CP Violation caused by another symmetry

Andreas Trautner

based on:

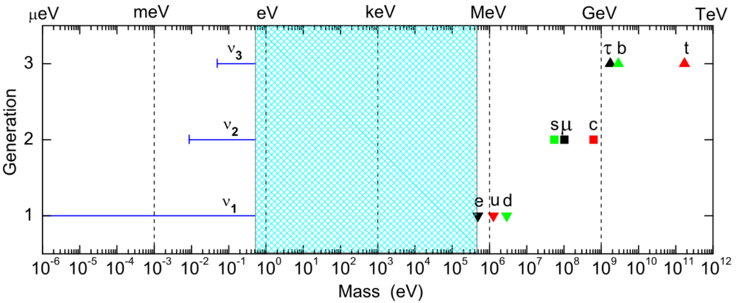
NPB883 (2014) 267-305	1402.0507	w/ M.-C. Chen, M. Fallbacher, K.T. Mahanthappa and M. Ratz
NPB894 (2015) 136-160	1502.01829	w/ M. Fallbacher
JHEP 1702 (2017) 103	1612.08984	w/ M. Ratz
arXiv	1808.07060	w/ H.P. Nilles, M. Ratz., P. Vaudrevange

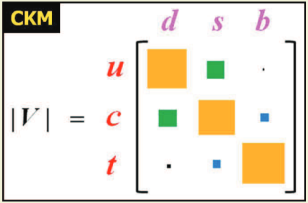
Inνisibles18 Workshop
Karlsruhe

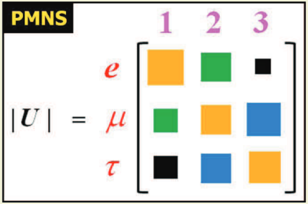
7.9.18

Motivation

- Standard Model flavor puzzle. Observed patterns:



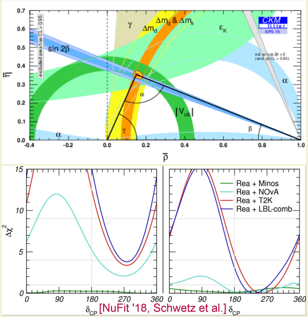




[Xing '14]

Motivation

- Standard Model flavor puzzle.
  4x 3 masses, 2x 3 angles, 2x 1 CP violating phase(+2).
- Origin of CP violation?
  - CP violation established in quark sector, consistent with SM (CKM). ✓
  - open question: CP violation in lepton sector ?
  - open question: Why $\overline{\theta} = (\theta + \arg\det y_u y_d) < 10^{-10}$? Why CPV *only* in FV processes?



- Flavor and CP are intertwined.

↷ *The* theory of flavor should also be a theory of CPV.

Goal: Understand origin of CPV ⇒ hints for origin of flavor.

Outline

- Standard Model CP: a special outer automorphism
- What is an outer automorphism?
- CP violation as consequence of certain symmetries
- Example (toy-)model: $\mathrm{SU}(3) \to \mathrm{T}_7$ with CPV and $\overline{\theta} = 0$
- Conclusion

Physical CP transformations

Physical observable: Asymmetry $\Leftrightarrow$ Basis–invariants, e.g. J.

$$\varepsilon_{i\to f} \;=\; \frac{\left|\Gamma(i\to f)\right|^2 - \left|\Gamma(\bar{\imath}\to\overline{f})\right|^2}{\left|\Gamma(i\to f)\right|^2 + \left|\Gamma(\bar{\imath}\to\overline{f})\right|^2} \;\Leftrightarrow\; J \;=\; \det\left[M_u M_u^\dagger, M_d M_d^\dagger\right]$$

CP conservation: $\varepsilon, J \stackrel{!}{=} 0$. see also [Bernabéu, Branco, Gronau '86], [Botella, Silva '94]

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see also [Bernabéu, Branco, Gronau '86], [Botella, Silva '94]

To warrant this: **need** a map $M_{u/d} \to M^*_{u/d}$.
Equivalently:

$$\mathscr{L} \;\supset\; c\;\mathcal{O}\,(x) + c^*\;\mathcal{O}^\dagger(x) \qquad \Rightarrow \qquad \text{Fields} \xrightarrow{\;\mathcal{CP}\;} (\text{Fields})^*$$

$\mathcal{CP}$

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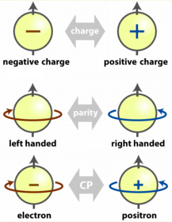
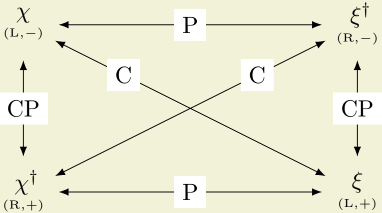


$$\Psi_{\text{Dirac}} = \begin{pmatrix}\chi_{\text{L}}\\ \xi_{\text{R}}^\dagger\end{pmatrix}$$



CP transformation in the Standard Model

In the Standard Model

$$\mathrm{SU}(3)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1) \qquad \text{and} \qquad \mathrm{SO}(3,1)\;,$$

physical CP is described by a *simultaneous* outer automorphism transformation of all symmetries which maps

$$\boldsymbol{r}_i \;\longleftrightarrow\; \boldsymbol{r}_i^{\,*}\;,$$
$$\left(\text{e.g. } (\mathbf{3},\mathbf{2})^{\mathrm{L}}_{1/6} \;\longleftrightarrow\; (\overline{\mathbf{3}},\overline{\mathbf{2}})^{\mathrm{R}}_{-1/6}\right)\;,$$

for *all* representations of *all* symmetries.

[Grimus, Rebelo '95]
[Buchbinder et al. '01]
[AT '16]

Conservation of such a transformation warrants $\overline{\theta},\ \delta_{\not{CP}} = 0$.

Violation of such a transformation is implied by experiment, and necessary requirement for baryogenesis. [Sakharov '67]

However: Why $\delta_{\mathrm{CKM}}\sim\mathcal{O}(1)$ while $\overline{\theta}_{\mathrm{exp}} \;<\; 10^{-10}$?

What is an outer automorphism?

Example: $\mathbb{Z}_3$ symmetry, generated by $\mathsf{a}^3 = \mathsf{id}$.

- All elements of $\mathbb{Z}_3$: $\{\mathsf{id}, \mathsf{a}, \mathsf{a}^2\}$.
- Outer automorphism group ("Out") of $\mathbb{Z}_3$: generated by

$$u(\mathsf{a}) : \mathsf{a} \mapsto \mathsf{a}^2. \quad \big(\text{think: } \mathsf{u\,a\,u}^{-1} = \mathsf{a}^2\big)$$

$\mathbb{Z}_3$	id	a	a^2
$\mathbf{1}$	1	1	1
$\mathbf{1}'$	1	ω	ω^2
$\mathbf{1}''$	1	ω^2	ω

$(\omega := \mathrm{e}^{2\pi\,\mathrm{i}/3})$

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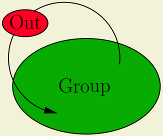

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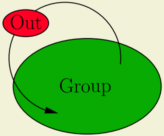

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In words: Out is a **"symmetry of the symmetry"**.

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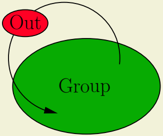

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Concrete: Out is a 1:1 mapping of representations $\boldsymbol{r} \mapsto \boldsymbol{r'}$.
Comes with a transformation matrix U, which is given by

$$U\rho_{\boldsymbol{r'}}(\mathrm{g})U^{-1} = \rho_{\boldsymbol{r}}(u(\mathrm{g}))\,, \qquad \forall \mathrm{g} \in G\,.$$

(consistency condition)

[Fallbacher, AT, '15]
[Holthausen, Lindner, Schmidt, '13]

- $\rho_{\boldsymbol{r}}(g)$: representation matrix for group element $g \in G$
- $u : g \mapsto u(g)$: **outer** automorphism

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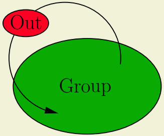

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Physical CP trafo
$\boldsymbol{r} \mapsto \boldsymbol{r}' = \boldsymbol{r}^*$
is a special case of this!

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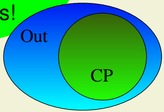


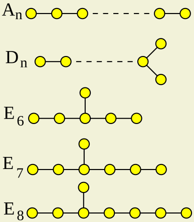
Outer automorphisms of groups

Outer automorphisms exist for continuous & discrete groups.

There are easy ways to depict this:

Continuous groups:

Outer automorphisms of a simple Lie algebra are the symmetries of the corresponding Dynkin diagram.



	Lie Group	Out	Action on reps
$A_{n>1}$	$\mathrm{SU}(N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$D_{n>4}$	$\mathrm{SO}(2N)$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
E_6	E_6	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$D_{n=4}$	$\mathrm{SO}(8)$	S_3	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
all others		/	/

Outer automorphisms of groups

Discrete groups:

Outer automorphisms of a discrete group are symmetries of the character table (not 1:1).

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
$\overline{\mathbf{3}}_1$	3	0	0	η^*	η

$\mathbb{Z}_2$

$\Delta(54)$	C_{1a}	C_{3a}	C_{3b}	C_{3c}	C_{3d}	C_{2a}	C_{6a}	C_{6b}	C_{3e}	C_{3f}
$\mathbf{1}_0$	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}_1$	1	1	1	1	1	-1	-1	-1	1	1
$\mathbf{2}_1$	2	2	-1	-1	-1	0	0	0	2	2
$\mathbf{2}_2$	2	-1	2	-1	-1	0	0	0	2	2
$\mathbf{2}_3$	2	-1	-1	2	-1	0	0	0	2	2
$\mathbf{2}_4$	2	-1	-1	-1	2	0	0	0	2	2
$\mathbf{3}_1$	3	0	0	0	0	1	ω^2	ω	3ω	$3\omega^2$
$\overline{\mathbf{3}}_1$	3	0	0	0	0	1	ω	ω^2	$3\omega^2$	3ω
$\mathbf{3}_2$	3	0	0	0	0	-1	$-\omega^2$	$-\omega$	3ω	$3\omega^2$
$\overline{\mathbf{3}}_2$	3	0	0	0	0	-1	$-\omega$	$-\omega^2$	$3\omega^2$	3ω

s

The outer automorphisms group of any ("small") discrete group can easily be found with GAP [GAP] .

Group	Out	Action on reps
$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$A_{n\neq 6}$	$\mathbb{Z}_2$	$\boldsymbol{r} \to \boldsymbol{r}^*$
$S_{n\neq 6}$	/	/
$\Delta(27)$	$\mathrm{GL}(2,3)$	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
$\Delta(54)$	S_4	$\boldsymbol{r}_i \to \boldsymbol{r}_j$
$\ldots$		

Not this talk

Outer automorphisms by themself have interesting features:

- Allow to understand origin of “geometrical T violation”. [Branco, Gerard, Grimus, ’83], [Fallbacher, AT, ’15]
- Deep connection to RGE flow of theories.
- Very useful tool to compute stationary points of potentials. [Fallbacher, AT, ’15]
- Systematic origin of emergent symmetries. [AT ’16]

Physical CP transformation

We *extrapolate* from the SM to possible symmetries in BSM.

$\Rightarrow$ “Definition” of CP in words:

CP is **a** special outer automorphism transformation which maps *all present* symmetry representations (global, local, space-time) to their complex conjugates.

[AT ’16]

This definition is consistent with the definitions in [Buchbinder et al. ’01] & [Grimus, Rebelo ’95]

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Any such transformation:

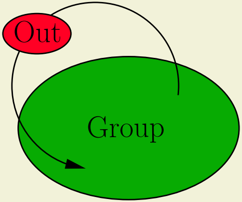
- warrants *physical* CP conservation (if conserved),

$\Rightarrow$ must be broken (by observation).

Note that a physical CP transformation:

- does not have to be unique,
- does not have to be of order 2, [Ecker, Grimus, Neufeld '87], [Weinberg '05] [Chen, Fallbacher, Mahanthappa, Ratz, AT '14] [Ivanov, Silva '15], [Ferreira et al. '17]
- is, in general, not guaranteed to exist for a given symmetry group. (It *does* exist for G_{SM}!)

Two types of groups (without mathematical rigor)



List of representations: $\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_k, \boldsymbol{r}_k{}^*, \ldots$

Out in general : $\boldsymbol{r}_i \mapsto \boldsymbol{r}_j$ $\forall$ irreps i,j $(1:1)$

Criterion:

Is there an (outer) automorphism transformation that maps

$$\boldsymbol{r}_i \mapsto \boldsymbol{r}_i{}^* \quad \text{for all irreps } i\,?$$

No	**Yes**
$\Rightarrow$ Group of "**type I**"	$\Rightarrow$ Group of "**type II**"

This tells us whether a CP transformation is possible, or not!

Do CP transformations exist for all symmetries?

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General answer: **No.**

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For example: Discrete groups of **type I**:

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\ldots$
SG id	$(20,3)$	$(21,1)$	$(27,3)$	$(27,4)$	

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- These are **inconsistent** with the trafo $\boldsymbol{r}_i \mapsto \boldsymbol{r}_i^* \; \forall\, i$.

$\Rightarrow$ CP transformation is inconsistent with a type I symmetry.
(assuming sufficient # of irreps are in the model)

There are models in which CP is violated *as a consequence* of another symmetry.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]

The corresponding CPV phases are calculable and quantized (e.g. $\delta_{\not{CP}} = 2\pi/3, \ldots$) stemming from the necessarily complex Clebsch-Gordan coefficients of the "type I" group. This has been termed "explicit geometrical" CP violation.

[Chen, Fallbacher, Mahanthappa, Ratz, AT '14]
[Branco, '15], [de Medeiros Varzielas, '15]

Do CP transformations exist for all symmetries?

On the contrary:

Semi-simple Lie groups are all of type II.

- There always exists an (outer) automorphism transformation that maps all $\boldsymbol{r} \mapsto \boldsymbol{r}^*$ simultaneously.
  [Grimus, Rebelo '95]

$\Rightarrow$ CP can only be violated (explicitly) if the number of rephasing degrees of freedom is less than the number of complex parameters. cf. e.g. [Haber, Surujon '12]

This is the case in the Standard Model.

☹ This just parametrizes CPV, there is no way to predict $\delta_{\not{CP}}$.

Aside: There are models with higher-order CP transformations which allow for complex couplings, yet conserve CP (groups of type II B).
[Chang, Mohapatra '01], [Chen, Fallbacher, Mahanthappa, Ratz, AT '14], [Ivanov, Silva '15]

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  - ✗ no type I subgroups of $\mathrm{SU}(2)$.
  - ✗ no type I subgroups of the Lorentzgroup.
    (Open question: Type I “spacetime crystals”? [Wilczek ’12]).
  - ✓ In $\geq 4\mathrm{D}$: crystals with type I point groups
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- Discrete flavor symmetries?
  - Many models with type I groups:

$$\mathrm{T}_7, \Delta(27), \Delta(54), \mathcal{PSL}_2(7), ...$$

    e.g. [Björkeroth, Branco, Ding, de Anda, Ishimori, King, Medeiros Varzielas, Neder, Stuart et al. '15-'18]
    [Chen, Pérez, Ramond '14], [Krishnan, Harrison, Scott '18]
  - These can originate from extra dimensions, e.g. in string theory. [Kobayashi et al. '06], [Nilles, Ratz, Vaudrevange '12]
  - Semi-realistic heterotic orbifold model with $\Delta(54)$ flavor symmetry and geometrical CP violation.
    [Nilles, Ratz, AT, Vaudrevange '18]

Example toy model:

CP violation with an unbroken CP transformation

An interesting observation

Observation:

Type I groups can arise as subgroups of type II groups.

For example: small finite subgroups of simple Lie groups.

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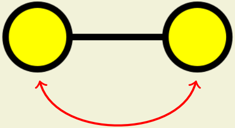
$$\mathrm{Out}(\,\mathfrak{su}(3)\,) \cong \mathbb{Z}_2$$



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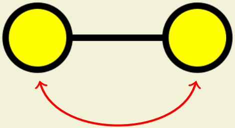

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$$\boldsymbol{r}_i \not\leftrightarrow \boldsymbol{r}_i^{\,*} \quad \forall i \;✗$$

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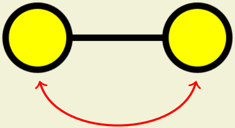
Structure of outer automorphisms:

$$\mathrm{Out}(\mathfrak{su}(3)) \cong \mathbb{Z}_2$$



$$\boldsymbol{r}_i \leftrightarrow \boldsymbol{r}_i^* \quad \forall i \text{ ✓}$$

$$\mathrm{Out}(\mathrm{T}_7) \cong \mathbb{Z}_2$$

T_7	C_{1a}	C_{3a}	C_{3b}	C_{7a}	C_{7b}
$\mathbf{1}_0$	1	1	1	1	1
$\mathbf{1}_1$	1	ω	ω^2	1	1
$\overline{\mathbf{1}}_1$	1	ω^2	ω	1	1
$\mathbf{3}_1$	3	0	0	η	η^*
$\overline{\mathbf{3}}_1$	3	0	0	η^*	η

$$\boldsymbol{r}_i \not\leftrightarrow \boldsymbol{r}_i^* \quad \forall i \text{ ✗}$$

Note: $\mathrm{Out}(\mathfrak{su}(3))$ acts on the $\mathrm{T}_7 \subset \mathrm{SU}(3)$ subgroup as $\mathrm{Out}(\mathrm{T}_7)$!

Toy model overview

Facts:

- $\mathrm{SU}(3)$ is **consistent** with a physical CP transformation.
- The T_7 subgroup of $\mathrm{SU}(3)$ is **inconsistent** with a physical CP transformation.

Question: How is CP violated in a breaking $\mathrm{SU}(3) \to \mathrm{T}_7$?

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Toy model: gauged $\mathrm{SU}(3)$ + complex scalar $\mathrm{SU}(3)$ $\mathbf{15}$-plet ϕ. [Ratz, AT '16]

$$\mathscr{L} \;=\; (D_\mu \phi)^\dagger \, (D^\mu \, \phi) - \frac{1}{4} \, G^a_{\mu\nu} \, G^{\mu\nu,a} - V(\phi) \;,$$

$$V(\phi) \;=\; -\,\mu^2 \phi^\dagger \phi + \sum_{i=1}^{5} \lambda_i \, \mathcal{I}_i^{(4)}(\phi) \;. \qquad \text{with } \lambda_i \in \mathbb{R}$$

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calculation enabled by SUSYNO [Fonseca '11]

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- VEV of the $\mathbf{15}$-plet $\langle\phi\rangle$ breaks $\mathrm{SU}(3) \to \mathrm{T}_7$. [Luhn, '11], [Merle, Zwicky '11]
- $\mathrm{Out}(\,\mathfrak{su}(3)\,) \cong \mathbb{Z}_2 \;\to\; \mathrm{Out}(\,\mathrm{T}_7\,) \cong \mathbb{Z}_2$; Out **unbroken** by VEV.

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2 \xrightarrow{\langle\phi\rangle} \mathrm{T}_7 \rtimes \mathbb{Z}_2;.$$

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

[Ratz, AT '16]

Name	$\mathrm{SU}(3)$	$\xrightarrow{\langle\phi\rangle}$ Name	T_7	mass
A_μ	$\mathbf{8}$	Z_μ	$\mathbf{1_1}$	$m_Z^2 = 7/3\, g^2\, v^2$
		W_μ	$\mathbf{3}$	$m_W^2 = g^2\, v^2$
ϕ	$\mathbf{15}$	$\mathrm{Re}\,\sigma_0$	$\mathbf{1_0}$	$m_{\mathrm{Re}\,\sigma_0}^2 = 2\,\mu^2$
		$\mathrm{Im}\,\sigma_0$	$\mathbf{1_0}$	$m_{\mathrm{Im}\,\sigma_0}^2 = 0$
		σ_1	$\mathbf{1_1}$	$m_{\sigma_1}^2 = -\mu^2 + \sqrt{15}\,\lambda_5\, v^2$
		τ_1	$\mathbf{3}$	$m_{\tau_1}^2 = m_{\tau_1}^2(\mu, \lambda_i)$
		τ_2	$\mathbf{3}$	$m_{\tau_2}^2 = m_{\tau_2}^2(\mu, \lambda_i)$
		τ_3	$\mathbf{3}$	$m_{\tau_3}^2 = m_{\tau_3}^2(\mu, \lambda_i)$

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			τ_3	$\mathbf{3}$	$m^2_{\tau_3} = m^2_{\tau_3}(\mu, \lambda_i)$

The action is invariant under the $\mathbb{Z}_2 - \mathrm{Out}$ transformation:

SU(3)	T_7
$A^a_\mu(x) \mapsto R^{ab}\,\mathcal{P}_\mu^{\ \nu}\,A^b_\nu(\mathcal{P}\,x)\,,$ $\phi_i(x) \mapsto U_{ij}\,\phi^*_j(\mathcal{P}\,x)\,.$	$W_\mu(x) \mapsto \mathcal{P}_\mu^{\ \nu}\,W^*_\nu(\mathcal{P}\,x)\,,$ $\sigma_0(x) \mapsto \sigma_0(\mathcal{P}\,x)\,,$ $\tau_i(x) \mapsto \tau^*_i(\mathcal{P}\,x)\,,$ $Z_\mu(x) \mapsto -\mathcal{P}_\mu^{\ \nu}\,Z_\nu(\mathcal{P}\,x)\,,$ $\sigma_1(x) \mapsto \sigma_1(\mathcal{P}\,x)\,.$
physical CP ✓	physical CP ✗

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

- The VEV does **not** break the CP transformation, $U\langle\phi\rangle^* = \langle\phi\rangle$.
- However, at the level of T_7, the $\mathrm{SU}(3)$-CP transformation merges to $\mathrm{Out}(\,\mathrm{T}_7\,)$:

$$\mathbb{Z}_2 - \mathrm{Out}: \quad \begin{array}{ccccccccccccccc} \mathbf{15} & \to & \mathbf{1_0} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{3} & \oplus & \mathbf{3} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} \\ \downarrow & & & & & & & & & & & & & & \\ \overline{\mathbf{15}} & \to & \mathbf{1_0} & \oplus & \overline{\mathbf{1}}_\mathbf{1} & \oplus & \mathbf{1_1} & \oplus & \overline{\mathbf{3}} & \oplus & \overline{\mathbf{3}} & \oplus & \mathbf{3} & \oplus & \mathbf{3} \end{array}$$

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⇒ The $\mathbb{Z}_2$-Out is conserved at the level of T_7, but it is **not** interpreted as a physical CP trafo,

$$\mathrm{SU}(3) \rtimes \mathbb{Z}_2^{(\mathrm{CP})} \xrightarrow{\langle\phi\rangle} \mathrm{T}_7 \rtimes \mathbb{Z}_2^{\not{(\mathrm{CP})}} .$$

- There is no other possible allowed CP transformation at the level of T_7 (type I).
- Imposing a transformation $\boldsymbol{r}_{\mathrm{T}_7,i} \leftrightarrow \boldsymbol{r}_{\mathrm{T}_7,i}{}^*$ enforces decoupling, $g = \lambda_i = 0$.

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

Explicit crosscheck: compute decay asymmetry.

$$\varepsilon_{\sigma_1 \to W\,W^*} \;:=\; \frac{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 - \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}{\left|\mathscr{M}(\sigma_1 \to W\,W^*)\right|^2 + \left|\mathscr{M}(\sigma_1^* \to W\,W^*)\right|^2}\;.$$

CP violation in $\mathbf{SU(3)} \to \mathbf{T_7}$ toy model

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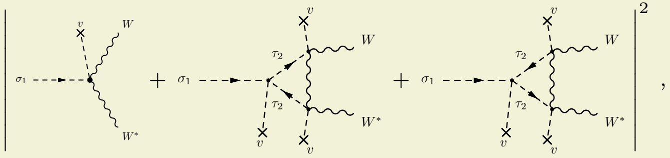
Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ from interference terms, e.g.



corresponding to non-vanishing CP-odd basis invariants

$$\mathcal{I}_1 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{imk} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jm\ell}\;,$$

$$\mathcal{I}_2 \;=\; \left[Y^\dagger_{\sigma_1 W W^*}\right]_{k\ell} \left[Y_{\sigma_1 \tau_2 \tau_2^*}\right]_{ij} \left[Y_{\tau_2^* W W^*}\right]_{i\ell m} \left[\left(Y_{\tau_2^* W W^*}\right)^*\right]_{jkm}\;.$$

- ✓ Contribution to $\varepsilon_{\sigma_1 \to W\,W^*}$ is proportional to $\operatorname{Im}\mathcal{I}_{1,2} \neq 0$.
- ✓ All CP odd phases are geometrical, $\mathcal{I}_1 = \mathrm{e}^{2\,\pi\,\mathrm{i}/3}\,\mathcal{I}_2$.
- ✓ $\left(\varepsilon_{\sigma_1 \to W\,W^*}\right) \to 0$ for $v \to 0$, i.e. CP is restored in limit of vanishing VEV.

Natural protection of $\theta = 0$

Topological vacuum term of the gauge group

$$\mathscr{L}_\theta \;=\; \theta\,\frac{g^2}{32\pi^2}\,G^a_{\mu\nu}\,\widetilde{G}^{\mu\nu,a}\;,$$

is forbidden by $\mathbb{Z}_2 - \mathrm{Out}$ (the $\mathrm{SU}(3)$-CP transformation).

The unbroken Out

$$\mathbb{Z}_2 - \mathrm{Out}\;:\;W_\mu(x)\;\mapsto\;\mathcal{P}_\mu^{\;\nu}\,W_\nu^*(\mathcal{P}\,x)\;,\quad Z_\mu(x)\;\mapsto\;-\,\mathcal{P}_\mu^{\;\nu}\,Z_\nu(\mathcal{P}\,x)\;,$$

still enforces $\theta = 0$ even though CP is violated for the physical T_7 states.

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Physical scalars (T_7 singlets and triplets):

$$\mathrm{Re}\,\sigma_0\;=\;\frac{1}{\sqrt{2}}\,(\phi_1+\phi_1^*)\;,\qquad \mathrm{Im}\,\sigma_0\;=\;-\,\frac{\mathrm{i}}{\sqrt{2}}\,(\phi_1-\phi_1^*)\;,$$

$$\sigma_1\;=\;\phi_2\;,$$

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau_3\end{pmatrix}\;=\;\begin{pmatrix}V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\begin{pmatrix}T_2\\ \overline{T}^*_3\\ T_1\end{pmatrix}\;.$$

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Possible application to strong CP problem?

- Starting point: CP conserving theory based on

$$[G_{\mathrm{SM}}\times G_{\mathrm{F}}]\rtimes\mathrm{CP}\;.$$

- break $G_{\mathrm{F}}\rtimes\mathrm{CP}\;\longrightarrow\;\mathrm{Type\,I}\rtimes\mathrm{Out}.$

↷ CP broken in flavor sector but not in strong interactions.

- Main problem: finding realistic model based on Type I group allowing for outer automorphism.

Summary

- Outer automorphisms are **symmetries of symetries** ($\rightarrow$ think of them as mappings among the irreps).
- CP is **a** special outer automorphism which maps *all* present representations to their complex conjugate.
- There are "**type I**" groups, they are inconsistent with CP transformations.
  $\Rightarrow$ CPV (explicit/spontaneous) with quantized phases.
- Example for appearance of **type I** symmetries: potentially realistic heterotic orbifold string theories.
  [Nilles, Ratz, AT, Vaudrevange '18]
- Explicit toy model: **type I** as subgroup of **type II** group

$$\text{gauged} \quad \mathrm{SU}(3) \xrightarrow{\langle \mathbf{15} \rangle} \mathrm{T}_7 \quad \text{with weak CPV but} \quad \theta_{\mathrm{SU}(3)} = 0.$$

Thank You

Backup slides

CP as a special outer automorphism

One generation of (chiral) fermion fields with gauge symmetry $[T_a, T_b] = \mathrm{i} f_{abc}\, T_c$

$$\mathscr{L} \;=\; \mathrm{i}\,\overline{\Psi}\,\gamma^\mu \left(\partial_\mu - \mathrm{i}\, g\, T_a\, W_\mu^a\right) \Psi - \frac{1}{4}\, G_{\mu\nu}^a\, G^{\mu\nu,a} \;.$$

The most general possible CP transformation:

$$\begin{aligned} W_\mu^a(x) \;&\mapsto\; R^{ab}\, \mathcal{P}_\mu^{\;\nu}\, W_\nu^b(\mathcal{P}\,x) \;, \\ \Psi_\alpha^i(x) \;&\mapsto\; \eta_{\mathsf{CP}}\, U^{ij}\, \mathcal{C}_{\alpha\beta}\, {\Psi^*}_\beta^j(\mathcal{P}\,x) \;. \end{aligned}$$

[Grimus, Rebelo,'95]

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[Grimus, Rebelo,'95]

For this to be a conserved symmetry of the *action*, require:

$$\begin{aligned} &\text{(i)} & R_{aa'}\,R_{bb'}\,f_{a'b'c} \;&=\; f_{abc'}\,R_{c'c}\;, \\ &\text{(ii)} & U\,(-T_a^{\mathrm{T}})\,U^{-1} \;&=\; R_{ab}\,T_b\;, \\ &\text{(iii)} & \mathcal{C}\,(-\gamma^{\mu\,\mathrm{T}})\,\mathcal{C}^{-1} \;&=\; \gamma^{\mu}\;. \end{aligned}$$

Meaning of these equations:

(i) CP is an (outer) automorphism of the gauge group.

(ii) CP maps representations to their complex conjugate representations. $\left(T_a \mapsto -T_a^{\mathrm{T}}\right)$

(iii) CP is an outer automorphism of the Lorentz group which maps representations to their complex conjugate representation. $\left(\chi_{\mathrm{L}} \mapsto (\chi_{\mathrm{L}})^{\dagger}\right)$

Basically two types of discrete groups

- Groups which **do not** allow for CP transformation: **Type I**
  Fine print: assuming sufficient # of irreps are there

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$	$\ldots$
SG id	$(20, 3)$	$(21, 1)$	$(27, 3)$	$(27, 4)$	

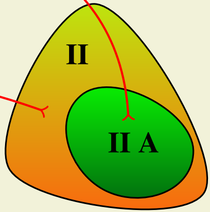
- Groups which **do** allow for CP transformation: **Type II**

Among those: all groups which allow for real CG's: Type II **A**

G	S_3	A_4	T'	S_4	A_5
SG id	$(6, 1)$	$(12, 3)$	$(24, 3)$	$(24, 12)$	$(60, 5)$

But also: CP trafo w/o real CG's: Type II **B**

G	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG id	$(72, 41)$	$(144, 120)$



Type II A groups: CP violation completely analogue to well–known case: $\mathbf{SU}(N)$
(i.e. it depends on # of rephasing d.o.f.'s vs # complex couplings)
Type II B groups: CP violation tied to certain operators

“Physical” CP transformation

Recall: e.g. complex scalar field σ, with field operator

$$\widehat{\boldsymbol{\sigma}}(x) \;=\; \int \widetilde{\mathrm{d}p}\,\left\{\widehat{\boldsymbol{a}}(\vec{p})\,\mathrm{e}^{-\mathrm{i}\,p\,x} + \widehat{\boldsymbol{b}}^{\dagger}(\vec{p})\,\mathrm{e}^{\mathrm{i}\,p\,x}\right\}\;.$$

Physical CP transformation of the complex scalar field

$$\mathrm{CP}\;:\quad \sigma(x)\;\mapsto\;\mathrm{e}^{\mathrm{i}\,\varphi}\,\sigma^{*}(\mathcal{P}\,x)\;,$$

corresponds to

$$\mathrm{CP}\;:\quad \widehat{\boldsymbol{a}}(\vec{p})\;\mapsto\;\mathrm{e}^{\mathrm{i}\,\varphi}\,\widehat{\boldsymbol{b}}(-\vec{p}) \qquad \text{and} \qquad \widehat{\boldsymbol{b}}^{\dagger}(\vec{p})\;\mapsto\;\mathrm{e}^{\mathrm{i}\,\varphi}\,\widehat{\boldsymbol{a}}^{\dagger}(-\vec{p})\;.$$

Note:

“matter”: $\widehat{\boldsymbol{a}}^{(\dagger)}$ “anti-matter”: $\widehat{\boldsymbol{b}}^{(\dagger)}$.

Toy model details

Complex scalar ϕ in T_7-diagonal basis of $\mathrm{SU}(3)$: (in unitary gauge)

$$\phi \;=\; \left(v+\phi_1,\,\frac{\phi_2}{\sqrt{2}},\,\frac{\phi_2^*}{\sqrt{2}},\,\phi_4,\,\phi_5,\,\phi_6,\,\frac{\phi_7}{\sqrt{2}},\,\frac{\phi_8}{\sqrt{2}},\,\frac{\phi_9}{\sqrt{2}},\,\phi_{10},\,\phi_{11},\,\phi_{12},\,\frac{\phi_7^*}{\sqrt{2}},\,\frac{\phi_8^*}{\sqrt{2}},\,\frac{\phi_9^*}{\sqrt{2}}\right)\;.$$

T_7 representations of the components:

$$\phi_1 \;\widehat{=}\; \mathbf{1}_0\;, \qquad \phi_2 \;\widehat{=}\; \mathbf{1}_1\;,$$
$$T_1 \;:=\; (\phi_4,\,\phi_5,\,\phi_6) \;\widehat{=}\; \mathbf{3}\;, \qquad T_2 \;:=\; (\phi_7,\,\phi_8,\,\phi_9) \;\widehat{=}\; \mathbf{3}\;,$$
$$\overline{T}_3 \;:=\; (\phi_{10},\,\phi_{11},\,\phi_{12}) \;\widehat{=}\; \overline{\mathbf{3}}\;.$$

The physical scalars are

$$\mathrm{Re}\,\sigma_0 \;=\; \frac{1}{\sqrt{2}}\left(\phi_1+\phi_1^*\right)\;, \qquad \mathrm{Im}\,\sigma_0 \;=\; -\,\frac{\mathrm{i}}{\sqrt{2}}\left(\phi_1-\phi_1^*\right)\;,$$
$$\sigma_1 \;=\; \phi_2\;,$$
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The physical vectors are

$$Z^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_7^\mu-\mathrm{i}\,A_8^\mu\right)\;, \qquad W_1^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_4^\mu-\mathrm{i}\,A_1^\mu\right)\;,$$
$$W_2^\mu \;=\; \frac{1}{\sqrt{2}}\left(A_5^\mu-\mathrm{i}\,A_2^\mu\right)\;, \qquad W_3^\mu \;=\; \frac{\mathrm{i}}{\sqrt{2}}\left(A_6^\mu-\mathrm{i}\,A_3^\mu\right)\;.$$

Toy model details

The VEV in this basis is simply

$$\langle\phi\rangle_1 \;=\; v \quad \text{and} \quad \langle\phi\rangle_i \;=\; 0 \quad \text{for} \quad i=2,\dots,15\;,$$

where

$$|v| \;=\; \mu\times 3\sqrt{\frac{7}{2}}\left(-7\sqrt{15}\,\lambda_1+14\sqrt{15}\,\lambda_2+20\sqrt{6}\,\lambda_4+13\sqrt{15}\,\lambda_5\right)^{-1/2}\;.$$

The masses of the physical states are

$$m_Z^2 \;=\; \frac{7}{3}\,g^2\,v^2 \quad \text{and} \quad m_W^2 \;=\; g^2\,v^2\;.$$

$$\begin{aligned} m^2_{\mathrm{Re}\,\sigma_0} \;&=\; 2\,\mu^2\;, \qquad m^2_{\mathrm{Im}\,\sigma_0} \;=\; 0\;,\\ m^2_{\sigma_1} \;&=\; -\,\mu^2+\sqrt{15}\,\lambda_5\,v^2\;. \end{aligned}$$

The massless mode is the goldstone boson of an additional $\mathrm{U}(1)$ symmetry of the potential. It can be avoided by either

- gauging the additional $\mathrm{U}(1)$,
- or breaking it softly by a cubic coupling of ϕ.

Toy model details

T_7 invariant couplings ($\omega := \mathrm{e}^{2\pi \mathrm{i}/3}$)

$$Y_{\sigma_1 W W^*} \;=\; \frac{v\,g^2}{\sqrt{6}}\,\mathrm{e}^{-\pi\,\mathrm{i}/6}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;, \quad Y_{\sigma_1 \tau_2 \tau_2^*} \;=\; v\,y_{\sigma_1 \tau_2 \tau_2^*}\,\mathrm{diag}(1,\,\omega,\,\omega^2)\;,$$

$$\begin{aligned}\left[Y_{\tau_2^* W W^*}\right]_{121} \;&=\; \left[Y_{\tau_2^* W W^*}\right]_{232} \;=\; \left[Y_{\tau_2^* W W^*}\right]_{313} \;=\; v\,g^2\,y_{\tau_2^* W W^*}\;,\\ \left[Y_{\tau_2^* W W^*}\right]_{ijk} \;&=\; 0 \qquad \text{(else)}\;.\end{aligned}$$

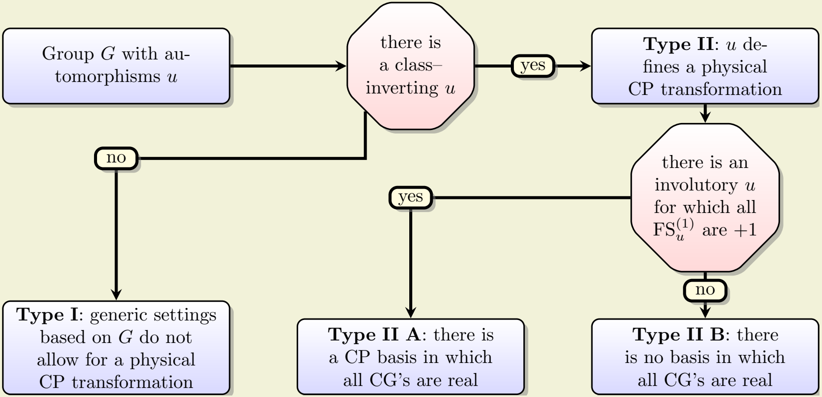
Toy model details

$$\begin{aligned}
y_{\sigma_1\tau_2\tau_2^*} \;=\; \frac{1}{504\sqrt{3}} \Big\{ & V_{21}^2 \,\Big[-14\sqrt{10}\left(17+5\sqrt{3}\,\mathrm{i}\right)\lambda_1 + 84\sqrt{30}\left(\sqrt{3}-\mathrm{i}\right)\lambda_2 \\
&\quad - 240\left(1+\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(197-55\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{22}^2\,\Big[28\sqrt{10}\left(1-\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 + 112\sqrt{3}\,\mathrm{i}\,\lambda_3 \\
&\quad - \left(30-26\sqrt{3}\,\mathrm{i}\right)\lambda_4 + \sqrt{10}\left(20-\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{23}^2\,\Big[28\sqrt{10}\left(1+\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 - 168\,\lambda_3 \\
&\quad + \left(6+65\sqrt{3}\,\mathrm{i}\right)\lambda_4 - 4\sqrt{10}\left(1-2\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 8\,V_{21}\,V_{22}\,\Big[-35\sqrt{10}\left(1-\sqrt{3}\,\mathrm{i}\right)\lambda_1 + 21\sqrt{30}\left(\sqrt{3}+\mathrm{i}\right)\lambda_2 \\
&\quad - 56\left(3+\sqrt{3}\,\mathrm{i}\right)\lambda_3 + 6\left(1+17\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(67+19\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& + 4\,V_{21}\,V_{23}\,\Big[-28\sqrt{10}\left(2+\sqrt{3}\,\mathrm{i}\right)\lambda_1 - 42\sqrt{30}\left(\sqrt{3}+\mathrm{i}\right)\lambda_2 \\
&\quad + 30\left(11+3\sqrt{3}\,\mathrm{i}\right)\lambda_4 - \sqrt{10}\left(31+11\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big] \\
& - 8\,V_{22}\,V_{23}\,\Big[14\sqrt{10}\,\lambda_1 - 14\sqrt{30}\,\mathrm{i}\,\lambda_2 \\
&\quad + 10\left(3+5\sqrt{3}\,\mathrm{i}\right)\lambda_4 + \sqrt{10}\left(1-3\sqrt{3}\,\mathrm{i}\right)\lambda_5\Big]\Big\}
\end{aligned}$$

and

$$y_{\tau_2^* W W^*} \;=\; -\,\frac{\sqrt{2}}{3}\,\left(2\,V_{21} + V_{22} + 2\,V_{23}\right)\;.$$

CP symmetries in settings with discrete G



(For details see [Chen, Fallbacher, Mahanthappa, Ratz, AT, '14])

Mathematical tool to decide: Twisted Frobenius-Schur indicator FS_u(Backup slides)

Twisted Frobenius–Schur indicator

Criterion to decide: existence of a CP outer automorphism.
$\curvearrowright$ can be probed by computing the

“twisted Frobenius–Schur indicator” $\mathbf{FS}_{\boldsymbol{u}}$

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;:=\; \frac{1}{|G|}\sum_{g\in G}\chi_{\boldsymbol{r}_i}(g\,u(g))$$

($\chi_{\boldsymbol{r}_i(g)}$: Character)
[Chen, Fallbacher, Mahanthappa, Ratz, AT, 2014]

$$\mathrm{FS}_u(\boldsymbol{r}_i) \;=\; \begin{cases} +1 \text{ or } -1 \quad \forall\, i, & \Rightarrow\; u \text{ is good for CP,} \\ \text{different from } \pm 1, & \Rightarrow\; u \text{ is no good for CP.} \end{cases}$$

In analogy to the Frobenius–Schur indicator

$\mathrm{FS}_{\not{u}}(\boldsymbol{r}_i) = +1, -1, 0$ for real / pseudo–real / complex irrep.

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