Production of Feebly Interacting Particles at Finite Temperature

in collaboration with

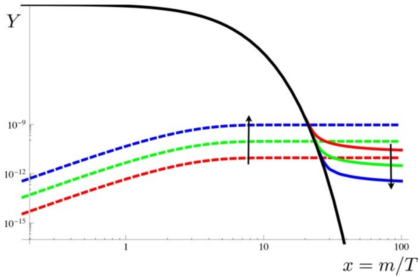
Emanuele Copello, Julia Harz, and Carlos Tamarit

based on ongoing work

supported by DFG Emmy Noether Grant No. HA 8555/1-1.

Mathias Becker

Motivation: Relevant Temperatures



Freeze-In

$$T_{f.i} \sim \frac{M}{2} - \frac{M}{5}$$

Freeze-Out

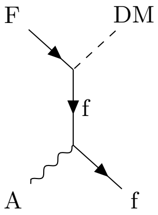
$$T_{f.o} \sim \frac{M}{25} - \frac{M}{30}$$

→ Finite Temperature Corrections relevant for Freeze-In

Motivation: Infrared Divergencies at NLO

Example: η (scalar DM), F (gauge charged Parent)

$$\mathcal{L}_{\text{int}} = \mathrm{y_{DM}}\, \bar{\mathrm{f}}_{\mathrm{SM}}\, \mathrm{F}\, \eta + \mathrm{h.c.}$$



$$\to \sigma \sim \int \mathrm{dt} |\mathcal{M}|^2 \sim \int \frac{\mathrm{dt}}{\mathrm{t}} \sim \ln\left(\frac{\mathrm{m_f}}{\mathrm{T}}\right) \Rightarrow \text{divergent for } \mathrm{m_f} \ll \mathrm{T}$$

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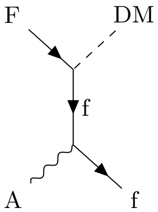

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Common Treatment: Thermal masses $\mathrm{m_f} \rightarrow \mathrm{m_f(T)} \sim \mathrm{T}$

see for instance [Belanger et. al(2020)],[No et. al(2020)],[Calibbi et. al(2021)]

What do we do?

$\rightarrow$ Calculate the DM production rate in the real time formalism of thermal QFT

$\rightarrow$ We consider DM feebly coupled to a gauge charged Parent (neglecting potential Yukawa or quartic interactions)

$\rightarrow$ Compare our results to:

- Thermal QFT calculations in Hard Thermal Loop approximation
- Boltzmann approach employing scattering rates regulated with thermal masses

Closed Time Path (Keldyish-Schwinger/real time) formalism

Construct 2 particle irreducible (2PI) effective action Γ

Find equations of motion $\frac{\delta\Gamma}{\delta\Delta} = \frac{\delta\Gamma}{\delta S} = 0$

Kadanoff-Baym equations

Kadanoff-Baym Ansatz

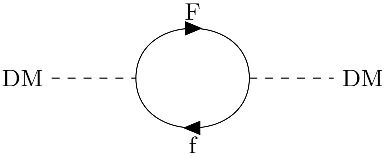
Boltzmann-like evolution equation

for details see [Garbrecht(2018)],[Berges(2004)],[Prokopec et. al(2003)]

DM Time Evolution

$$\dot{n}_{\mathrm{DM}} + 3Hn_{\mathrm{DM}} = \gamma_{\mathrm{DM}} \sim \int d^3p \frac{\Pi^{\mathcal{A}}_{\mathrm{DM}}}{E_{\mathrm{DM}}} f_{\mathrm{DM}}(E_{\mathrm{DM}})$$

Spectral Self-Energy $\Pi^{\mathcal{A}}_{\mathrm{DM}} = -\mathrm{Im}\Pi^{R}_{\mathrm{DM}} =$



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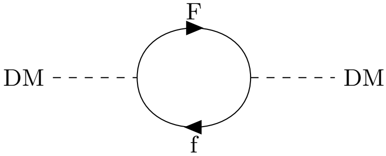

DM Self-Energy

$$\Pi^{\mathcal{A}}_{\mathrm{DM}}\left(P\right) \sim \int dK \, S^{\mathcal{A}}_{F}\left(K\right) S^{\mathcal{A}}_{f}\left(K-P\right) + \text{higher order contributions}$$

Level of approximation depends on

- Loop order at which $\Pi^{\mathcal{A}}_{\mathrm{DM}}$ is evaluated
  $\rightarrow$ LO (this work), NLO, ..
- Which propagators are used to derive $\Pi^{\mathcal{A}}_{\mathrm{DM}}$
  $\rightarrow$ Tree Level, perturbative 1-Loop, HTL approximated resummed, fully resummed (this work)

Example: Tree-Level Propagators

Spectral Propagator(Tree-Level)

$$\not{S}^{\mathcal{A}}_{\mathrm{F/f}} \sim (\not{\mathrm{k}} + \mathrm{m}_{\mathrm{F/f}})\delta\left(\mathrm{k}^2 - \mathrm{m}^2_{\mathrm{F/f}}\right)$$

Implies a dispersion relation $\mathrm{k}^0 = \pm\sqrt{|\vec{\mathrm{k}}|^2 + \mathrm{m}^2_{\mathrm{F/f}}}$

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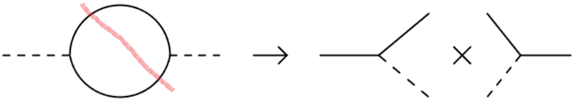
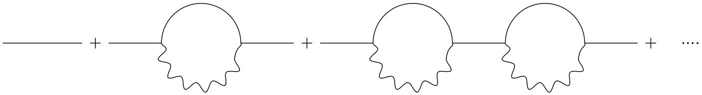


$\Rightarrow$ recovers Boltzmann equation for a tree-level decay!

Resummed Propagators

We resum the gauge boson contribution (remember $y_{\mathrm{DM}} \ll g$)



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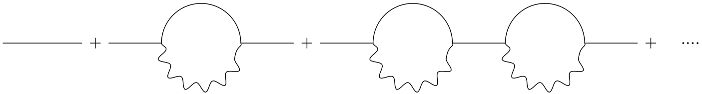
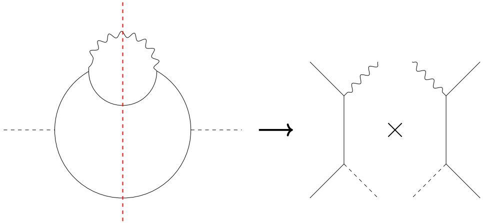
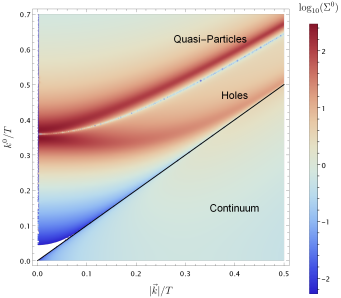


$\rightarrow$ scattering contributions also arise at leading order



Form of the spectral propagator with a narrow width

$$\$^{\mathcal{A}} \overset{\Gamma \ll k, \Sigma^{\mathcal{H}}}{\sim} \frac{\Gamma}{\left((k-\Sigma^{\mathcal{H}})^2 - m^2\right)^2 + \Gamma^2} \overset{\Gamma \to 0}{\sim} \delta\left((k-\Sigma^{\mathcal{H}})^2 - m^2\right)$$



- Much simpler analytic form in the Hard Thermal Loop (HTL) approximation. But only reliable for $T \gg m$!
- Previous calculations assume the HTL [Garbrecht et. al(2019)] or interpolate between HTL and non-relativistic results [Biondini et. al(2020)]

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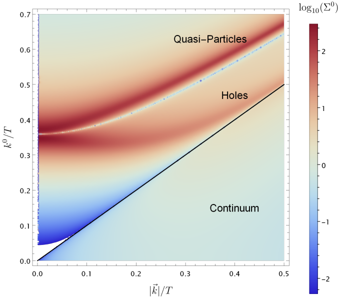

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Remember: Freeze-In occurs around $T \sim M \Rightarrow$ Accuracy in intermediate regime required.

Results

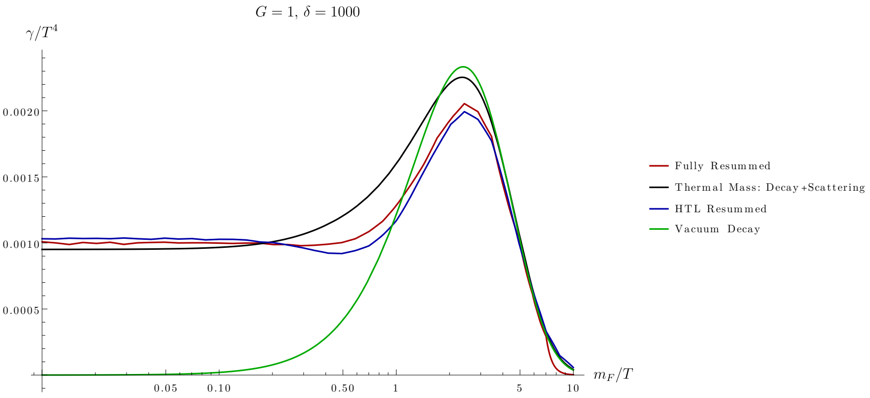
We compare

- Fully resummed results (our work in progress)
- Hard Thermal Loop resummed results
- Boltzmann Equations with decays and scatterings regulated by thermal masses
- Boltzmann Equations with only decays and in vacuum masses (Tree-Level Vacuum)

in terms of two relevant parameters

- the effective gauge coupling G
- the mass splitting between the parent F and DM $\delta = 1 - \mathrm{m_{DM}}/\mathrm{m_F}$

Preliminary Results (Interaction Rate)



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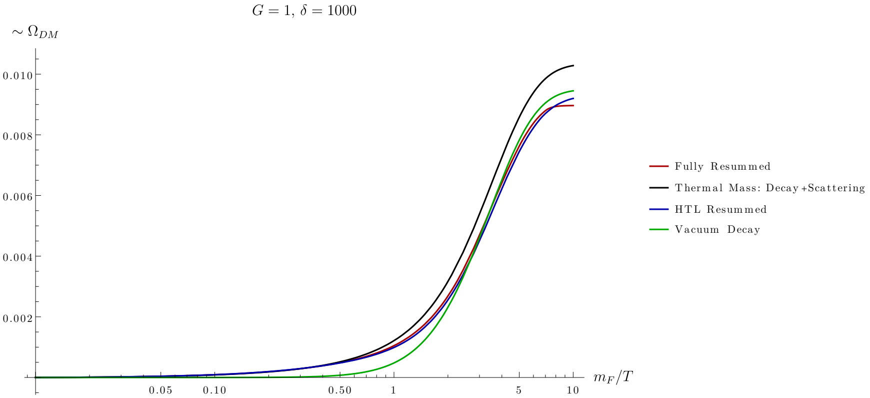

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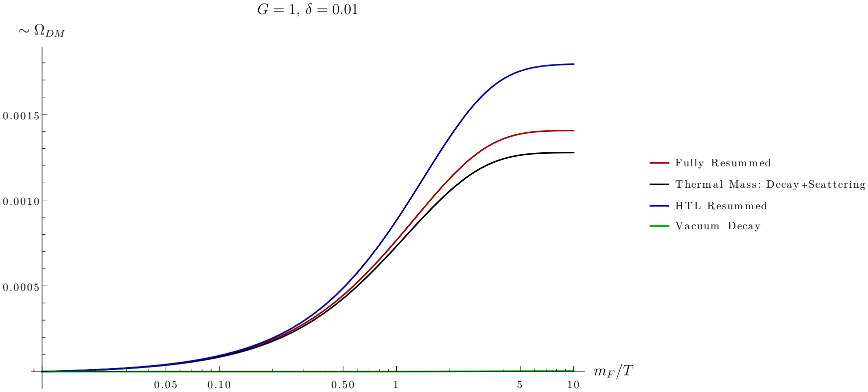

Upcoming Workshop

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Conclusions

Finte temperature corrections are relevant for freeze-in as $T_{f.i} \sim M$.

$\rightarrow$ We compare finite temperature QFT results using complete 1PI resummed propagators with

- a Boltzmann approach regularizing IR divergencies with thermal masses
- finite temperature results using the Hard Thermal Loop approximation

Our preliminary results indicate:

- For large mass splittings between Dark Matter and its Parent we find a $\sim +15\,\%$ difference of the thermal mass regulated and our result.
- For small mass splittings the thermal mass approach only differs by $\sim -10\,\%$