Renormalization group effects in QCD axion phenomenology

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—> KEK, Japan (11.2023 -)

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Based on collaboration with

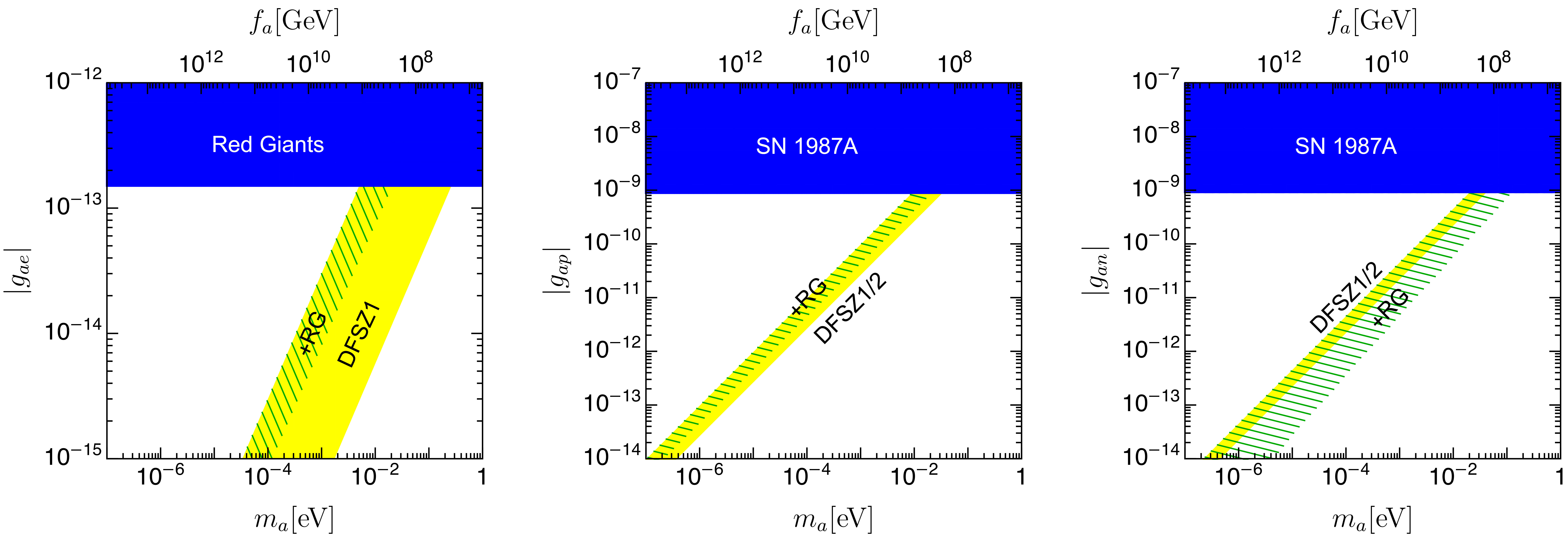
Luca Di Luzio, Federico Mescia, Enrico Nardi [arXiv: 2205.15326]

+ Maurizio Giannotti, Gioacchino Piazza [arXiv: 2305.11958]

Light Dark World 2023 @ KIT, 19.09.2023

Today's talk

Renormalization group corrections to axion couplings are **not negligible**



Yellow: Tree-level prediction in DFSZ models, Green: +RG effects

A flash review of PQ mechanism and axion

- Axion is predicted in a solution to the Strong CP problem

$$\mathscr{L}_{\rm QCD} \supset \theta \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \longrightarrow \frac{a(x)}{f_a} \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \Rightarrow \langle a/f_a \rangle = 0 \quad \text{at the QCD vacuum}$$

  - a(x) = NG boson from SSB of a chiral U(1) PQ symmetry

- **Two classes of benchmark (invisible) axion models** $(v_{\rm EW} \ll f_a)$**:**

DFSZ axion: SM quarks and Higgses charged under PQ. [Zhitnitsky (1980), Dine, Fischler, Srednicki (1981)]

Minimally requires 2HDM + 1 scalar singlet. SM leptons are also PQ charged.

KSVZ axion: All SM fields are neutral under PQ. [Kim (1979), Shifman, Vainshtein, Zakharov (1980)]

QCD anomaly induced by new quarks that are vector-like under SM and chiral under PQ. Singlet scalar breaks PQ.

Axion couplings to matter and radiation

$$\mathcal{L}_a^{\text{eff}} = \frac{\partial_\mu a}{2 f_a} \sum_{f=p,n,e} C_f \bar{f} \gamma^\mu \gamma_5 f + \frac{a}{f_a} \frac{e^2}{32\pi^2} \left(\frac{E}{N} - 1.92 \right) F \widetilde{F}$$

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a-π^0 mixing (aGG term)

PQ-EM anomaly

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PQ-EM anomaly

$$C_{p,n} = \Delta_u C_{u,d} + \Delta_d C_{d,u} + \Delta_s C_s - \left(\frac{\Delta_{u,d}}{1+z} + \frac{z\,\Delta_{d,u}}{1+z} \right)$$

- $C_f = C_f(\mu_{\text{QCD}}) \quad (f = u, d, s, e)$
- $s^\mu \Delta_{u,d,s} = \langle N | \bar{q} \gamma^\mu \gamma_5 q | N \rangle$
- $z = m_d / m_u \simeq 2$

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since $\mu_{\rm QCD} \ll f_a$, $C_f(\mu_{\rm QCD}) = C_f(f_a) + \Delta C_f(\mu_{\rm QCD}; f_a)$

corrections from RG evolution

Running of DFSZ axion couplings

μ

f_a — $c_{\psi,\Phi} = \mathcal{X}_{\psi,\Phi}$ (PQ charge)

[Bauer et al. 2012.12272; Choi et al. 2106.05816]

✓ Matching with axion effective Lagrangian in the GKR basis

$$\mathcal{L}_a^{\rm GKR} = \frac{\partial_\mu a}{f}\left(\sum_{\psi=q_L,\ell_L,\ldots} c_\psi \bar{\psi}\gamma^\mu\psi + \sum_{\Phi=H_1,H_2,\ldots} c_\Phi \Phi^\dagger i\overleftrightarrow{D}^\mu\Phi\right) + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2}\frac{a}{f}F_A\widetilde{F}_A$$

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No running

$(\because \partial_\mu J^\mu_{\rm PQ} = 0)$

$m_{\rm BSM}$ — $c_\psi(m_{\rm BSM}) = c_\psi(f_a)$

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↓

⋮

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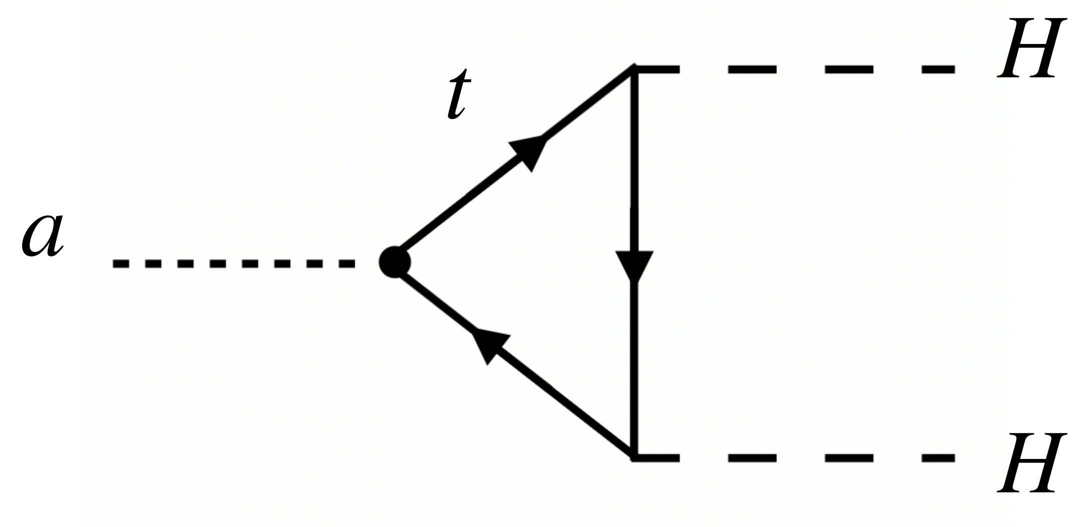
RG correction crucially depend on the heavy Higgs scale instead of PQ scale ($1\,{\rm TeV} \lesssim m_{\rm BSM} \leq f_a$)

RG corrections in DFSZ axion models

In the DFSZ models, the leading contribution arises from top loop diagrams induced by axion-top coupling C_t

$$C_f(2\text{GeV}) \simeq C_f(f_a) + r_f^t(m_{\text{BSM}})\, C_t(f_a)$$



$$\frac{\partial_\mu a}{f}(H^\dagger i D_\mu H) \to \frac{\partial_\mu a}{f} \sum_{\psi=q_L,u_R,\dots} \beta_\psi \overline{\psi}\gamma_\mu \psi \qquad (\beta_\psi = Y_\psi / Y_H)$$

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Analytical approximation

$$r_3^t = r_u^t - r_d^t \simeq -0.54 \ln(\sqrt{x} - 0.52)$$

$$r_0^t = r_u^t + r_d^t \simeq 3.8 \times 10^{-4} \ln^2(x - 1.25)$$

with $x = \log_{10}\left(\frac{m_{\text{BSM}}}{\text{GeV}}\right)$

$$r_e^t \simeq -\frac{r_3^t}{2}$$

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$$r_f^t \simeq T_{3,f}\, r_3^t$$

proportional to weak isospin!

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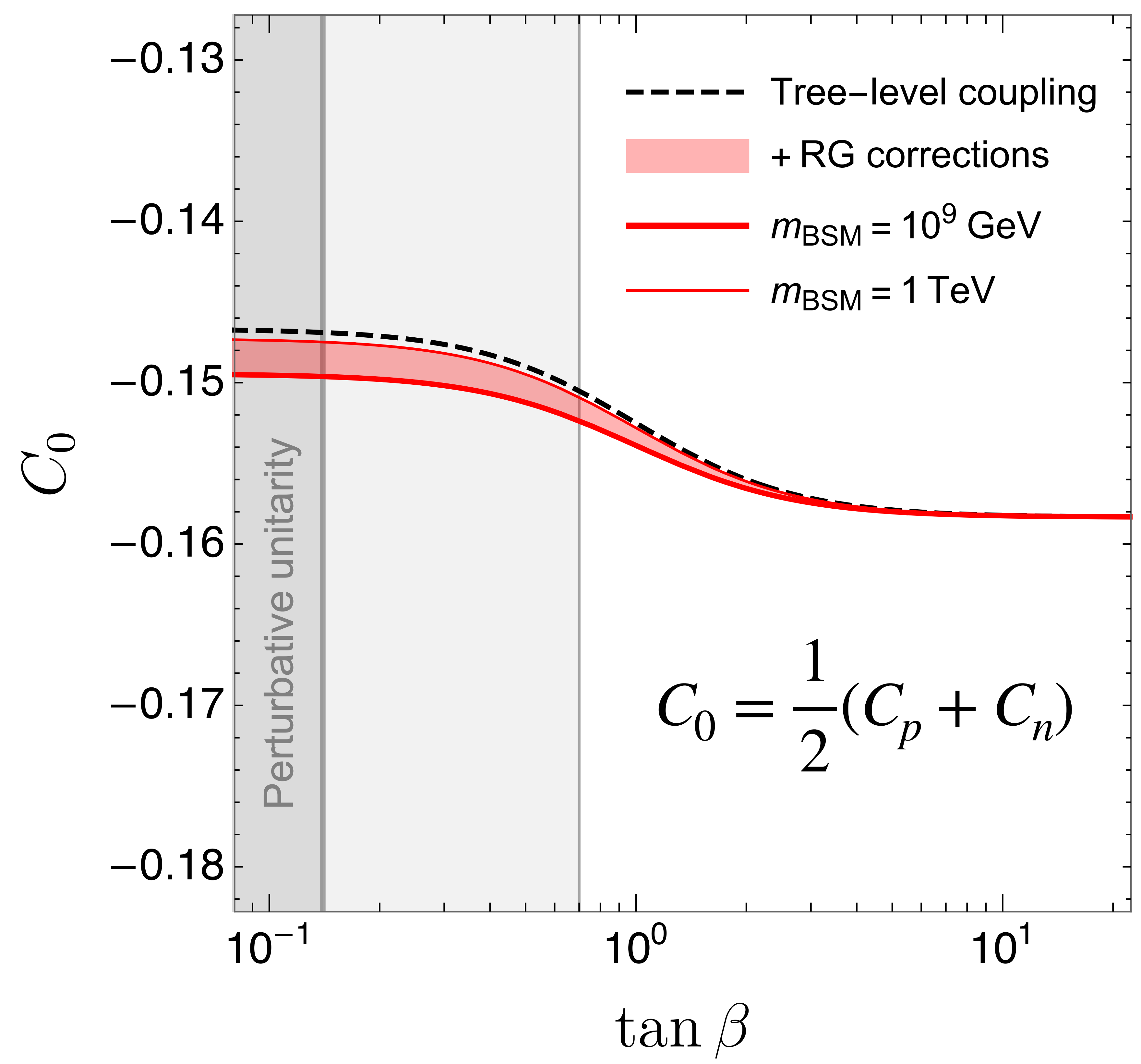
RG corrections in DFSZ axion models

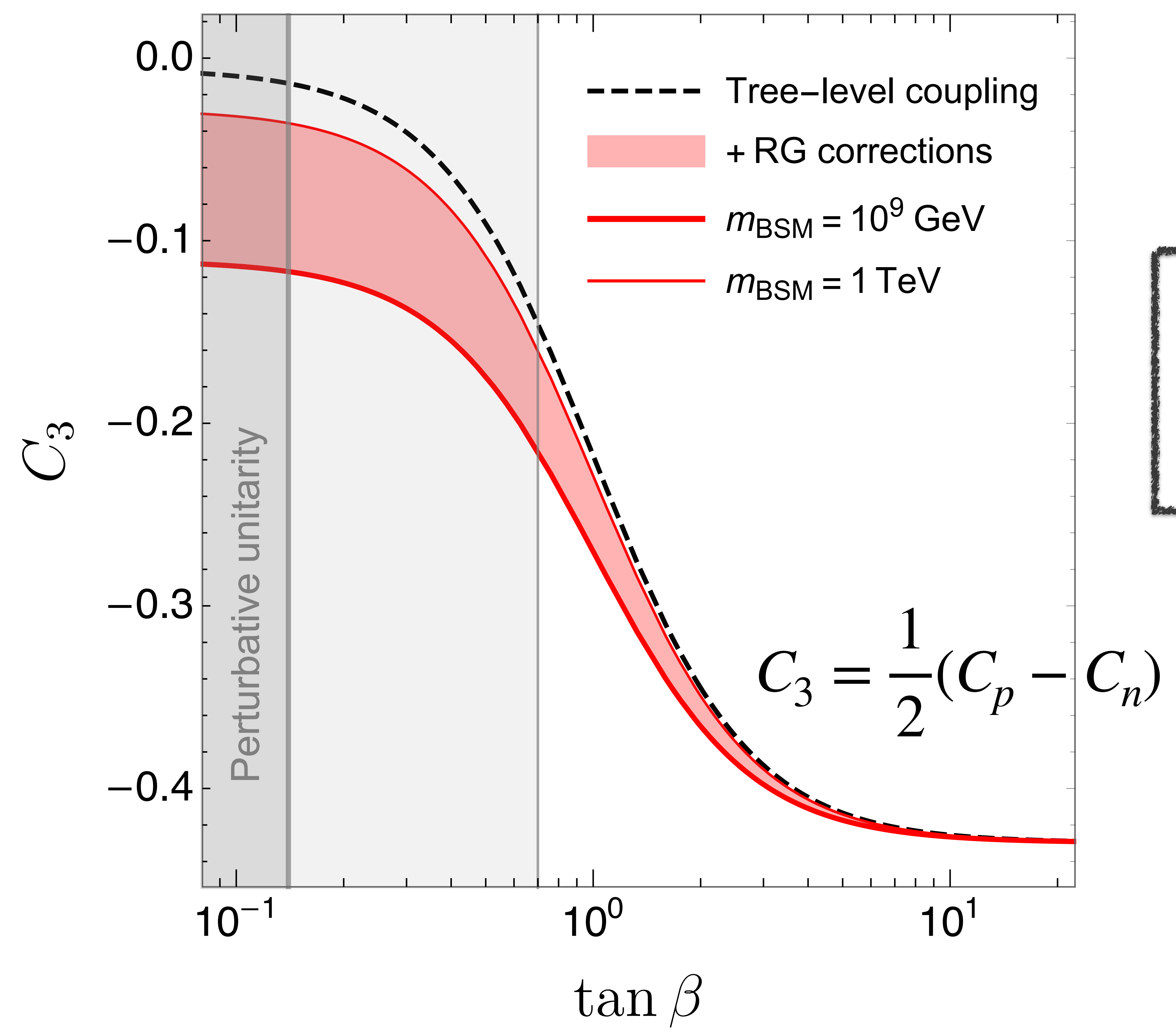
- DFSZ1: $\bar{q}_L H_1 u_R\,,\ \bar{q}_L H_2 d_R\,,\ \bar{l}_L H_2 e_R$
- DFSZ2: $\bar{q}_L H_1 u_R\,,\ \bar{q}_L H_2 d_R\,,\ \bar{l}_L \tilde{H}_1 e_R$

Coupling (DFSZ1)	Coupling (DFSZ2)	Approx. Correction
$C_0 \simeq -0.20$	$C_0 \simeq -0.20$	$\Delta C_0 \approx 0$
$C_3 \simeq -0.43\,\sin^2\beta$	$C_3 \simeq -0.43\,\sin^2\beta$	$\Delta C_3 \simeq -0.12\,l(x)\,\cos^2\beta$
$C_e = \frac{1}{3}\sin^2\beta$	$C_e = -\frac{1}{3}\cos^2\beta$	$\Delta C_e \simeq 0.094\,l(x)\,\cos^2\beta$
$C_\gamma = \frac{8}{3} - 1.92$	$C_\gamma = \frac{2}{3} - 1.92$	$\Delta C_\gamma = 0$

$l(x) = \ln(\sqrt{x} - 0.52)$

$\tan\beta = v_1/v_2$





Only $C_3\,, C_e$ receive large corrections at small tanβ

RG corrections in DFSZ axion models

- DFSZ1: $\bar{q}_L H_1 u_R\,,\ \bar{q}_L H_2 d_R\,,\ \bar{l}_L H_2 e_R$
- DFSZ2: $\bar{q}_L H_1 u_R\,,\ \bar{q}_L H_2 d_R\,,\ \bar{l}_L \tilde{H}_1 e_R$

RG corrections do not

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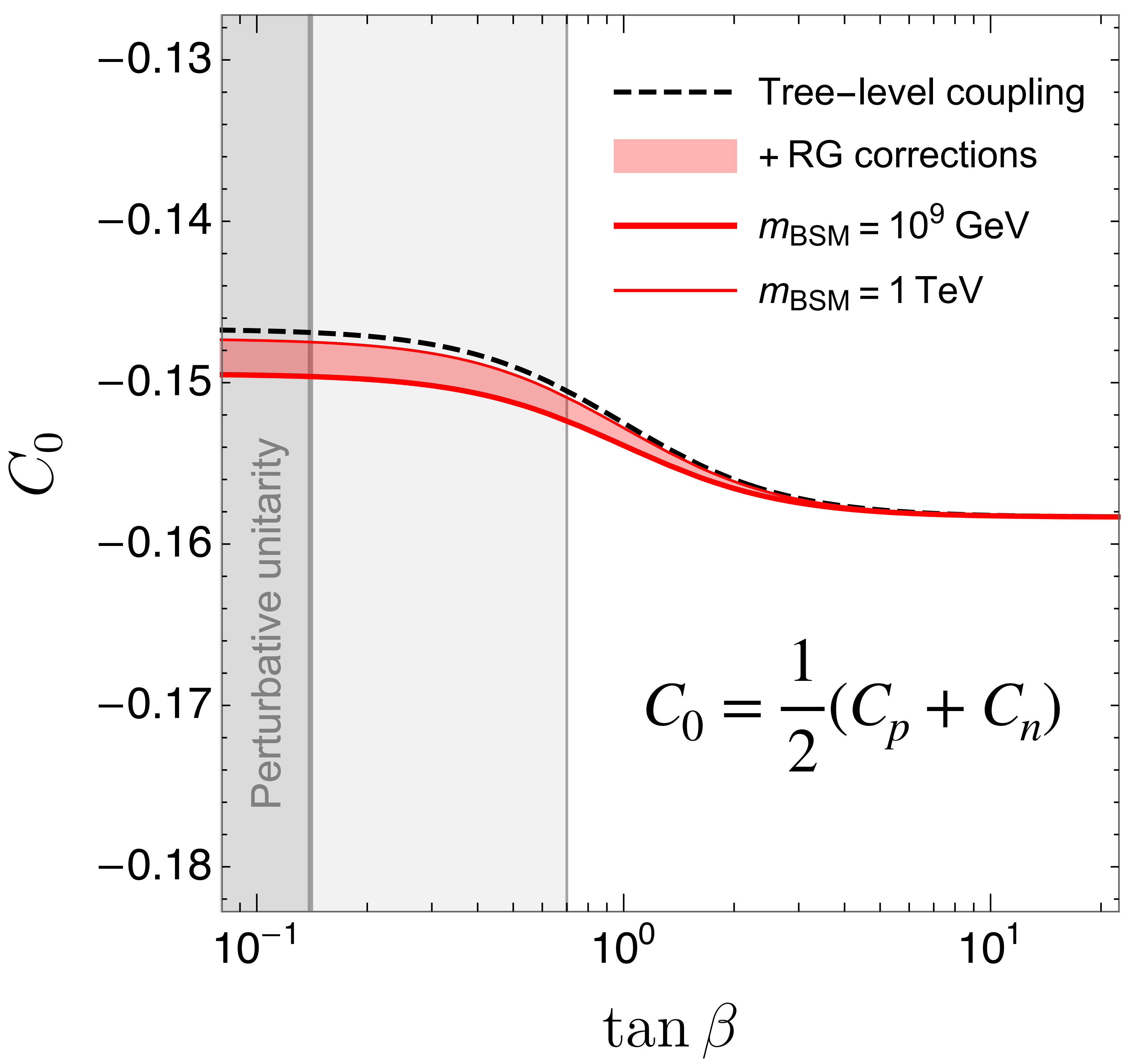
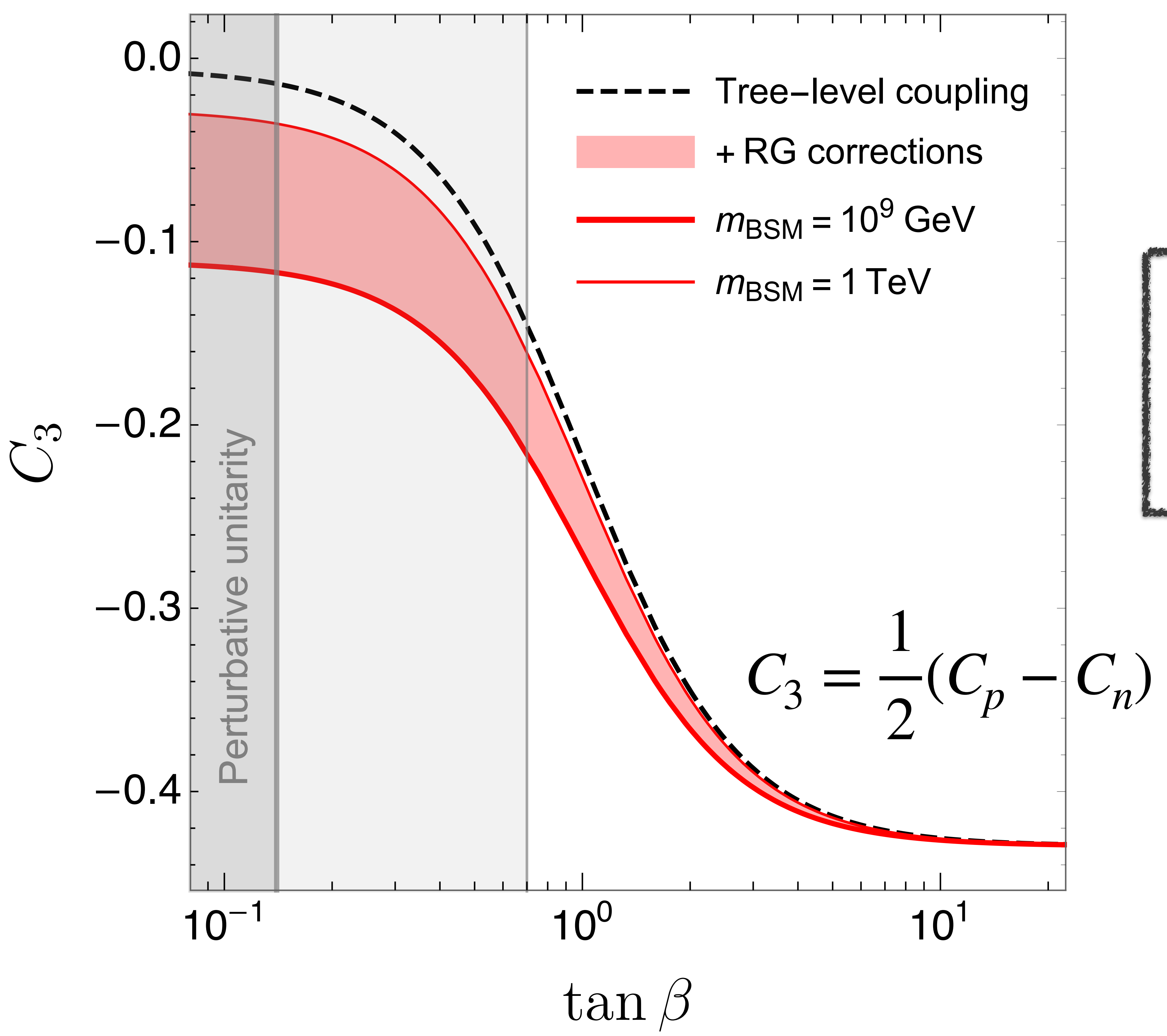
tree couplings vanish at β -> 0

$$l(x) = \ln(\sqrt{x} - 0.52)$$

$$\tan\beta = v_1/v_2$$





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Impact on Axion Phenomenology

- RGB bound: $|C_e| \leq 1.65 \times 10^{-3}\,(m_a/\mathrm{eV})^{-1}$

- SN1987A: $L_a \leq L_\nu = 3 \times 10^{52}\,\mathrm{erg/s}$

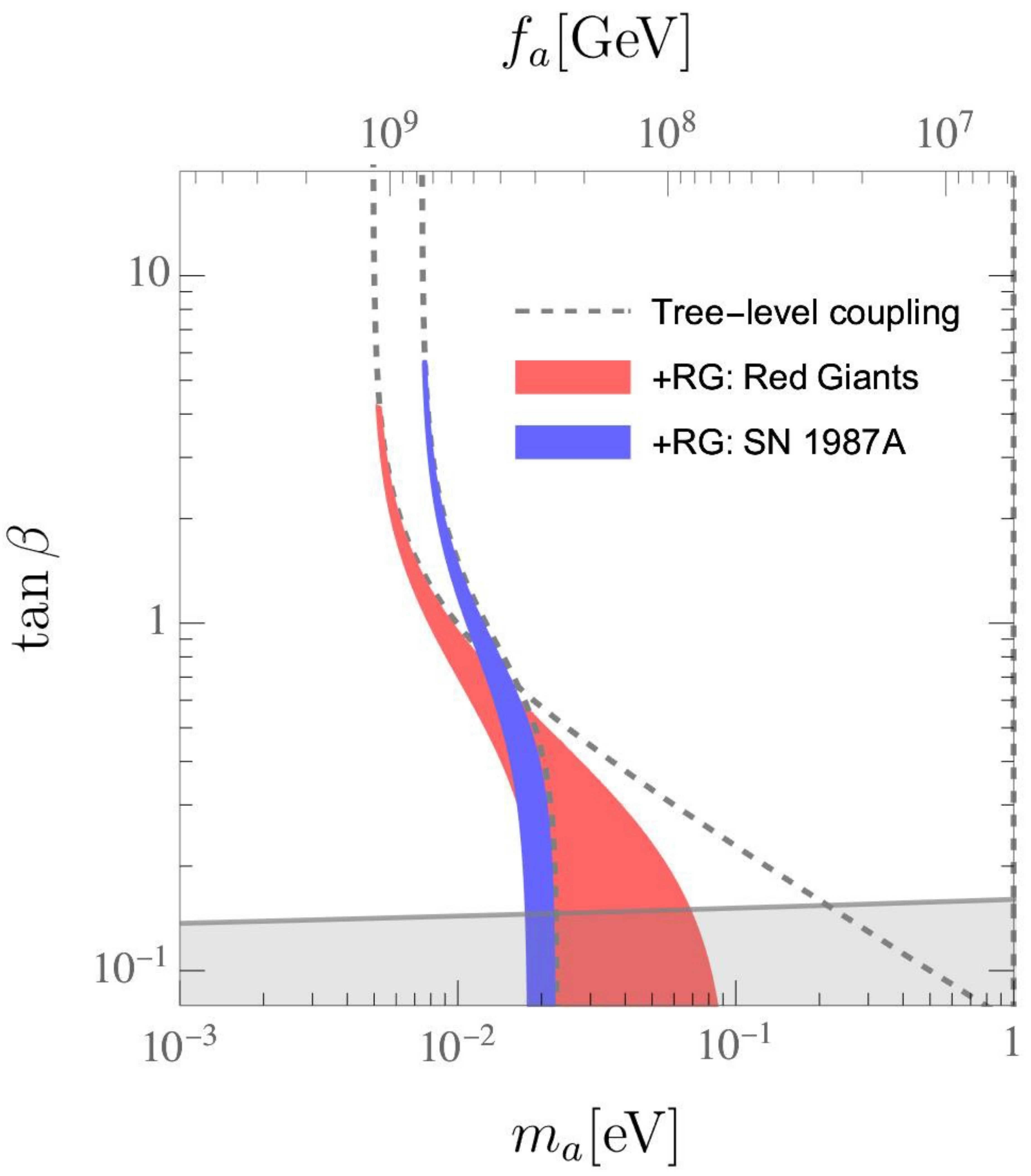
$$L_a = \epsilon_0 \left(\frac{m_N}{f_a}\right)^2 C_{\mathrm{SN}}^2 \times 10^{70}\,\mathrm{erg/s}$$ <- axion emission rate

$$C_{\mathrm{SN}} = 1.4\left(C_0^2 + 1.3C_3^2 + 0.11C_0C_3\right)$$ [Lella et al. 2211.13760]

$NN \to NNa$

$+\ \pi N \to Na$ [Carenza et al. 2010.02943; Choi et al. 2110.01972]

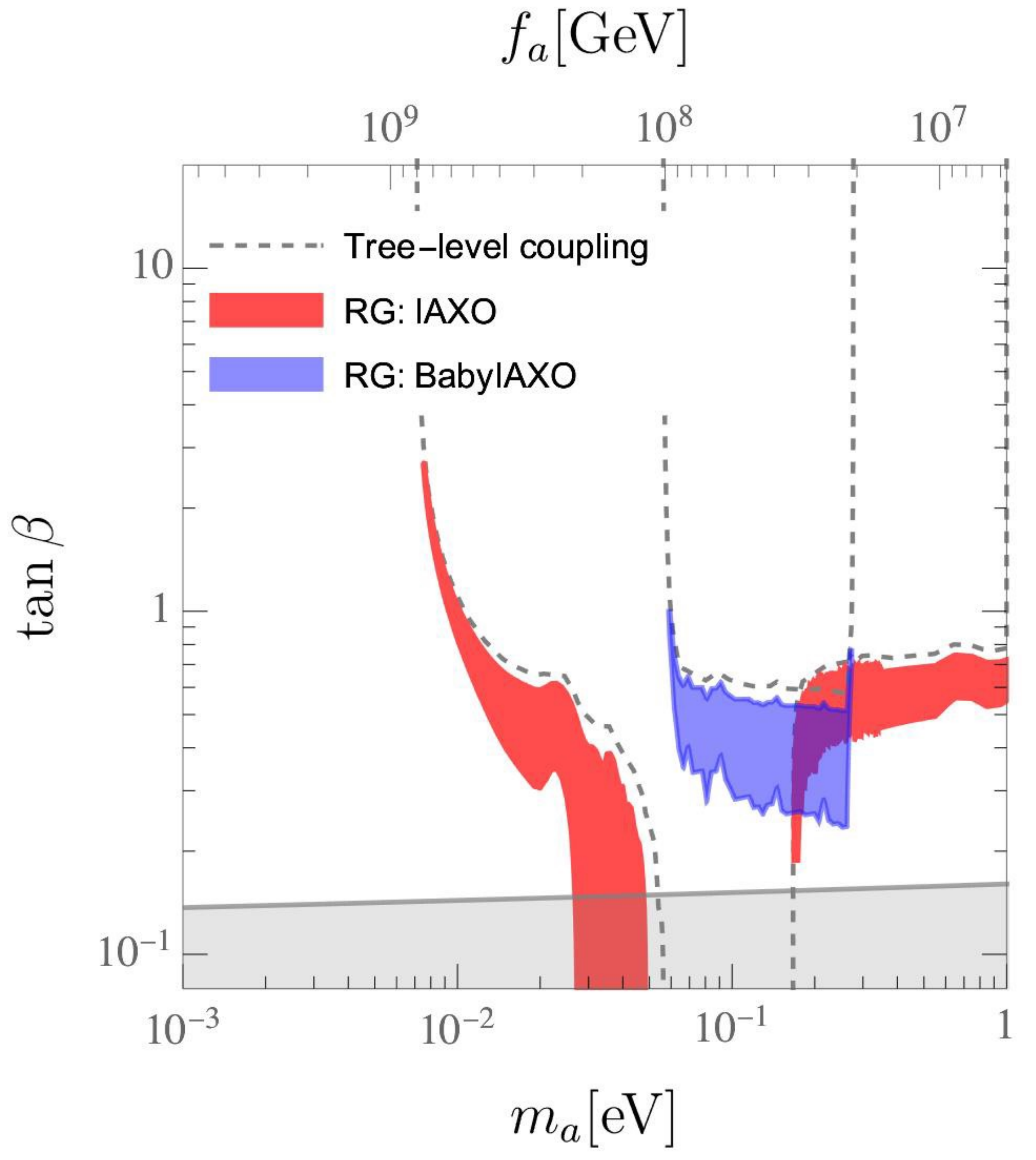
$+\ \Delta(1232)$ resonance [Ho, Kim, Ko, Park, 2212.01155]



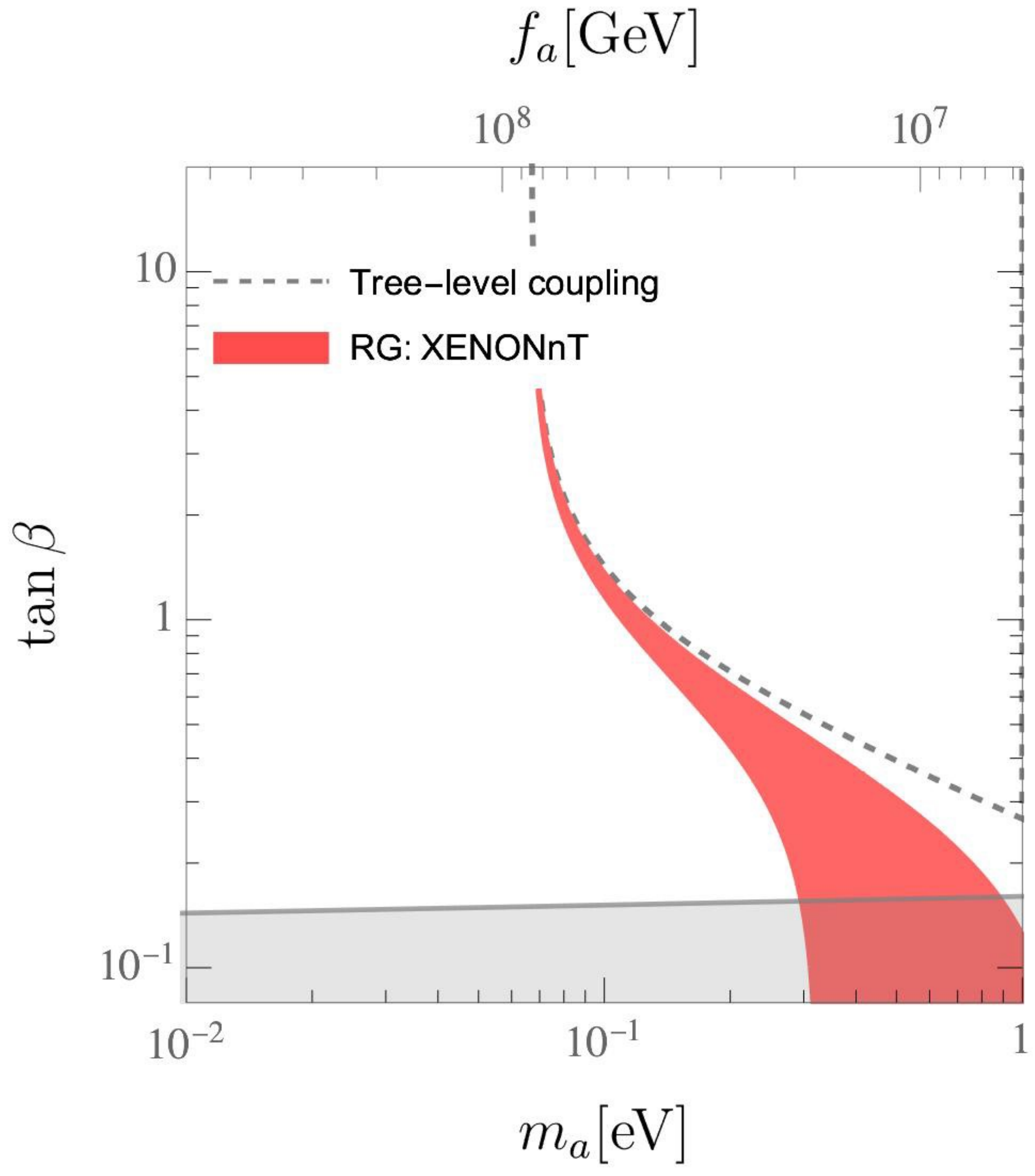
For RG effects, $m_{\mathrm{BSM}} \in [1\mathrm{TeV}, f_a]$

Impact on Axion Phenomenology

- (Baby)IAXO ($g_{a\gamma}, g_{ae}$)

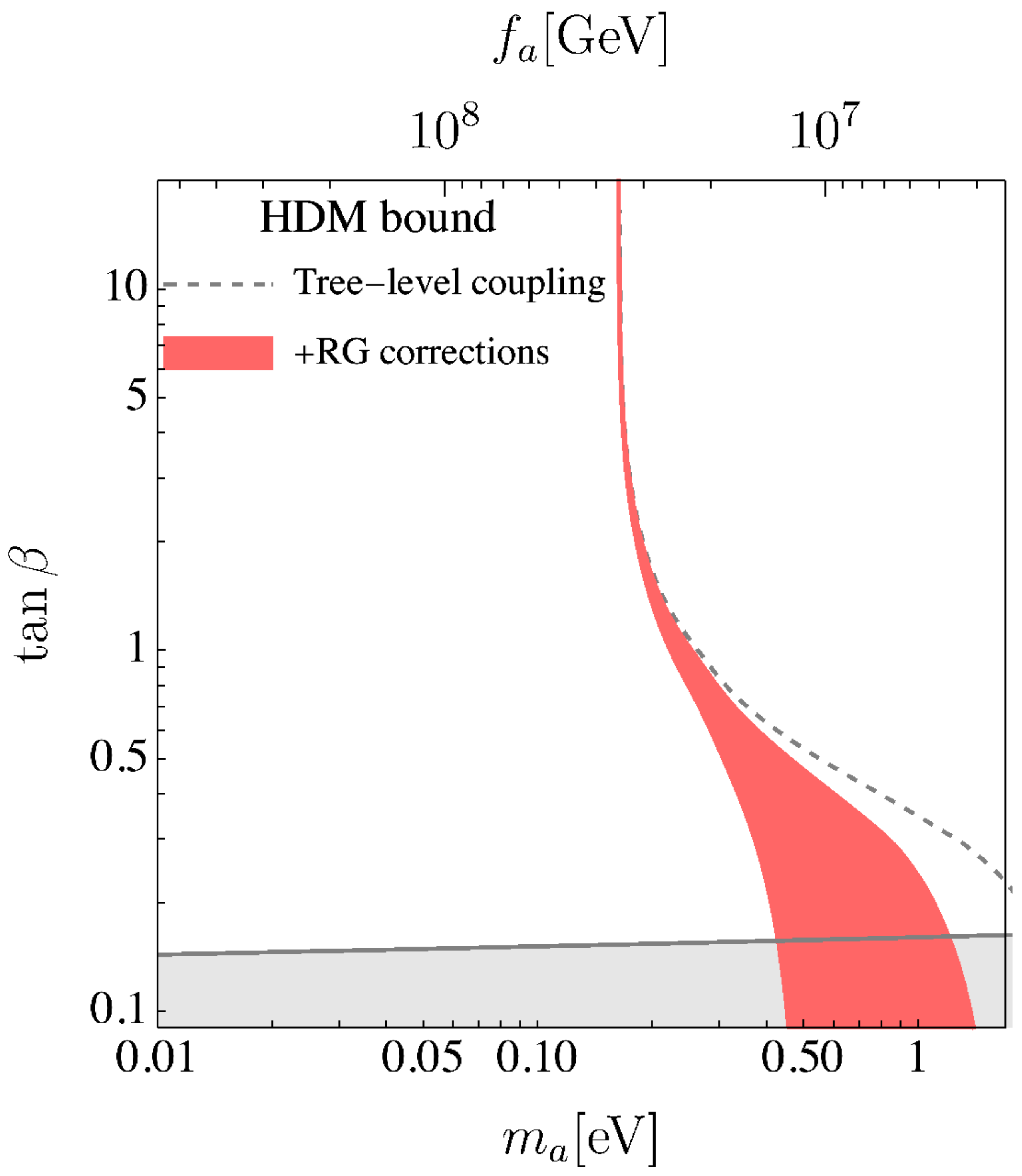


- XENON-nT (g_{ae})



- Hot Dark Matter ($g_{a\pi}$)

$a\pi \leftrightarrow \pi\pi$



For RG effects, $m_{\rm BSM} \in [1\text{TeV}, f_a]$

Application to other axion models

[Di Luzio, Mescia, Nardi, Panci, Ziegler, 1712.04940]
[Di Luzio, Mescia, Nardi, SO, 2205.15326]

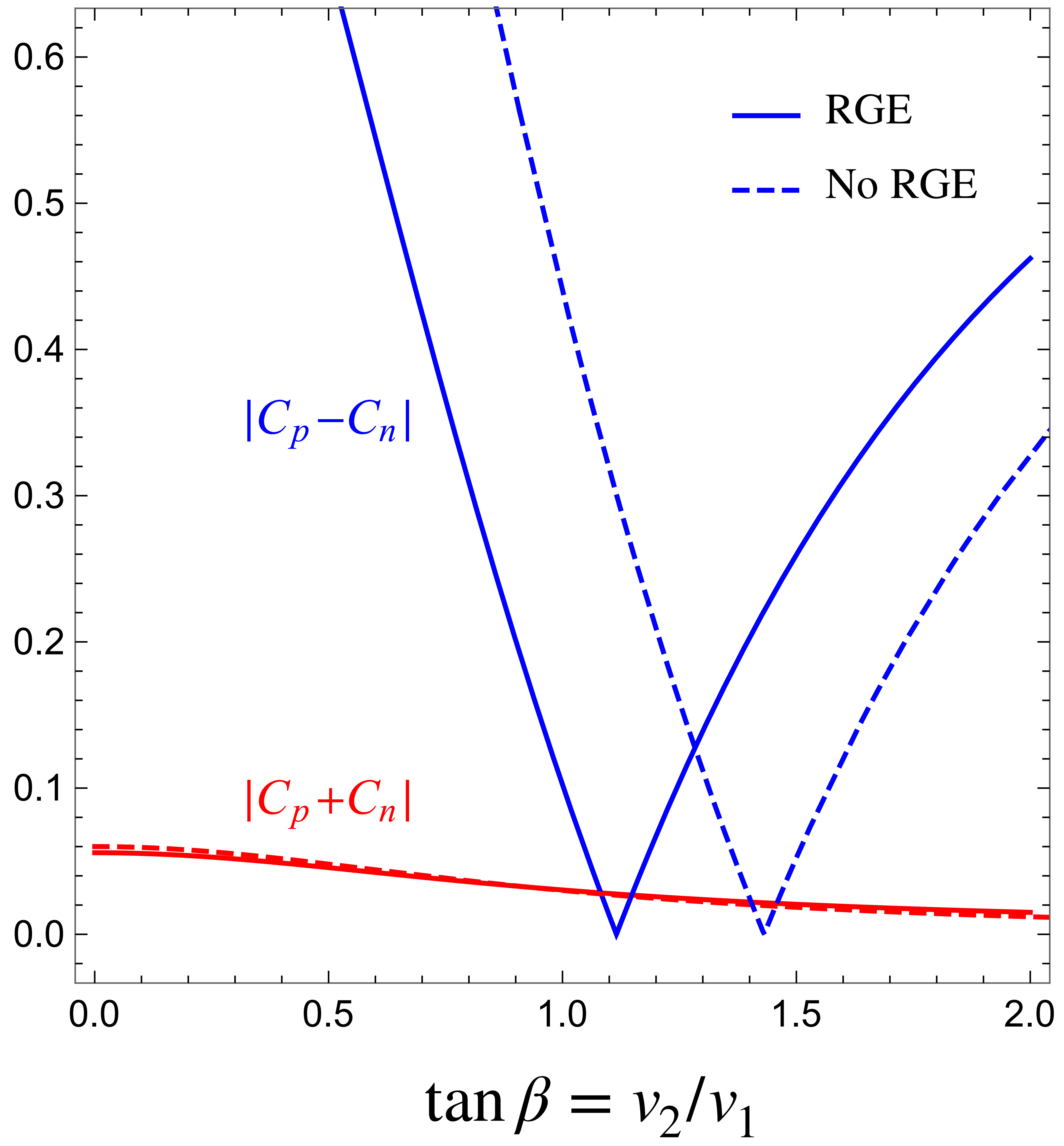
- Nucleophobic axion
  - Strong SN1987A bound is relaxed if $C_N \approx 0$
  - At the tree level, this suppression is realized by generation dependent PQ charge for quarks such that

    (i) $N = N_{1\mathrm{st}}\,,\ N_{2\mathrm{nd}} = -N_{3\mathrm{rd}}$ (N: PQ-QCD anomaly)

    (ii) $v_2^2/v_1^2 = \tan^2\beta = 2$ [Alves, Weiner (2017), Alves (2020)]

    → These relations can be modified by the RG effects at low energy scales (see figure)

  - *Axion nucleophobia keeps holding after including the RG effects, albeit with a fairly different VEV ratio*



Summary

- RG corrections to DFSZ axion couplings depend on heavy Higgs scale m_{BSM}
  - non-negligibly large even if $m_{\mathrm{BSM}} = 1\,\mathrm{TeV}$
  - $C_p + C_n\,, C_e\,, C_\pi$ receive large RG corrections, while $C_p + C_n$ doesn't
  - Some DFSZ axion bounds were underestimated

- Applications to other axion models
  - axion nucleophobia operates in a fairly difference parameter space before and after including the RG effects

Thanks for your attention

Back up

Axion interactions to matter and radiation

$$\mathcal{L}_a = C_\gamma \frac{\alpha}{8\pi}\frac{a}{f_a}F^{\mu\nu}\tilde{F}_{\mu\nu} + \sum_{f=p,n,e} C_f \frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f$$

$\leftarrow$ RGB bound (C_e), SN1987A ($NN \to NNa$)

$$+ C_\pi \frac{\partial_\mu a}{f_a f_\pi}(2\partial^\mu\pi^0\pi^+\pi^- - \pi^0\partial^\mu\pi^+\pi^- - \pi^0\pi^+\partial^\mu\pi^-)$$

$\leftarrow$ HDM bound ($a\pi \leftrightarrow \pi\pi$)

$$+ C_{\pi N}\frac{\partial_\mu a}{2f_a f_\pi}(i\pi^+\bar{p}\gamma^\mu n - i\pi^-\bar{n}\gamma^\mu p)$$

$\leftarrow$ SN1987A ($\pi N \to Na$)

$$+ C_{N\Delta}\frac{\partial^\mu a}{2f_a}\left(\bar{p}\,\Delta^+_\mu + \overline{\Delta^+_\mu}\,p + \bar{n}\,\Delta^0_\mu + \overline{\Delta^0_\mu}\,n\right) + \ldots,$$

$\leftarrow$ SN1987A (Δ-resonance)

$$C_\gamma = \frac{E}{N} - 1.92; \quad \frac{E}{N} = \frac{c_W + c_B}{c_G}$$

$$C_\pi = -\frac{2}{3}g_A^{-1}C_3; \quad C_{\pi N} = \sqrt{2}g_A^{-1}C_3; \quad C_{N\Delta} = -\sqrt{3}C_3$$

$$C_{0,3} = \frac{1}{2}(C_p \pm C_n)$$

Axion couplings to SM fields

- Axion effective Lagrangian in the Georgi-Kaplan-Randall basis:

$$\mathcal{L}_a^{\rm GKR} = \frac{\partial_\mu a}{f}\left(\sum_{\psi=q_L,\ell_L,\dots} c_\psi \bar{\psi}\gamma^\mu\psi + \sum_{\Phi=H_1,H_2,\dots} c_\Phi \Phi^\dagger i\overleftrightarrow{D}^\mu\Phi\right) + \sum_{A=G,W,B} c_A \frac{g_A^2}{32\pi^2}\frac{a}{f}F_A\widetilde{F}_A$$

  - $U(1)_{PQ}$ non-linearly realized: $a \to a + \alpha f$
  - Heavy $O(f)$ radial mode ignored
  - At $\mu = f$, one can take this basis by performing axion-dependent field redefinition $\psi \to e^{-i\mathcal{X}_\psi a/f}\psi$

  -> axion couplings correspond to PQ charge of fields and PQ-(gauge)^2 anomaly coefficients

$$c_{\psi,\Phi} = \mathcal{X}_{\psi,\Phi}$$

$$c_A = \sum_{\psi_R} 2\mathcal{X}_{\psi_R}\,{\rm Tr}\,T_A^2(\psi_R) - \sum_{\psi_L} 2\mathcal{X}_{\psi_L}\,{\rm Tr}\,T_A^2(\psi_L)$$

Axion couplings to nucleons

$$C_0 = \frac{1}{2}(C_p + C_n) = 0.22\,(C_u + C_d - 1) - 0.035 C_s$$

$$C_3 = \frac{1}{2}(C_p - C_n) = 0.64\left(C_u - C_d - \frac{m_d - m_u}{m_d + m_u}\right)$$

$$C_3 \simeq -0.43\sin^2\beta + 0.64\left(\frac{1}{3} - \frac{m_d - m_u}{m_d + m_u}\right)$$

$\simeq 0 \ (\because m_d/m_u \simeq 2)$

$C_u = \cos^2\beta/3$

$C_d = \sin^2\beta/3$

Renormalization group equations (1/2)

- $\mu_{EW} < \mu < m_{BSM}$

$$(4\pi)^2 \frac{dc'_{q_L}}{d\log\mu} = \frac{1}{2}\{c'_{q_L}, Y_u Y_u^\dagger + Y_d Y_d^\dagger\} - Y_u c'_{u_R} Y_u^\dagger - Y_d c'_{d_R} Y_d^\dagger$$
$$+ \left(8\alpha_s^2 \tilde{c}_G + \frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{1}{6}\alpha_1^2 \tilde{c}_B\right)\mathbf{1} - \beta_q \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{u_R}}{d\log\mu} = \{c'_{u_R}, Y_u^\dagger Y_u\} - 2Y_u^\dagger c'_{q_L} Y_u - \left(8\alpha_s^2 \tilde{c}_G + \frac{8}{3}\alpha_1^2 \tilde{c}_B\right)\mathbf{1}$$
$$- \beta_u \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{d_R}}{d\log\mu} = \{c'_{d_R}, Y_d^\dagger Y_d\} - 2Y_d^\dagger c'_{q_L} Y_d - \left(8\alpha_s^2 \tilde{c}_G + \frac{2}{3}\alpha_1^2 \tilde{c}_B\right)\mathbf{1}$$
$$- \beta_d \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{\ell_L}}{d\log\mu} = \frac{1}{2}\{c'_{\ell_L}, Y_e Y_e^\dagger\} - Y_e c'_{e_R} Y_e^\dagger + \left(\frac{9}{2}\alpha_2^2 \tilde{c}_W + \frac{3}{2}\alpha_1^2 \tilde{c}_B\right)\mathbf{1}$$
$$- \beta_\ell \gamma_H \mathbf{1},$$

$$(4\pi)^2 \frac{dc'_{e_R}}{d\log\mu} = \{c'_{e_R}, Y_e^\dagger Y_e\} - 2Y_e^\dagger c'_{\ell_L} Y_e - 6\alpha_1^2 \tilde{c}_B \mathbf{1} - \beta_e \gamma_H \mathbf{1},$$

where

$$\gamma_H = -2\,\mathrm{Tr}\left(3Y_u^\dagger c'_{q_L} Y_u - 3Y_d^\dagger c'_{q_L} Y_d - Y_e^\dagger c'_{\ell_L} Y_e\right)$$
$$+ 2\,\mathrm{Tr}\left(3Y_u c'_{u_R} Y_u^\dagger - 3Y_d c'_{d_R} Y_d^\dagger - Y_e c'_{e_R} Y_e^\dagger\right),$$
$$\tilde{c}_G = c_G - \mathrm{Tr}\left(c'_{u_R} + c'_{d_R} - 2c'_{q_L}\right),$$
$$\tilde{c}_W = c_W + \mathrm{Tr}\left(3c'_{q_L} + c'_{\ell_L}\right),$$
$$\tilde{c}_B = c_B - \mathrm{Tr}\left(\frac{1}{3}(8c'_{u_R} + 2c'_{d_R} - c'_{q_L}) + 2c'_{e_R} - c'_{\ell_L}\right).$$

- Axion-Higgs coupling c_H is removed at any scale by performing an axion-dependent hypercharge rotation:

$$\psi \to e^{-ic_H \beta_\psi a/f}\psi \quad (\beta_\psi = Y_\psi / Y_H)$$

- This redefines all axion-fermion couplings as $c_\psi \to c'_\psi = c_\psi - \beta_\psi c_H$, which you see in the RGEs

Renormalization group equations (2/2)

- $\mu_{QCD} < \mu < \mu_{EW}$

$$(4\pi)^2 \frac{d(C_u^A)_{ii}}{d\log\mu} = -16\alpha_s^2 \tilde{c}_G - \frac{8}{3}\alpha_{\rm em}^2 \tilde{c}_\gamma\,,$$

$$(4\pi)^2 \frac{d(C_d^A)_{ii}}{d\log\mu} = -16\alpha_s^2 \tilde{c}_G - \frac{2}{3}\alpha_{\rm em}^2 \tilde{c}_\gamma\,,$$

$$(4\pi)^2 \frac{d(C_e^A)_{ii}}{d\log\mu} = -6\alpha_{\rm em}^2 \tilde{c}_\gamma\,,$$

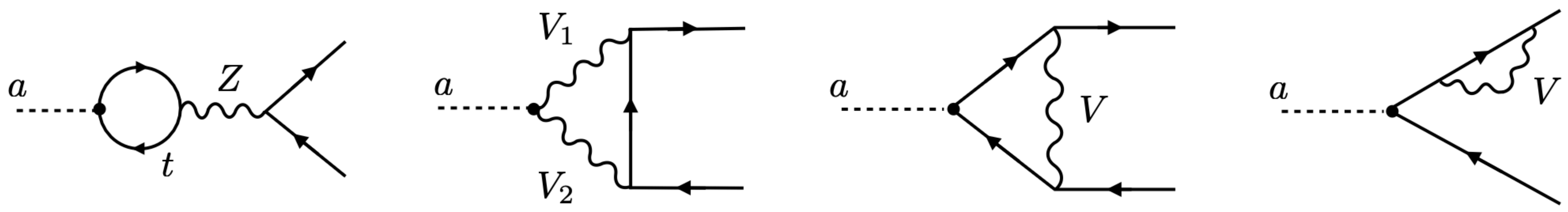
where

$$\tilde{c}_G(\mu) = 1 - \sum_q C_q^A(\mu)\Theta(\mu - m_q)\,,$$

$$\tilde{c}_\gamma(\mu) = \frac{c_\gamma}{c_G} - 2\sum_f N_c^f Q_f^2 C_f^A(\mu)\Theta(\mu - m_f)\,,$$

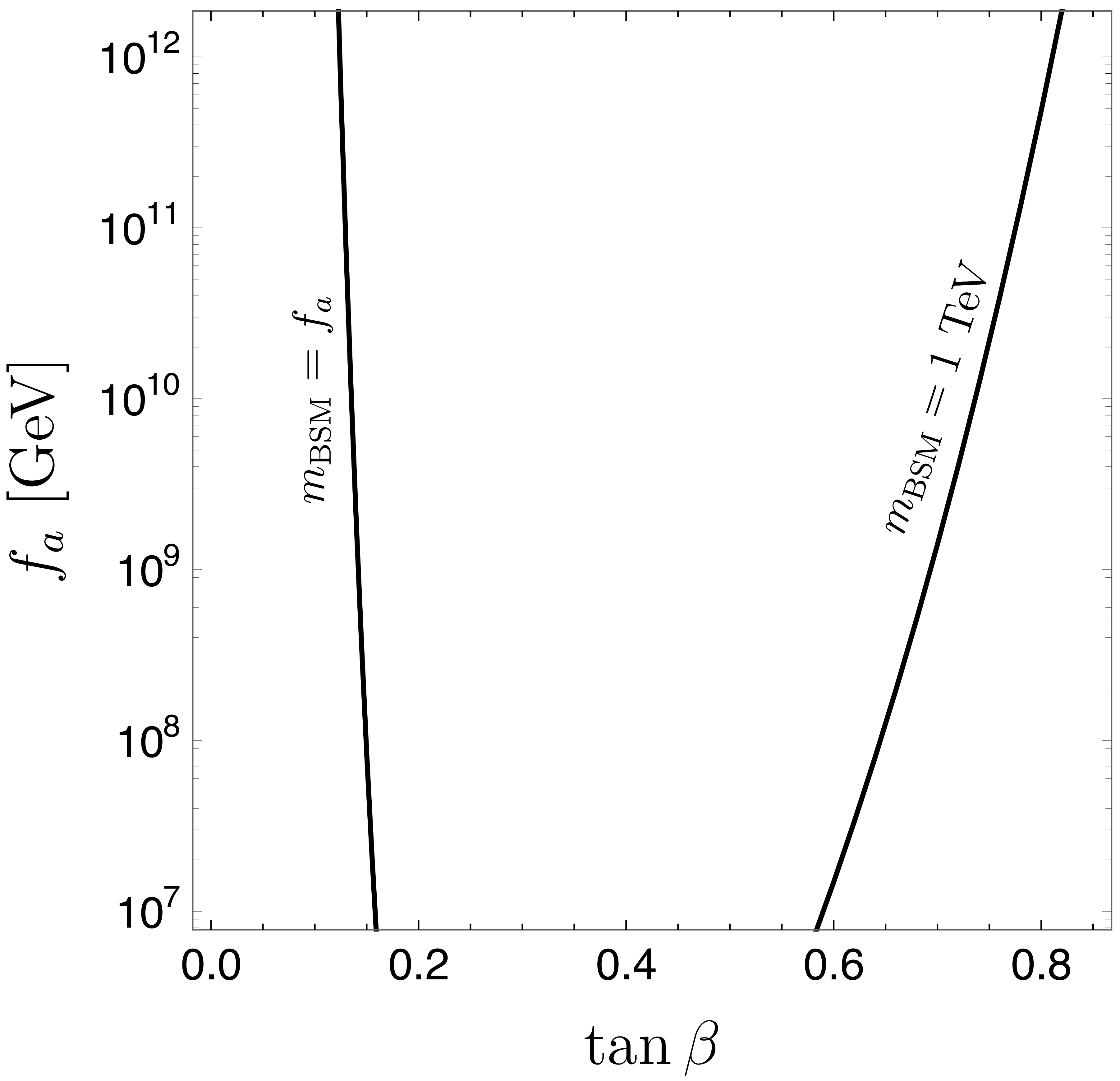
- add threshold corrections at the EW scale

$$C_f^A(\mu_{\rm EW}) = c_{f_R}(\mu_{\rm EW}) - c_{f_L}(\mu_{\rm EW}) + \underline{\Delta c_{f_R} - \Delta c_{f_L}}$$



[Bauer, Neubert, Renner, Schnubel, Thamm, 2012.12272]

Perturbative Unitarity Bounds



- Small tanβ leads low Landau pole scales
  - impose perturbative unitarity on Higgs-mediated fermion 2 → 2 scattering up to $\mu = f_a$:

    $\Rightarrow Y_{t,b}^{\rm 2HDM} \leq \sqrt{16\pi/3}$ [Di Luzio, Kamenik, Nardecchia (2016)]

    (region left to the black line is excluded)
  - Yukawa couplings are RG-evolved from m_Z to f_a , while appropriately matching with the 2HDM at $\mu = m_{\rm BSM}$

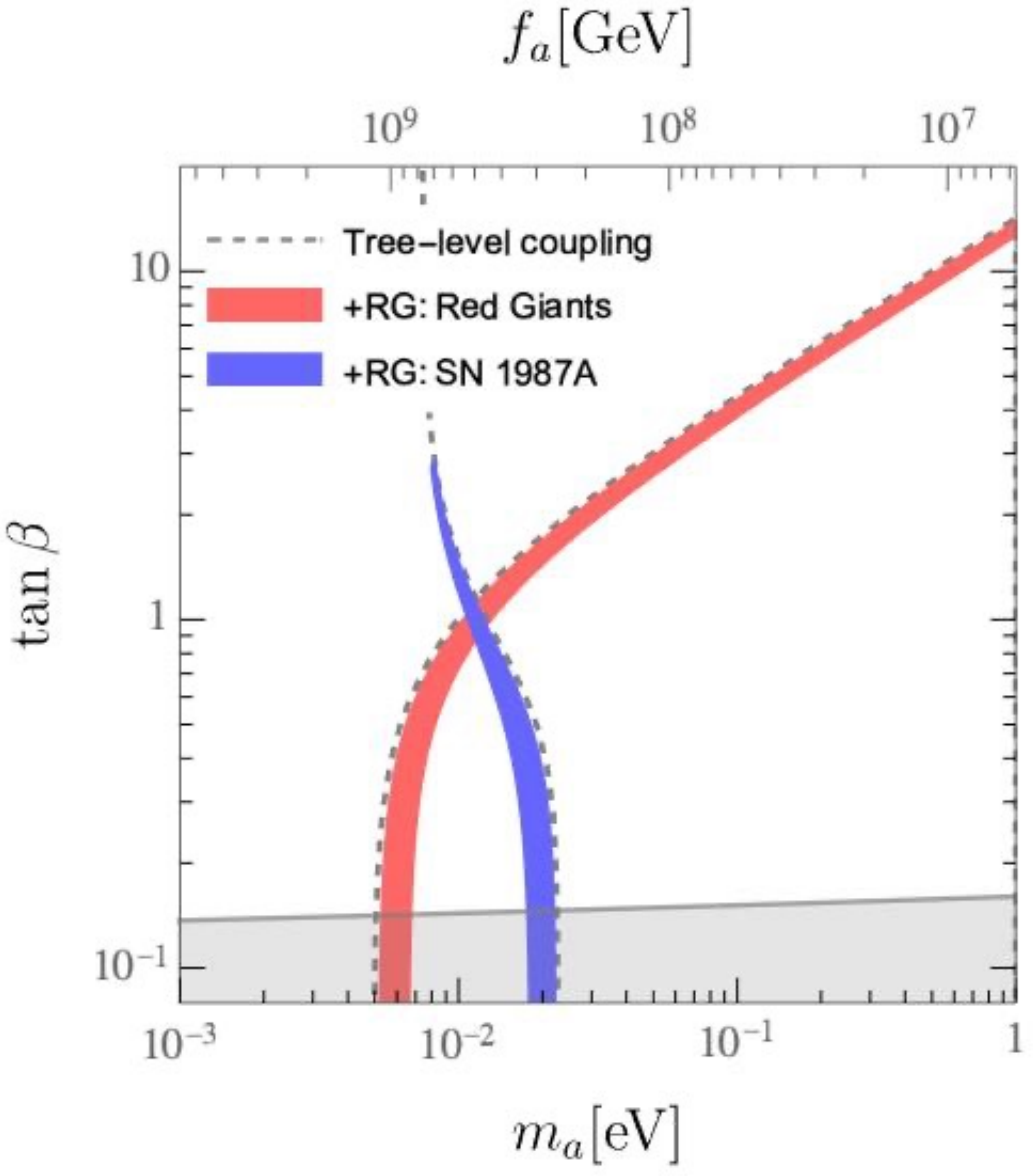
Astrophysical bounds in DFSZ2 model

- DFSZ2 model:
  - axion-electron coupling is not suppressed in the small tanβ region: $C_e(f_a) = -c_\beta^2/3$

    ⇒ RG effect less important for the RGB bound than in the DFSZ1 model

  - Hadronic couplings are unchanged

    ⇒ RG effect is unchanged for the SN1987A bound



For RG effects, $m_{\rm BSM} \in [1{\rm TeV}, f_a]$