$SO(10) \times S_4$ Grand Unified Theory of Flavour and Leptogenesis

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in collaboration with:

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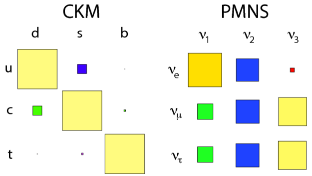
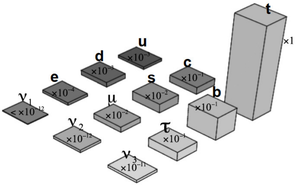
- **Flavour problem**:
  The origin of the three families of quarks and leptons with their pattern of masses and mixing.





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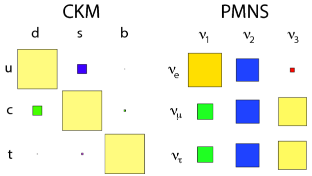

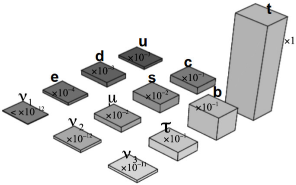

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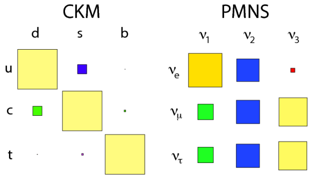

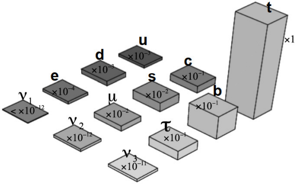

- **Neutrino mass mechanism**

- **Baryon Asymmetry of the Universe**:

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = (6.1 \pm 0.1) \times 10^{-10}$$

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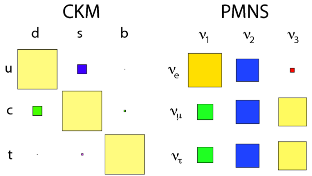

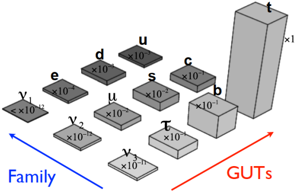

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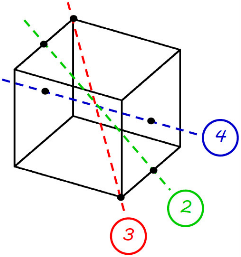
SUSY FLAVOUR GUTS

- S_4: Symmetric group of permutations of 4 objects $\cong$ group of rotational symmetries of a cube.
  Broken spontaneously by scalar fields, **flavons** ϕ



$$\mathcal{L} \sim \frac{1}{\lambda}\phi H\bar{\psi}\psi \to \frac{\langle\phi\rangle}{\lambda}H\bar{\psi}\psi \to yH\bar{\psi}\psi$$

Yukawa parameters are given a dynamical origin.

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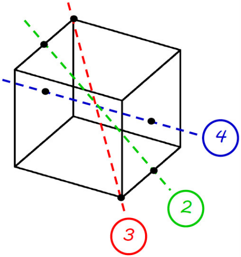

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- $SO(10)$: A complete family of quarks and leptons fits into a single **16** representation

  Including the Right Handed Neutrino (RHN) $\to$ type-I seesaw mechanism

The flavon superpotential fixes the symmetry breaking flavon VEVs

$$\langle \phi_1 \rangle = v_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \langle \phi_2 \rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle \phi_3 \rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

"CSD2 vacuum alignment"

VEVs are driven to scales with the hierarchy

$$v_1 \ll v_2 \ll v_3$$

$$v_1 \simeq 0.001 M_{GUT}, \quad v_2 \simeq 0.1 M_{GUT}, \quad v_3 \simeq M_{GUT}$$

CP spontaneously broken by the complex VEVs of the flavons.

Up-type quarks and neutrinos Yukawa matrices $Y_{ij} \sim \langle\phi_i\rangle\langle\phi_j\rangle^T$ with an universal structure

$$Y^{u,\nu} = y_1^{u,\nu} e^{i\eta} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + y_2^{u,\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{u,\nu} e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The right-handed (RH) neutrino mass M^R has the same structure as the Y^ν.

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- **Natural** understanding of the hierarchical Yukawa couplings:

$$y_u \simeq v_1^2/M_{\rm GUT}^2 \simeq 10^{-6} \quad y_c \simeq v_2^2/M_{\rm GUT}^2 \simeq 10^{-2} \quad y_t \simeq v_3^2/M_{\rm GUT}^2 \simeq 1$$

RH neutrino parameters

$$M_1^R \simeq 10^7 \text{GeV} \quad M_2^R \simeq 10^{11} \text{GeV} \quad M_3^R \simeq 10^{13} \text{GeV}$$

Down-type quarks and charged leptons couple to a second Higgs H_{10}^d, with a new mixed term involving $Y_{12} \sim \langle\phi_1\rangle\langle\phi_2\rangle^T$

$$Y^{d,e} = y_{12}^{d,e} e^{i\frac{\eta}{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 0 \end{pmatrix} + y_2^{d,e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{d,e} e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y^P e^{i\gamma} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

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- It leads to a **milder hierarchy** in the down and charged lepton sectors.

$$y_{12}^{d,e} \simeq \cos\beta \frac{v_1 v_2}{M_{\mathrm{GUT}}^2} \simeq 10^{-5} \quad y_2^{d,e} \simeq \cos\beta \frac{v_2^2}{M_{\mathrm{GUT}}^2} \simeq 10^{-2} \quad y_3^{d,e} \simeq \cos\beta \frac{v_3^2}{M_{\mathrm{GUT}}^2} \simeq 1$$

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- This new term enforces a **texture zero** in the (1,1) element of Y^d, giving the GST relation for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$.

- The light neutrino Majorana matrix, after **seesaw**, will also have the CSD2 structure

$$m^{\nu} = \mu_1^{\nu} e^{i\eta} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \mu_2^{\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_3^{\nu} e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where the parameters μ_i are given by

$$\mu_i = v_u^2 \frac{(y_i^{\nu})^2}{M_i^R}$$

- Flavons yield to **normal hierarchy** $m_1 \ll m_2 \ll m_3$.

- χ^2 test function to find the best fit

$$\chi^2 = \sum_n \left(\frac{P_n(x) - P_n^{\text{obs}}}{\sigma_n} \right)^2$$

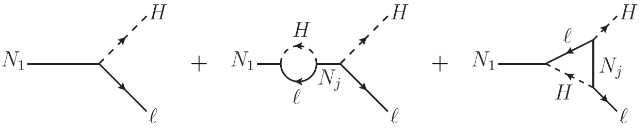
19 observables given by $\{\theta_{ij}^q, \delta^q, y_{u,c,t}, y_{d,s,b}, \theta_{ij}^\ell, \delta^l, y_{e,\mu,\tau}, \Delta m_{ij}^2\}$.

- After seesaw, **15 effective parameters** $x = \{y_i^u, y_i^d, y_i^e, y^P, \mu_i, \eta', \gamma\}$.
- The best fit found has a $\chi_\nu^2 = \chi^2/\nu \simeq 3$ where $\nu = n - n_i = 4$ d.o.f.

- Baryon Asymmetry of the Universe (BAU)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.1) \times 10^{-10}$$

- Asymmetry can be generated through CP breaking decays of heavy RHNs into leptons, then converted into baryons through sphalerons.



The baryon asymmetry is

$$\eta_B \simeq \sin\eta' \frac{3}{8\pi} \frac{\alpha_{sph}}{N_\gamma^{rec}} \; \kappa\left(\frac{\mu_2}{m_\star^{MSSM}}\right) \frac{\mu_3 M_2}{v^2}$$

Using the parameters from the fit, the correct BAU is generated when

$$M_2 \simeq 1.9 \times 10^{11} \text{ GeV}$$

This is the natural value for the second RHN mass: the model naturally explains the origin of the BAU through N_2 leptogenesis without any need of tunning!!

Matter fields unified into a single representation $(3', 16)$ of $S_4 \times SO(10)$

- **Simple**: minimal field content and low-dimensional representations.
- **Natural**: matter hierarchies explained by the flavon VEVs when setting all the GUT scale parameters to be $\sim \mathcal{O}(1)$.
- **Predictions**: neutrino masses, normal neutrino mass ordering, CP oscillation phase $\delta^l \sim 200°$.

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If you want to know more...

$\mathbf{SO(10)} \times \mathbf{S_4}$ Grand Unified Theory of flavour and Leptogenesis

arXiv:1710.03229

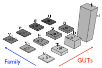
Francisco J. de Anda, Stephen F. King, **Elena Perdomo*** *e.perdomo-mendez@soton.ac.uk

Motivation

Flavour problem
Origin of the three families of quarks and leptons. Very hierarchical charged fermion masses, small and hierarchical quark mixing, small neutrino masses and large lepton mixing.

Family symmetry
A non-Abelian discrete symmetry imposes constraints on the Yukawa couplings and reproduces precise predictions for masses and mixing. S_4 enforces CSD(2).

Grand Unified Theory
Unifies fermions within each family and reproduces an universal mass matrix structure, predicting relationships between quark and lepton Yukawa matrices.



Unified model of flavour

- We present a model with quarks and leptons unified in a single ψ representation of $\mathbf{SO(10)} \times \mathbf{S_4}$.
- The essential superfields are given in the table below. We only allow small Higgs representations **10**, **16** and **45**.

Field	Representation S_4	$SO(10)$	$\mathbb{Z}_4^R$	
ψ	$3'$	16	1	Quarks and leptons
$H_{10}^{u,d}$	1	10	0	Break electroweak symmetry
$H_{\overline{16},16}$	1	$\overline{16}$	0	Break SO(10) and give RH Majorana masses
$H_{45}^{X,Y,W,Z}$	1	45	0	Separate quarks and lepton masses
H_{45}^{B-L}	1	45	2	Gives DT splitting via DW mechanism
ϕ_i	$3'$	1	0	Acquire CSD(2) vacuum alignments

- The discrete symmetry $\mathbb{Z}_4^R$ is broken at the GUT scale by the H_{45}^{B-L} VEV to $\mathbb{Z}_2^R$, the usual R parity in the MSSM.

CSD(2) flavon vacuum alignments

The Yukawa parameters are given a dynamical origin

$$\mathscr{L} \sim \frac{1}{\lambda}\phi H \bar{\psi}\psi \to \frac{\langle\phi\rangle}{\lambda} H \bar{\psi}\psi \to y H \bar{\psi}\psi,$$

where the flavon fields break S_4 with the CSD(2) vacuum alignment [1]

$$\langle\phi_1\rangle = v_1 \begin{pmatrix}1\\0\\2\end{pmatrix}, \quad \langle\phi_2\rangle = v_2 \begin{pmatrix}0\\1\\-1\end{pmatrix}, \quad \langle\phi_3\rangle = v_3 \begin{pmatrix}0\\1\\0\end{pmatrix}.$$

VEVs driven to scales with the hierarchy $v_1 \ll v_2 \ll v_3 \sim M_{\mathrm{GUT}}$.

Yukawa matrices

- Up-type quarks and neutrinos couple to one Higgs H_{10}^u, leading to Yukawa matrices $Y_{ij} \sim \langle\phi_i\rangle\langle\phi_j\rangle^T$ with an universal structure

$$Y^{u,\nu} = y_1^{u,\nu}\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + y_2^{u,\nu}\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + y_3^{u,\nu}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}.$$

Natural understanding of the hierarchical Yukawa couplings
$y_u \sim v_1^2/M_{\mathrm{GUT}}^2$, $y_c \sim v_2^2/M_{\mathrm{GUT}}^2$, $y_t \sim v_3^2/M_{\mathrm{GUT}}^2$.

- Down-type quarks and charged leptons couple to a second Higgs H_{10}^d, with a new mixed term involving $Y_{12} \sim \langle\phi_1\rangle\langle\phi_2\rangle^T$

$$Y^{d,e} = y_{12}^{d,e}\begin{pmatrix}0&1&1\\1&4&2\\1&2&0\end{pmatrix} + y_2^{d,e}\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + y_3^{d,e}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} + y^P\begin{pmatrix}0&0&1\\0&2&0\\1&0&0\end{pmatrix}.$$

This new term enforces a zero in the (1,1) element of Y^d, giving the GST relation [2] for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$. It also leads to a milder hierarchy in the down and charged lepton sectors.

Seesaw mechanism

The right-handed neutrino (RHN) mass M^R has the same structure as Y^ν. The light neutrino mass matrix is obtained by the **type-I seesaw mechanism** [3,4] and will also have the CSD(2) structure

$$m^\nu = \mu_1^\nu\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + \mu_2^\nu\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \mu_3^\nu\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}.$$

The parameters μ_i are given in terms of the parameters y_i^ν and M_i^R simply by

$$\mu_i = v_u^2\frac{(y_i^\nu)^2}{M_i^R}$$

The flavons yield a light neutrino mass matrix m^ν, where the normal hierarchy $m_1 \ll m_2 \ll m_3$ after seesaw is due to the very hierarchical RHN masses.

Numerical fit

The model accurately fits all available quark and lepton data, with 15 input parameters to fit 19 data points and a reduced $\chi_\nu^2 \approx 3$. It predicts **normal neutrino hierarchy** and a CP phase δ^l

$$\delta^l \sim 200^\circ$$

The neutrino masses are also predicted

$$m_1 \approx 10.94\ \mathrm{meV}, \quad m_2 \approx 13.95\ \mathrm{meV}, \quad m_3 \approx 51.42\ \mathrm{meV}.$$

The model predicts significant deviation from both zero and maximal CP violation.

N_2 Leptogenesis

- Baryon Asymmetry of the Universe (BAU)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.1) \times 10^{-10}$$

- Asymmetry generated through CP breaking decays of heavy RHNs into leptons, then converted into baryons through sphalerons [5].
- Leptogenesis generated mainly by the decays of the second RHN "N_2 leptogenesis".
- Using the parameters from the fit, the correct BAU is generated when

$$M_2 \simeq 1.9 \times 10^{11}\ \mathrm{GeV},$$

natural expected value for the second RHN mass.

Conclusion

Simple	**Natural**	**Complete**
Minimal field content	No tunning of $\mathcal{O}(1)$ parameters	Renormalisable
Low-dimensional representations	**Predictions**	Reduces to MSSM
CSD(2) from S_4	Neutrino masses	μ term of $\mathcal{O}(TeV)$
	Normal Hierarchy	DT splitting
	$\delta^l \sim 200^\circ$	Proton decay suppressed

[1] S. Antusch, S. F. King, C. Luhn and M. Spinrath [arXiv:1108.4278]
S. Antusch, S. F. King and M. Spinrath [arXiv:1301.6764]
[4] T. Yanagida [hep-ph/9809459]
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