# Bimetric Gravity and Dark Matter

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- Motivation
- Massless & Massive Spin-2 Fields
- The Ghost-Free Theory
- Physics of Massive Spin-2 Fields
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# Motivation

# Consistent Field Theories

## Standard Model of Particle Physics & General Relativity

| | | |
|---|---|---|
| Spin 0: | Higgs boson | $\phi$ |
| Spin 1/2: | leptons, quarks | $\psi^a$ |
| Spin 1: | gluons, photon, W- & Z-boson | $A_\mu$ |
| Spin 2: | graviton | $g_{\mu\nu}$ |

## Consistent Field Theories

### Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson $\phi$

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Spin 1: gluons, photon, W- & Z-boson $A_\mu$

Spin 2: graviton $g_{\mu\nu}$

+ Supersymmetry

## Consistent Field Theories

### Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson $\phi$

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Spin 1: gluons, photon, W- & Z-boson $A_\mu$

Spin 2: graviton $g_{\mu\nu}$

new models are usually built using more copies of these particles

less understood...

## Consistent Field Theories

### Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson $\phi$

Spin 1/2: leptons, quarks $\psi^a$

Spin 1: gluons, photon, W- & Z-boson $A_\mu$

massless & massive

Spin 2: graviton $g_{\mu\nu}$

MASSLESS !

## Consistent Field Theories

### Standard Model of Particle Physics & General Relativity

Spin 0: Higgs boson $\phi$

Spin 1/2: leptons, quarks $\psi^a$

Spin 1: gluons, photon, W- & Z-boson $A_\mu$

Spin 2: graviton $g_{\mu\nu}$

multiplets of gauge groups

just one field...

How do we make a spin-2 field massive ?

Can several spin-2 fields interact ?

# Massless & Massive Spin-2 Fields

# Massless Theory

## General Relativity

= classical nonlinear field theory for metric tensor $g_{\mu\nu}$

- Einstein-Hilbert action: $S_{\rm EH}[g] = M_{\rm P}^2 \int \mathrm{d}^4x \sqrt{g} \left( R(g) - 2\Lambda \right)$

- Einstein's equations: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0$

⇒ describes the two degrees of freedom
of a self-interacting, massless spin-2 particle

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two derivatives ⇨ kinetic term

⇨ describes the two degrees of freedom of a self-interacting, massless spin-2 particle

General Relativity

=

unique description of
self-interacting massless spin-2 field

# Mass Term

... should not contain derivatives nor loose indices.

Examples: scalar (spin 0)

$$-\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2$$

vector (spin 1)

$$-F^{\mu\nu} F_{\mu\nu} - m^2 A^\mu A_\mu$$

# Mass Term

... should not contain derivatives nor loose indices.

Examples: scalar (spin 0) vector (spin 1)

$$-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2 \qquad - g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu} - m^2 g^{\mu\nu}A_\mu A_\nu$$

For the spin-2 tensor contracting indices of the metric gives: $g^{\mu\nu}g_{\mu\nu} = 4$

This is not a mass term.

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This is not a mass term.

Simplest way out: Introduce second "metric" to contract indices:

$$g^{\mu\nu}f_{\mu\nu} = \mathrm{Tr}\,(g^{-1}f) \qquad f^{\mu\nu}g_{\mu\nu} = \mathrm{Tr}\,(f^{-1}g)$$

## Mass Term

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➪ Massive gravity action: $S_{\mathrm{MG}}[g] = \underbrace{S_{\mathrm{EH}}[g]}_{\text{kinetic term}} - \int \underbrace{\mathrm{d}^4x\; V(g,f)}_{\text{mass term}}$

## Mass Term

... should ... indices.

Examples: scalar (sp

$$-g^{\mu\nu}\partial_\mu\phi\partial_\nu$$

For the spin-2 tensor cont

This is not a mass term.

Simplest way out: Introduce se

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\,(g^{-} \quad \ldots \quad g)$$

What determines $f_{\mu\nu}$ ?
Shouldn't it be dynamical ?

Massive gravity action: $S_{\mathrm{MG}}[g] = S_{\mathrm{EH}}[g] - \int \mathrm{d}^4x\, V(g, f)$

kinetic term — mass term

# Bimetric Theory

Nonlinear action for two interacting tensors:

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\,\Big(R(g) - 2\Lambda\Big) \\
&+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\,\Big(R(f) - 2\tilde{\Lambda}\Big) \;-\; \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

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$$

- both metrics are dynamical and treated on equal footing
- should describe massive & massless spin-2 field (5+2 d.o.f.)

This looks good, but in general the theory has a ghost!

# The Ghost-Free Theory

#  - free Bimetric Theory

de Rham, Gabadadze, Tolley (2010); Hassan, Rosen (2011)

$$S_{\rm b}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f)$$

$$V(g,f) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n \left(\sqrt{g^{-1}f}\right)$$

- arbitrary spin-2 mass scale $m$
- 3 interaction parameters $\beta_n$
- square-root matrix $S$ defined through $S^2 = g^{-1}f$

#  - free Bimetric Theory

de Rham, Gabadadze, Tolley (2010); Hassan, Rosen (2011)

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] \;&=\; m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+\; m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \;-\; \int \mathrm{d}^4x\, V(g,f)
\end{aligned}
$$

$$
V\left(g,f\right) = m^4 \sqrt{g} \sum_{n=0}^{4} \beta_n\, e_n \left(\sqrt{g^{-1}f}\right) = m^4 \sqrt{f} \sum_{n=0}^{4} \beta_{4-n}\, e_n \left(\sqrt{f^{-1}g}\right)
$$

**elementary symmetric polynomials:**

$$
e_1(S) = \mathrm{Tr}[S] \qquad e_2(S) = \tfrac{1}{2}\left((\mathrm{Tr}[S])^2 - \mathrm{Tr}[S^2]\right)
$$

$$
e_3(S) = \tfrac{1}{6}\left((\mathrm{Tr}[S])^3 - 3\,\mathrm{Tr}[S^2]\mathrm{Tr}[S] + 2\,\mathrm{Tr}[S^3]\right)
$$

# Physics of
# Massive Spin-2 Fields

## Mass spectrum

Hassan, ASM, von Strauss (2012)

- Maximally symmetric solutions: $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with $c =$ const.

- Perturbations around proportional backgrounds:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \qquad f_{\mu\nu} = c^2 \bar{g}_{\mu\nu} + \delta f_{\mu\nu}$$

- Can be diagonalised into mass eigenstates:

$$\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$$ massless (2 d.o.f.)

$$\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$$ massive (5 d.o.f.)

with Fierz-Pauli mass $m_{\rm FP} = m_{\rm FP}(\alpha, \beta_n, c)$

Ghost-free bimetric theory

=

unique description of
massless + massive spin-2 field

What is the physical metric ?

How does matter couple
to the tensor fields ?

# Matter Coupling

Yamashita, de Felice, Tanaka;
de Rham, Heisenberg, Ribeiro (2015)

$$
\begin{aligned}
S_{\mathrm{b}}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) \\
&\qquad + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g,\phi)
\end{aligned}
$$

Absence of ghosts: only one metric can couple to matter!

⇨ $g_{\mu\nu}$ is gravitational metric

# Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
Hassan, ASM, von Strauss (2012/14);
Akrami, Hassan, Koennig, ASM, Solomon (2015)

$$\begin{aligned} S_{\rm b}[g,f] &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\ &+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\rm matter}(g,\phi) \end{aligned}$$

(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu} \qquad (\alpha \equiv m_f/m_g)$$

massless massive

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$$S_{\mathrm{b}}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_{\mathrm{matter}}(g,\phi)$$

(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu} \qquad (\alpha \equiv m_f / m_g)$$

massless — massive

The gravitational metric is not massless but a superposition of mass eigenstates.

Max, Platscher, Smirnov (2017): analysis of gravitational wave oscillations

# Mass Eigenstates

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(linearised) gravitational metric:

$$\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu} \qquad (\alpha \equiv m_f/m_g)$$

massless massive

- for small $\alpha = m_f/m_g$ gravity is dominated by the massless mode
- the massive spin-2 field interacts only weakly with matter

# Mass Eigenstates

Baccetti, Martin-Moruno, Visser (2012);
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$$S_\mathrm{b}[g,f] = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x\, V(g,f) + \int \mathrm{d}^4x \sqrt{g}\, \mathcal{L}_\mathrm{matter}(g,\phi)$$

$$\alpha = m_f / m_g \to 0$$

is the General Relativity limit of bimetric theory

(when all other parameters are fixed,
this makes the spin-2 mass $m_\mathrm{FP}$ infinitely large)

Ghost-free bimetric theory

=

General Relativity +
additional tensor field

# Structure of Vertices

(bimetric action expanded in mass eigenstates)

Quadratic (Fierz-Pauli)

| $\delta G^2$ | $\delta G \delta M$ | $\delta M^2$ |
|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\mathrm{FP}}^2$ |

what about higher orders?

$$S = \frac{1}{2}\int \mathrm{d}^4x \left[\delta G_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta G_{\rho\sigma} \;+\; \delta M_{\mu\nu}\mathcal{E}^{\mu\nu\rho\sigma}\delta M_{\rho\sigma} \right.$$
$$\left. -\; \frac{m_{\mathrm{FP}}^2}{2}\left(\delta M^{\mu\nu}\delta M_{\mu\nu} - \delta M^2\right)\right] \;+\; \mathcal{O}\left(\frac{1}{m_{\mathrm{Pl}}}\right)$$

# Structure of Vertices

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic (Fierz-Pauli)

| $\delta G^2$ | $\delta G \delta M$ | $\delta M^2$ |
|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\rm FP}^2$ |

Cubic (suppressed by $m_{\rm Pl}^{-1}$ )

| $\delta G^3$ | $\delta G^2 \delta M$ | $\delta G \delta M^2$ | $\delta M^3$ |
|---|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\rm FP}^2$ | $\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$<br>$\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$ |

$$m_{\rm Pl} = m_g\sqrt{1+\alpha^2}$$

# Structure of Vertices

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

Quadratic (Fierz-Pauli)

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| $\delta G^3$ | $\delta G^2 \delta M$ | $\delta G \delta M^2$ | $\delta M^3$ |
|---|---|---|---|
| $1\,,\Lambda$ | $0$ | $1\,,\Lambda\,,m_{\rm FP}^2$ | $\alpha\,,\alpha\Lambda\,,\alpha m_{\rm FP}^2$<br>$\frac{1}{\alpha}\,,\frac{1}{\alpha}\Lambda\,,\frac{1}{\alpha}m_{\rm FP}^2$ |

- self-interactions of massless spin-2 sum up to General Relativity
- no vertices giving rise to decay of massive into massless spin-2
- massive spin-2 particle gravitates like baryonic matter
- self-interactions of massive spin-2 are enhanced in the GR limit

# Could Dark Matter have spin-2 ?

# Cosmology

# The cosmological cake

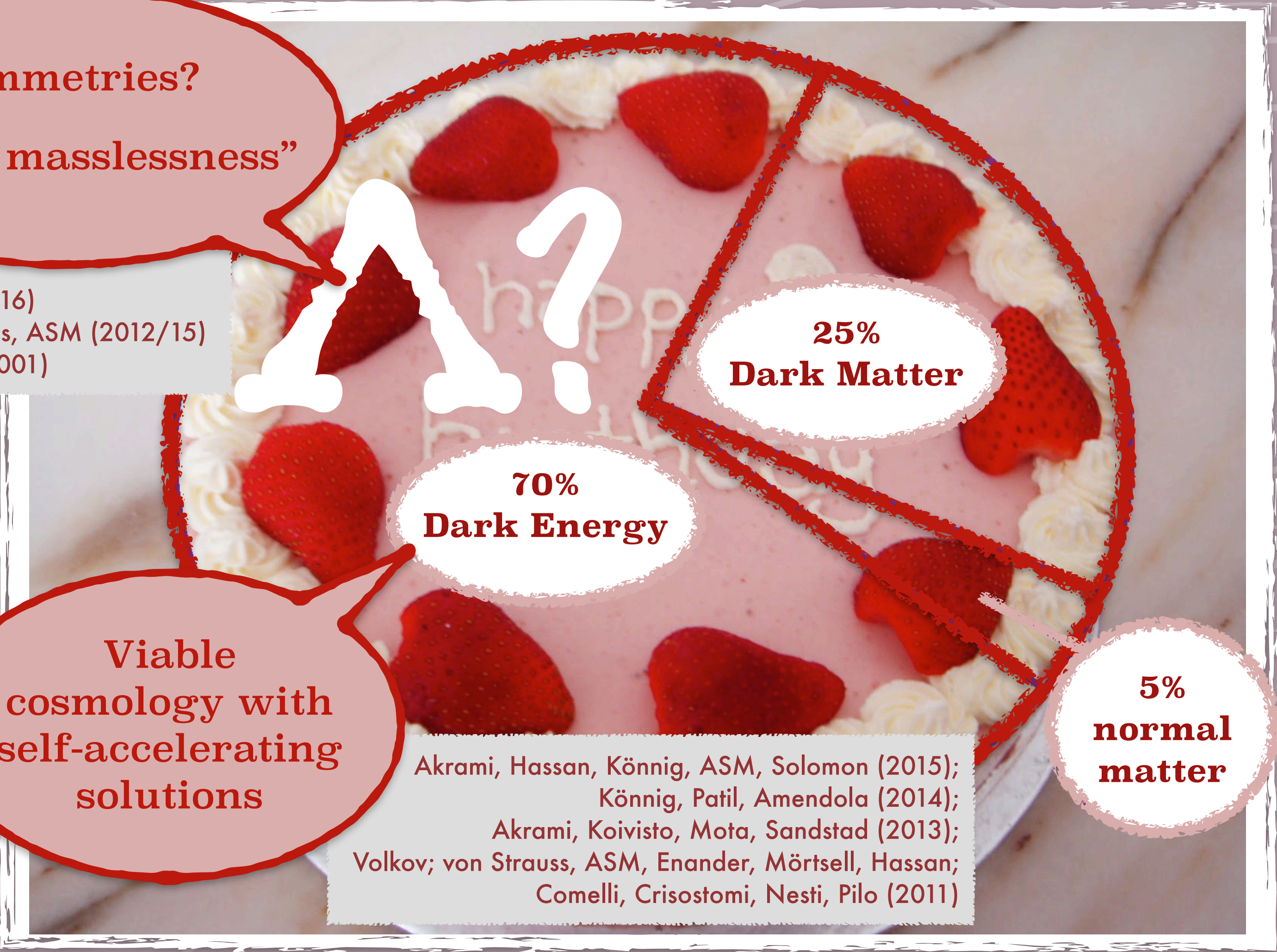
# The cosmological cake


Λ?
25%
Dark Matter
70%
Dark Energy
5%
normal
matter
Viable cosmology with self-accelerating solutions
Akrami, Hassan, Könnig, ASM, Solomon (2015);
Könnig, Patil, Amendola (2014);
Akrami, Koivisto, Mota, Sandstad (2013);
Volkov; von Strauss, ASM, Enander, Mörtsell, Hassan;
Comelli, Crisostomi, Nesti, Pilo (2011)

Symmetries?

“partial masslessness”

Apolo, Hassan (2016)
Hassan, von Strauss, ASM (2012/15)
Deser, Waldron (2001)

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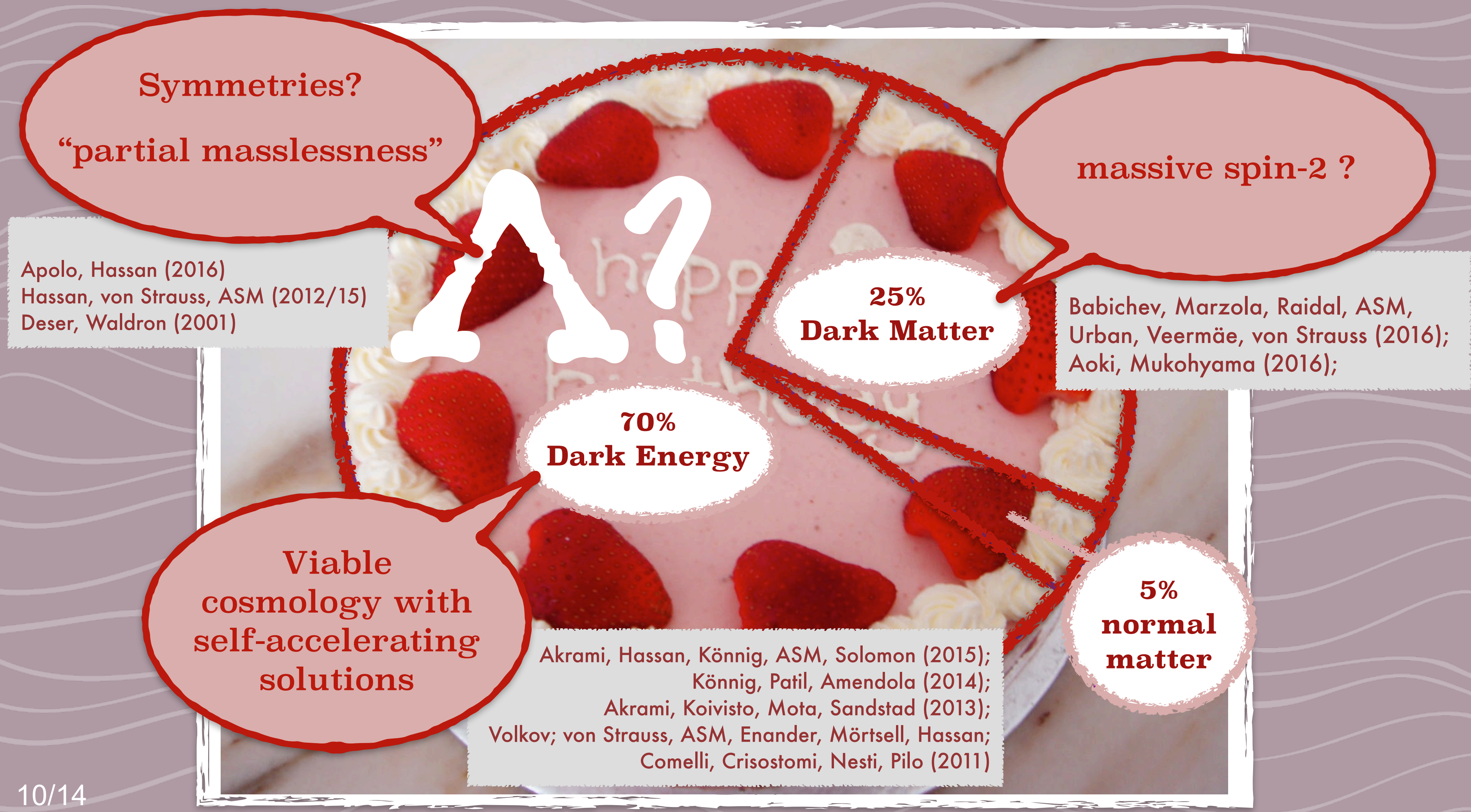
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massive spin-2 ?
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Dark Matter
Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016);
Aoki, Mukohyama (2016);
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Comelli, Crisostomi, Nesti, Pilo (2011)
5%
normal matter

# Spin-2 Dark Matter

Babichev, Marzola, Raidal, ASM, Urban, Veermäe, von Strauss (2016)

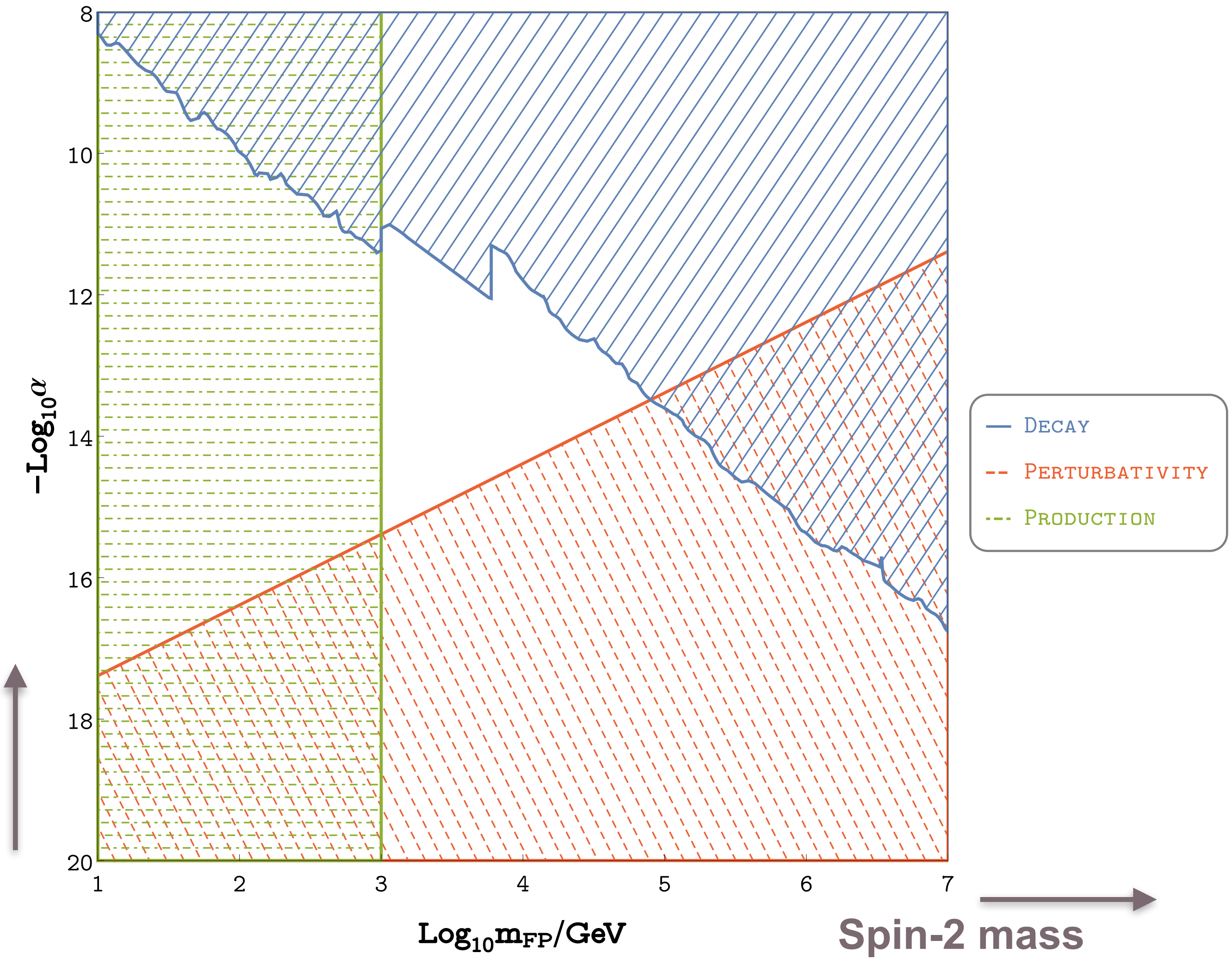
Recall the (linearised) gravitational metric: $\delta g_{\mu\nu} \propto \delta G_{\mu\nu} - \alpha^2 \delta M_{\mu\nu}$

massless massive

and the General Relativity limit of bimetric theory: $\alpha = m_f / m_g \to 0$

- gravity is weak because the physical Planck mass is large ( $m_{\rm Pl} = m_g \sqrt{1 + \alpha^2}$ )
- massive spin-2 field decouples from matter, interacts only with gravity

# Constraints




ratio of Planck masses

Spin-2 mass

# Features

- heavy spin-2 field automatically resembles dark matter when gravity resembles general relativity
- interactions with baryonic matter are suppressed by the Planck mass
- spin-2 mass and interaction scale are on the order of a few TeV

## Features

- heavy spin-2 field automatically resembles dark matter when gravity resembles general relativity
- interactions with baryonic matter are suppressed by the Planck mass
- spin-2 mass and interaction scale are on the order of a few TeV

Chu & Garcia-Cely (2017): may be lowered to MeV by taking into account self-interactions of massive spin-2

Gonzalez, ASM, von Strauss (2017): interesting new effects for more than one massive spin-2 field

# Features

- heavy spin-2 field automatically resembles dark matter when gravity resembles general relativity
- interactions with baryonic matter are suppressed by the Planck mass
- spin-2 mass and interaction scale are on the order of a few TeV

---

- no need for extra fields, artificial symmetries or fine tuning
- bimetric theory could explain dark matter in the context of gravity
- massive spin-2 field is a natural addition to the Standard Models

# Summary

# Massive spin-2 fields...

review: ASM, Mikael von Strauss; 1512.00021

- provide one of the few known consistent modifications of GR
- are uniquely described by ghost-free bimetric theory
- could be a dark matter candidate whose coupling to baryonic matter is suppressed by the Planck scale

---

- Larger theoretical framework: String Theory ?
- Additional symmetries ? Quantum gravity ?
- Can we detect/observe the massive spin-2 ?


Thank you for your attention!

# Back-up slides

# Linear Constraints

$$\bar{\mathcal{E}}_{\mu\nu}^{\ \ \rho\sigma}\delta g_{\rho\sigma} + \frac{m_{\rm FP}^2}{2}\left(\delta g_{\mu\nu} - \mathbf{a}\,\bar{g}_{\mu\nu}\delta g\right) = 0$$

$\mathbf{a} = 1$

- Divergence of equation gives $\nabla^\mu\left(\delta g_{\mu\nu} - \bar{g}_{\mu\nu}\delta g\right) = 0$

  Trace gives $\delta g = 0$

- 5 contraints on 10 components, equation propagates <u>5 degrees of freedom</u>

---

$\mathbf{a} \neq 1$

- Trace equation contains two derivatives, not a constraint
- 

# Proportional solutions

Hassan, ASM, von Strauss (2012)

Ansatz: $\bar{f}_{\mu\nu} = c^2 \bar{g}_{\mu\nu}$ with $c =$ const.

- gives two copies of Einstein's equations $(\alpha \equiv m_f/m_g)$ :

$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_g(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$
$$R_{\mu\nu}(\bar{g}) - \tfrac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) + \Lambda_f(\alpha, \beta_n, c)\bar{g}_{\mu\nu} = 0$$

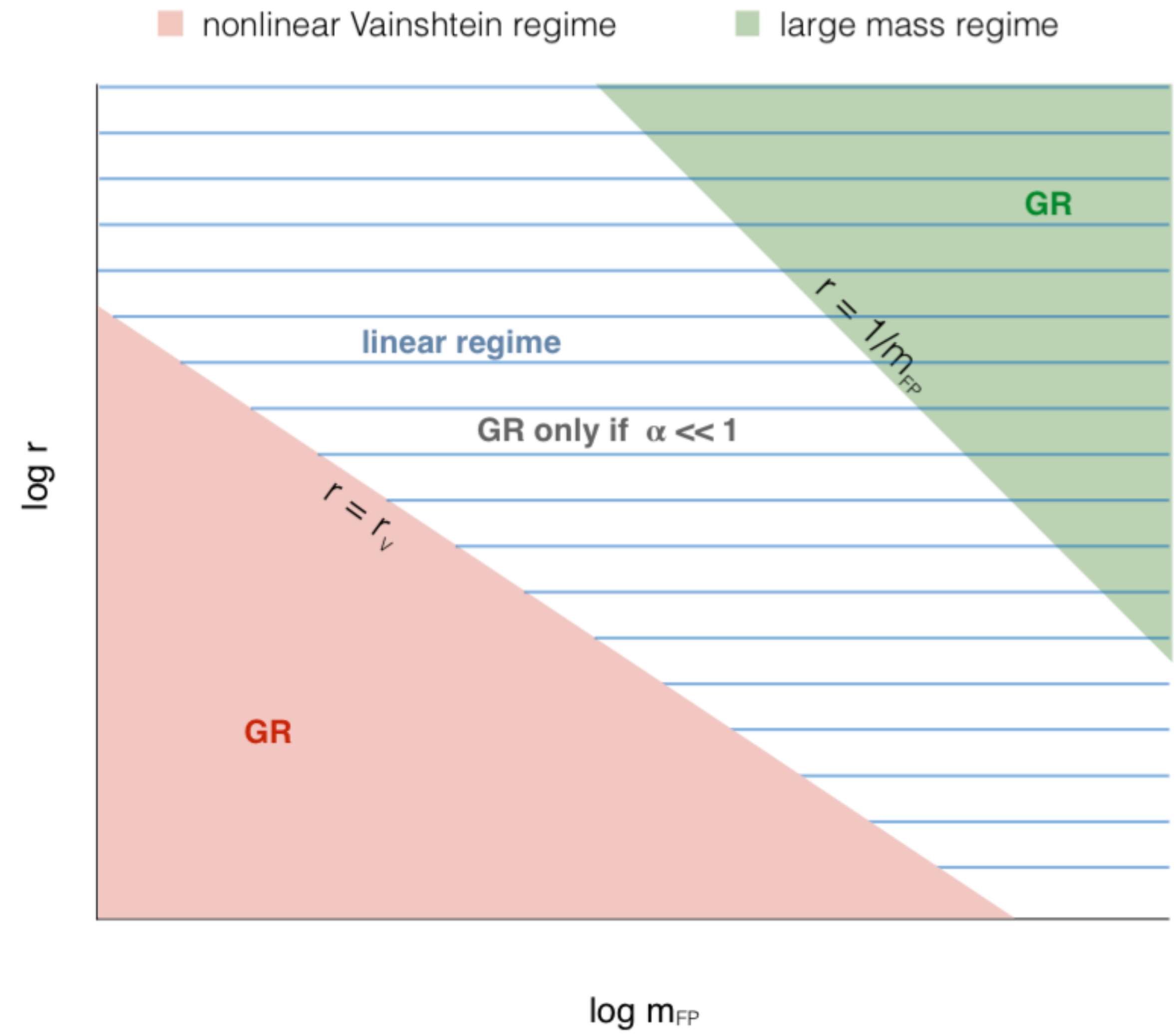
- consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines $c$

⇒ Maximally symmetric backgrounds with $R_{\mu\nu}(\bar{g}) = \Lambda_g \bar{g}_{\mu\nu}$

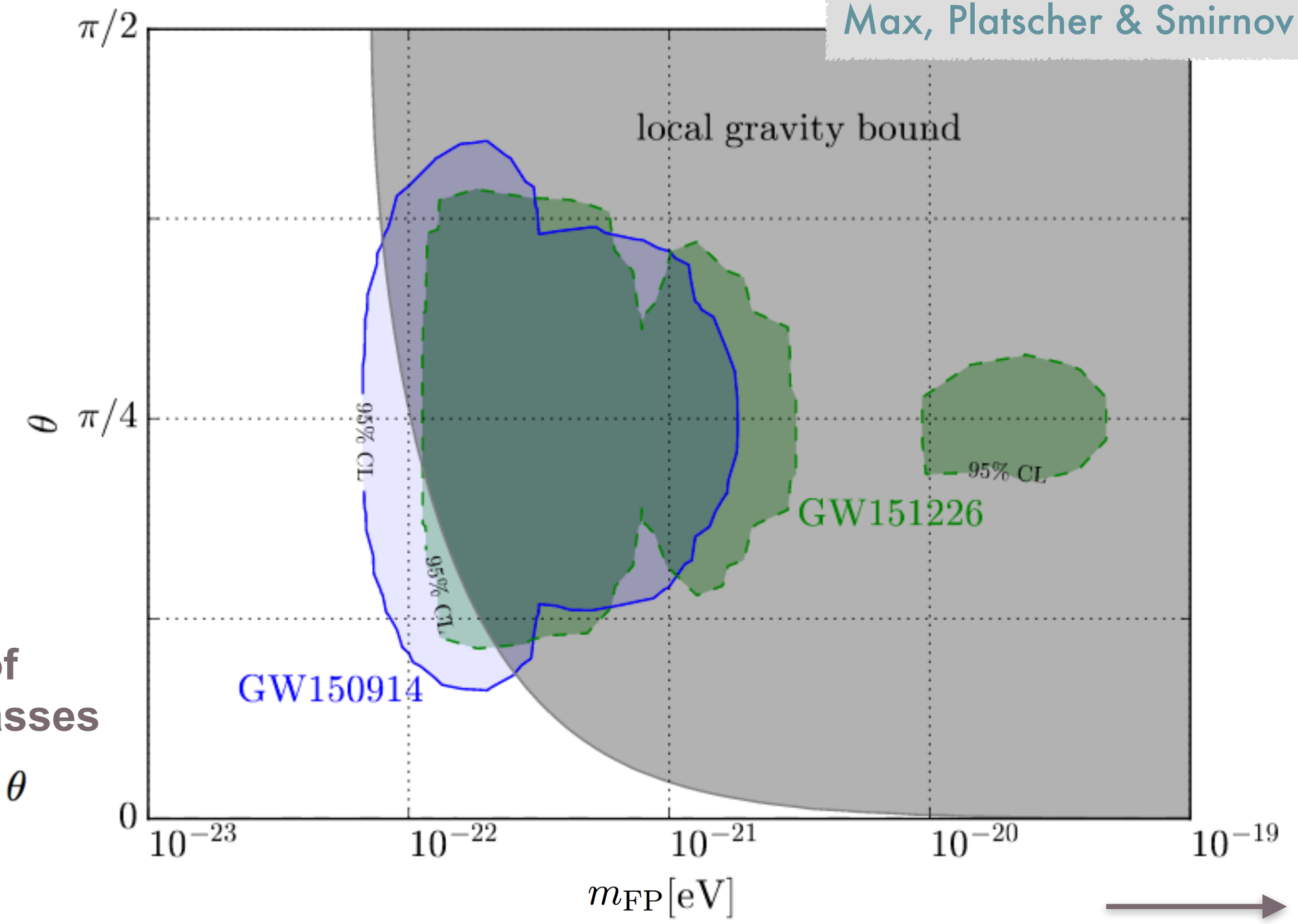
# Recovery of GR

$$r_V = \left(\frac{r_S}{m_{FP}^2}\right)^{1/3}$$

$$V_{Yuk} \sim \frac{e^{-m_{FP} r}}{r}$$



Max, Platscher & Smirnov (2017)



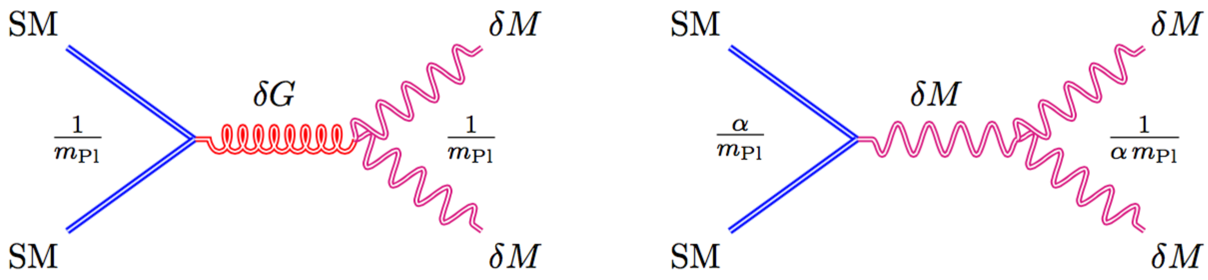

ratio of Planck masses

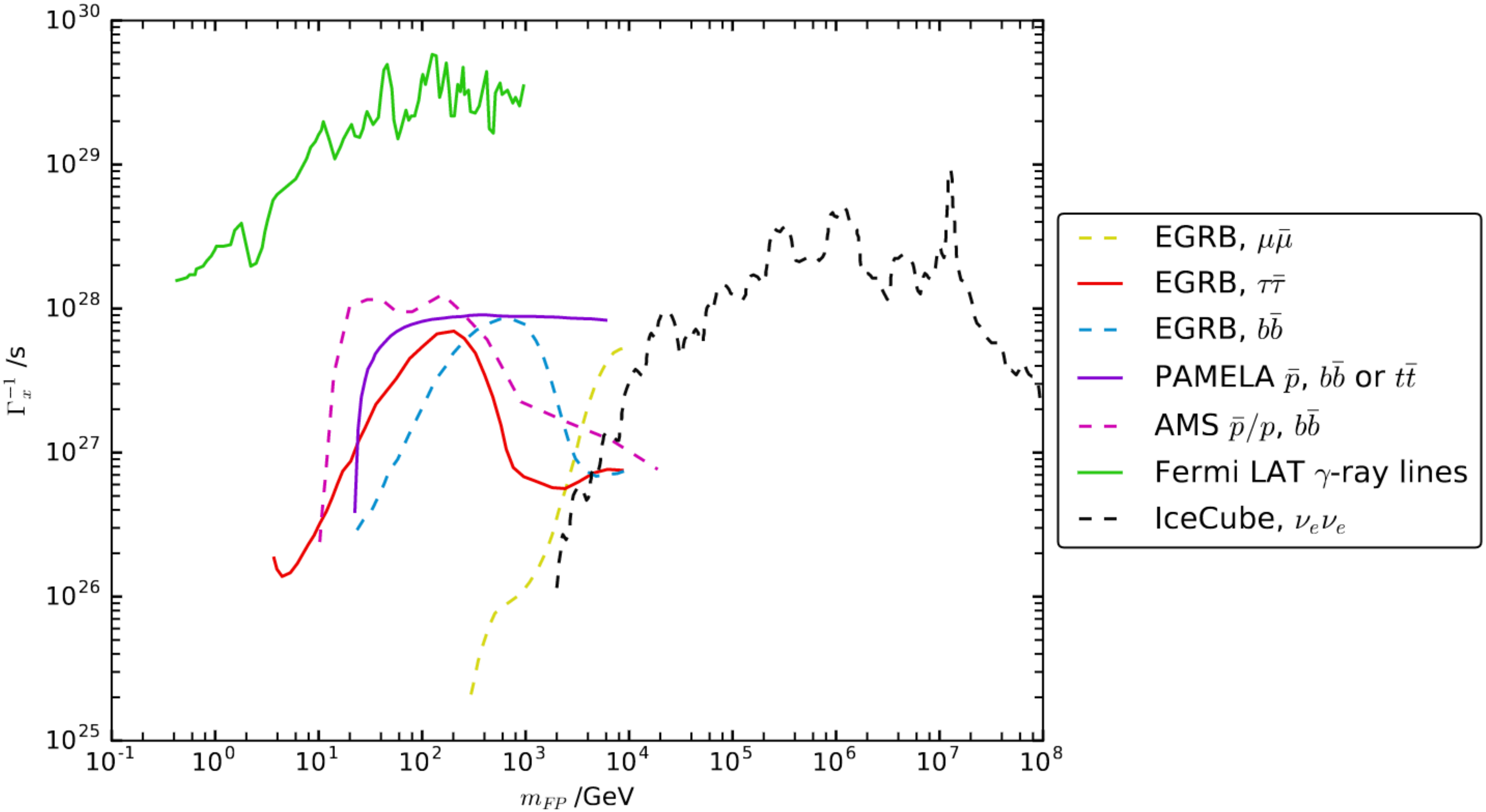
$\alpha^2 = \tan^2\theta$

Spin-2 mass

# DM Production

- standard freeze-out does not work: no thermal equilibrium, expansion rate always dominates over interaction rate
- gravitational production does not work: required DM mass is too large and violates perturbativity bound
- freeze-in mechanism works: gives lower bound on DM mass





$10^{30}$
$10^{29}$
$10^{28}$
$10^{27}$
$10^{26}$
$10^{25}$
$\Gamma_x^{-1}$ /s
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
$10^{4}$
$10^{5}$
$10^{6}$
$10^{7}$
$10^{8}$
$m_{FP}$ /GeV
EGRB, $\mu\bar{\mu}$
EGRB, $\tau\bar{\tau}$
EGRB, $b\bar{b}$
PAMELA $\bar{p}$, $b\bar{b}$ or $t\bar{t}$
AMS $\bar{p}/p$, $b\bar{b}$
Fermi LAT $\gamma$-ray lines
IceCube, $\nu_e\nu_e$

# Higher-Derivative Action

Hassan, ASM, von Strauss (2013); Gording & ASM (2018)

Equations for $f_{\mu\nu}$ can be solved perturbatively in $\alpha = m_f/m_g$

➡ Higher-derivative action for $g_{\mu\nu}$ :

$$S_{\text{eff}}[g] = \int \mathrm{d}^4x\,\sqrt{-g}\,\Big[\underbrace{m_{\text{Pl}}^2(R-2\Lambda)}_{\text{general relativity}} + \frac{\alpha^4 c_{RR}}{m^2}\underbrace{\Big(\frac{1}{3}R^2 - R^{\mu\nu}R_{\mu\nu}\Big)}_{(\text{Weyl tensor})^2\ +\ \text{total derivative}}\Big] + \mathcal{O}(\alpha^6)$$

➡ curvature corrections to GR capture effects of heavy spin-2 field

# Vierbein Formulation

Hinterbichler & Rosen (2012)

$$g_{\mu\nu} = e^a{}_\mu \eta_{ab} e^b{}_\nu \qquad f_{\mu\nu} = \tilde{e}^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu$$

Einstein-Hilbert terms: $S_{\rm EH} = m_g^2 \epsilon_{abcd} \int (R^{ab} - \Lambda\, e^a \wedge e^b) \wedge e^c \wedge e^d$

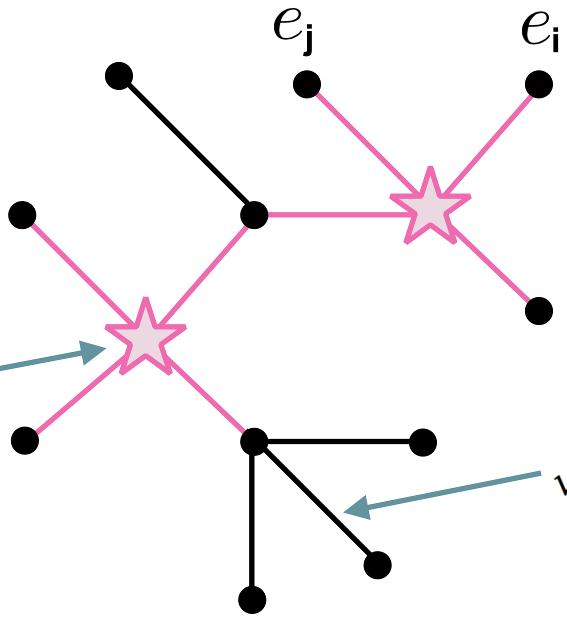
interaction terms:

$$S_{\rm int} = -m^4 \epsilon_{abcd} \int \left[\bar{\beta}_1\, e^a \wedge e^b \wedge e^c \wedge \tilde{e}^d + \bar{\beta}_2\, e^a \wedge e^b \wedge \tilde{e}^c \wedge \tilde{e}^d + \bar{\beta}_3\, e^a \wedge \tilde{e}^b \wedge \tilde{e}^c \wedge \tilde{e}^d\right]$$

- equivalent to bimetric theory and ghost-free only if $e^a{}_\mu \eta_{ab} \tilde{e}^b{}_\nu = e^a{}_\nu \eta_{ab} \tilde{e}^b{}_\mu$
  - ⇨ existence of square-root and intersection of light cones (Hassan & Kocic, 2017)
- natural generalization to other spacetime dimensions

# General interaction graph

Hassan & ASM (2018)




$$\det\left(\sum_{I=1}^{\mathcal{N}} \beta^I e_I\right)$$

$$\sqrt{g}\sum_{n=0}^{4} \beta_n\, e_n\left(\sqrt{g^{-1}f}\right)$$

☆ used to be a (nondynamical) ● which has been integrated out