Strong CP Problem: Models with Exotic Color Groups

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Invisibles18 Workshop

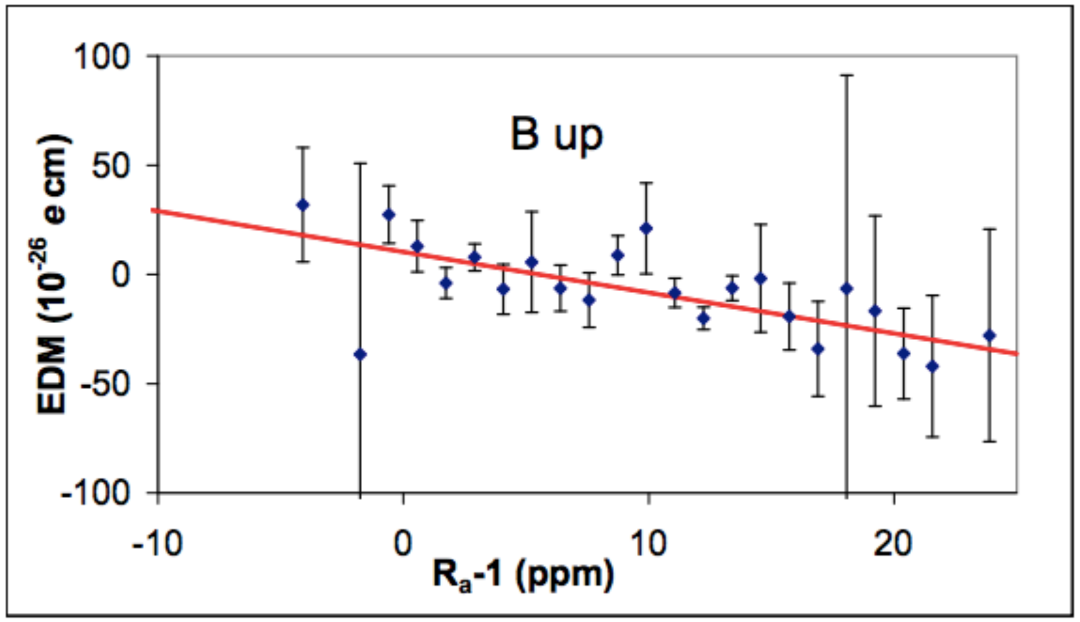
Color Unified Dynamical Axion

M. K. Gaillard, M. B. Gavela, R. del Rey, P. Quilez

arXiv:1805.06465

The Strong CP Problem and Axions

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \frac{g^2\theta}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \bar{q}Mq$$



C. A. Baker *et al.* "An improved experimental limit on the electric dipole moment of the neutron." Phys. Rev. Lett. 97 (2006). hep-ex/0602020

$$\bar{\theta} = \theta + \arg\det M \qquad \bar{\theta} < 10^{-10}$$

- Dynamical $U(1)_{PQ}$ solution results in the axion

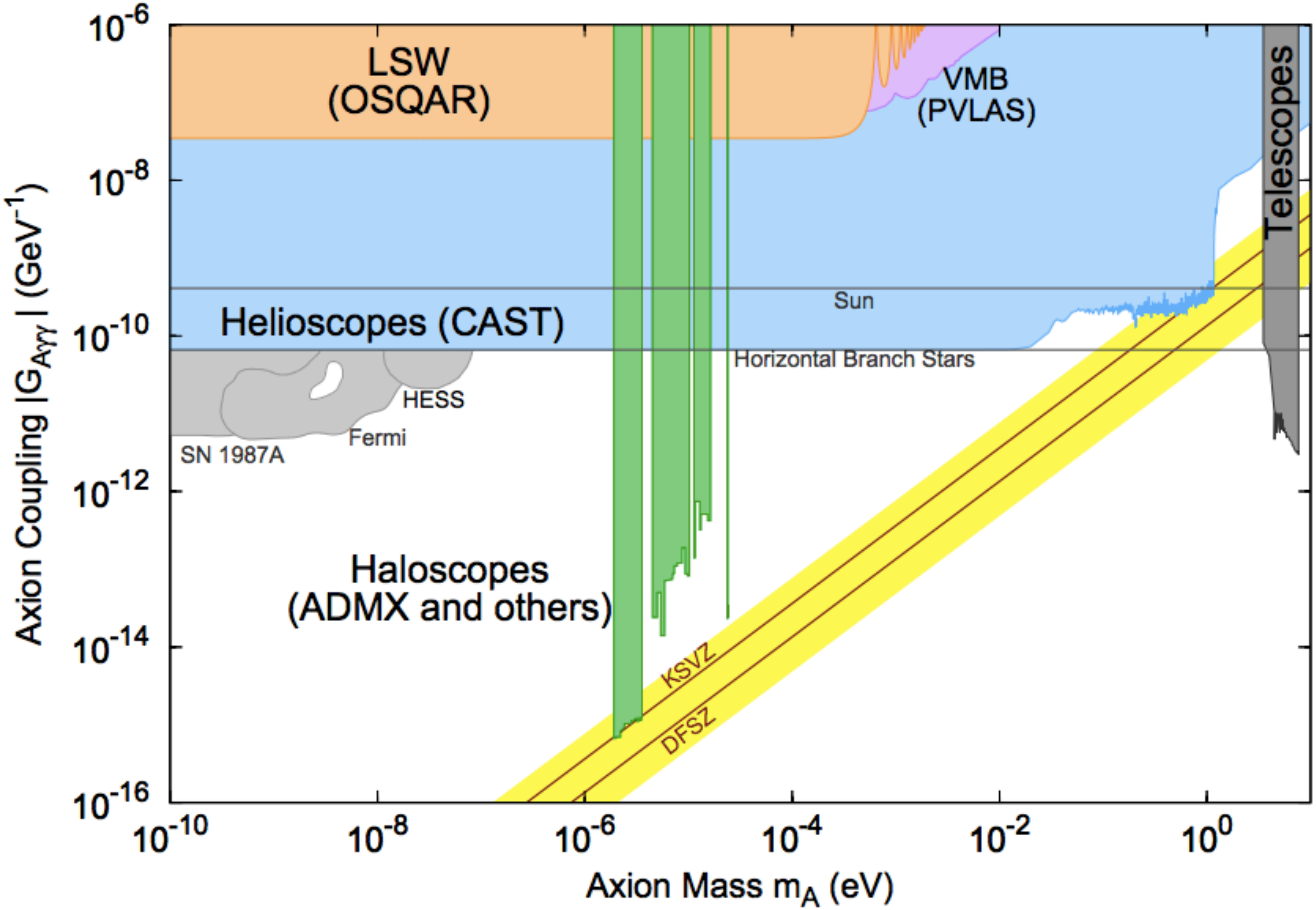
R. Peccei, H. Quinn, "CP Conservation in the Presence of Instantons," Phys. Rev. Lett. 38 (1977)

$$\mathcal{L} \ni \frac{g^2}{32\pi^2}\frac{a}{f_a}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

F. Wilczek, "The Problem of Strong P and T Invariance in the Presence of Instantons," Phys. Rev. Lett. 40 (1978)

S. Weinberg, "A New Light Boson?" Phys. Rev. Lett. 40 (1978)

Invisible Axion Parameter Space

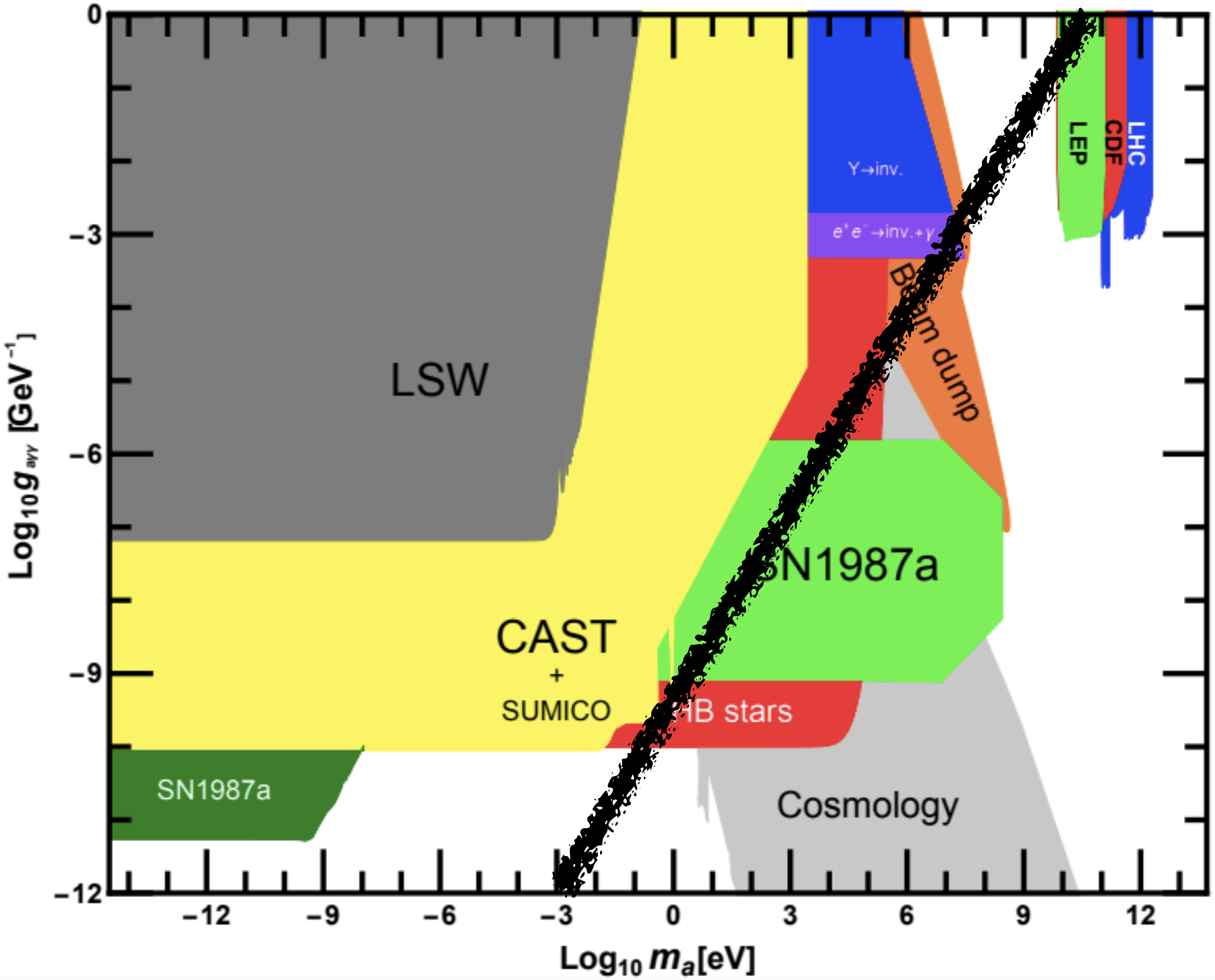


J. Kim, Phys. Rev. Lett. 43 (1979), M. Shifman, A. Vainstein, V. Zakharov, Nucl. Phys. B166 (1980)

M. Dine, W. Fischler, M. Srednicki, Phys. Lett. 104B (1981), Zhitnitsky, Sov. J. Nucl. Phys. 31 (1980)

A. Ringwald, L. J. Rosenberg, G. Rybka, “Axions and Other Similar Particles,” Particle Data Group (2017)

Invisible Axion Parameter Space



Sketch of the invisible axion expected region

J. Jaeckel, M. Spannowsky, "Probing MeV to 90 GeV axion-like particles with LEP and LHC," Phys. Lett. B753 (2016)

I. Brivio, "Axion-Like-Particles EFT & collider signals," HEFT 2017

Strong CP Problem and Massless Quarks

Under a chiral rotation:

$$\mathcal{L} \ni -m_q \bar{q} q - \theta \frac{g^2}{32\pi^2} G\tilde{G}$$

$$\rightarrow -m_q \bar{q} e^{2i\gamma_5 \alpha} q - (\theta - 2\alpha) \frac{g^2}{32\pi^2} G\tilde{G}$$

If $m_q = 0$, then this rotation is just a shift $\theta \rightarrow \theta - 2\alpha$

G. 't Hooft. "Computation of the Quantum Effects Due to a Four-Dimensional Pseudoparticle" Phys. Rev. D14 (1976)

➡ θ can be removed by field redefinition

➡ No longer physical

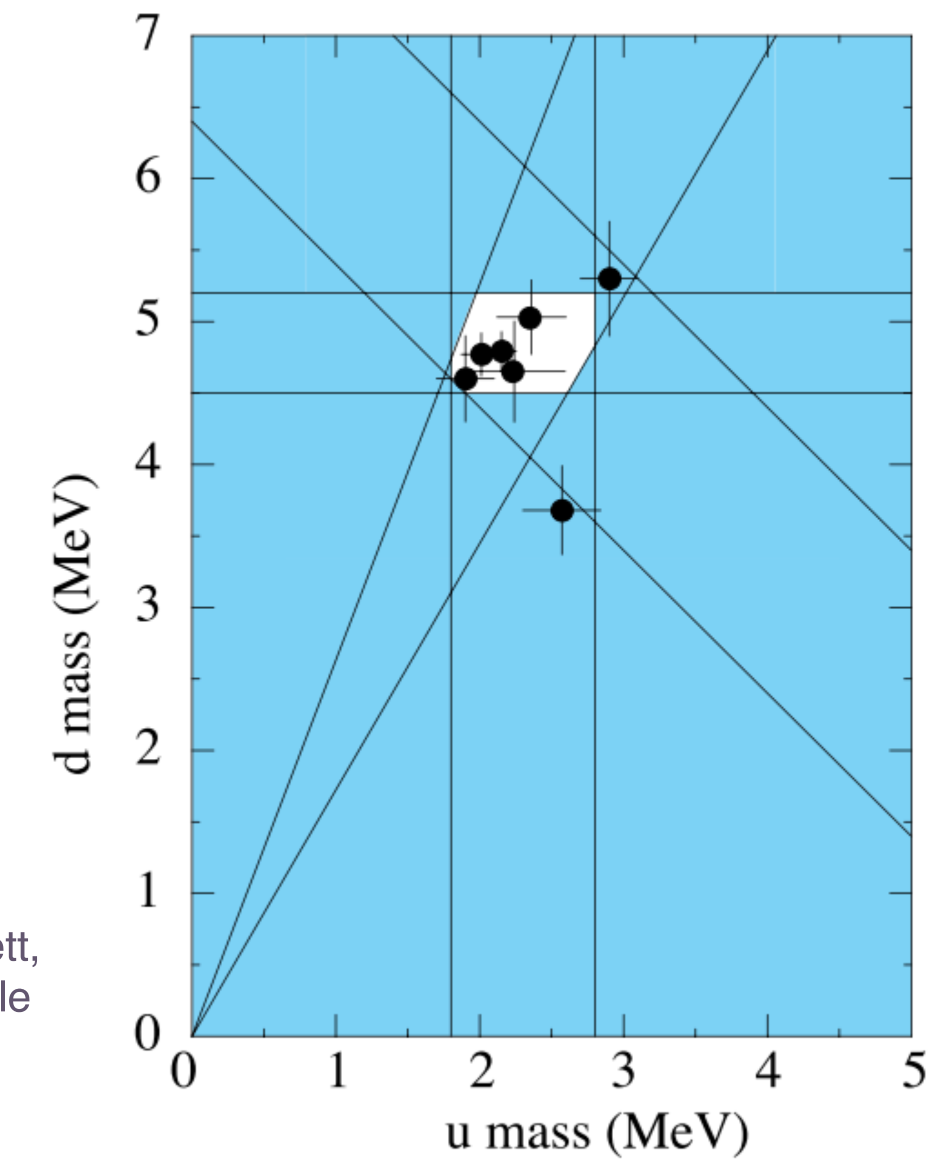
➡ Strong CP Problem solved

Massless Quark Solution

- The simplest solution is to make the up quark massless in the UV, and have it obtain its apparent mass via QCD confinement
- Disfavored by lattice data

S. Aoki, *et al*, “Review of lattice results concerning low-energy particle physics,” Eur. Phys. J. C77 (2017), arXiv/1607.00299

- Why not an exotic quark?
- Must be somehow hidden



A. V. Manohar, C. T. Sachrajda, R. M. Barnett, “Quark Masses,” Particle Data Group (2016)

Reasons to Extend the Strong Sector

(1) Additional color interactions can alter the m_a, f_a relationship

$$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\text{new}}^4$$

Dimopoulos, Susskind, “A Technicolored Solution to the Strong CP Problem” (1979)

Tye, “A Superstrong Force With a Heavy Axion” (1981)

Rubakov, “Grand unification and heavy axion” (1997)

Berezhiani, Gianfagna, Giannotti, “Strong CP problem and mirror world: The Weinberg-Wilczek axion revisited” (2000)

Fakuda, Hariyaga, Ibe, Yanagida, “Model of visible QCD axion” (2015)

Ghergetta, Nagata, Shifman, “A Visible Axion from an Enlarged Color Group” (2016)

Dimopoulos, Hook, Huang, Marques-Tavares, “A collider observable QCD axion” (2016)

Agrawal, Howe. “Factoring the Strong CP Problem” (2017)

(2) Hide massless quark in bound states

$$\sim \Lambda_{\text{new}}$$

$$m_\psi = 0$$

Choi, Kim, “Dynamical Axion” (1985)

Hook, “Anomalous solutions to the strong CP problem” (2015)

Gaillard, Gavela, Houtz, Quilez, del Rey, “Color Unified Dynamical Axion” (2018)

Axicolor

- Add a massless quark and a new gauge group

$$\tilde{\Lambda} >> \Lambda_{QCD}$$

Massless quark content

	$SU(3)_{QCD}$	$SU(\tilde{N})$
ψ	$\square$	$\square$

K. Choi, J. E. Kim, "Dynamical Axion," Phys. Rev. D32 (1985)

- When $SU(\tilde{N})$ confines, ψ forms bound states $\sim \tilde{\Lambda}$
- Problem: The new group has its own CP violating angle $\tilde{\theta}$
- Solution: Absorb $\tilde{\theta}$ with a new massless quark

Axicolor

- Add another massless quark χ charged under $SU(\tilde{N})$

- When $SU(\tilde{N})$ confines:

$$SU(4)_L \times SU(4)_R \to SU(4)_V$$

$$15 = 8 + 3 + \bar{3} + 1$$

The η becomes the **dynamical axion**

Massless quark content

	$SU(3)_{QCD}$	$SU(\tilde{N})$
ψ	$\Box$	$\Box$
χ	1	$\Box$

- The decay constant is near the very high confinement scale
- This is then a UV completion for an invisible axion

Are there mass sources for all the axions?

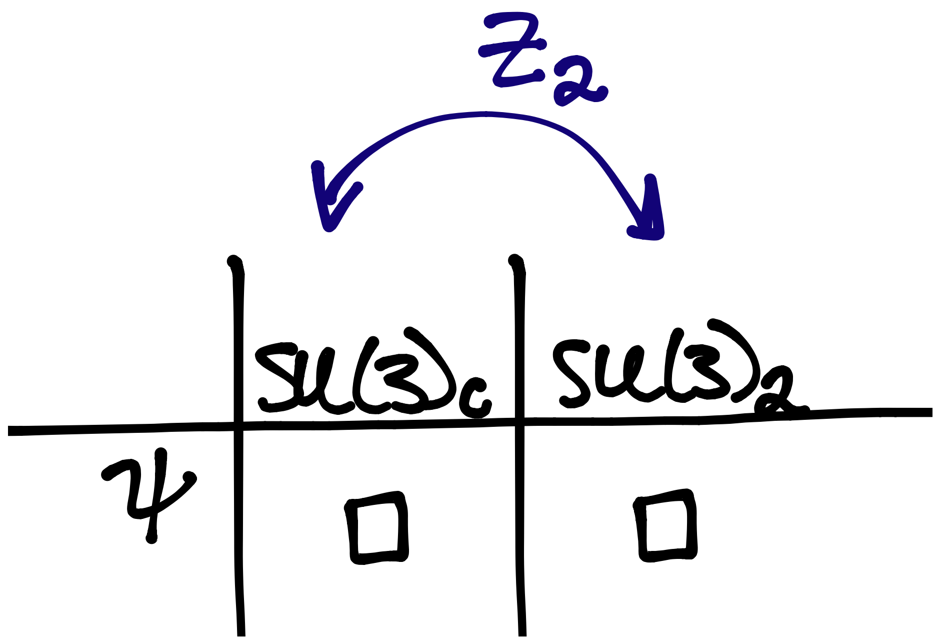
1. Add an additional color group to contribute to axion mass
2. Introduce a new θ angle
3. Introduce a new axion
4. Get stuck with a light axion again

	$SU(3)_{QCD}$	$SU(N)_1$	$SU(N)_2$	$\cdots$
ψ	$\square$	$\square$	1	$\cdots$
χ_1	1	$\square$	$\square$	$\cdots$
χ_2	1	1	$\square$	$\cdots$
$\vdots$	$\vdots$			

$$\mathcal{L}_{\text{eff}} \ni \Lambda_{\text{QCD}} \cos\left(\frac{\eta'_{QCD}}{f_\pi} + \frac{a_1}{f_1}\right) + \Lambda_1 \cos\left(\frac{a_2}{f_2}\right) + \Lambda_2 \cos\left(\frac{a_3}{f_3}\right) + \ldots$$

➡ You can avoid this problem if you relate two of the θ angles

Massless Quark and a $\mathbf{Z_2}$



- Only one massless quark
- Complete Z_2 copy of the SM
- The $SU(3)_2$ θ-angle doesn't introduce new CP violating effects

A. Hook, "Anomalous solutions to the strong CP problem," Phys. Rev. Lett. 114 (2015)

- Set up one Higgs VEV to be very large:

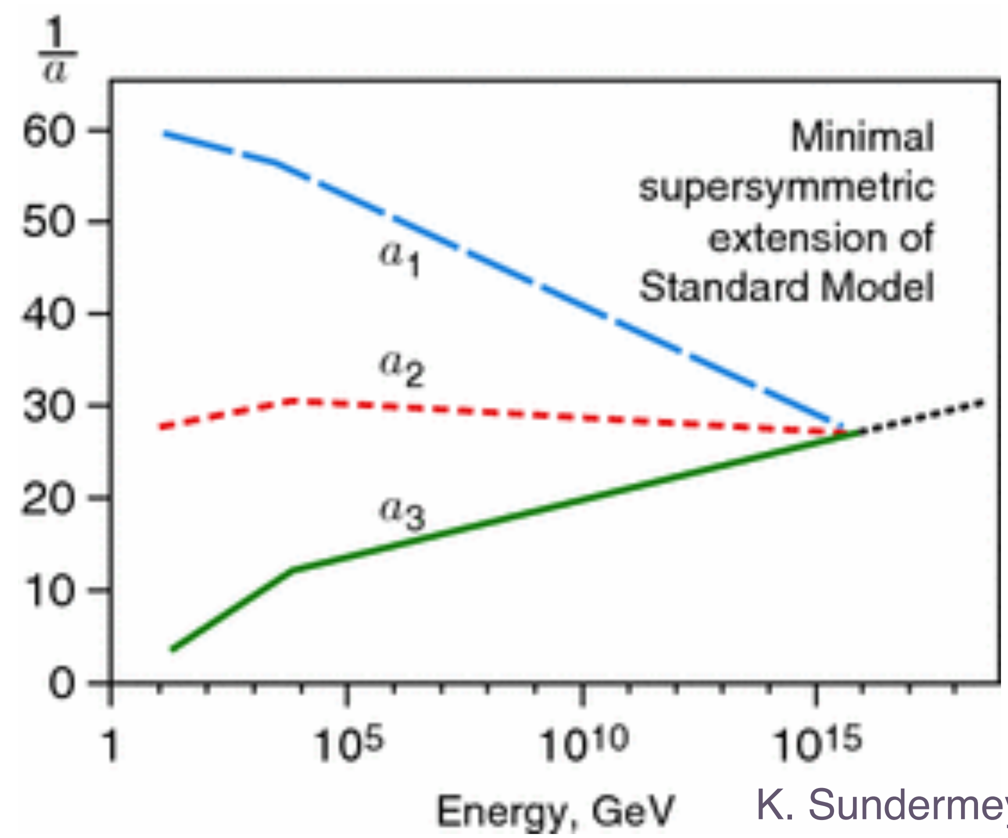
$$v_2 \gg v \quad \Rightarrow \quad m'_q \gg m_q \quad \Rightarrow \quad \Lambda'_{QCD} \gg \Lambda_{QCD}$$

➡ Bound states composed of massless quarks live at Λ'_{QCD}

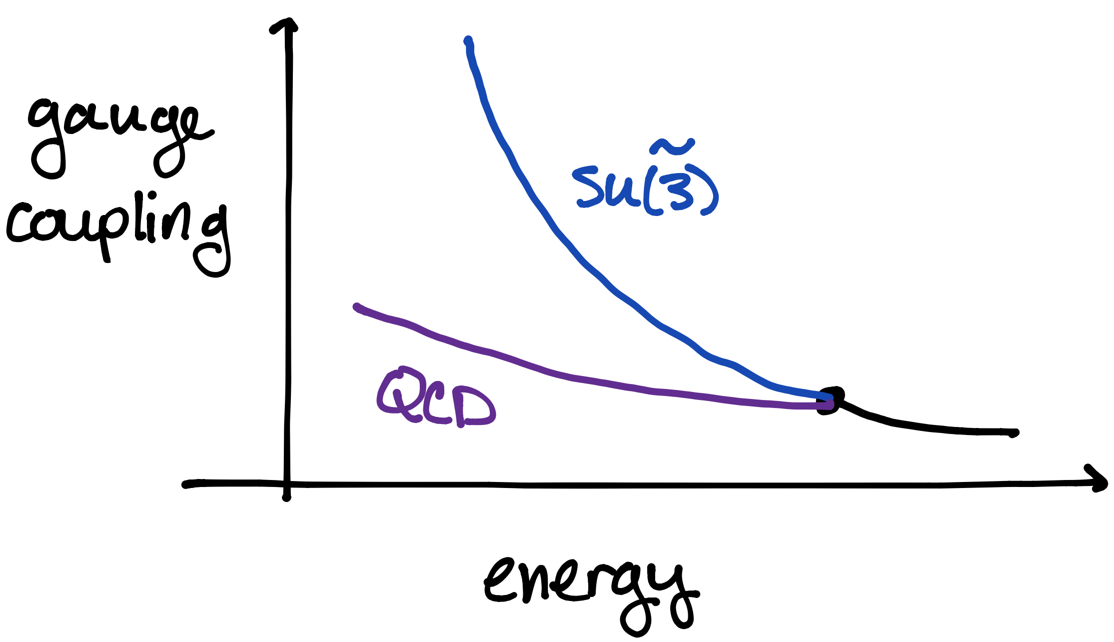
Z. Berezhiani, L. Gianfagna, M. Giannotti, "Strong CP problem and mirror world: the Weinberg Wilczek axion revisited," Phys.Lett.B500 (2001)

Color Unification

- Instead of a discrete symmetry, two color groups can be related by unification
- A single unified color sector with a single CP violating angle in the UV



K. Sundermeyer "Unified Field Theories," Springer (2014)



T. Ghergetta, N. Nagata, M. Shifman, "A Visible Axion from an Enlarged Color Group," Phys. Rev. D93 (2016)

V.A. Rubakov, "Grand unification and heavy axion," JETP Lett. 65 (1997)

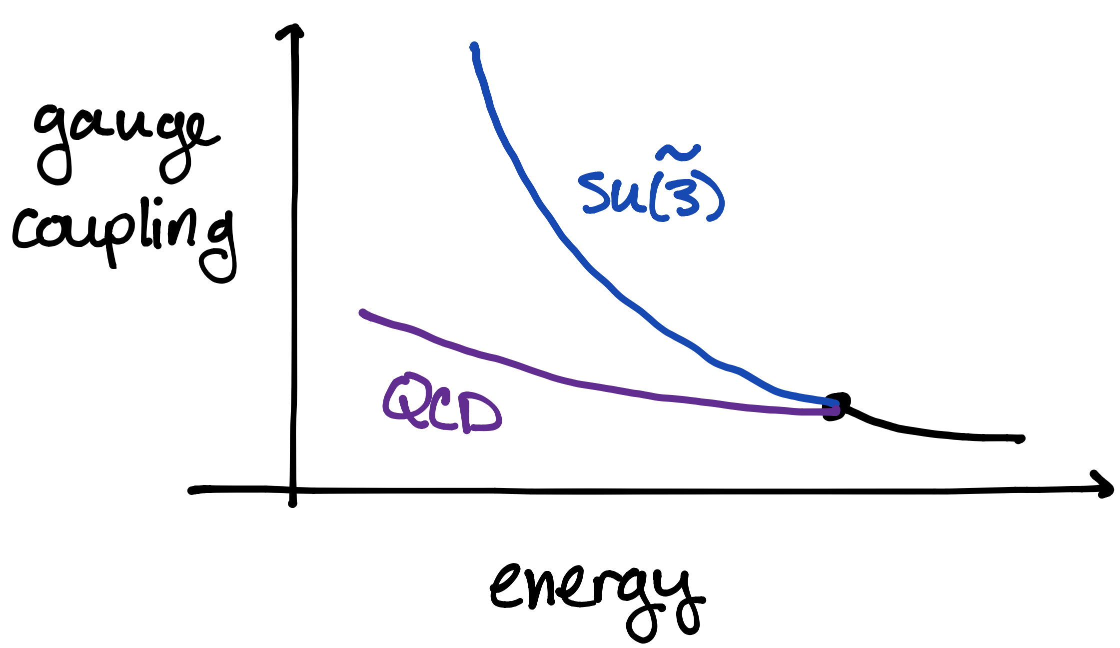
Color Unified Dynamical Axion

- Introduce massless quark $m_\Psi = 0$

M.K. Gaillard, M.B. Gavela, R. Houtz, P. Quilez, R. Del Rey, “Color Unified Dynamical Axion,” arXiv/1805.06465

- QCD lives inside a larger unified color group, which breaks to QCD and $SU(\tilde{3})$ at some high unification scale

- The new color group to allows Ψ to form bound states near its confinement ~TeV



- A bound state will be a dynamical axion, with its own m_a, f_a relationship

Massless Quark and Unification

- The massless quark to absorb the unified group's θ_6

	$SU(6)$	$SU(2)_L$	$U(1)_Y$
Ψ_L	20	1	0

$$SU(6) \xrightarrow{\Lambda_{\text{CUT}}} SU(3)_{\text{QCD}} \times SU(\tilde{3}) \times U(1)$$

- Below unification scale:

	$SU(3)_{QCD}$	$SU(\tilde{3})$
ψ_L	$\square$	$\bar{\square}$
$(\psi^c)_L$	$\bar{\square}$	$\square$
ψ_{V_1}	1	1
ψ_{V_2}	1	1

$$\Psi(20) \to (1,1)(-3) + (1,1)(+3) + (3,\bar{3})(-1) + (\bar{3},3)(+1)$$

Matter Content Above and Below CUT Breaking

$$SU(6) \xrightarrow{\Lambda_{\rm CUT}} SU(3)_{\rm QCD} \times SU(\tilde{3})$$

	$SU(6)$	$SU(2)_L$
Q_L	$\square$	$\square$
$\overline{U}_R$	$\overline{\square}$	$\mathbb{1}$
$\overline{D}_R$	$\overline{\square}$	$\mathbb{1}$
Φ	20	$\mathbb{1}$

$\xrightarrow{\Lambda_{\rm CUT}}$

	$SU(3)$	$SU(\tilde{3})$	$SU(2)_L$
q_L	$\square$	$\mathbb{1}$	$\square$
$\overline{u}_R$	$\overline{\square}$	$\mathbb{1}$	$\mathbb{1}$
$\overline{d}_R$	$\overline{\square}$	$\mathbb{1}$	$\mathbb{1}$
$\tilde{q}_L$	$\mathbb{1}$	$\square$	$\square$
$\overline{\tilde{u}}_R$	$\mathbb{1}$	$\overline{\square}$	$\mathbb{1}$
$\overline{\tilde{d}}_R$	$\mathbb{1}$	$\overline{\square}$	$\mathbb{1}$
ψ	$\square$	$\overline{\square}$	$\mathbb{1}$
$2\psi_\nu$	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$

Goal: provide a mechanism for these fields to form mass terms

Decoupling the SU(6) Partner Fields

$$SU(6) \xrightarrow{\Lambda_{\mathrm{CUT}}} SU(3)_{\mathrm{QCD}} \times SU(\tilde{3})$$

$$SU(6) \times SU(3)' \xrightarrow{\Lambda_{\mathrm{CUT}}} SU(3)_{\mathrm{QCD}} \times SU(\tilde{3}) \times SU(3)'$$

- Add another color group
- Matter in the $SU(3)'$ sector can form mass terms with the unification partners. Then both are removed from the low energy spectrum.

Decoupling the SU(6) Partner Fields

$$SU(6) \xrightarrow{\Lambda_{\mathrm{CUT}}} SU(3)_{\mathrm{QCD}} \times SU(\tilde{3})$$

$$SU(6) \times SU(3)' \xrightarrow{\Lambda_{\mathrm{CUT}}} SU(3)_{\mathrm{QCD}} \times SU(\tilde{3}) \times SU(3)'$$

$$SU(6) \times SU(3)' \xrightarrow{\Lambda_{\mathrm{CUT}}} SU(3)_{\mathrm{QCD}} \times SU(3)_{\mathrm{diag}}$$

- Similar gauge structure below Λ_{CUT}
- Below Λ_{CUT}, our strong sector has:
  - SM quarks
  - Massless quarks

Decoupling the SU(6) Partner Fields

- Problem: We introduced another CP phase
- Solution: Add another massless quark

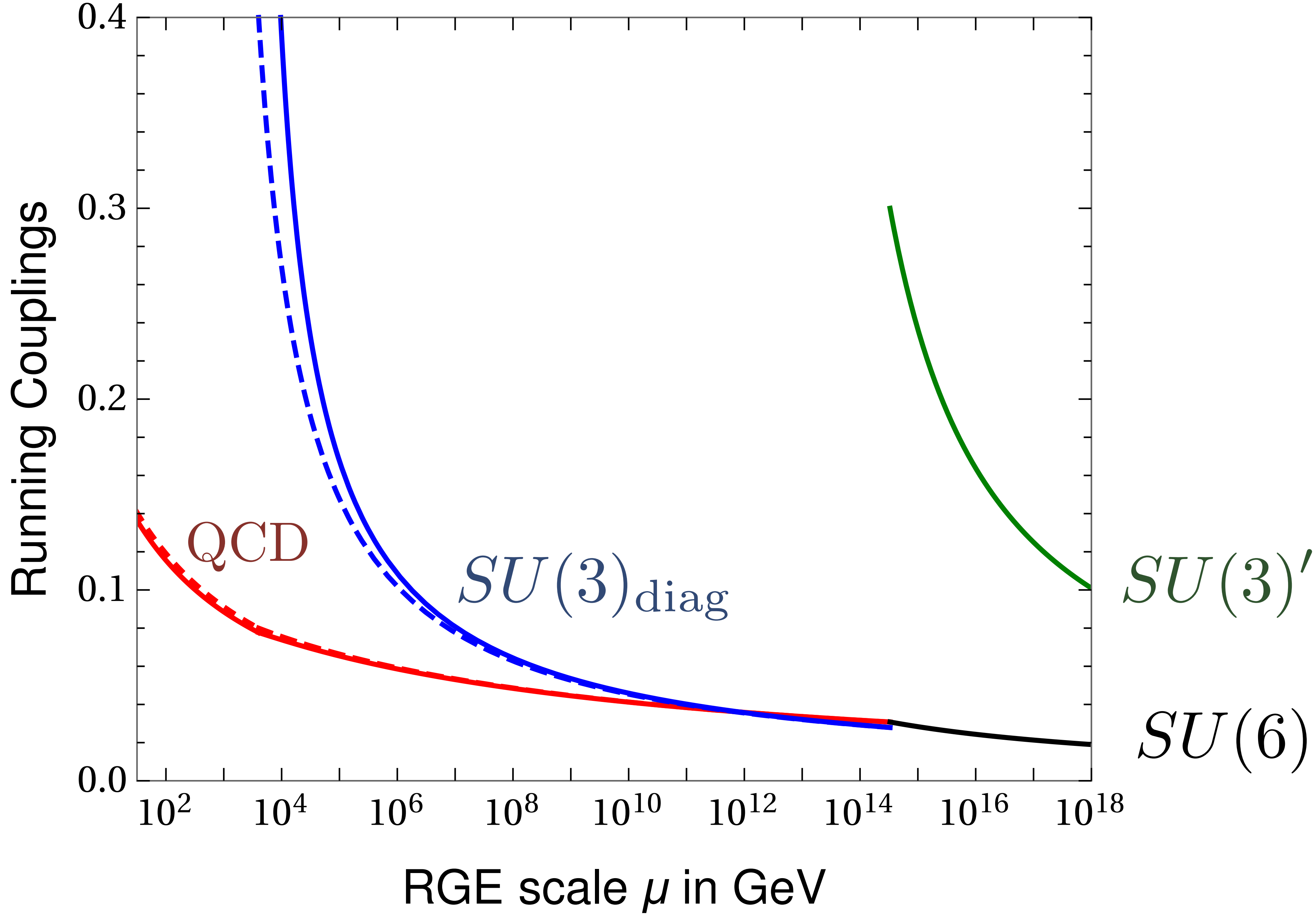
	SU(6)	SU(3')
Ψ	20	$\mathbb{1}$
χ	$\mathbb{1}$	$\square$

The new massless quark

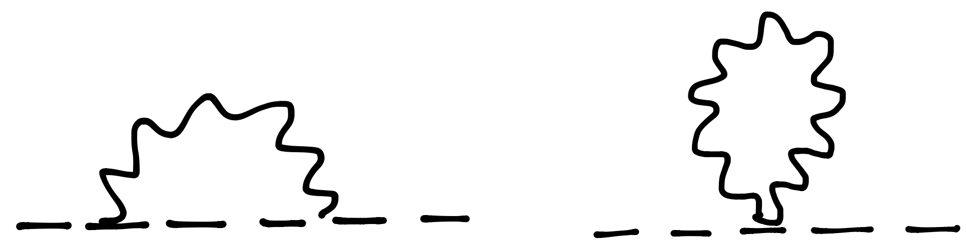
$$SU(6) \times SU(3)' \xrightarrow{\Lambda_{\rm CUT}} SU(3)_{\rm QCD} \times SU(3)_{\rm diag}$$

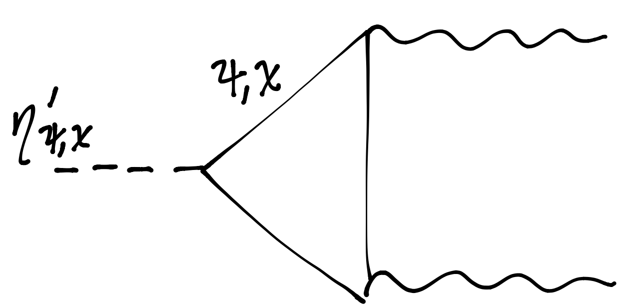
- Similar gauge structure below Λ_{CUT}
- Below Λ_{CUT}, our strong sector has:
  - SM quarks
  - Massless quarks

Unification and Confinement



$\mathbf{SU(3)_{diag}}$ Confinement

- Chiral symmetry breaking: $U(4)_L \times U(4)_R \to U(4)_V$
- This results in 16 pGB's: $16 = 8_c + \bar{3}_c + 3_c + 1_c + 1_c$
- The "pion" masses:  $\sim \Lambda^4_{\text{diag}}$
- The η' masses:



$$\mathcal{L}_{eff} = \Lambda^4_{\text{diag}} \cos\left(\frac{2\eta'_\chi}{f_d} + \sqrt{6}\frac{\eta'_\psi}{f_d}\right) + \Lambda^4_{\text{QCD}} \cos\left(\frac{2\eta'_{\text{QCD}}}{f_\pi} + \sqrt{6}\frac{\eta'_\psi}{f_d}\right)$$

→ This looks like **two** masses for **three** η' pseudoscalars

Small Size Instantons and Axion Mass

- Typically, at high scales $\alpha_{\rm QCD}$ is very small
- If new physics alters RG flow, large couplings can induce new instanton corrections to the axion mass

J. Flynn, L. Randall, "A computation of the small instanton contribution to the axion potential." Nucl. Phys. B208 (1987)

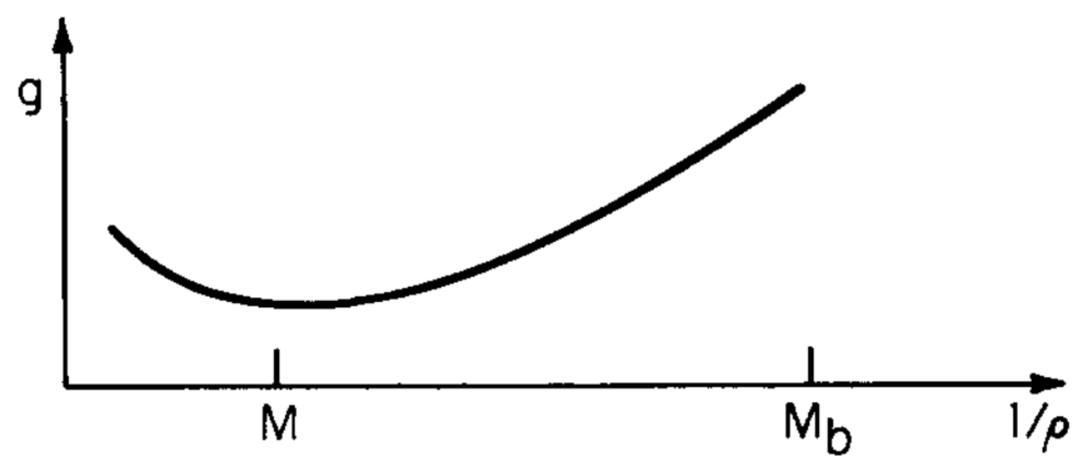


Fig. 1. Coupling g as a function of $1/\rho$.

P. Agrawal, K. Howe. "Factoring the Strong CP Problem," arXiv/1710.04213

$$SU(3) \times ... \times SU(3) \to SU(3)_c$$

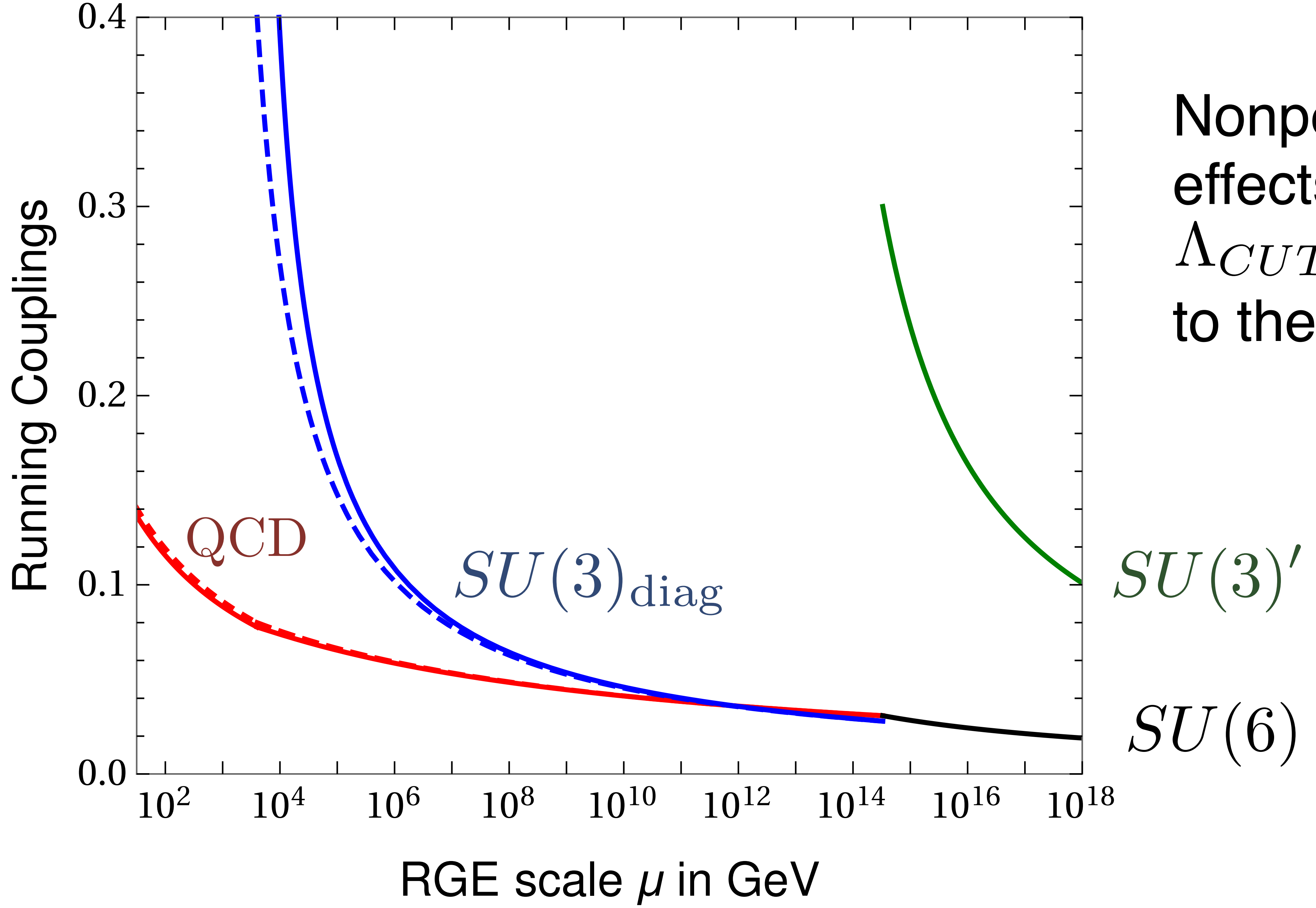
$$\frac{1}{\alpha_{\rm QCD}(\mu)} = \frac{1}{\alpha_1(\mu)} + \frac{1}{\alpha_2(\mu)} + ... + \frac{1}{\alpha_N(\mu)}$$

$$\mu = M_b$$

M. Dine, N. Seiberg, "String Theory and the Strong CP Problem," Nucl. Phys. B273 (1986)

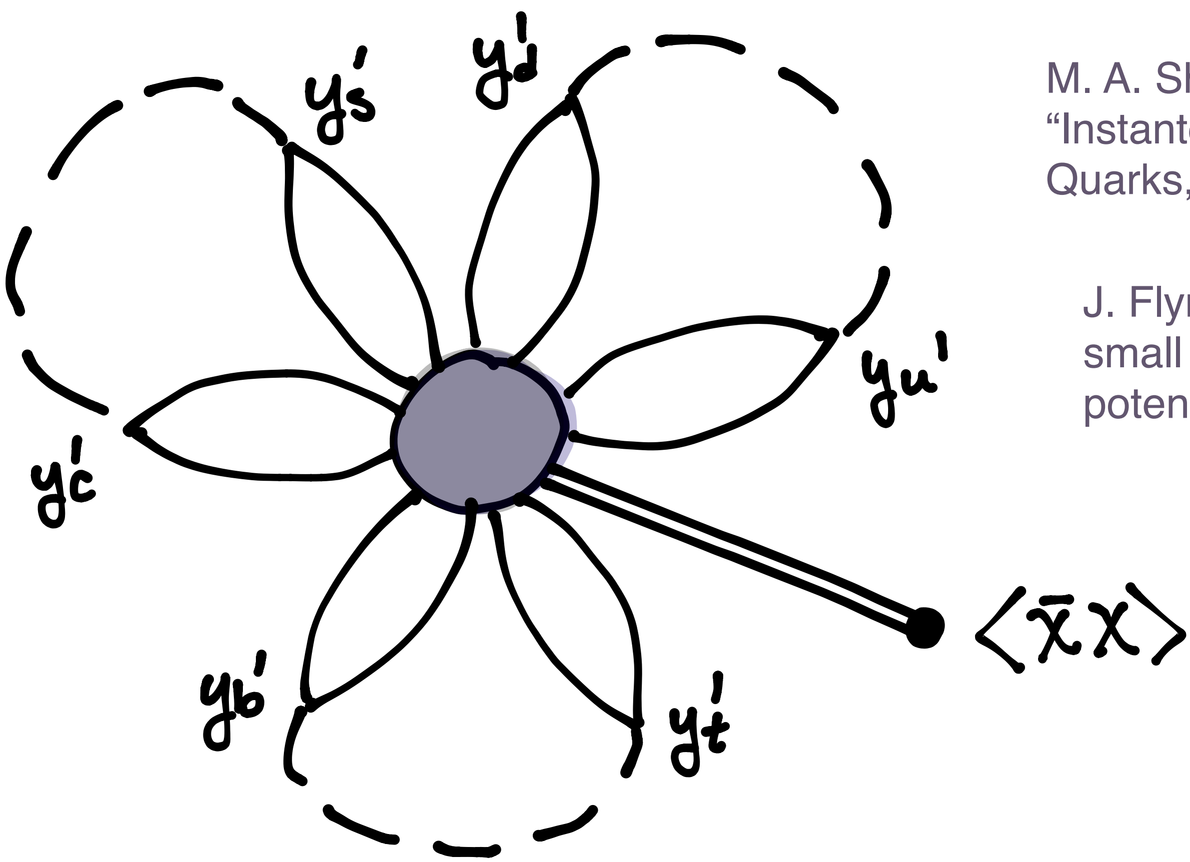
B. Holdom, M. Peskin, "Raising the Axion Mass," Nucl. Phys. B208 (1982)

Small Size Instanton Effects



Nonperturbative effects just above Λ_{CUT} can contribute to the axion potential

Small Size Instantons Effects

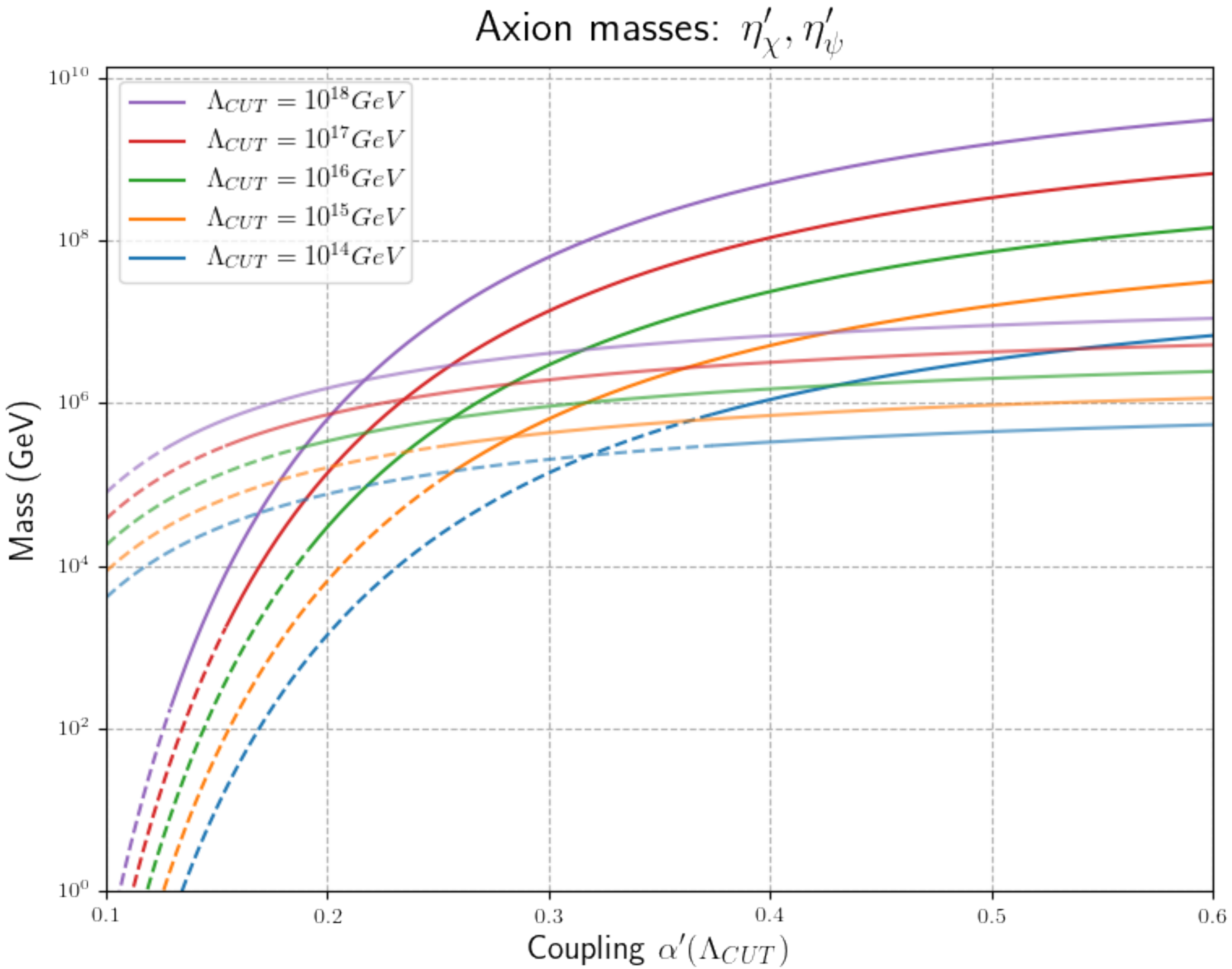


M. A. Shifman, A. I. Vainshtein, V. I. Sakharov, “Instanton Density in a Theory with Massless Quarks,” Nucl. Phys. B163 (1980)

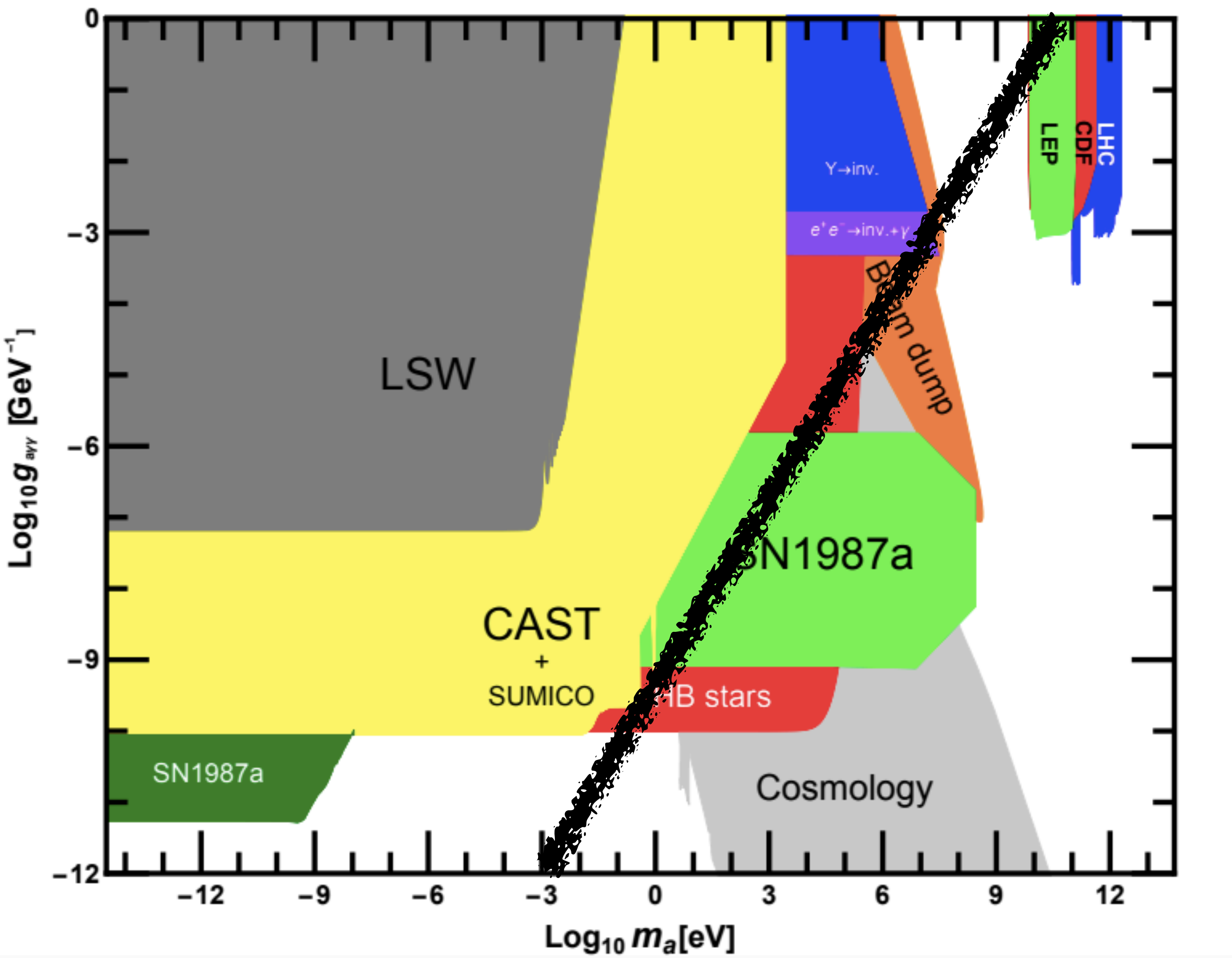
J. Flynn, L. Randall, “A computation of the small instanton contribution to the axion potential.” Nucl. Phys. B208 (1987)

Typical contributions from Λ_{diag} confinement

$$\mathcal{L}_{eff} = \Lambda^4_{SSI} \cos\left(2\frac{\eta'_\chi}{f_d}\right) + \Lambda^4_{\text{diag}} \cos\left(2\frac{\eta'_\chi}{f_d} + \sqrt{6}\frac{\eta'_\psi}{f_d}\right) + \Lambda^4_{\text{QCD}} \cos\left(\sqrt{6}\frac{\eta'_\psi}{f_d}\right)$$


Axion masses: η'_χ, η'_ψ
$\Lambda_{CUT} = 10^{18} GeV$
$\Lambda_{CUT} = 10^{17} GeV$
$\Lambda_{CUT} = 10^{16} GeV$
$\Lambda_{CUT} = 10^{15} GeV$
$\Lambda_{CUT} = 10^{14} GeV$
Mass (GeV)
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
0.1
0.2
0.3
0.4
0.5
0.6
Coupling $\alpha'(\Lambda_{CUT})$

Very Heavy Axions



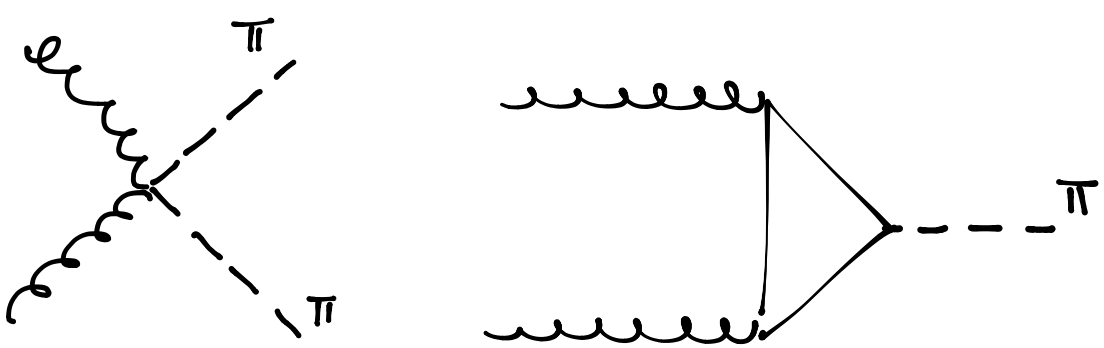
Sketch of the invisible axion expected region

J. Jaeckel, M. Spannowsky, "Probing MeV to 90 GeV axion-like particles with LEP and LHC," Phys. Lett. B753 (2016)

I. Brivio, "Axion-Like-Particles EFT & collider signals," HEFT 2017

Collider Phenomenology

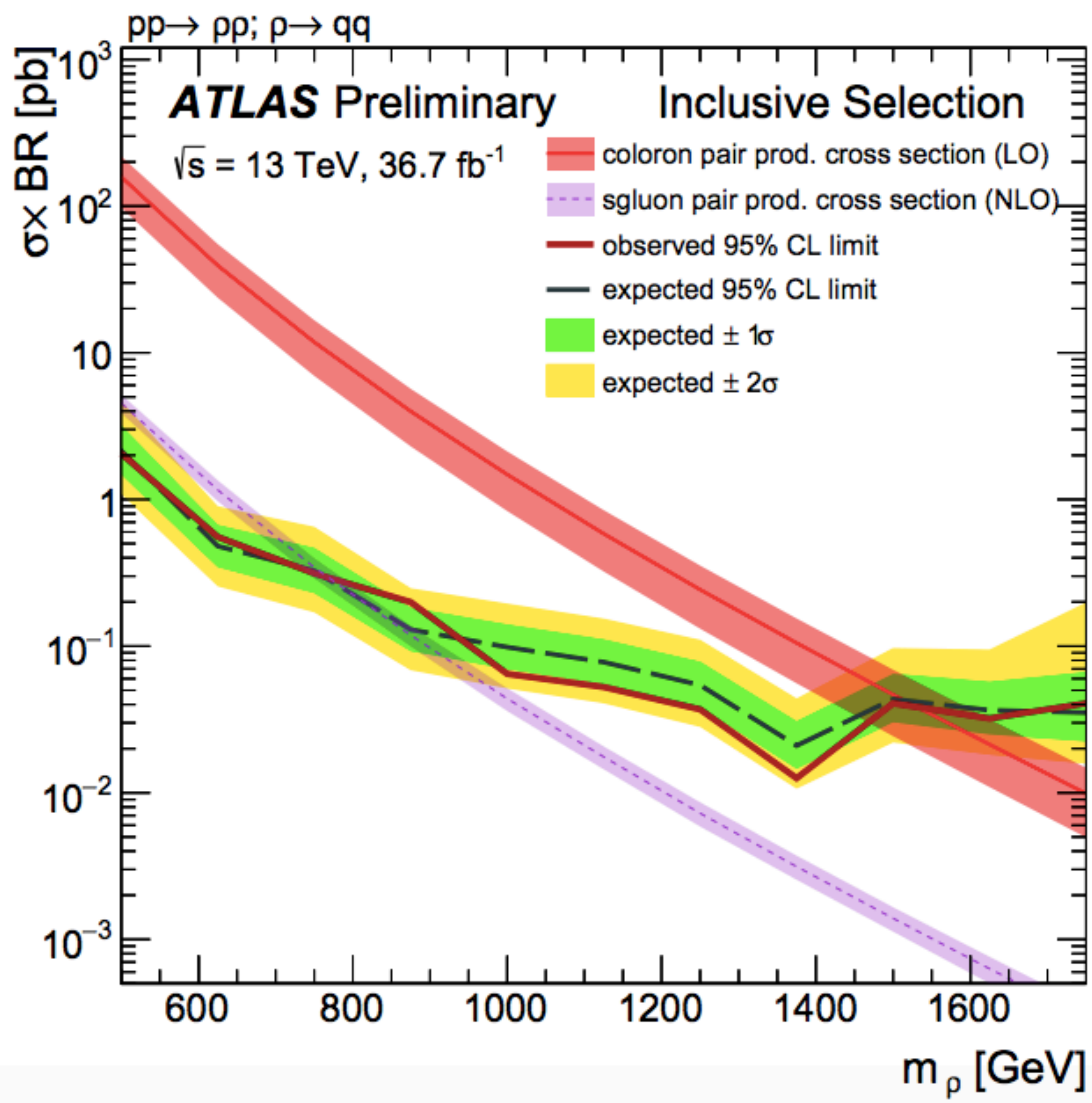
- Collider accessible states are QCD colored "pions"



- We have a bound on color octet scalars

$$m^2(8_c) \approx \frac{9}{4\pi}\alpha_{\rm QCD}\Lambda^2_{\rm diag}$$

$$\Lambda_{\rm diag} \gtrsim 2.9 \text{ TeV}$$



Summary

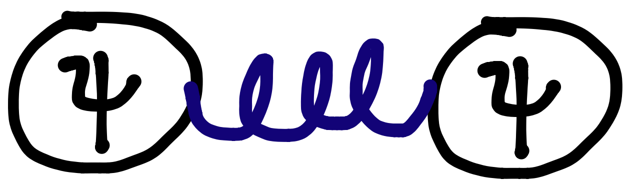
- The Strong CP Problem can be solved using massless quarks and unification

$$SU(6) \times SU(3)' \xrightarrow{\Lambda_{\text{CUT}}} SU(3)_{\text{QCD}} \times SU(3)_{\text{diag}}$$

- Unification and confinement are naturally separated by RG flow
    - Decoupling the unification partner fields requires additional model building
- Small size instantons provided an extra source of mass for axions
- Most promising collider signals come from the exotic pions

Conclusions

- Extra color groups provide a window to richer phenomenology
- Visible axions $$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 \longrightarrow + \sim \Lambda_{\mathrm{new}}^4$$
- Massless quarks still viable



- Symmetries are usually needed to simplify the CP violating phases
- Nonperturbative effects of extra color groups can provide another source of axion mass

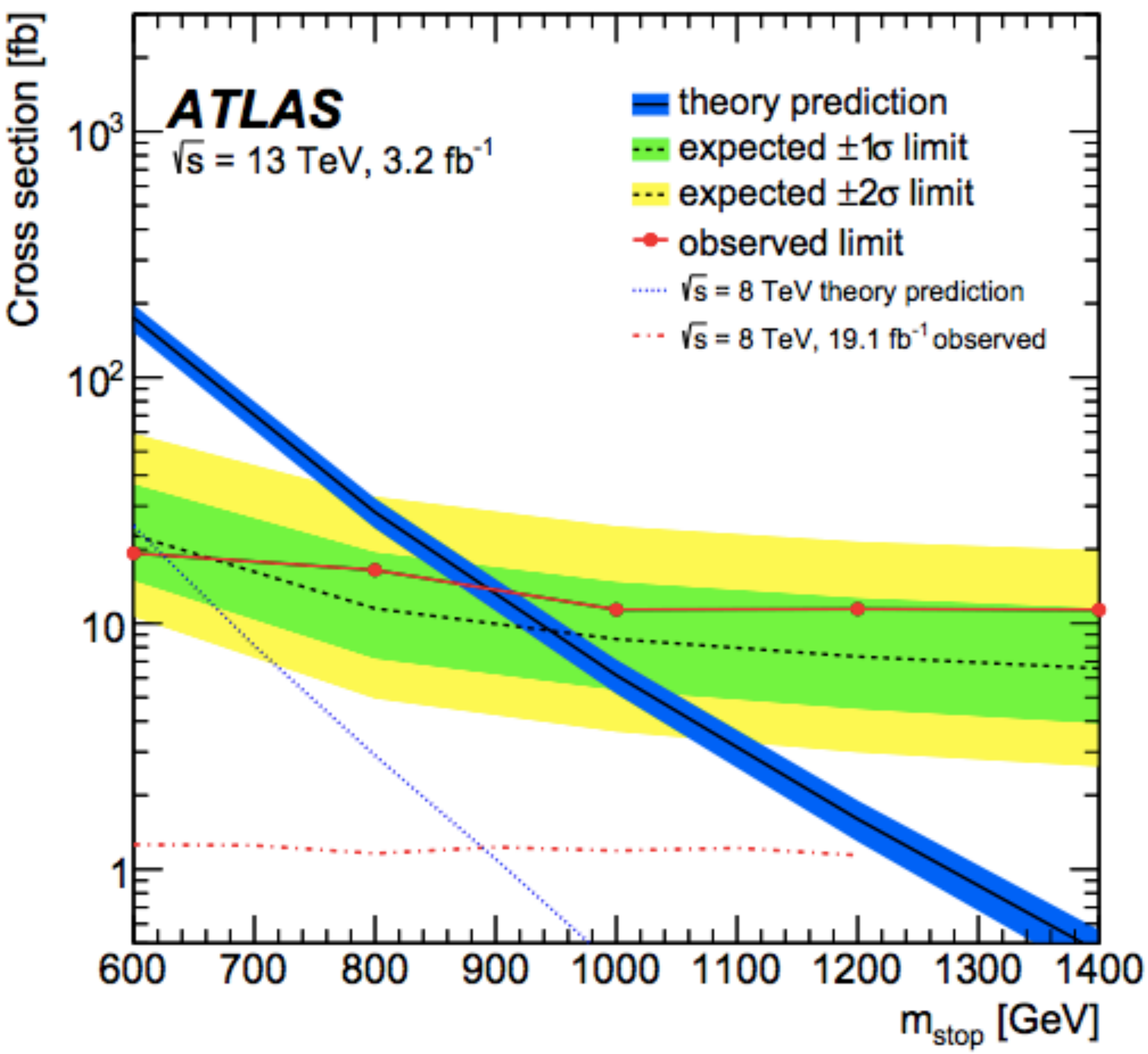
Thank you!

This project has received funding/support from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No 674896.

Back-up Slides

Collider Phenomenology: R-Hadron Searches

arXiv:1606.05129



- We have a bound on color triplet scalars

$$m(\pi_d) \gtrsim 890\ \mathrm{GeV}$$

$$m^2(3_c) \approx \frac{\alpha_c}{\pi}\Lambda_{\mathrm{diag}}$$

$$\Lambda_{\mathrm{diag}} \gtrsim 3\ \mathrm{TeV}$$

Matter Content Above and Below CUT Breaking

	$SU(6)$	$SU(3')$	$SU(2)_L$
Q_L	$\square$	$\mathbb{1}$	$\square$
$\bar{U}_R$	$\bar{\square}$	$\mathbb{1}$	$\mathbb{1}$
$\bar{D}_R$	$\bar{\square}$	$\mathbb{1}$	$\mathbb{1}$
$\bar{q}'_R$	$\mathbb{1}$	$\bar{\square}$	$\square$
u'_L	$\mathbb{1}$	$\square$	$\mathbb{1}$
d'_L	$\mathbb{1}$	$\square$	$\mathbb{1}$
Φ	20	$\mathbb{1}$	$\mathbb{1}$
Δ	$\square$	$\bar{\square}$	$\mathbb{1}$

$\xrightarrow{\Lambda_{\mathrm{CUT}}}$

	$SU(3)$	$SU(3)_{diag}$	$SU(2)_L$
q_L	$\square$	$\mathbb{1}$	$\square$
$\bar{u}_R$	$\bar{\square}$	$\mathbb{1}$	$\mathbb{1}$
$\bar{d}_R$	$\bar{\square}$	$\mathbb{1}$	$\mathbb{1}$
ψ	$\square$	$\bar{\square}$	$\mathbb{1}$
$2\psi_\nu$	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$
$\tilde{q}_L$	$\mathbb{1}$	$\square$	$\square$
$\bar{\tilde{u}}_R$	$\mathbb{1}$	$\bar{\square}$	$\mathbb{1}$
$\bar{\tilde{d}}_R$	$\mathbb{1}$	$\bar{\square}$	$\mathbb{1}$
$\bar{q}'_R$	$\mathbb{1}$	$\bar{\square}$	$\square$
u'_L	$\mathbb{1}$	$\square$	$\mathbb{1}$
d'_L	$\mathbb{1}$	$\square$	$\mathbb{1}$

Massless quark sector (ψ, $2\psi_\nu$)

Obtain mass near the CUT breaking scale ($\tilde{q}_L$ through d'_L)

Prime sector ($\bar{q}'_R$, u'_L, d'_L)

The pseudoscalar mixing in QCD

$$M^2_{\pi_3,\pi_8,\eta',a} = \begin{pmatrix} & \text{meson} & & 0 \\ & \text{mixing} & & 0 \\ & & & \frac{\sqrt{6}\Lambda^4}{f_9 f_a} \\ 0 & 0 & \frac{\sqrt{6}\Lambda^4}{f_9 f_a} & \frac{\Lambda^4}{f_a^2} \end{pmatrix}$$

Choi, Kang, Kim, "Effects of eta' in Low Energy Axion Physics," (1986)

$$m_a^2 f_a^2 = \frac{\Lambda^4}{1 + \frac{\Lambda^4}{2m_q\langle\bar{\psi}\psi\rangle}} \xrightarrow{\Lambda \gg m_q} m_a^2 f_a^2 \approx m_q \langle\bar{\psi}\psi\rangle$$

$$\sim m_\pi^2 f_\pi^2$$

Another Massless Quark

	$SU(6)$	$SU(3')$	$SU(2)_L$
Ψ	20	$\mathbb{1}$	$\mathbb{1}$
χ	$\mathbb{1}$	$\square$	$\mathbb{1}$

$\xrightarrow{\Lambda_{\mathrm{CUT}}}$

	$SU(3)$	$SU(3)_{\mathrm{diag}}$	$SU(2)_L$
ψ	$\square$	$\overline{\square}$	$\mathbb{1}$
χ	$\mathbb{1}$	$\square$	$\mathbb{1}$
$2\psi_L$	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$

The new massless quark (χ)

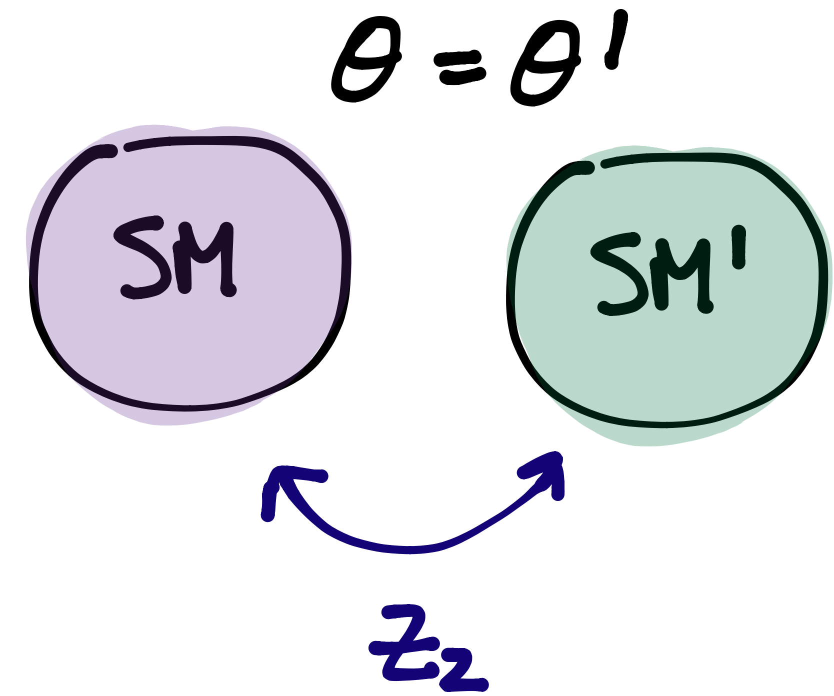
- Goal: $SU(3)_{\mathrm{diag}}$ confines at a higher scale than $SU(3)_c$

$$\frac{1}{\alpha_{\mathrm{diag}}(\mu)} = \frac{1}{\alpha_6(\mu)} + \frac{1}{\alpha'(\mu)} \qquad \mu = \Lambda_{CUT}$$

$$\alpha_c(\Lambda_{CUT}) = \alpha_6(\Lambda_{CUT})$$

Axion and a $\mathbf{Z_2}$

- Complete Z_2 copy of the SM
- A single axion VEV can cancel both θ angles
- Set up one Higgs VEV to be very large:



$$v' \gg v \rightarrow m_q' \gg m_q \rightarrow \Lambda'_{QCD} \gg \Lambda_{QCD}$$

Z. Berezhiani, L. Gianfagna, M. Giannotti, “Strong CP problem and mirror world: the Weinberg Wilczek axion revisited,” Phys.Lett.B500 (2001)

➡ axion mass is $\sim\Lambda'_{\mathrm{QCD}}$

$$m_a^2 f_a^2 \approx m_\pi^2 f_\pi^2 + {\Lambda'}^4_{\mathrm{QCD}}$$

Check in: Solution to the Strong CP Problem from the Low-E EFT

$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos\left(2\frac{\eta'_\chi}{f_d} - \bar{\theta}'\right) + \Lambda_{\text{diag}}^4 \cos\left(2\frac{\eta'_\chi}{f_d} + \sqrt{6}\frac{\eta'_\psi}{f_d} - \bar{\theta}' - \bar{\theta}_6\right) + \Lambda_{\text{QCD}}^4 \cos\left(\sqrt{6}\frac{\eta'_\psi}{f_d} - \bar{\theta}_6\right)$$

- The CP-Conserving minimum is

$$\left\langle \bar{\theta}' - 2\frac{\eta'_\chi}{f_d} \right\rangle = 0 \qquad \left\langle \bar{\theta}_6 - \sqrt{6}\frac{\eta'_\psi}{f_d} \right\rangle = 0$$

- Rewriting with the axion fields expanded about their minima:

$$\mathcal{L}_{eff} = \Lambda_{SSI}^4 \cos\left(2\frac{\eta'_\chi}{f_d}\right) + \Lambda_{\text{diag}}^4 \cos\left(2\frac{\eta'_\chi}{f_d} + \sqrt{6}\frac{\eta'_\psi}{f_d}\right) + \Lambda_{\text{QCD}}^4 \cos\left(\sqrt{6}\frac{\eta'_\psi}{f_d}\right)$$