Light Dark World 2023

19-21 September 2023
KIT
Europe/Berlin timezone

Hot QCD axion abundance

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International Centre
for Theoretical Physics

2211.03799 with A. Notari and F. Rompineve

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

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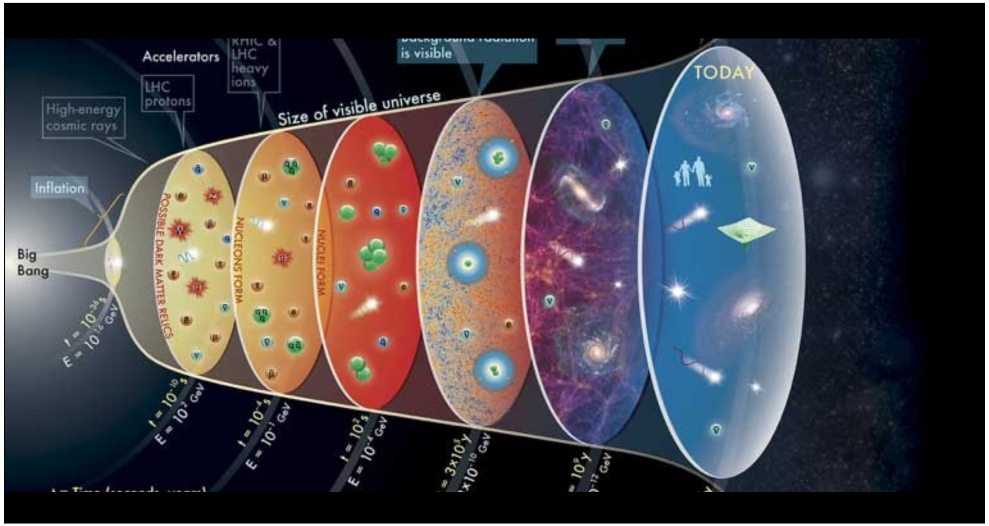

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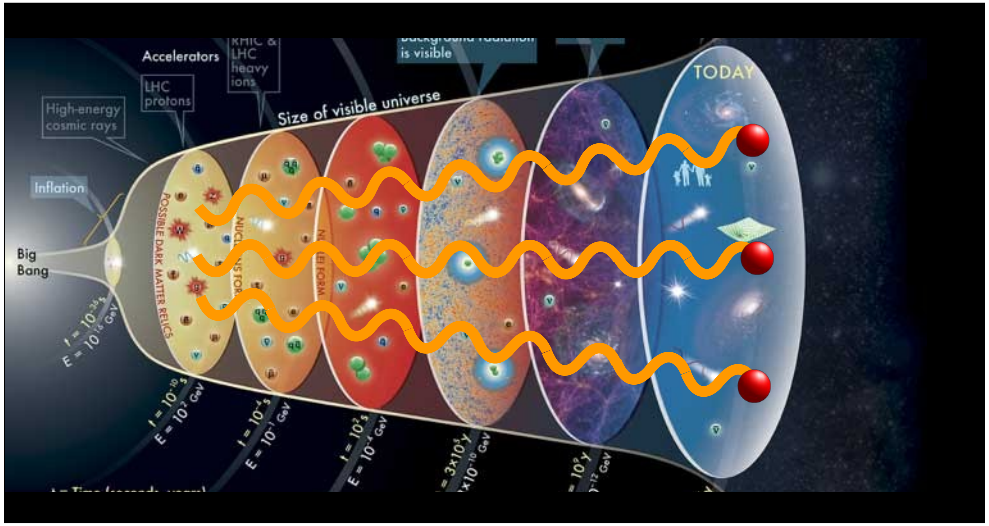

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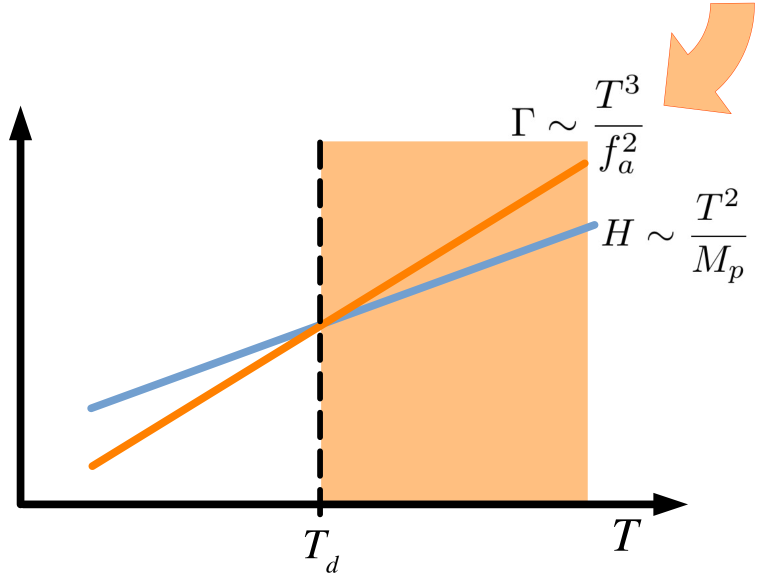

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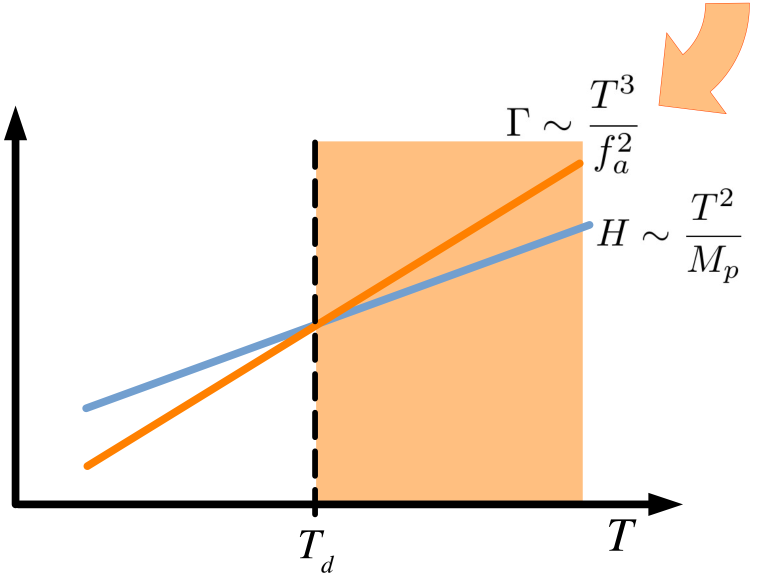

$$\Gamma \sim \frac{T^3}{f_a^2}$$

$$H \sim \frac{T^2}{M_p}$$

$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \text{GeV}}\right)^2$$

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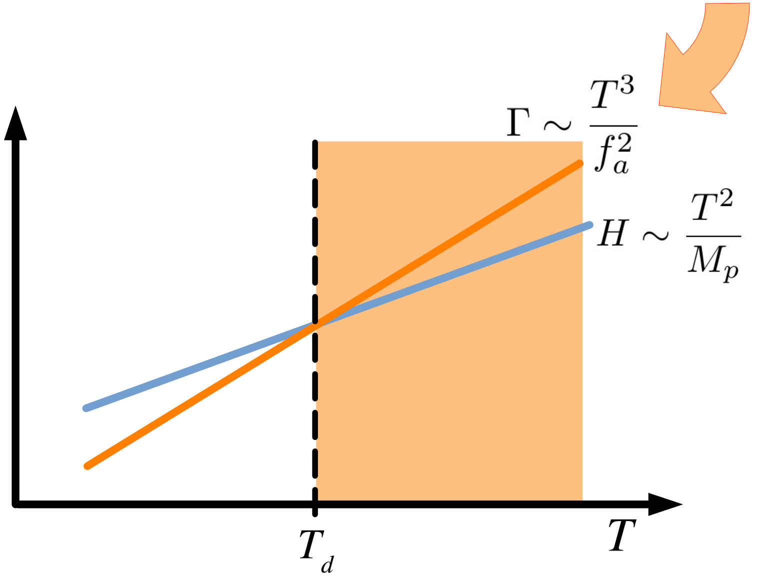

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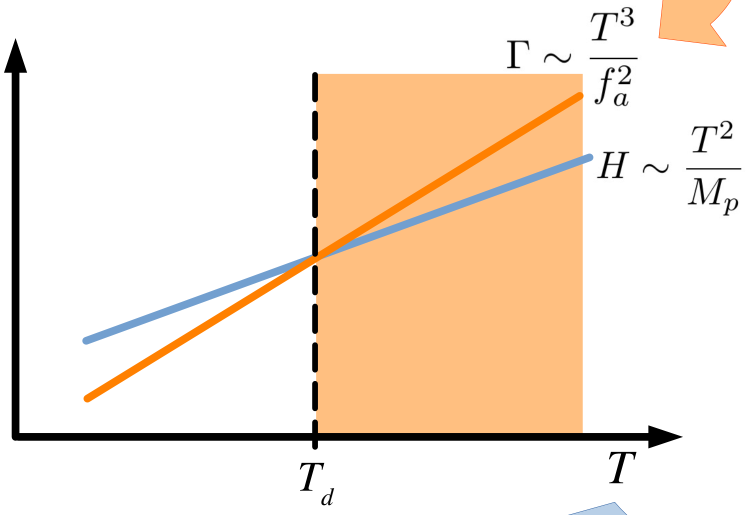

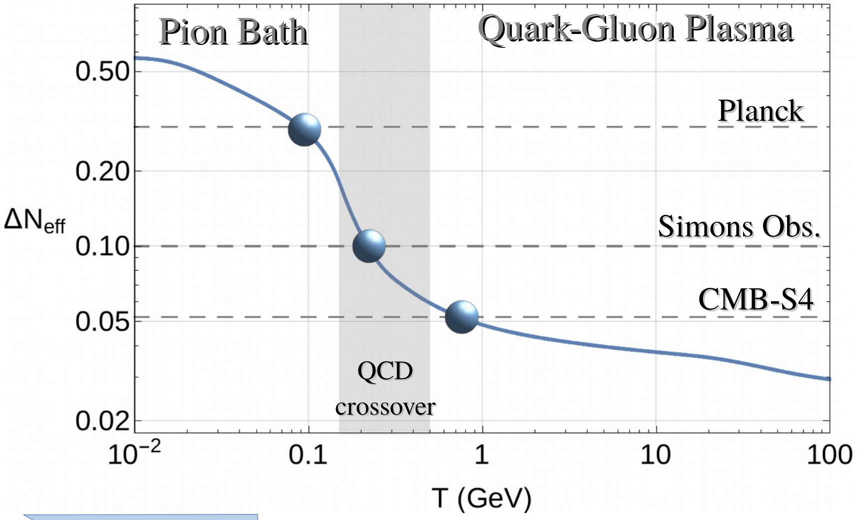

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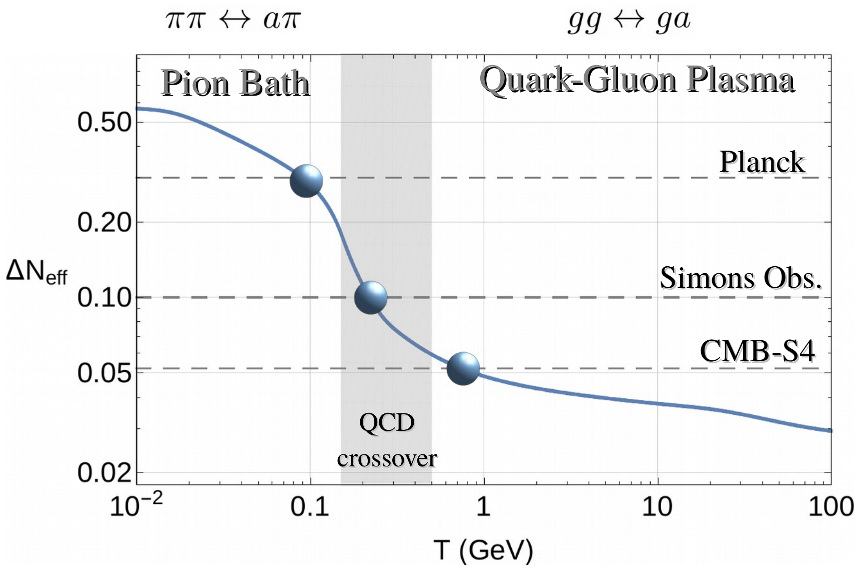
Axion $\Delta N_{\rm eff}$ has long history:



Arias-Arogon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, Notari, Piazza, Raffelt, Rompineve, Rota, Sakharov, Silk, Slosar, Steffen, Wallisch, Wong, Yun, Zsembinszki ...

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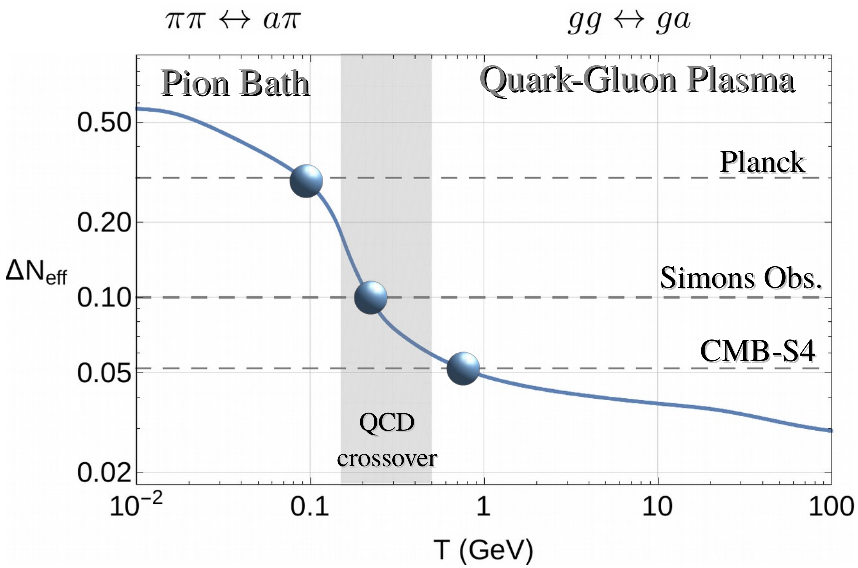
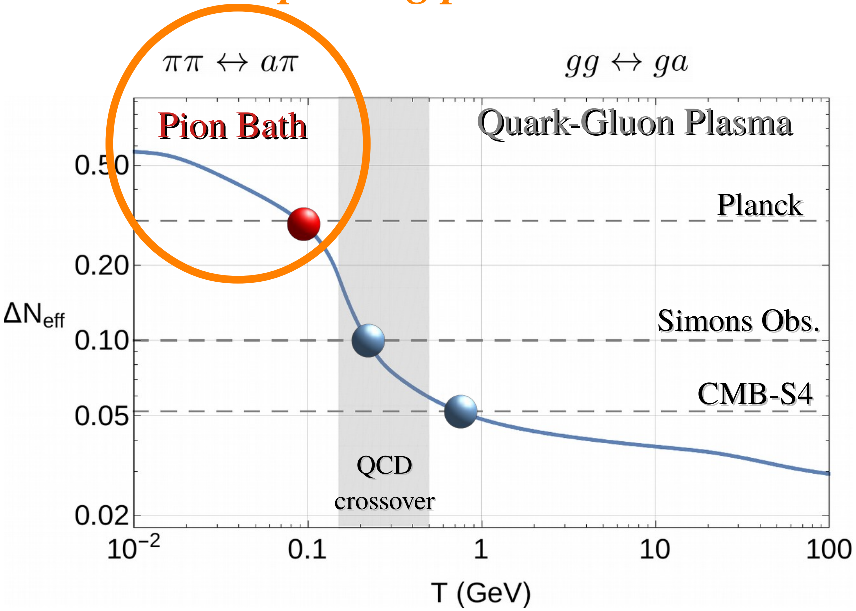


Boltzmann Eq.

$$\frac{dY}{d\log x} = (Y^{\text{eq}} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right)$$

Axion ΔN_{eff} has long history:

Improving present bounds



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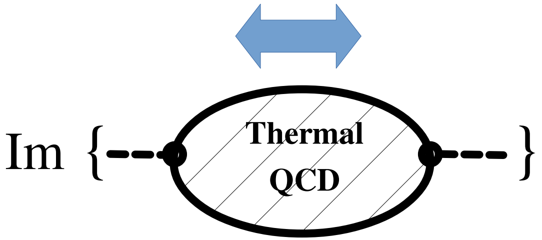

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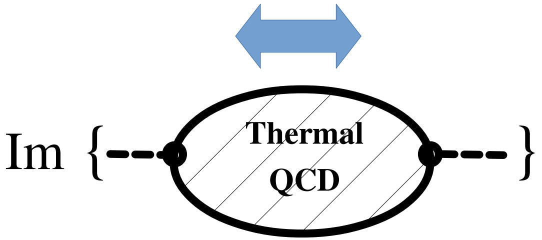

$$\Gamma^{<} = \frac{1}{2E} \int \left(\prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i} \right) f_1^{\text{eq}} f_2^{\text{eq}} (1 + f_3^{\text{eq}}) (2\pi)^4 \delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu) |\mathcal{M}|^2_{2\,\leftrightarrow\,2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{f_\pi}{2f_a}\left(\frac{m_d - m_u}{m_d + m_u} + c_u - c_d\right)$$

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<u>→ breaks down at T ~ 60 MeV</u>

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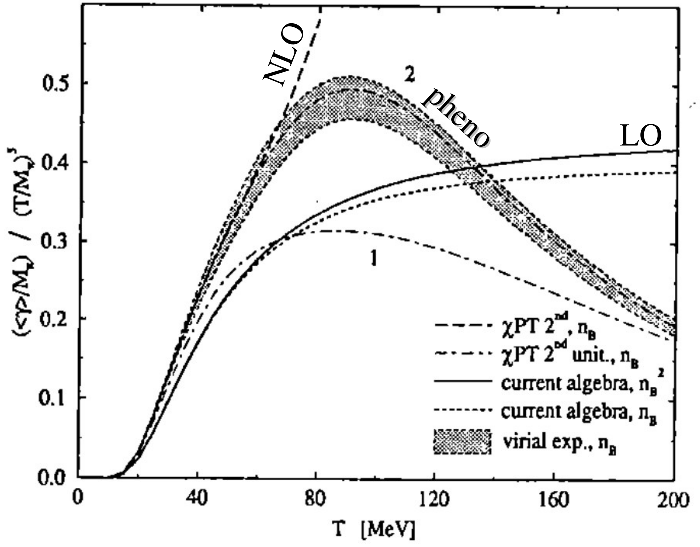

Schenk '94

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Strategy:

@ all orders in χPT

$$\mathcal{M}_{a\pi^i \to \pi^j \pi^k} = \theta_{a\pi} \cdot \mathcal{M}_{\pi^0 \pi^i \to \pi^j \pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$|\mathcal{M}_{\pi\pi}^{\rm LO}|^2 = \frac{s^2 + t^2 + u^2 - 4m_\pi^4}{f_\pi^4}$$

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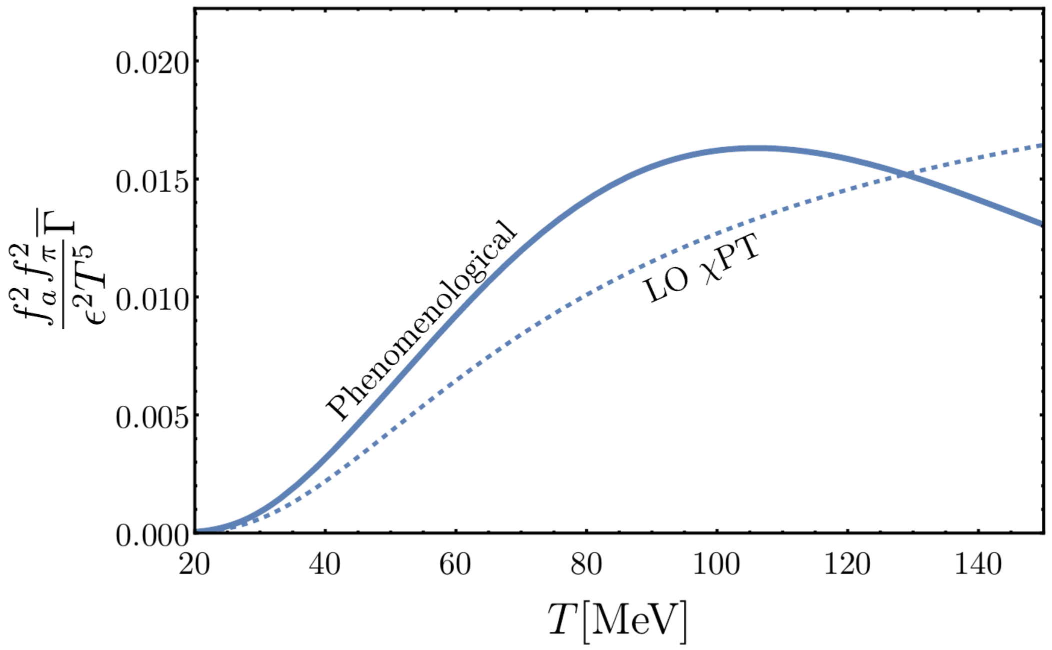

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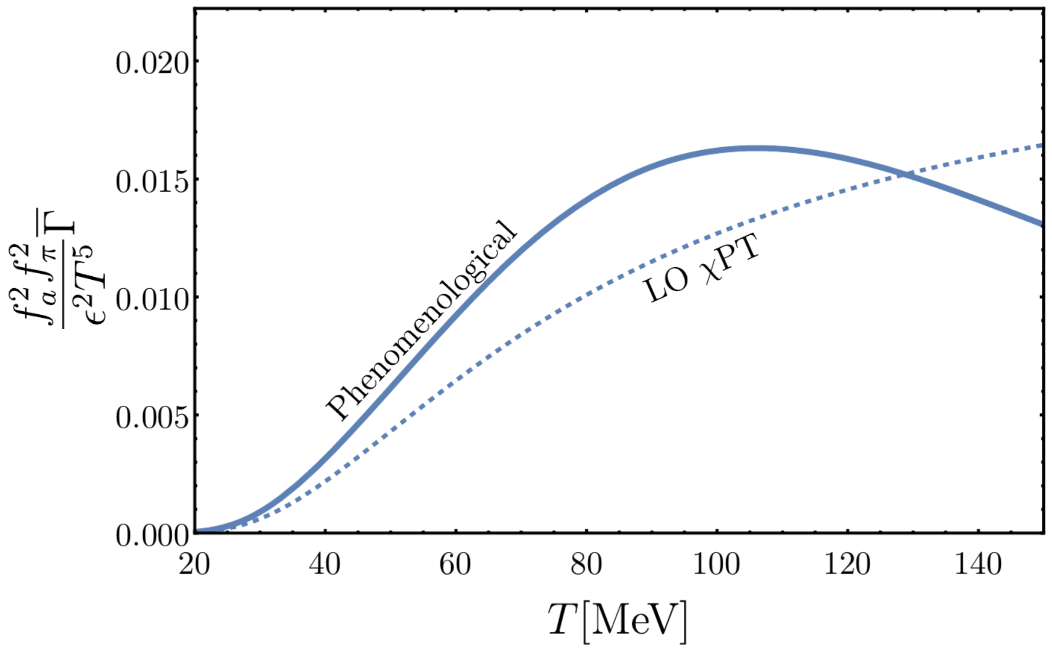

In reasonable agreement with:

Di Luzio, Camalich, Martinelli, Oller, Piazza '22 (using NLO+unitarization)

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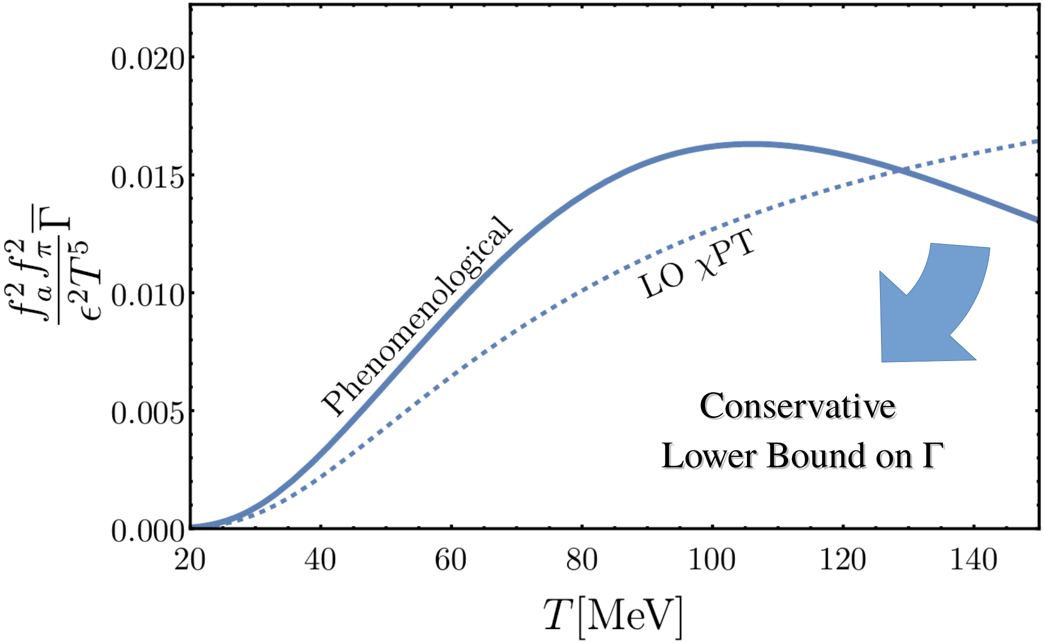

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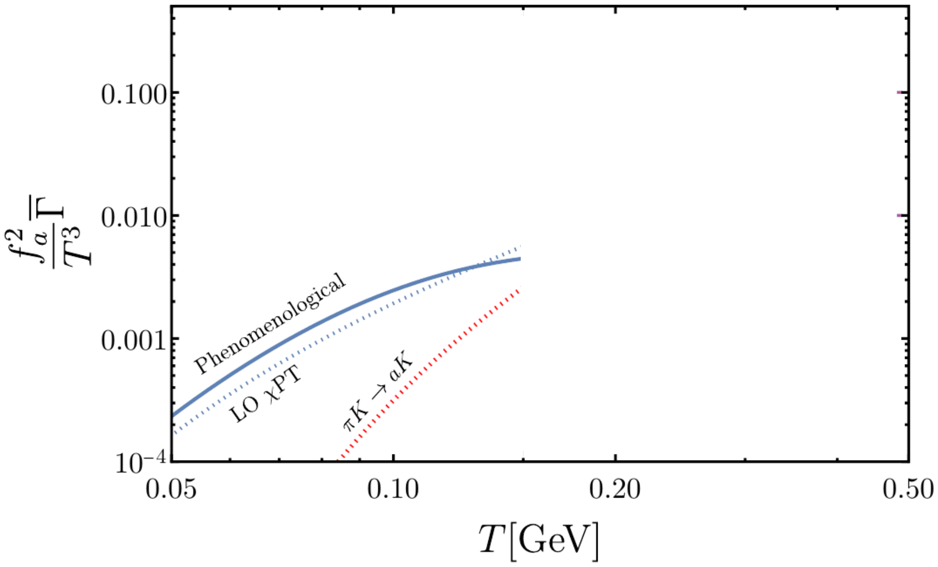

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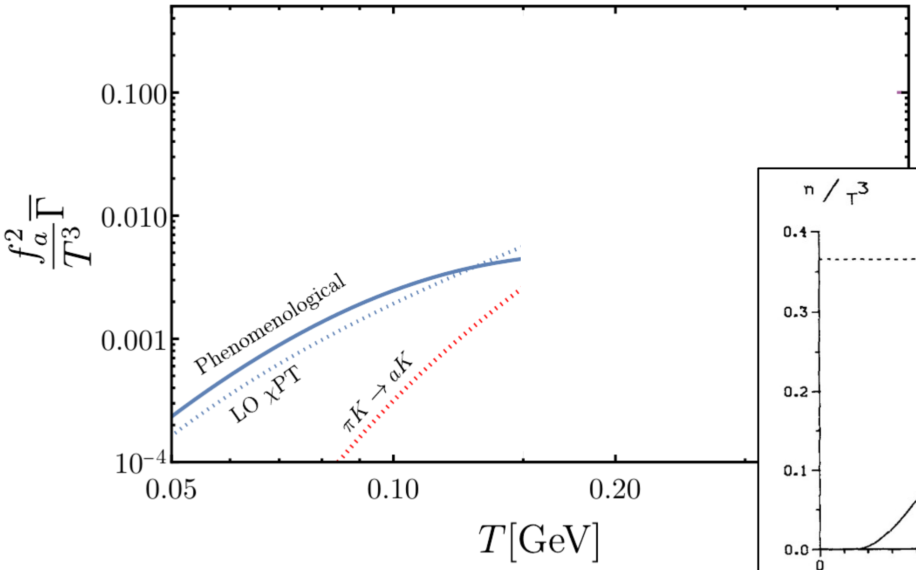
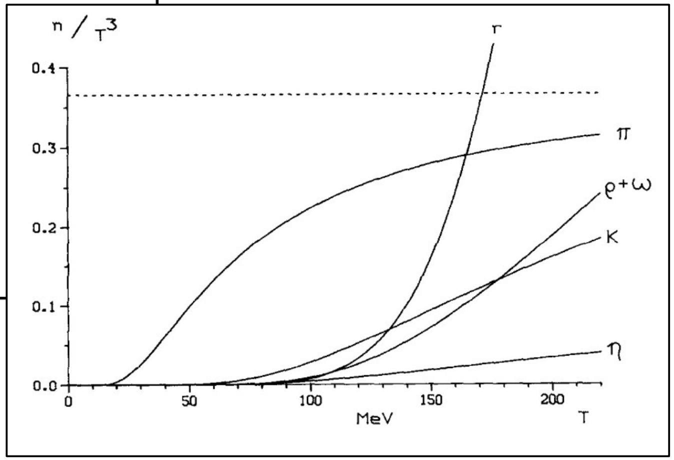
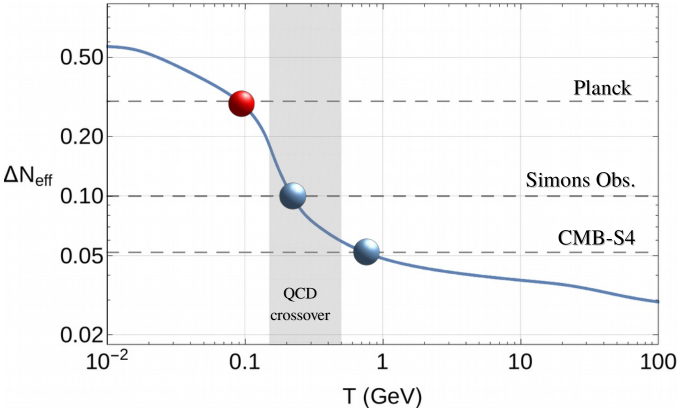




Gerber Leutwyler ‘89

2. Momentum Dependence



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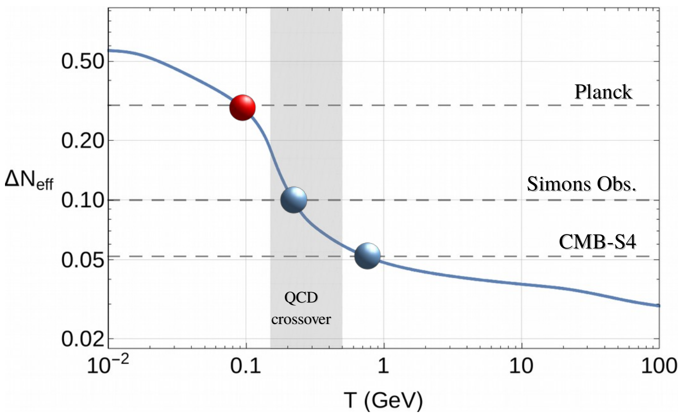

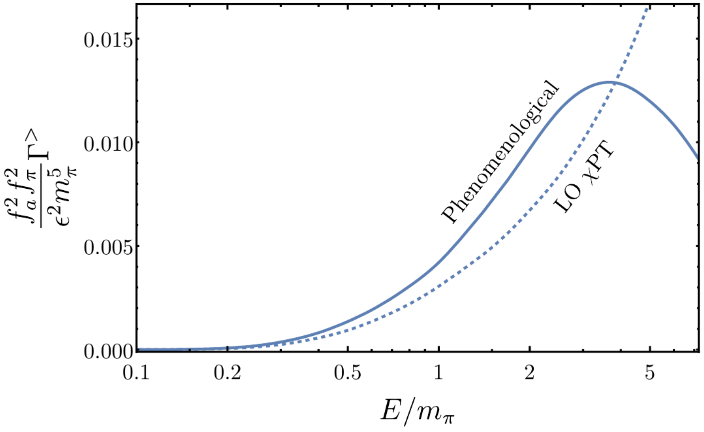

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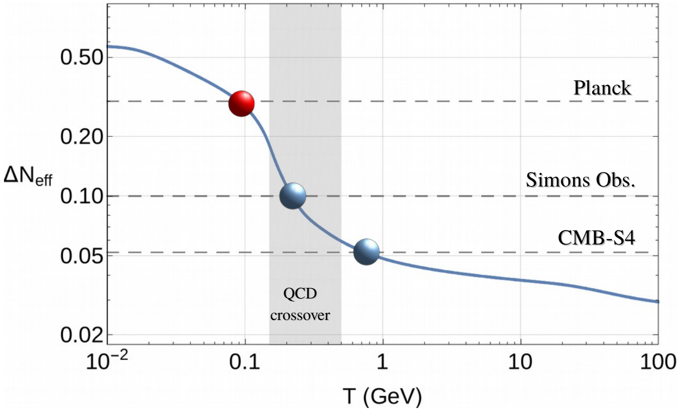

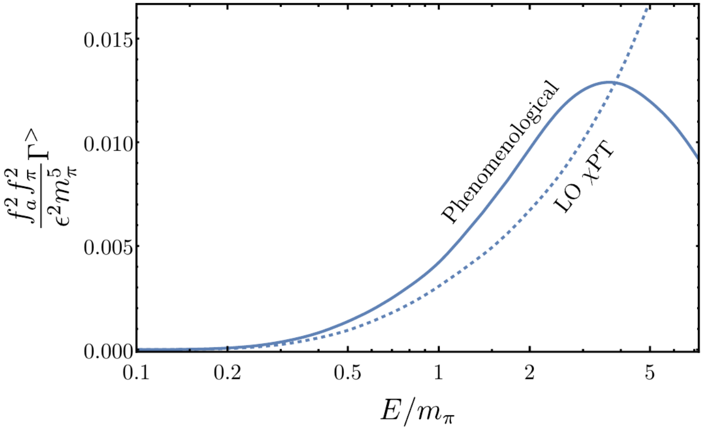

Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

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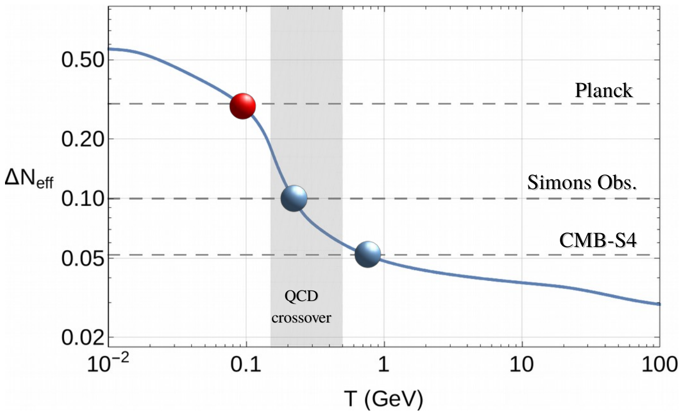

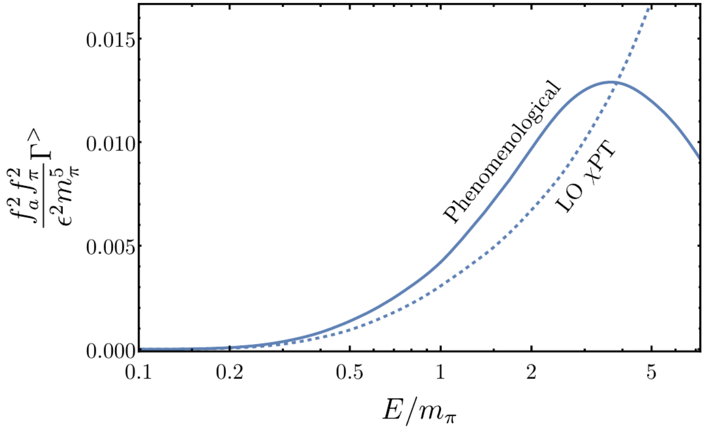

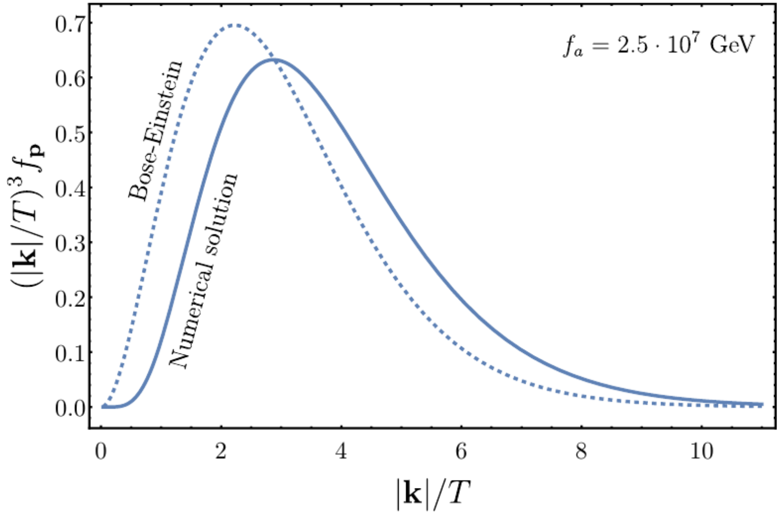

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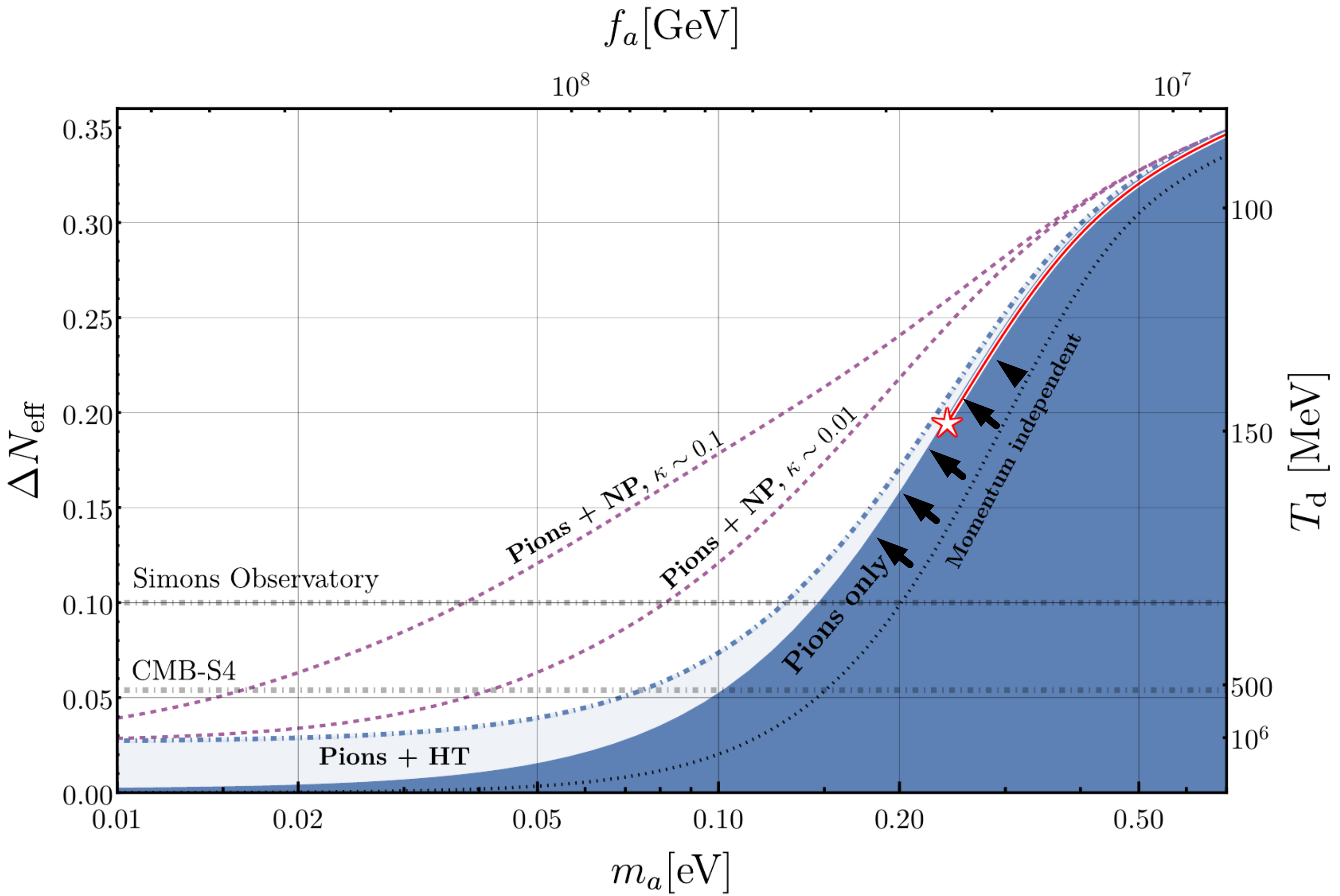
~ 40% enhancement

Future Reach



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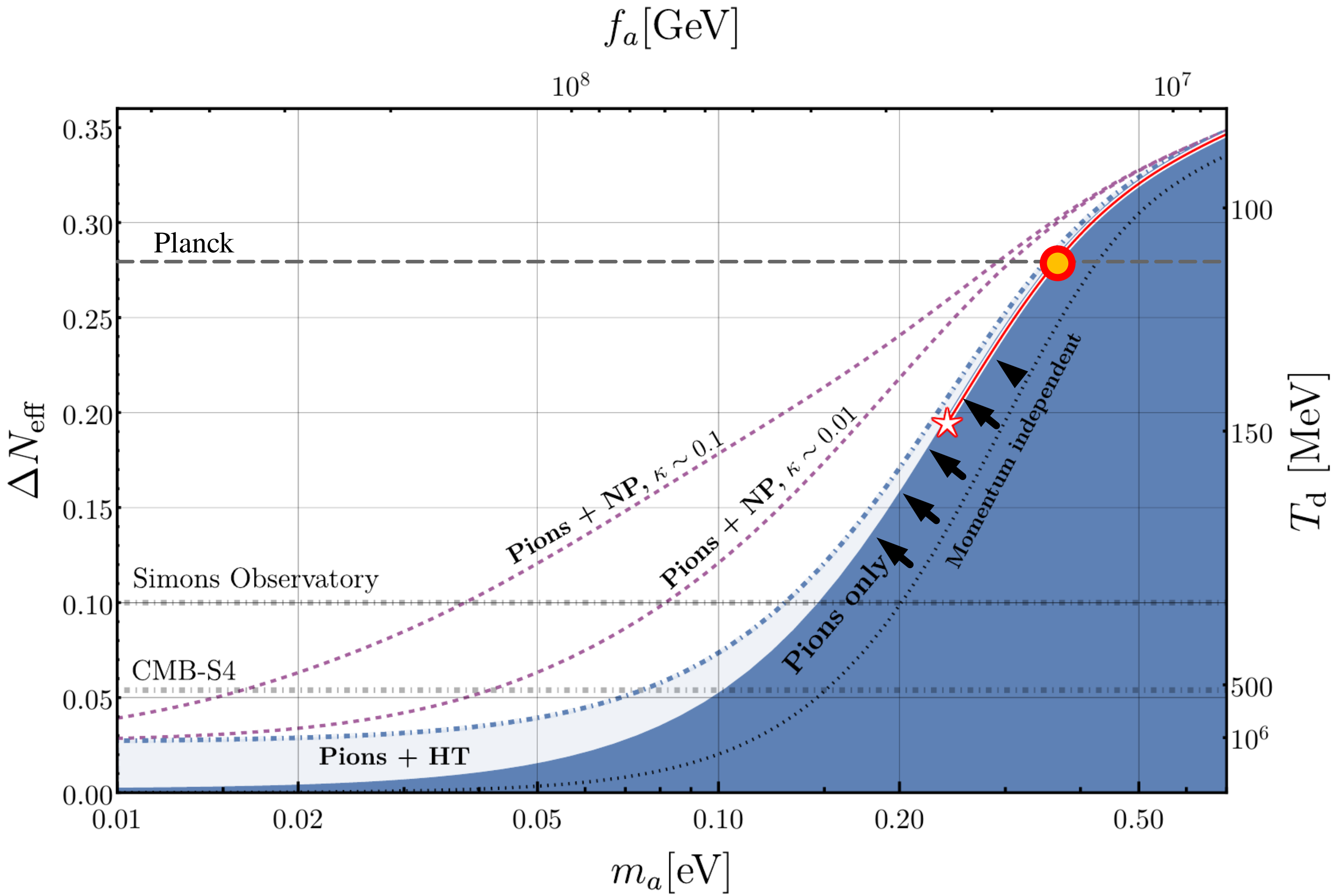

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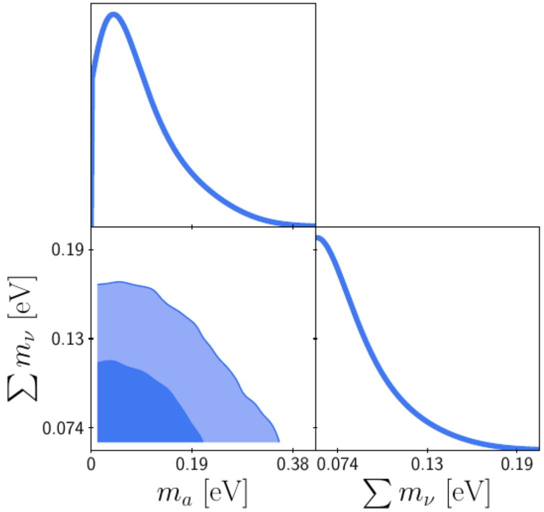

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$f_a \geqslant 2.4 \cdot 10^7$ GeV

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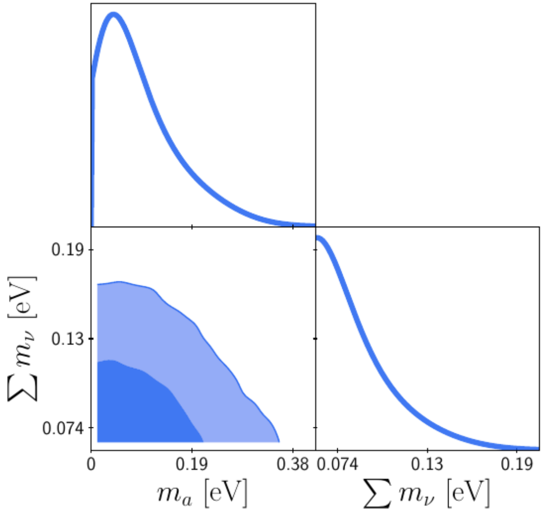

$m_a \leq 0.24$ eV $\qquad f_a \geqslant 2.4 \cdot 10^7$ GeV

$\Leftrightarrow$

$\Delta N_{\text{eff}} \lesssim 0.19$

finite mass effect

~ 40% enhancement

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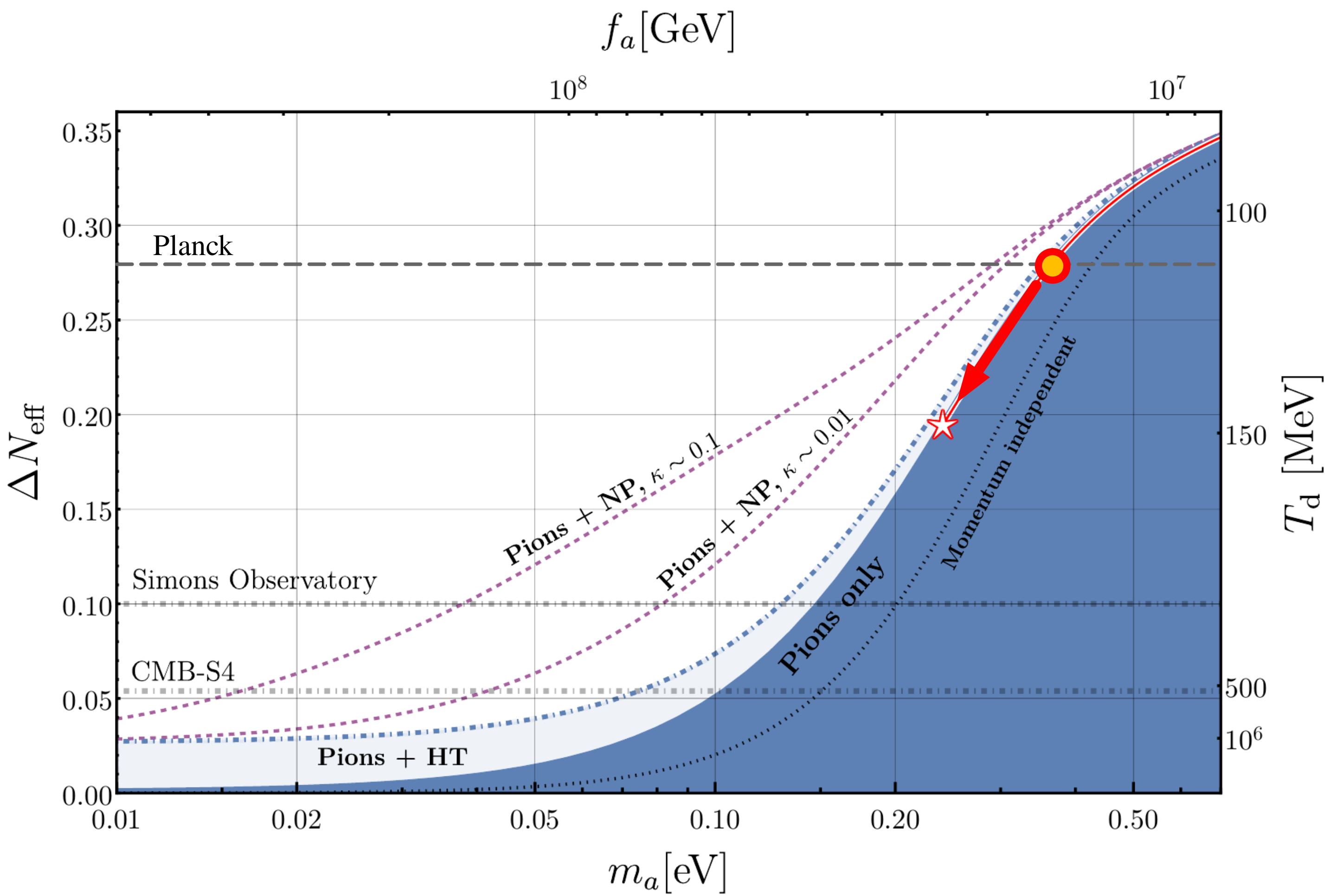

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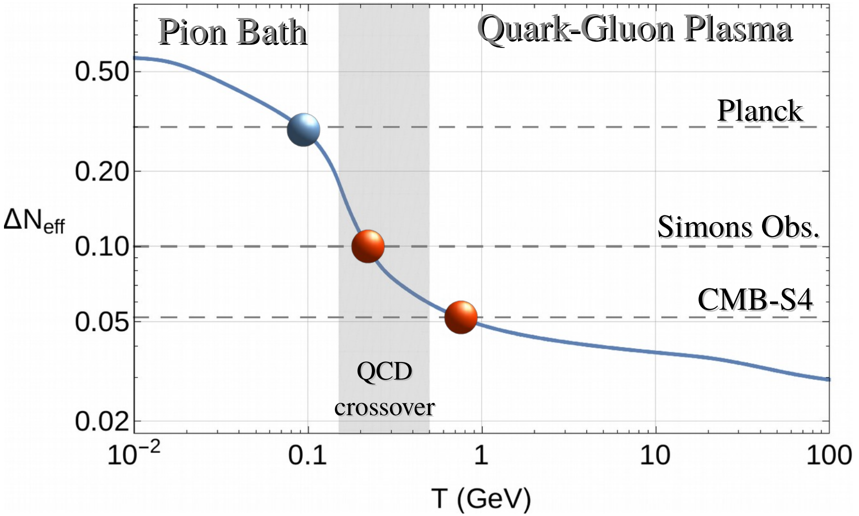

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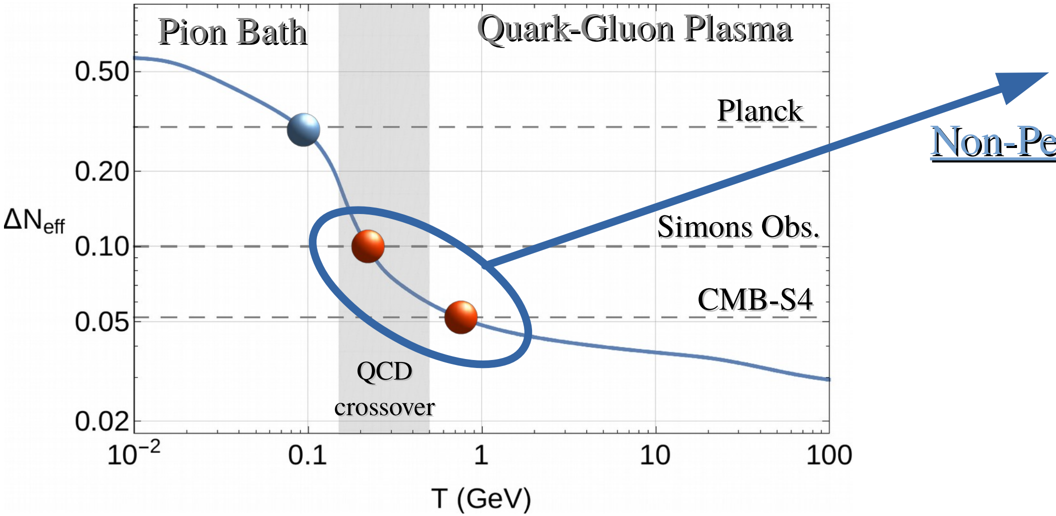

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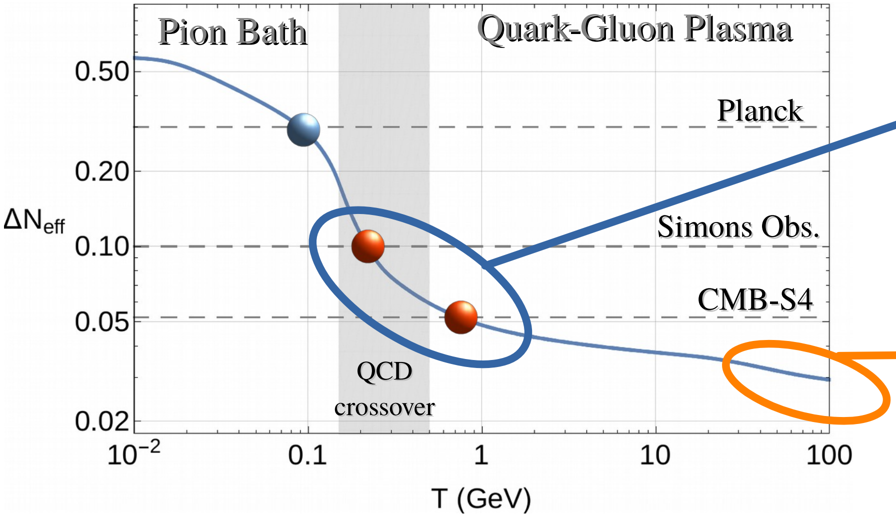
Non-Perturbative

Future Reach



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Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

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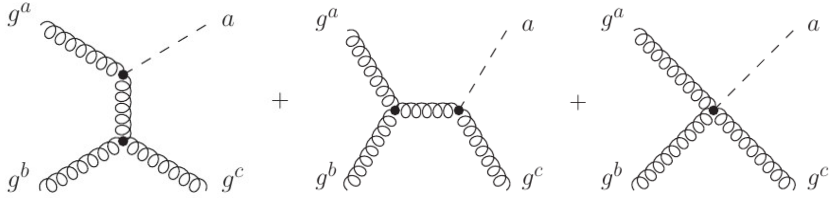
High Temperatures Regime



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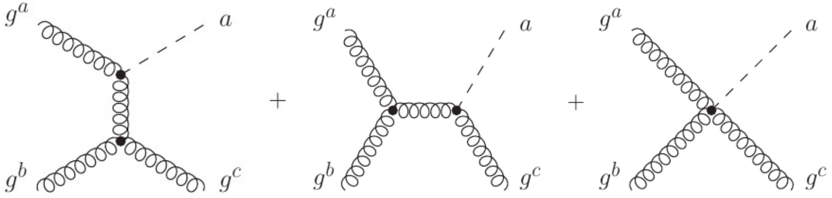

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Masso, Rota, Zsembinszki '02
Graf, Steffen '10

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for $g_s \ll 1$

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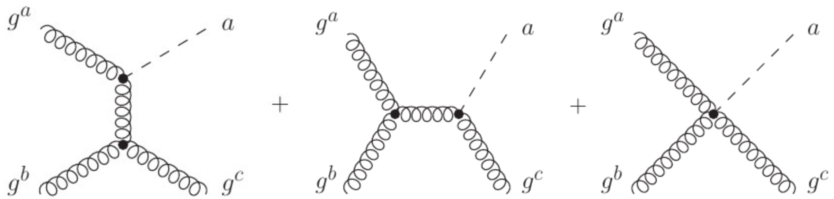

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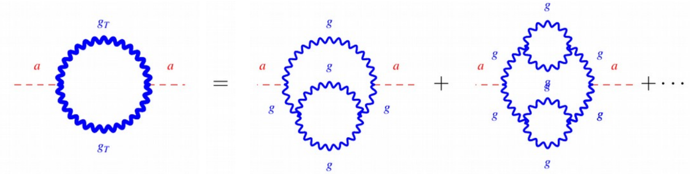
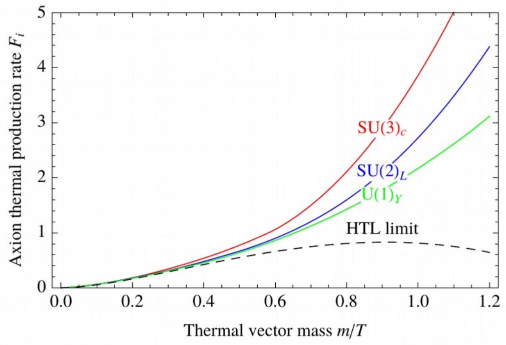
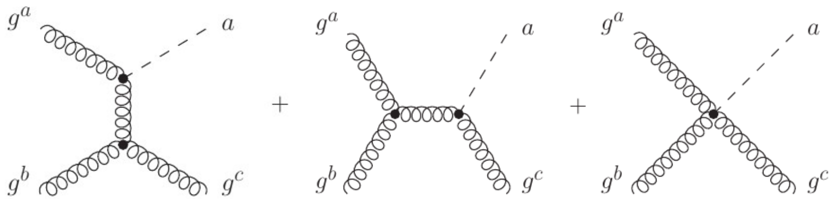
for $g_s \ll 1$

Salvio, Strumia, Xue ‘13





High Temperatures Regime



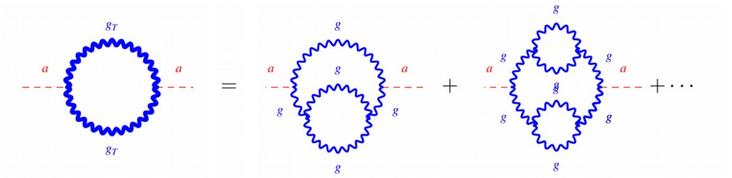
$$\overline{\Gamma}_{\text{pert}} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

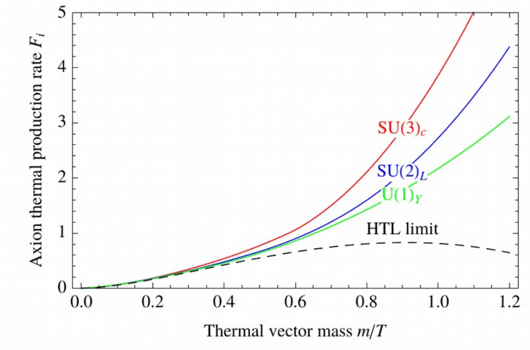
Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

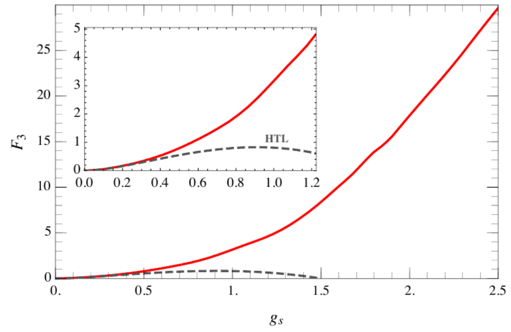
for $g_s \ll 1$

Salvio, Strumia, Xue ‘13





D’Eramo, Hajkarim, Yun ‘21



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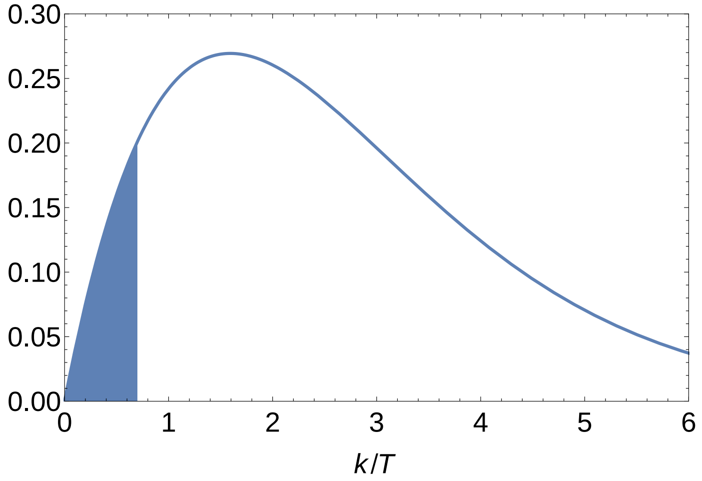

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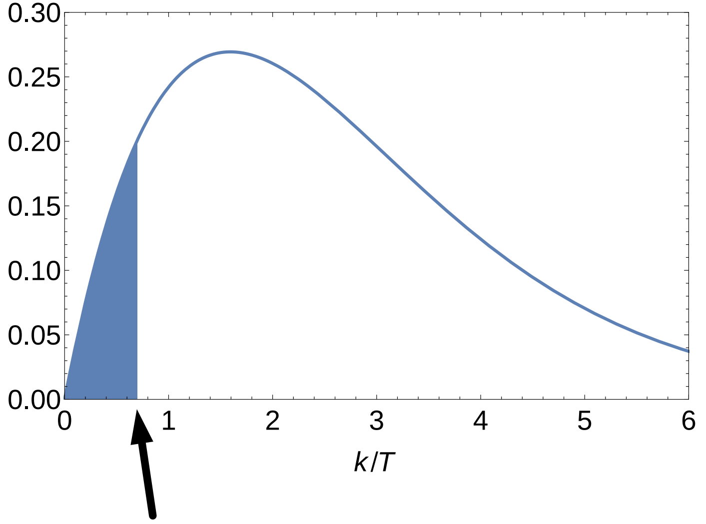
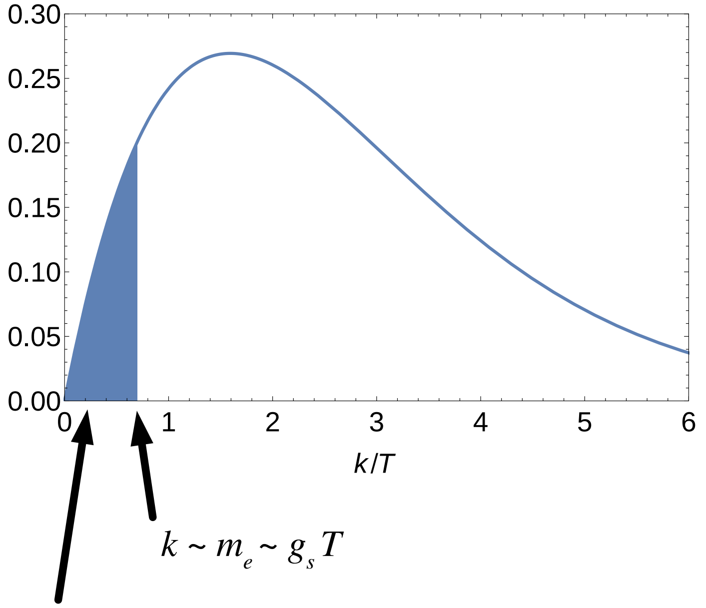


$k \sim m_e \sim g_s T$

High Temperatures Regime



$k \sim m_e \sim g_s T$

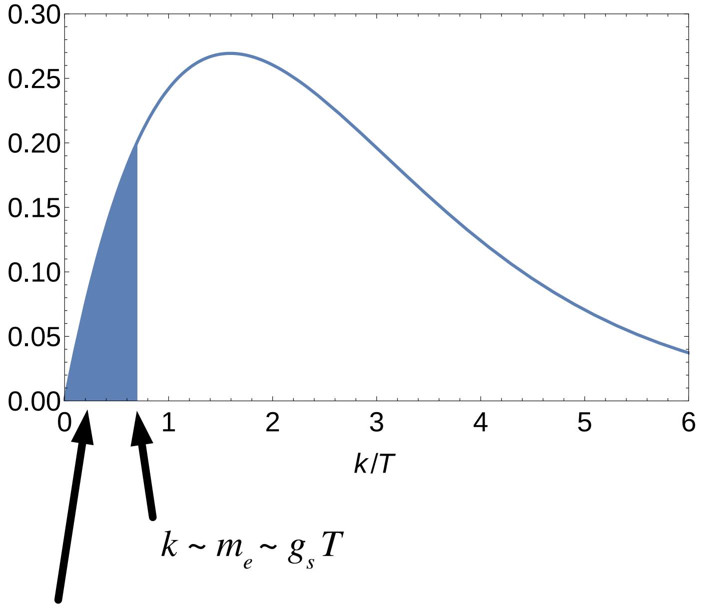
$k \sim m_m \sim g_s^2 T$

$\# \sim 1 / g_s^2$

Linde '80

Gross, Pisarski, Yaffe '81

High Temperatures Regime



$k \sim m_m \sim g_s^2 T$ Linde '80

$\# \sim 1/g_s^2$ Gross, Pisarski, Yaffe '81

@ $g_s \ll 1$:

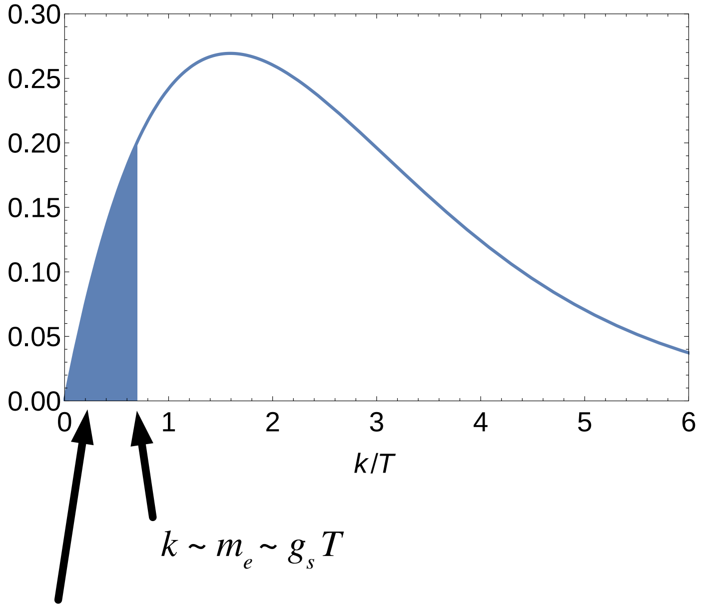
collective effects are phase-space suppressed $O(g_s^n)$

[e.g. for free energy $O(g_s^6)$]

large occupation numbers → dominated by semi-classic

[non-linear YM equations - e.g. strong sphalerons]

High Temperatures Regime



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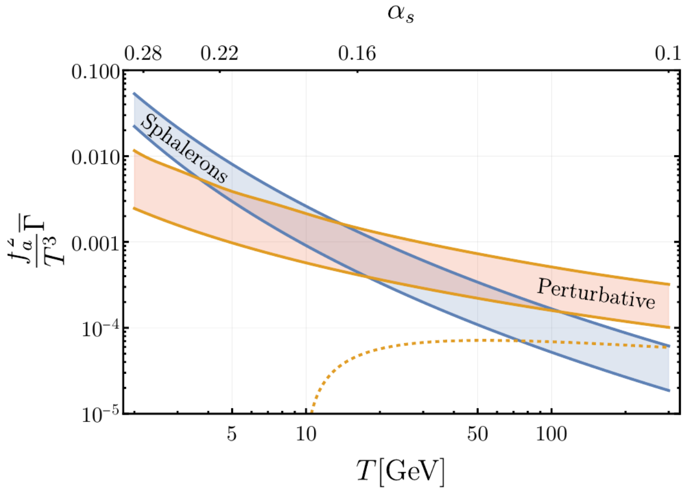
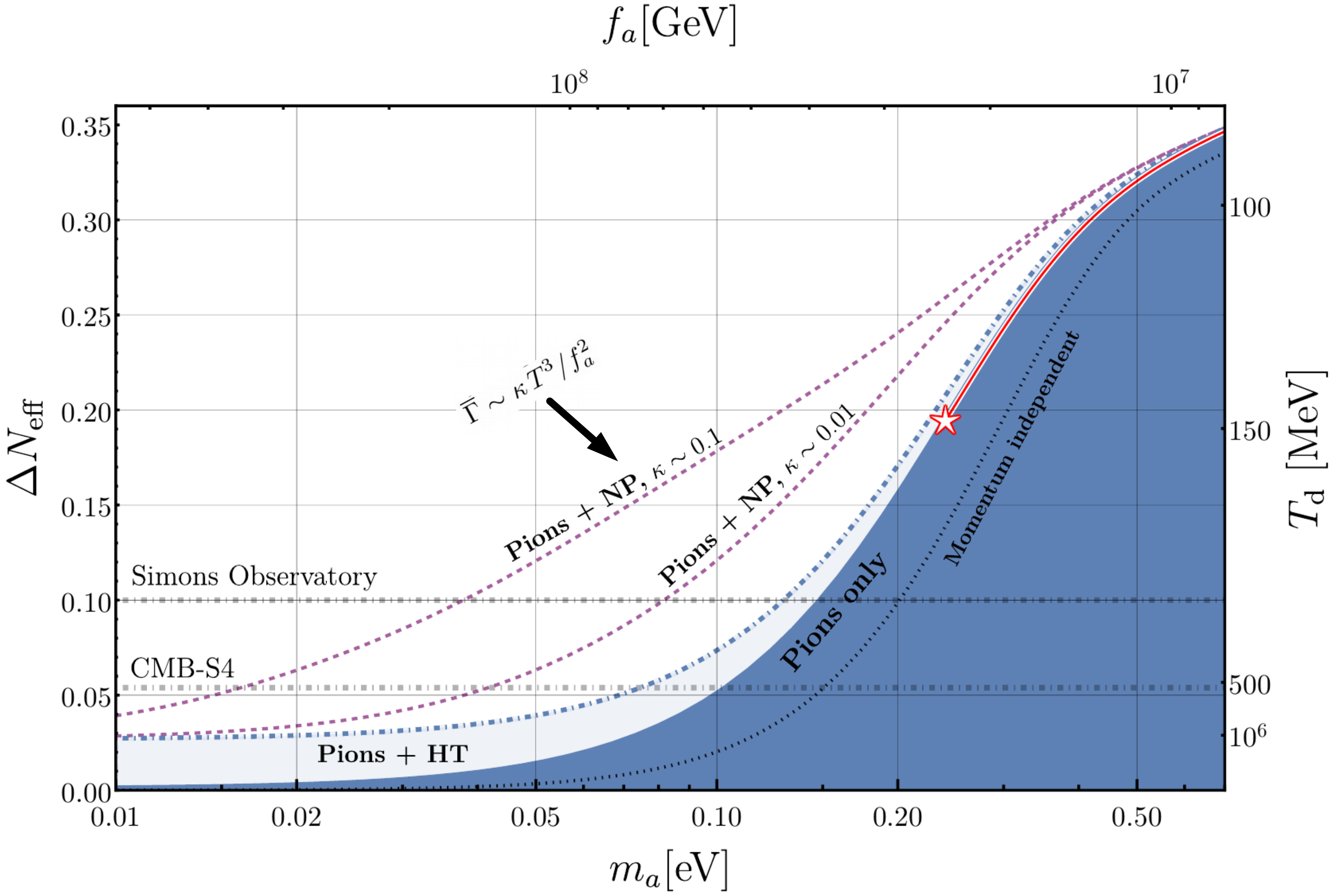


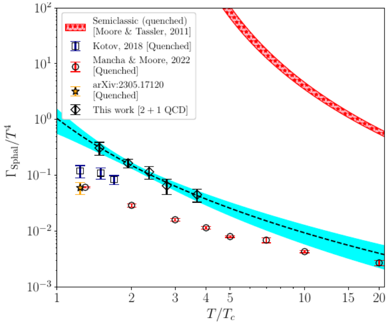
$\Gamma_{\text{sphal}} \simeq (N_c \alpha_s)^5 T^4$

Moore, Tassler ‘10

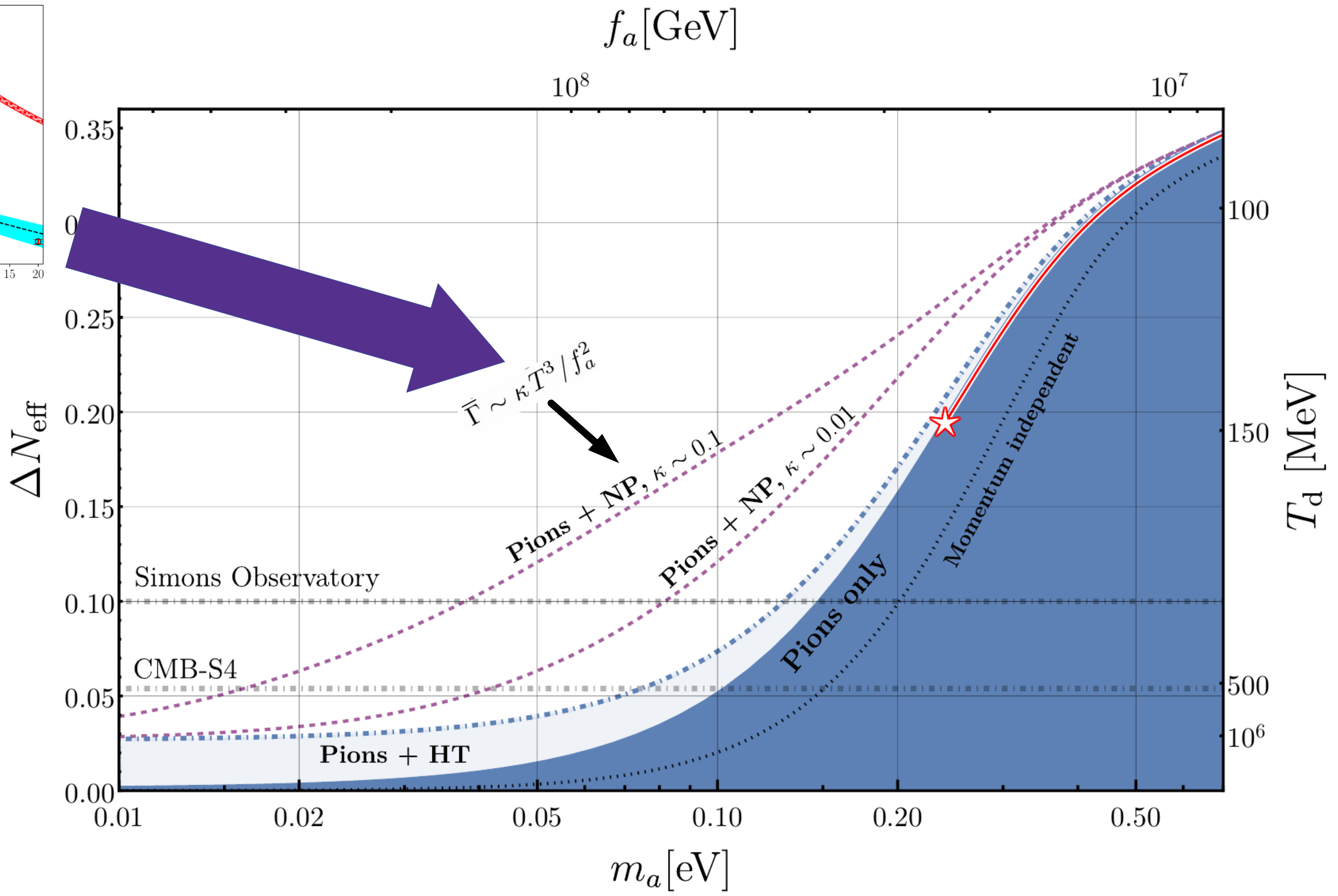
Future Reach



Future Reach



Bonanno et al.
2308.01287



Conclusions:

- More reliable upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Doubts about reliability of perturbative rates above T_c
- Non-perturbative rates crucial for interpreting upcoming CMB experiments

  Promising preliminary results from Lattice QCD...

Thanks!

Back Up

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}(1+c_3\sigma^3)}$$

$$\frac{\partial_\mu a}{2f_a} j^\mu_A \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\rm phys} + \sin(\theta_{a\pi})\,a_{\rm phys} \simeq \pi^0_{\rm phys} + \theta_{a\pi} a_{\rm phys}$$

@ all orders in χPT

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

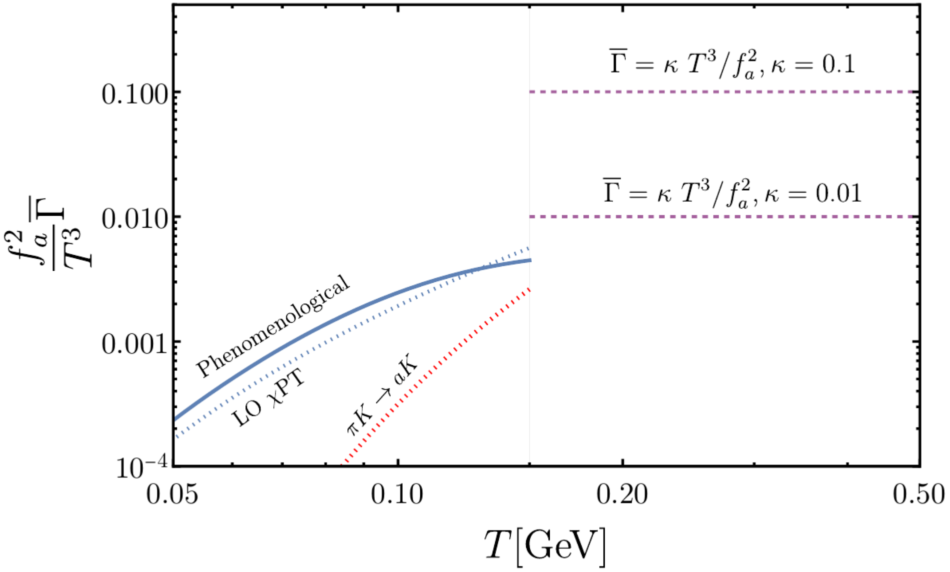
$\lesssim 10\%$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta^2_{a\pi}\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4} \qquad |\mathcal{M}^{\rm LO}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c \alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: <u>Compute Γ_k for $T > T_c$</u>

Existing Attempts (at k=0) e.g.

- Moore, Tassler '10 : Classical SU(N) simulations
- Kotov '18 : Quantum Euclidean (anal. cont.)
- Altenkort et al. '20 : Quantum Euclidean (anal. cont.)
- Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\text{sphal}} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0},0) q(\vec{x},\tau)\rangle = -\int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \rho(\omega) \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!