Gravitational corrections to the beta functions

(Unimodular Gravity versus General Relativity)

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① Background

Unimodular Gravity is a theory of gravitation where the spacetime metric is restricted to be of unit determinant.

- Vacuum energy does not couple to gravity in Unimodular Gravity. Therefore, it does not predict a huge value of the cosmological constant.
- At the classical level, it gives the same physics that General Relativity.
- So far, quantum effects are also the same in both theories.

Our goal is to find any difference that can tell General Relativity from Unimodular Gravity.

② Motivation

- In **Phys.Rev.Lett. 104 (2010) 081301** the GR corrections to the beta functions for a scalar λ and a Yukawa g couplings are computed.

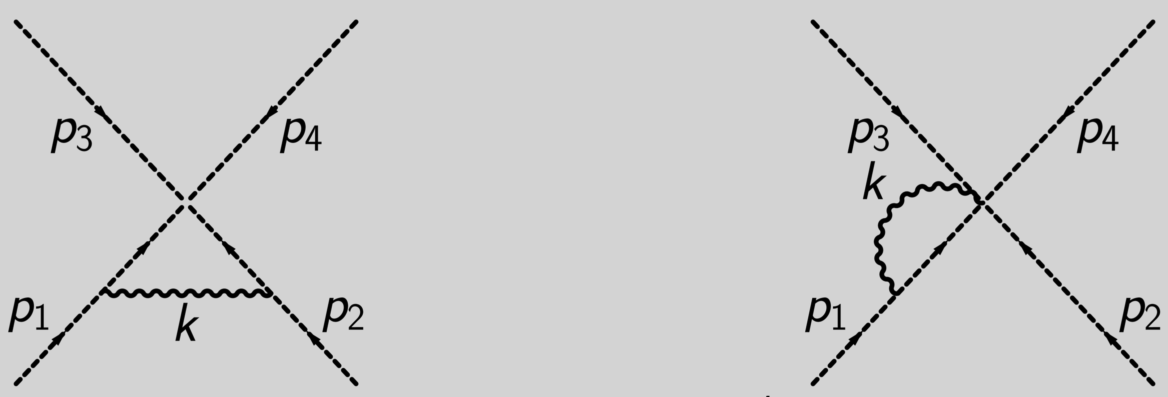
$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2\,\lambda, \quad \beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2\Big\{m_\phi^2\Big(\frac{1}{2}\Big) + m_\Psi^2\Big(-1\Big)\Big\}$$

m_ϕ = mass of the scalar, m_Ψ = mass of the fermion

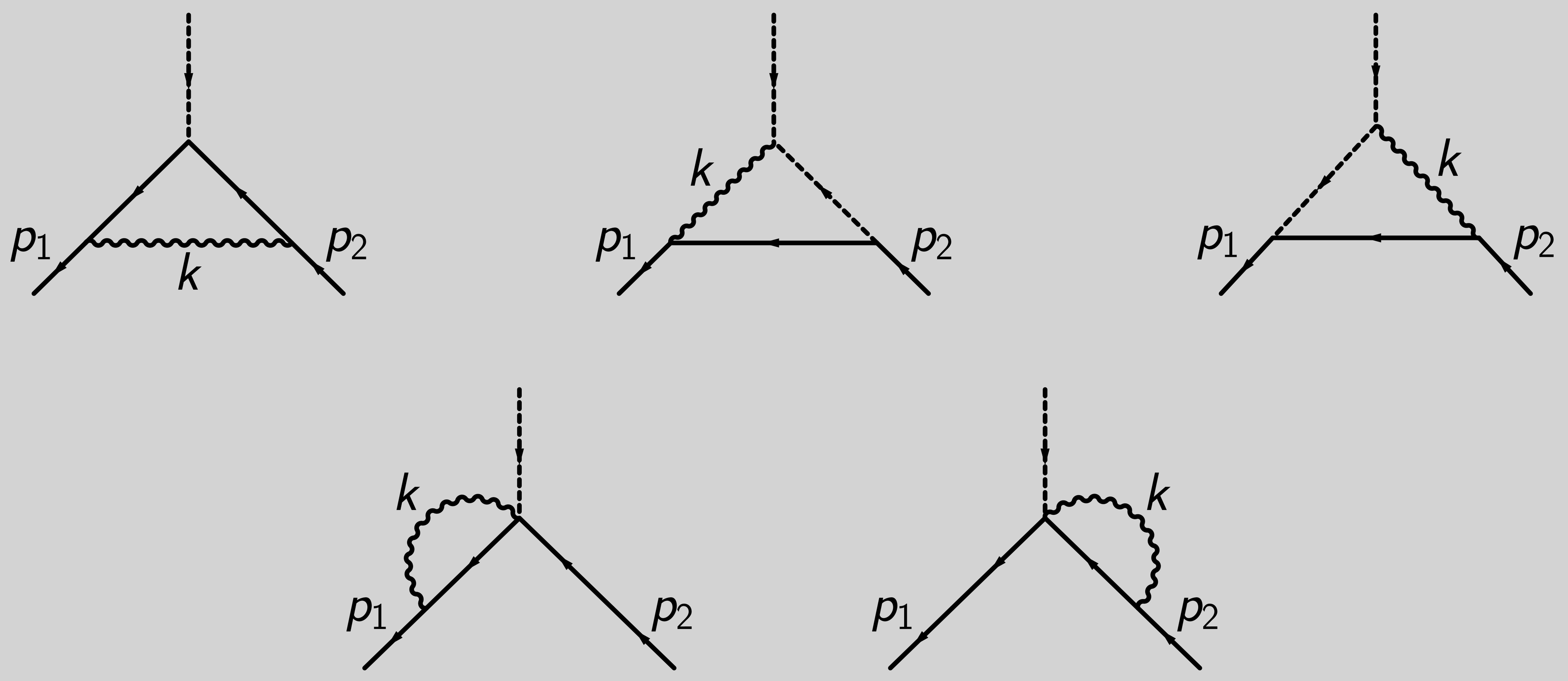
Their conclusion is that it can lead to an asymptotically free theory.

- We decided to carry out the same computations for Unimodular Gravity. For this we need to compute some diagrams.

③ Corrections to the vertices



Contributions to the ϕ^4 vertex.



Contributions to the Yukawa vertex.

④ Unimodular beta functions

The final result for Unimodular Gravity is

$$\beta_\lambda^{\mathrm{UG}} = 0, \quad \beta_g^{\mathrm{UG}} = \frac{1}{16\pi^2}\kappa^2 m_\Psi^2 \frac{3}{16}$$

Although the first conclusion could be that there is a difference between the two theories, this is not the case: the beta functions are gauge dependent. For a generalized De-Donder gauge

$$\alpha\Big(\partial^\mu h_{\mu\nu} - \frac{1}{2}\partial_\nu h\Big)^2,$$

The GR beta functions are now

$$\beta_\lambda^{\mathrm{GR}} = -\frac{1}{4\pi^2}\kappa^2 m_\phi^2\Big(\frac{3}{2}+\alpha\Big)\lambda$$

$$\beta_g^{\mathrm{GR}} = \frac{1}{16\pi^2}\kappa^2\Big\{m_\phi^2\Big[\frac{1}{2} - \Big(\frac{1}{2}+\alpha\Big)\Big] + m_\Psi^2\Big[-1-\Big(\frac{1}{2}+\alpha\Big)\frac{85}{16}\Big]\Big\}$$

No physical information from the gravitational contributions to the beta functions

⑤ Canceling the beta functions

In fact, we can even set the (gravitational) beta functions to zero.
Instead of the usual *multiplicative renormalization* we can do a non-multiplicative one (i.e. a field redefinition).

$$g_0 = \mu^{-\epsilon} Z_g Z_\psi^{-1} Z_\phi^{-1/2} g, \qquad \phi_0 = \phi + \frac{1}{2}\delta Z_\phi \phi,$$

$$\Psi_0 = \Psi + \frac{1}{2}\delta Z_\Psi \Psi + \frac{1}{2}a_1\kappa^2 m_\Psi^2 \Psi + \frac{1}{2}b_1\kappa^2 m_\phi^2\Psi, \qquad m_{\Psi_0} = (1+\delta Z_{m_\Psi})m_\Psi,$$

$$\bar\Psi_0 = \bar\Psi + \frac{1}{2}\delta Z_\Psi \bar\Psi + \frac{1}{2}a_1\kappa^2 m_\Psi^2 \bar\Psi + \frac{1}{2}b_1\kappa^2 m_\phi^2\bar\Psi, \qquad m_{\phi_0} = (1+\delta Z_{m_\phi})m_\phi.$$

By setting accordingly the values of a_1 and b_1 is easy to see that we end up with $\beta_g\Big|_{\text{gravitational}} = 0$.

- The same result holds for the scalar coupling (c.f. PLB 773 (2017) 585).
- A similar result –with similar background– happens for the coupling of the gauge field (c.f. J.Ellis & N. Mavromatos Phys.Lett. B711 (2012) 139 and references therein).

⑥ Conclusions

- Gravitational contributions to the beta functions are gauge dependent.
- The discrepancy between its value in General Relativity and Unimodular Gravity is therefore not physically relevant.
- By using a non-multiplicative wave renormalization the beta functions can be set to zero.
- So far no difference between General Relativity and Unimodular Gravity (further work in that direction: *JCAP*, 1801(01):028, 2018. and *Eur. Phys. J.*, C78(3):236, 2018).
- **Outlook**: Corrections to the Newton potential could give a difference at the one-loop level. Otherwise, it is needed to go to higher loops; computation of the Goroff & Sagnotti counterterm is the next "easiest" thing to figure out.

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