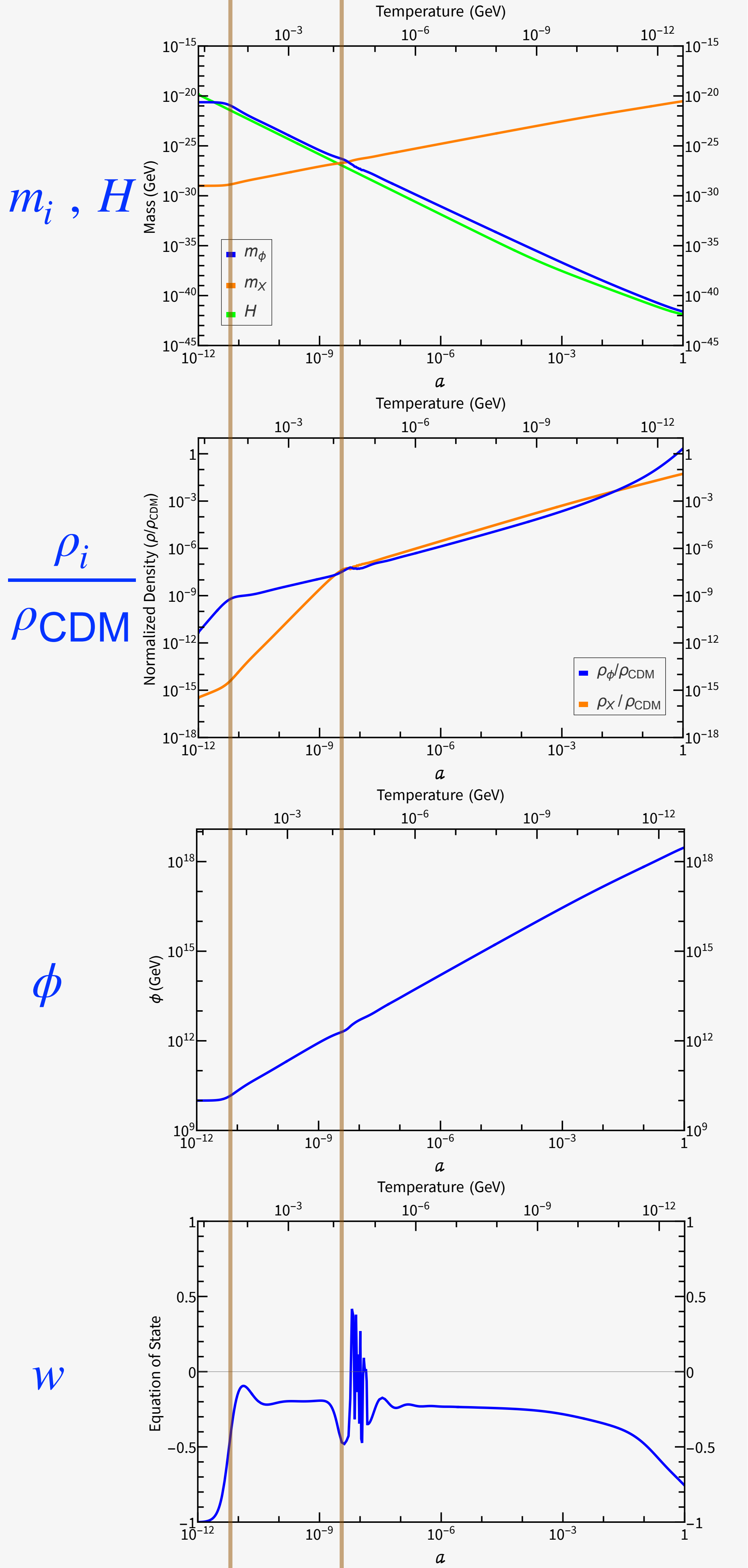
(ii) $m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

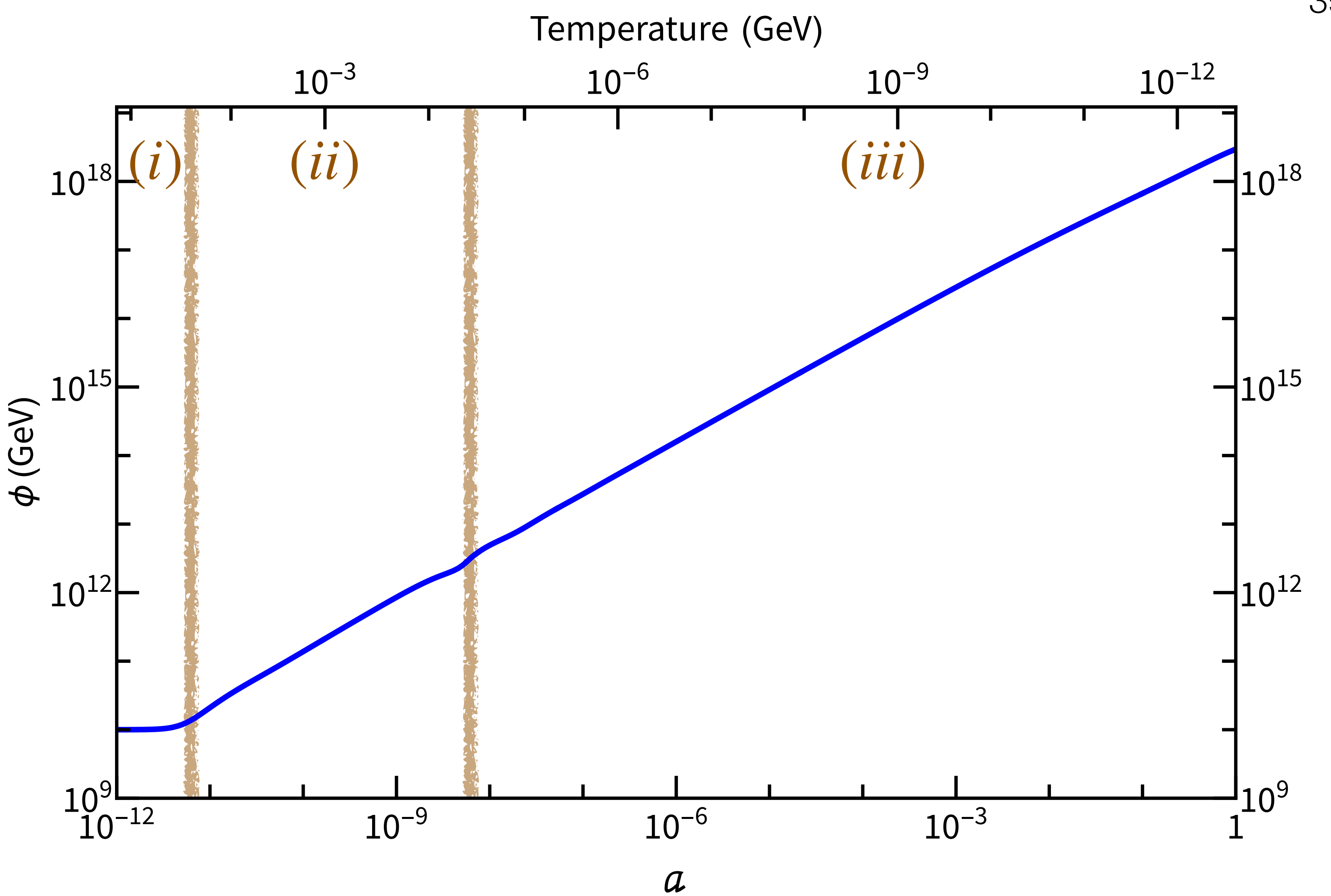
(iii) $m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

(i) $\rho_X \propto a^{-2}$: frozen.

(ii) $\rho_X \propto m_X^2 a^{-2}$: frozen. m_X changes.

(iii) $\rho_X \propto m_X a^{-3}$: m_X changes.





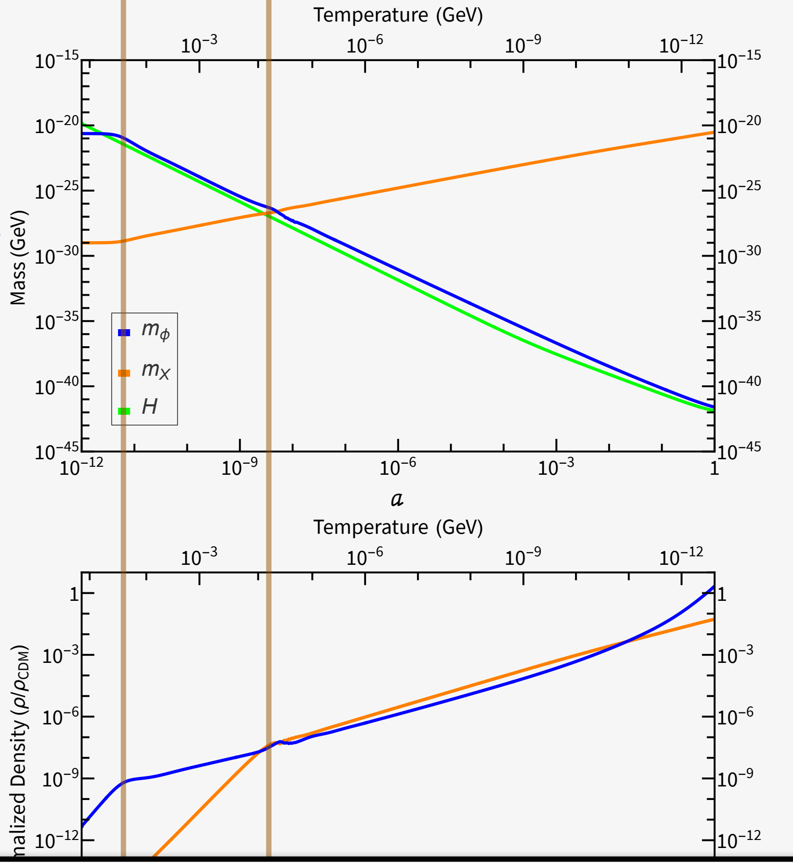
$(i)\ H > m_\phi, m_X$: Both ϕ and X are frozen by Hubble friction.

$(ii)\ m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

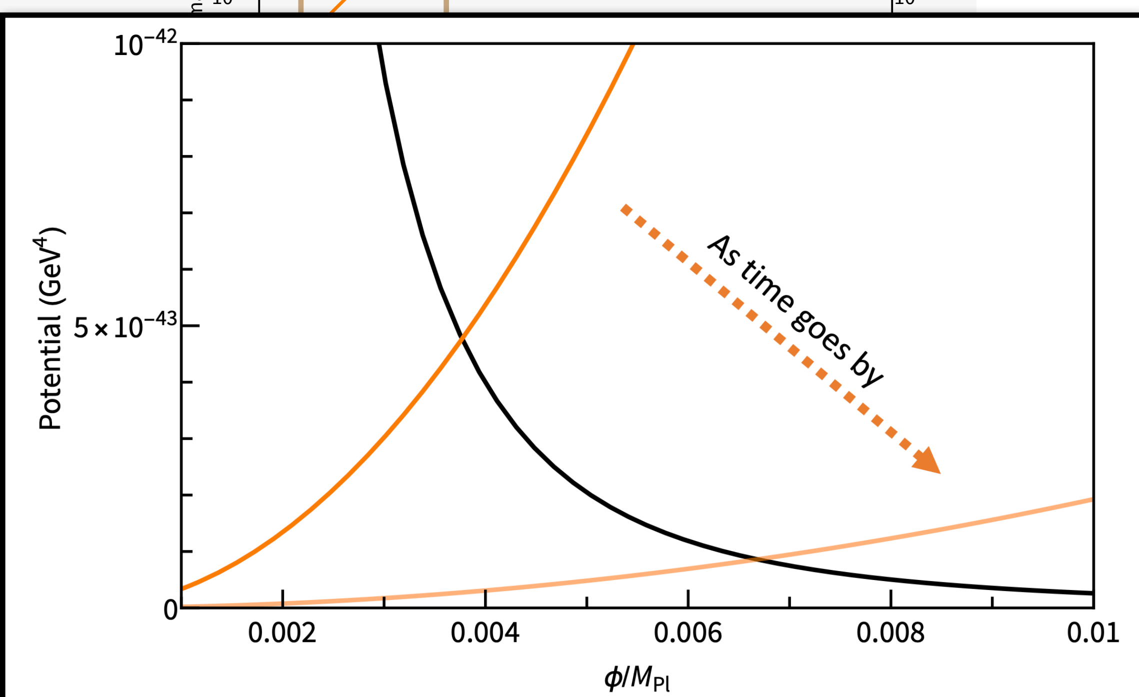
$(iii)\ m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

When $V_{\rm gauge} \sim \rho_X$ becomes sizable, it affects the quintessence dynamics. ϕ oscillates around the minimum of the potential until $V_{\rm gauge}$ becomes subdominant. ϕ gets back to the tracking solution.

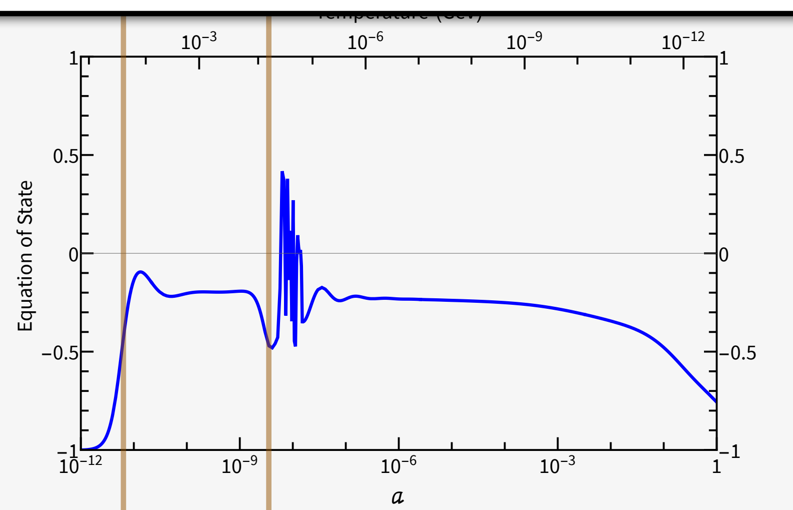
m_i , H

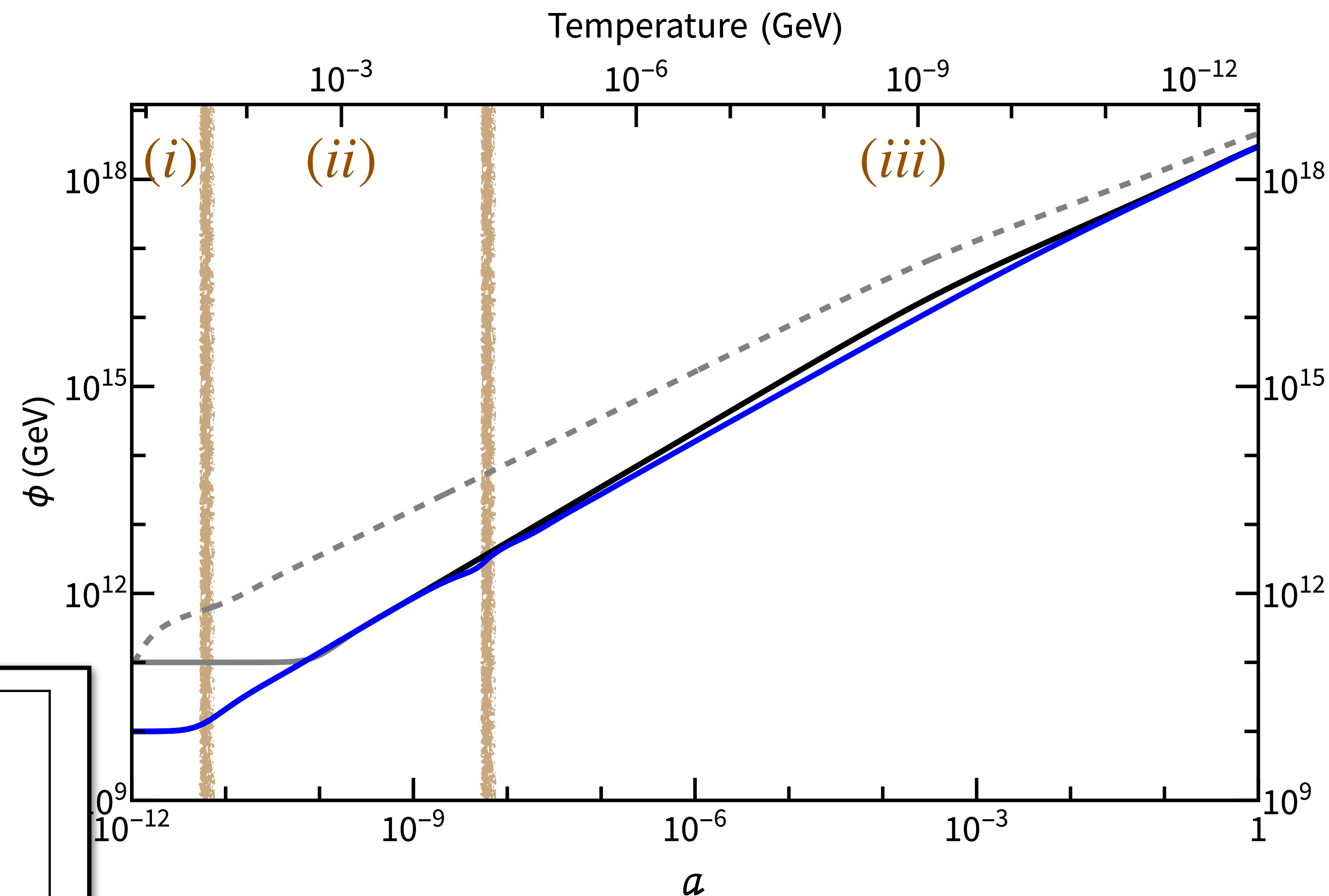


$\dfrac{\rho_i}{\rho_{\rm CDM}}$



w



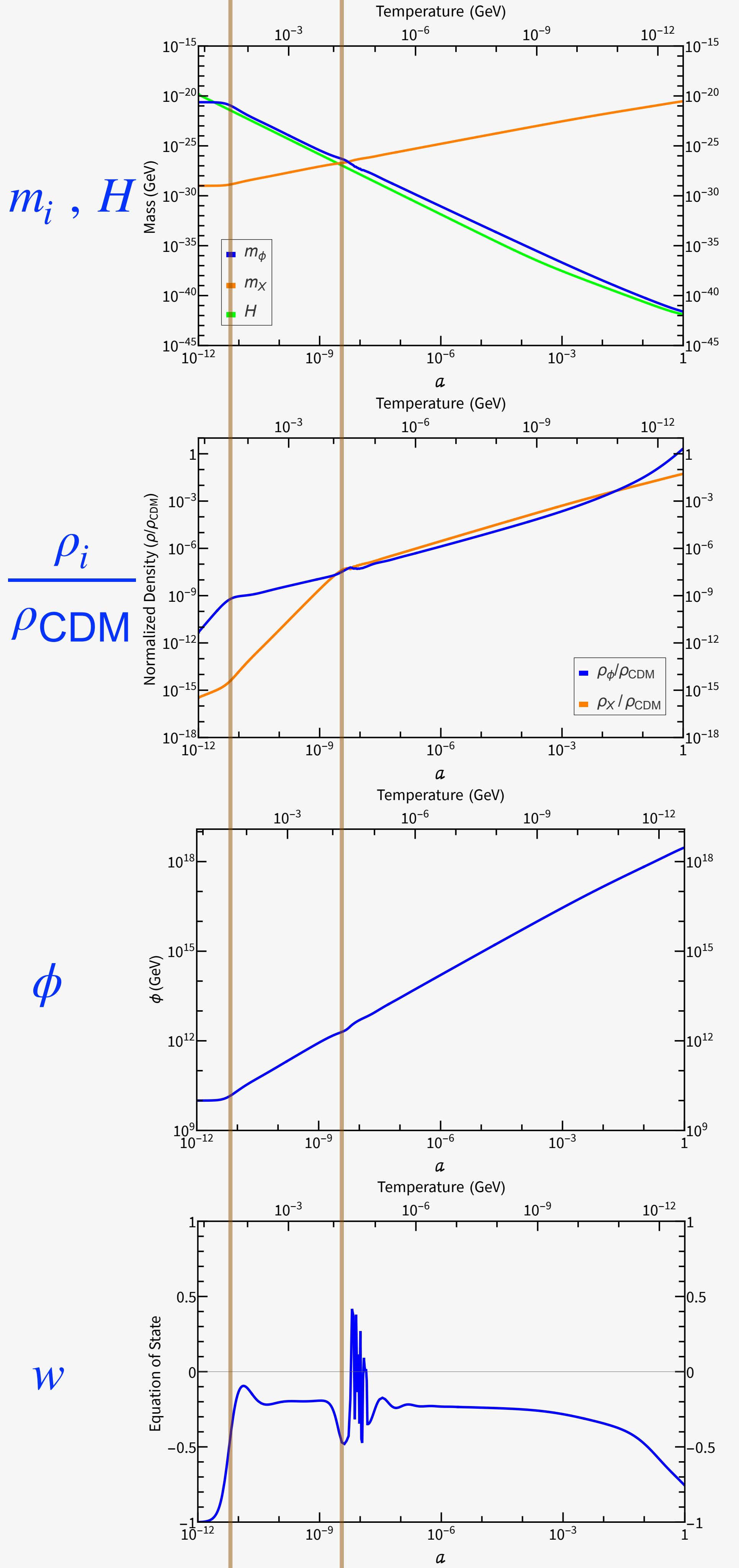


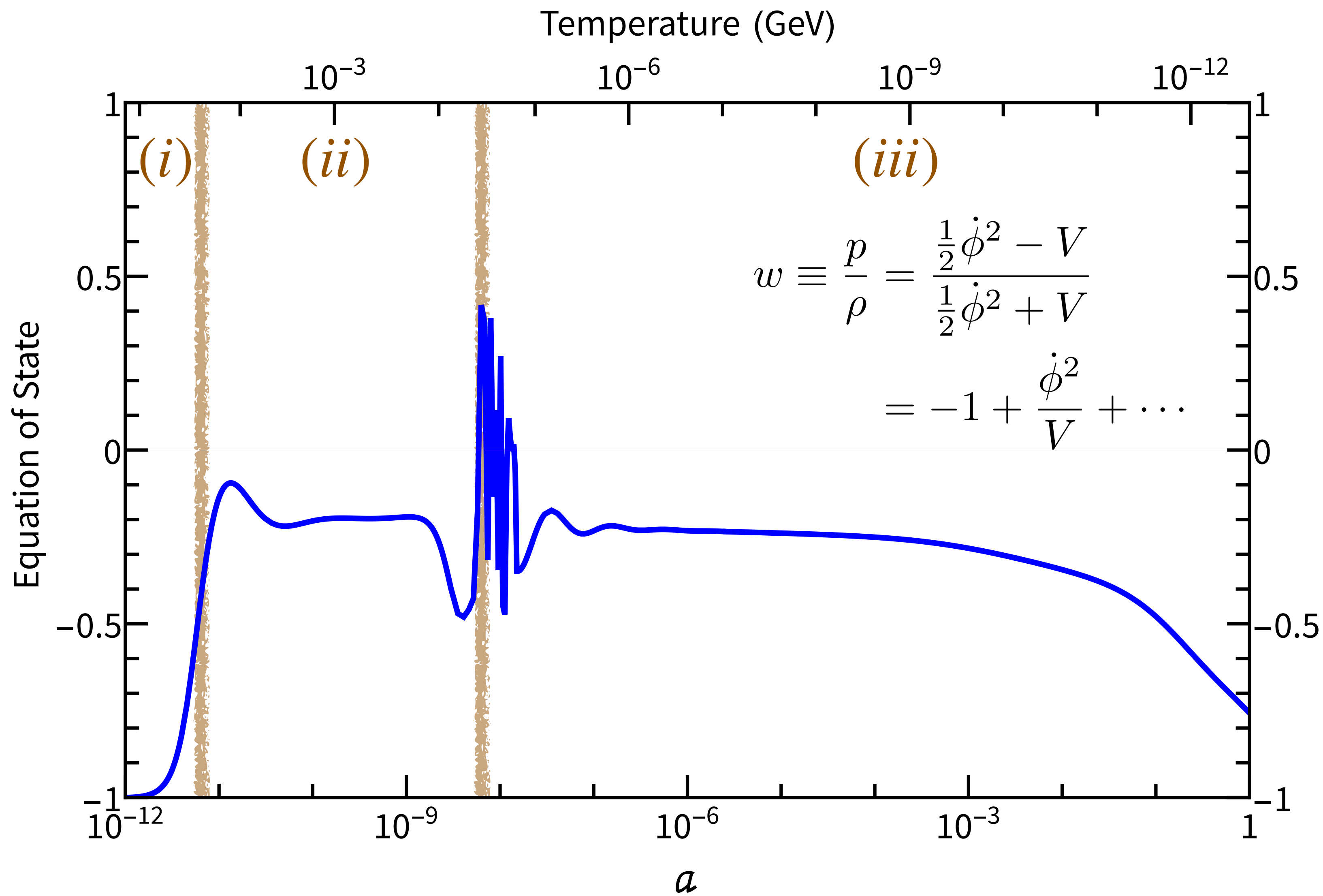
$H > m_\phi, m_X$: Both ϕ and X are frozen by Hubble friction.

$m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

) $m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

When $V_{\rm gauge} \sim \rho_X$ becomes sizable, it affects the quintessence dynamics. ϕ oscillates around the minimum of the potential until $V_{\rm gauge}$ becomes subdominant. ϕ gets back to the tracking solution.





(i) $H > m_\phi, m_X$: Both ϕ and X are frozen by Hubble friction.

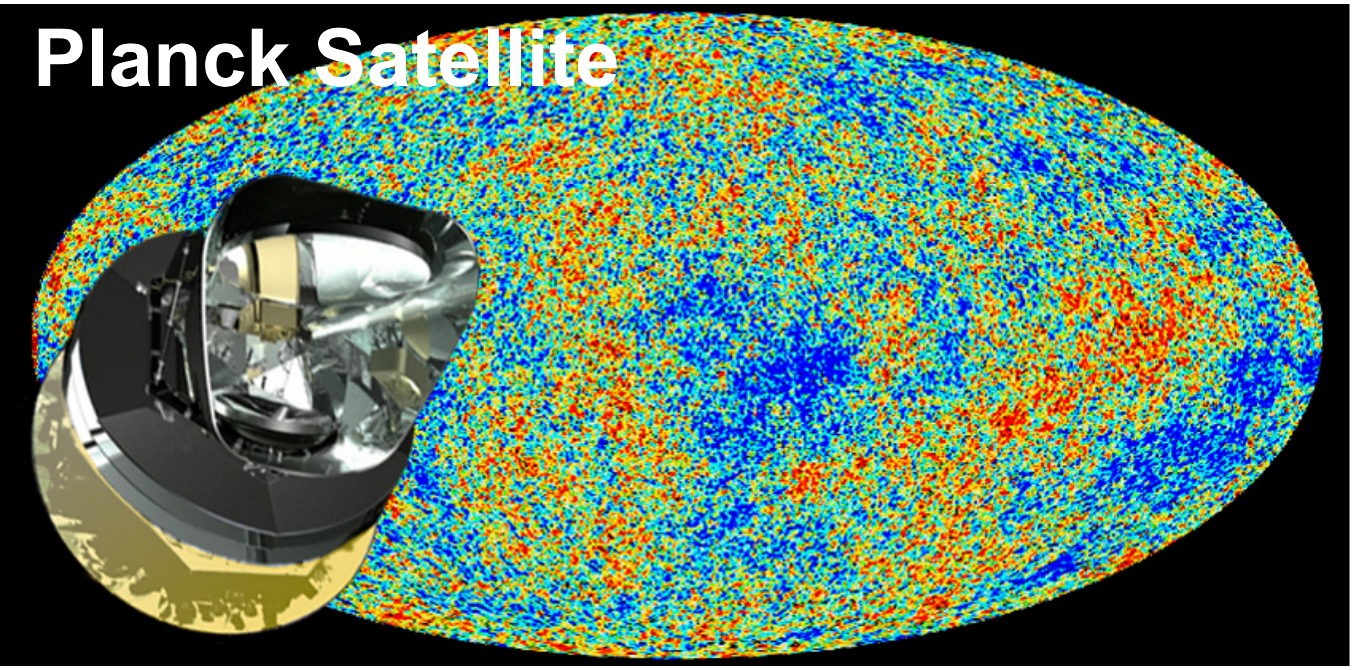
(ii) $m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

(iii) $m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

The oscillation in the ϕ equation of state reflects the ϕ oscillation around the minimum of the potential. After $V_{\rm gauge}$ becomes subdominant, it restores tracking solution.

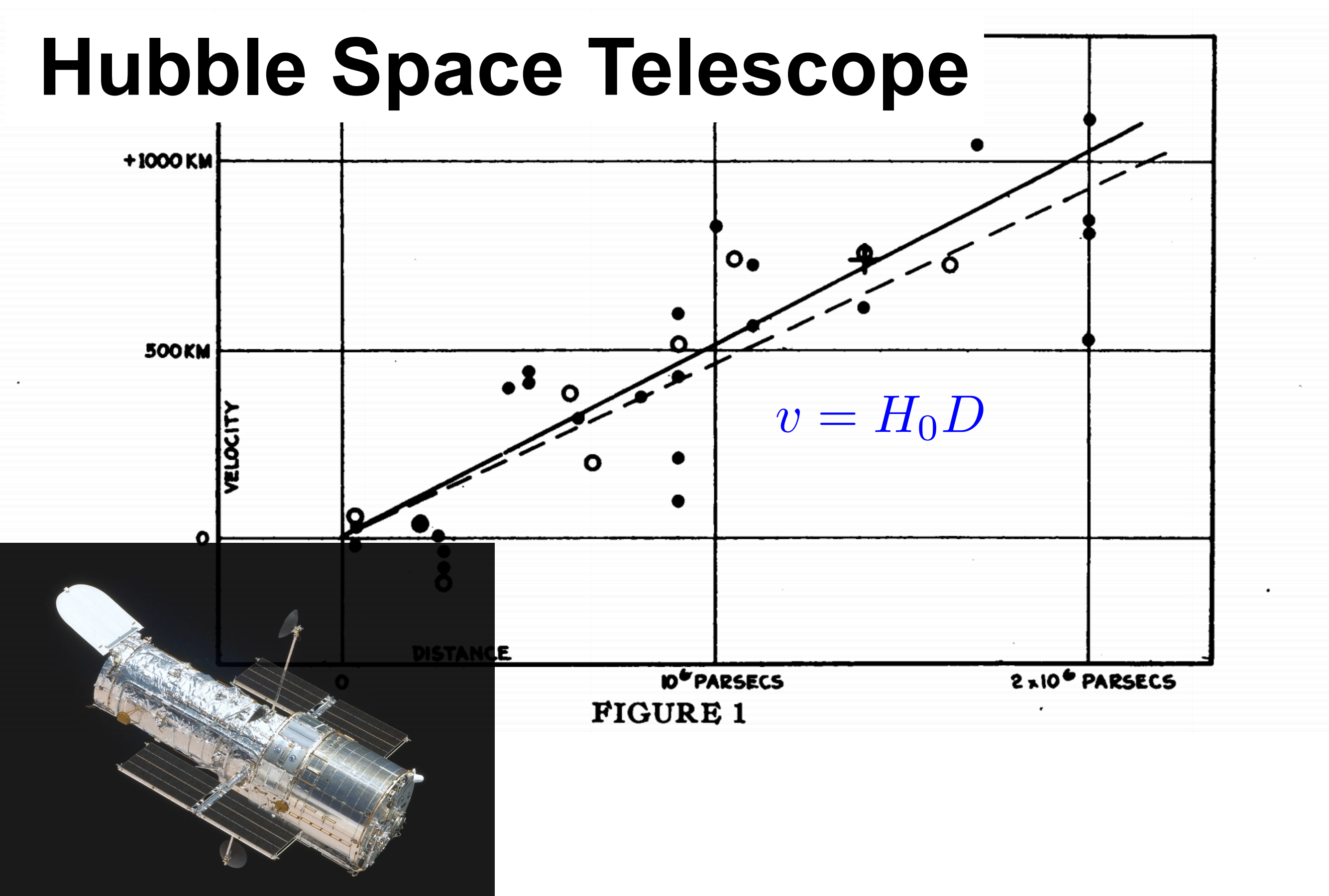
Hubble tension

Modern measurements of the Hubble constant



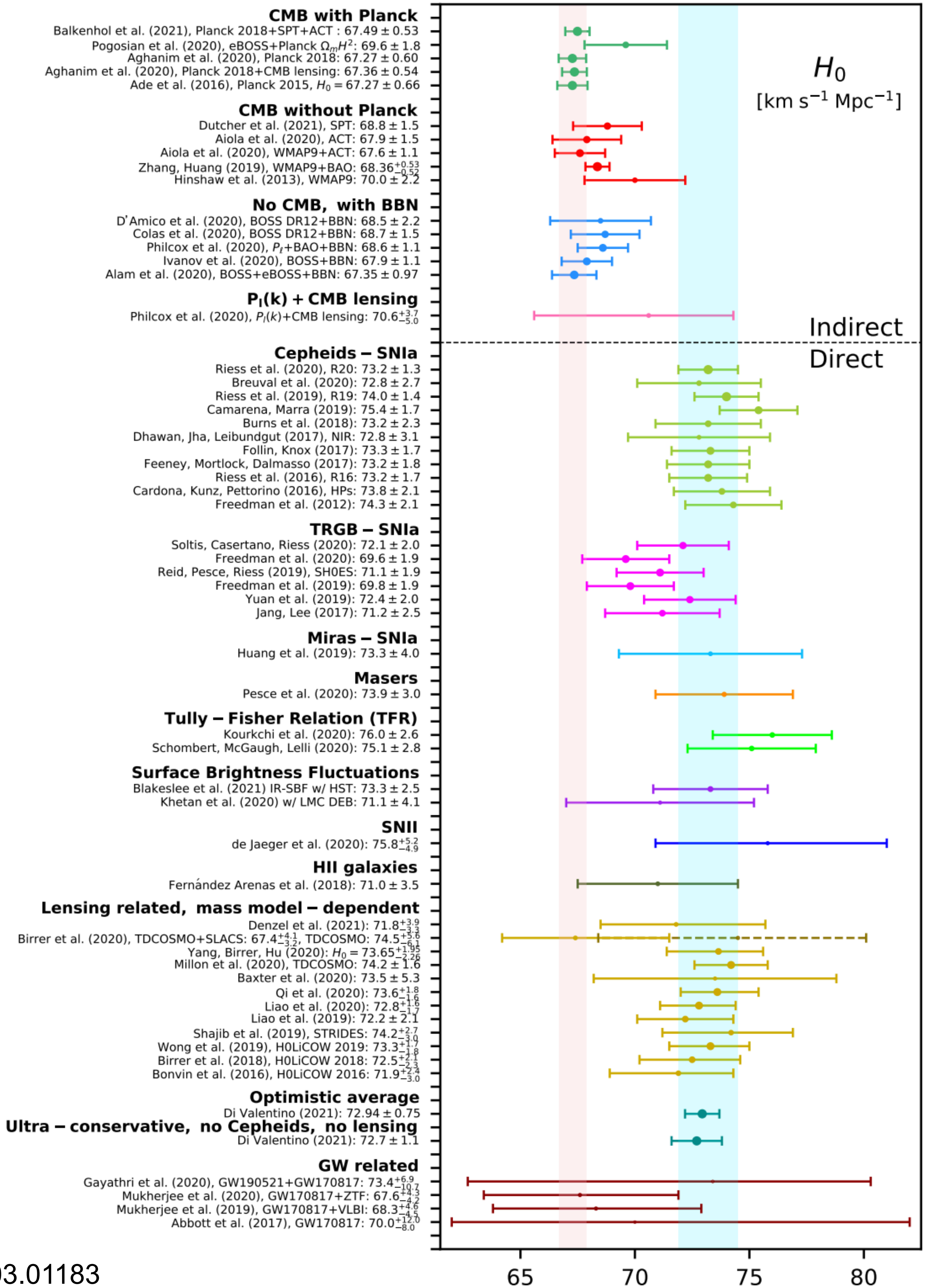
(early Universe)

(i) By fitting CMB data to the $\boldsymbol{\Lambda}$-CDM model



(late Universe)

(ii) With the observation of the expansion (standard candles: Cepheid variables + Supernovae)



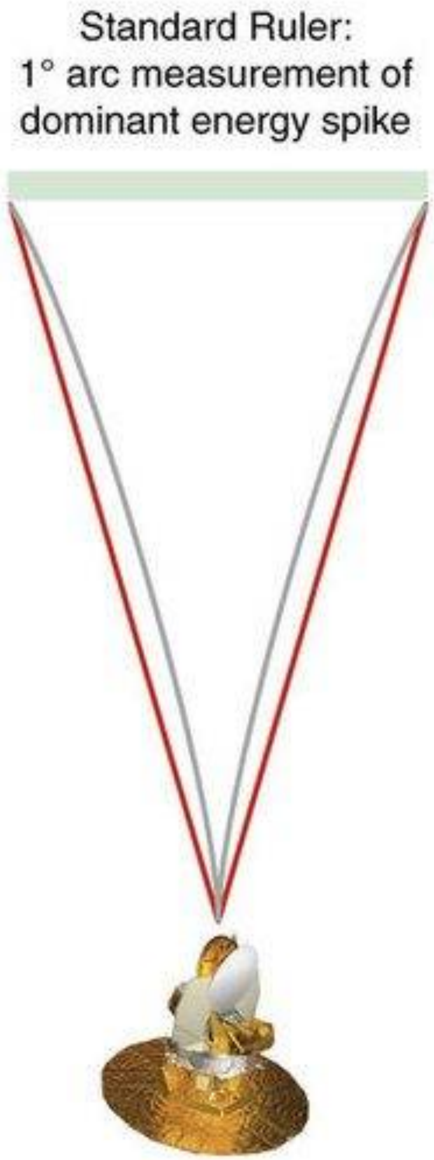
2103.01183

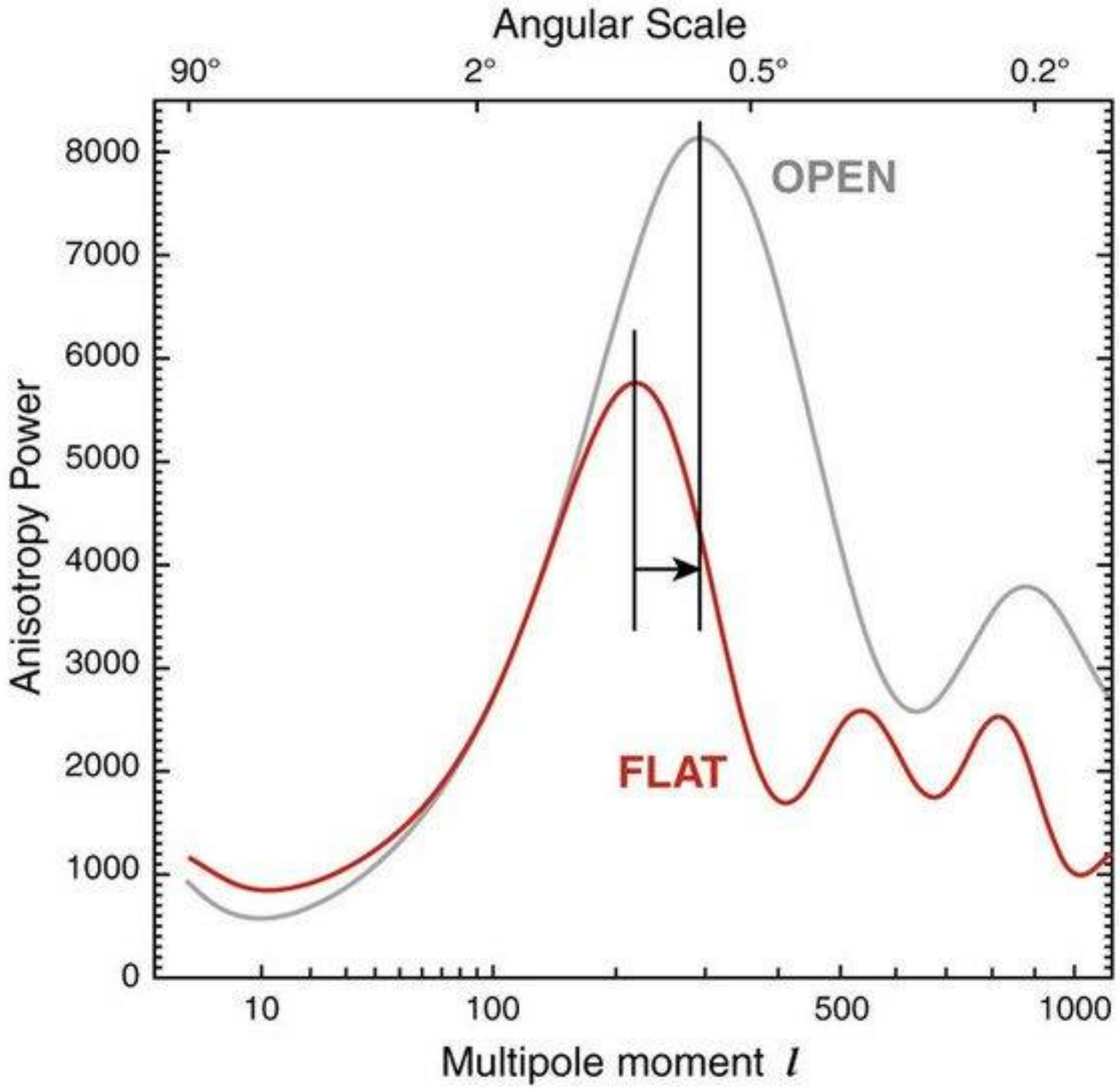
(early Universe). H_0 ~ 67

(late Universe). H_0 ~ 73

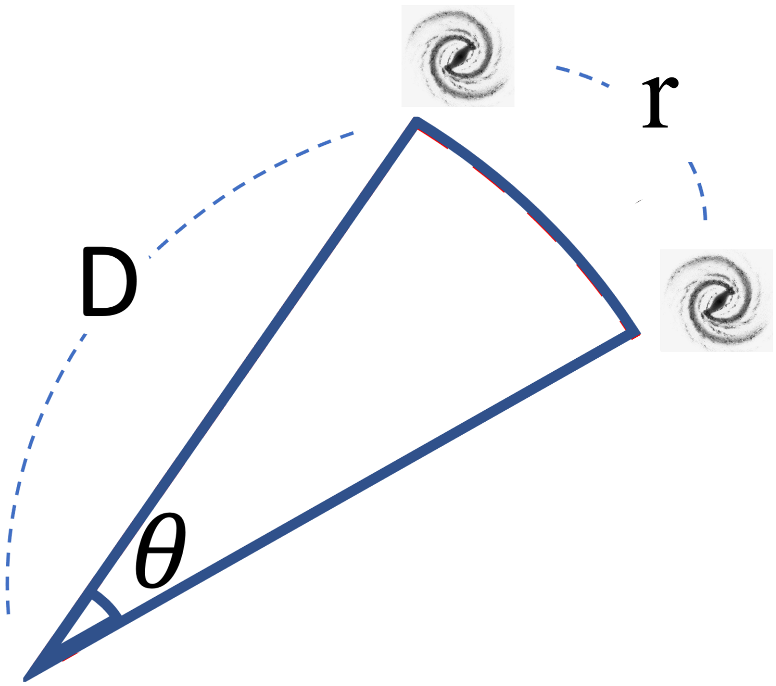
Hubble tension:
about 5σ difference in H_0 between the early and late Universe values.
(Potential hint of the new cosmology.)

Sound horizon in CMB





Baryon Acoustic Oscillations



$$(\textit{sound horizon}):\ r_s = \int_{z_s}^{\infty} dz\, \frac{c_s(z)}{H(z)}$$

$$(\textit{distance to the sound horizon}):\ D(z_s) = \int_{0}^{z_s} dz\, \frac{c}{H(z)}$$

(observed quantity)

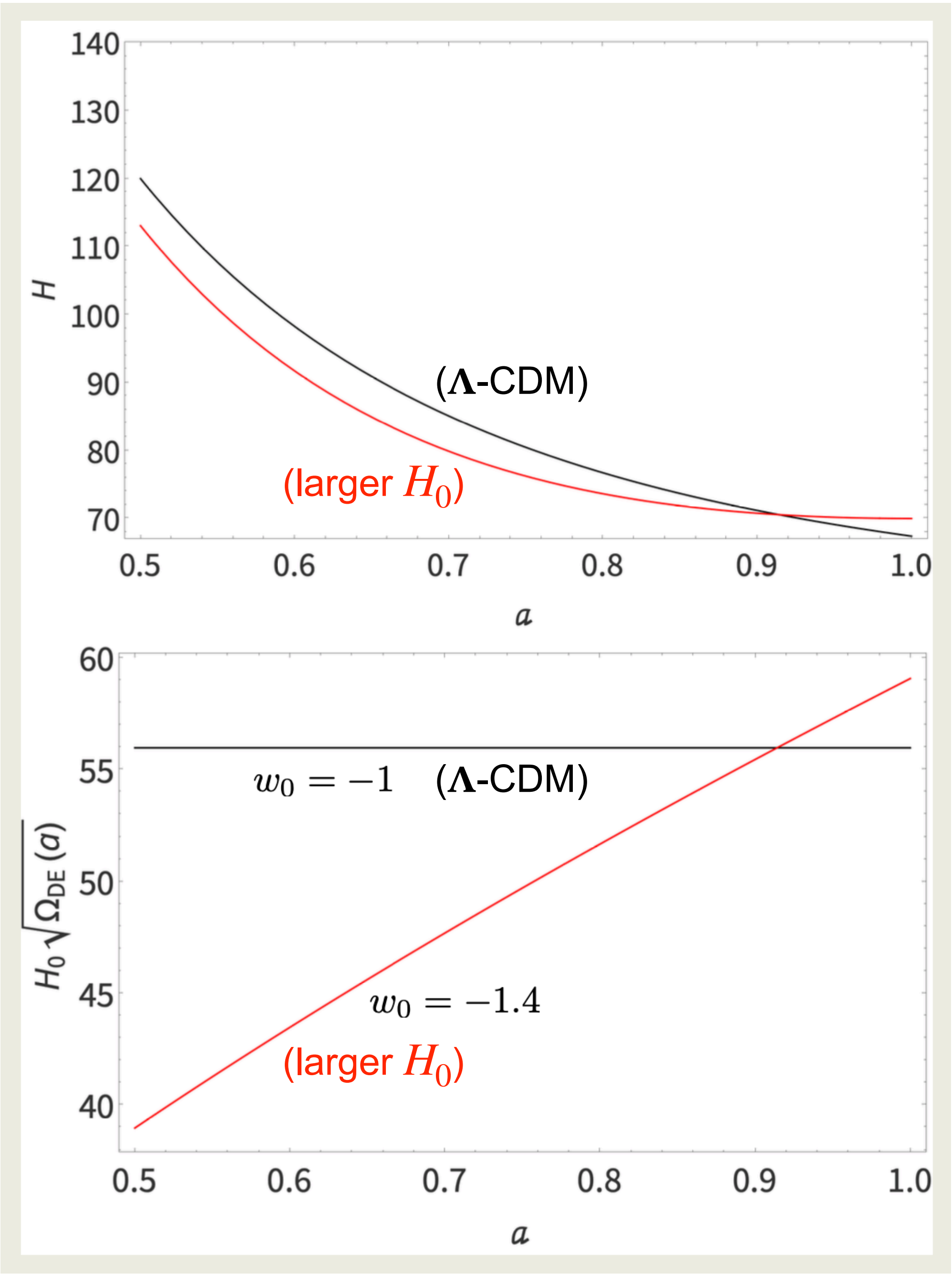
(early universe)

$$\boxed{(\textit{angular size of the sound horizon}) = \frac{(\textit{sound horizon})}{(\textit{distance to the sound horizon})}}$$

(late universe)

DE becomes dominant only in the late universe.
Assuming no change in the sound horizon (early universe physics), the comoving distance to the last scattering (D) should remain intact with a new DE model.

Hubble tension



$$D(z_s) = \int_0^{z_s} dz \, \frac{c}{H(z)}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_{\rm DE} + \Omega_{\rm matter}\right)$$

$$\rho \propto a^{-3(1+w)}$$

To keep D unchanged, a larger H_0 (resolving Hubble tension) should be compensated by a smaller H in the recent past
: It demands w(DE) < -1 ($\mathbf{\Lambda}$-CDM value).

In the uncoupled quintessence model, w(DE) > -1 (worsening Hubble tension).

If an interacting DE model with effective w < -1 is found, it may alleviate Hubble tension.

[Valentino, Melchiorri, Mina (2017)]
[Lee, Lee, Colgain, Sheikh-Jabbari, Thakur (2022)]

(quintessence) $$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} = -1 + \frac{\dot{\phi}^2}{V} + \cdots$$

Effective DE density

$$w_{\rm eff}(\widetilde{DE}) = -1 + \frac{1}{\rho_{\widetilde{DE}}}\left((1+w_0)\rho_\phi + \left(\frac{m_X}{m_X^0} - 1\right)\frac{\rho_X^0}{a^3}\right)$$

$\dot\rho + 3H(1+w)\rho = 0$

: effective w for the effective DE density in the gauged quintessence

Developed in the DE-DM interaction model. [Das, Corasaniti, Khoury (2006)]

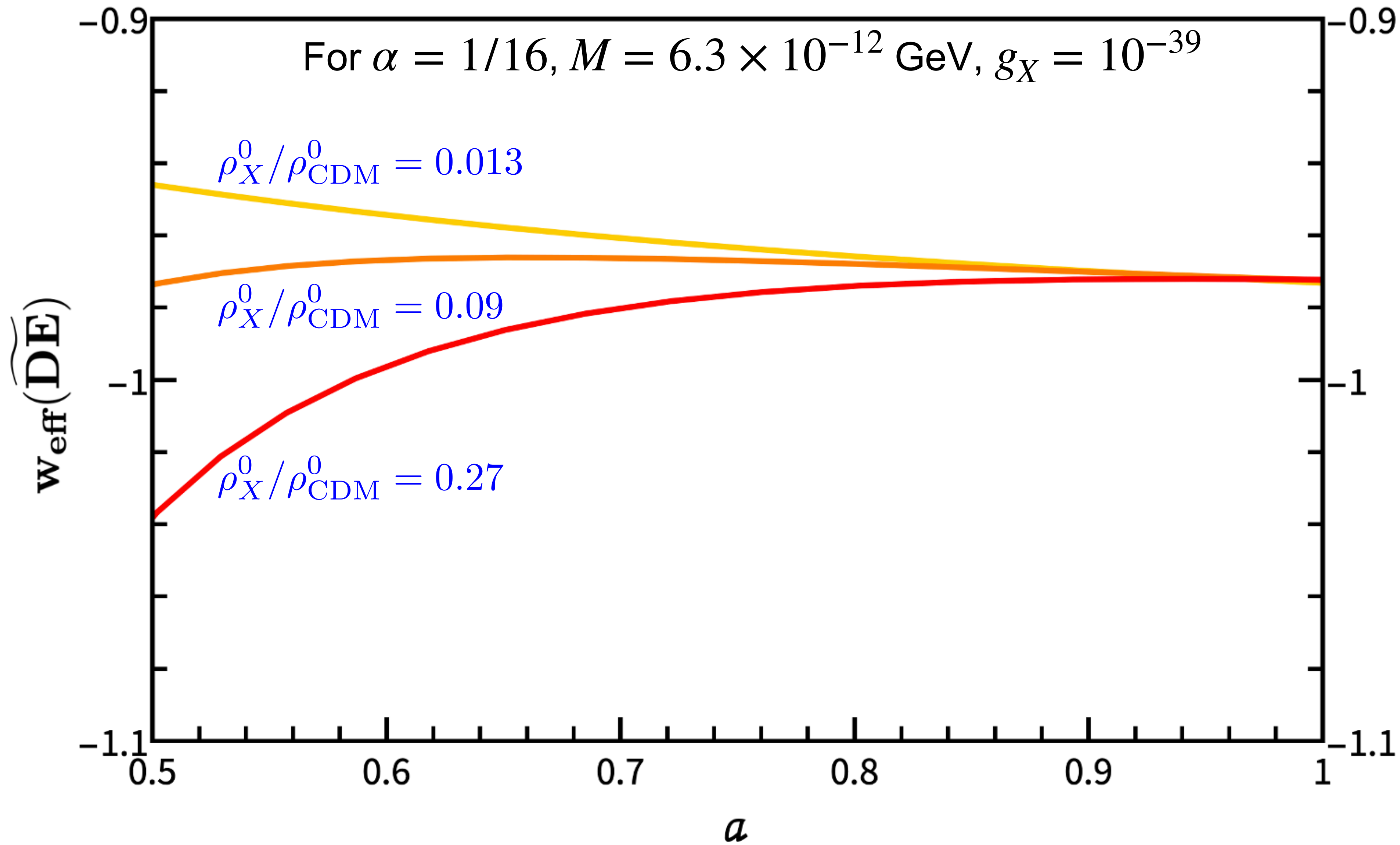
Take the effective DM density ($\rho_{\widetilde{CDM}}$) for the constant mass with a^{-3} scaling part.
The remaining mass-varying part is absorbed in the effective DE density ($\rho_{\widetilde{DE}}$).

$$\frac{a^3\rho_X}{m_X} = \frac{\rho_X^0}{m_X^0}$$

$$\begin{aligned}
\rho_{\rm CDM} + \rho_X + \rho_\phi &= \rho_{\rm CDM}^0 a^{-3} + \frac{m_X}{m_X^0}\rho_X^0 a^{-3} + \rho_\phi \\
&= \left[\left(\rho_{\rm CDM}^0 + \rho_X^0\right)a^{-3}\right] + \underbrace{\left[\left(\frac{m_X}{m_X^0} - 1\right)\rho_X^0 a^{-3} + \rho_\phi\right]}_{\rho_{\widetilde{DE}}} \\
&= \rho_{\widetilde{CDM}} + \rho_{\widetilde{DE}}
\end{aligned}$$

Effective DE density

$$w_{\rm eff}(\widetilde{DE}) = -1 + \frac{1}{\rho_{\widetilde{DE}}}\left((1+w_0)\rho_\phi + \left(\frac{m_X}{m_X^0} - 1\right)\frac{\rho_X^0}{a^3}\right)$$



For $\dot{m}_X > 0$, $w_{\rm eff}(\widetilde{DE})$ is lower than the uncoupled quintessence.
It can be even lower than the $\mathbf{\Lambda}$-CDM (w=-1).
Possibility of alleviating the Hubble tension. (It requires numerical fitting study.)