Karlsruhe Institute of Technology

Top-quark loops for precision Higgs physics

Young Scientists Meeting of the CRC TRR 257 – 18 Oct 2023

Marco Vitti – KIT, TTP and IAP

Outline

1. Precision Higgs Physics at the LHC
2. Example: $gg \to XY$ @ NLO QCD
3. Top-quark loops via pT expansion

Work in collaboration with:

L. Alasfar, L. Bellafronte, G. Degrassi, P.P. Giardino,
R. Gröber, X. Zhao

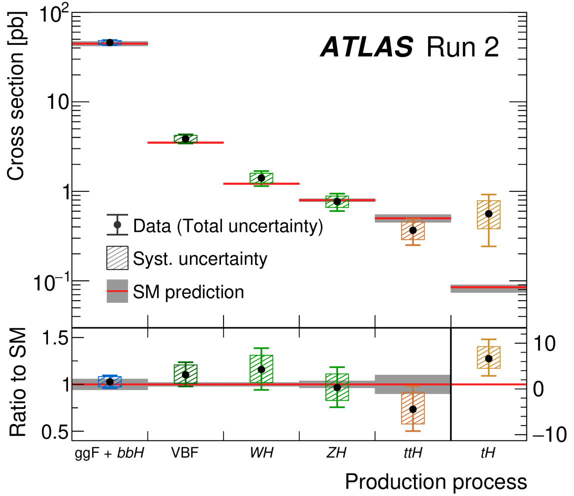
Higgs Physics at the LHC

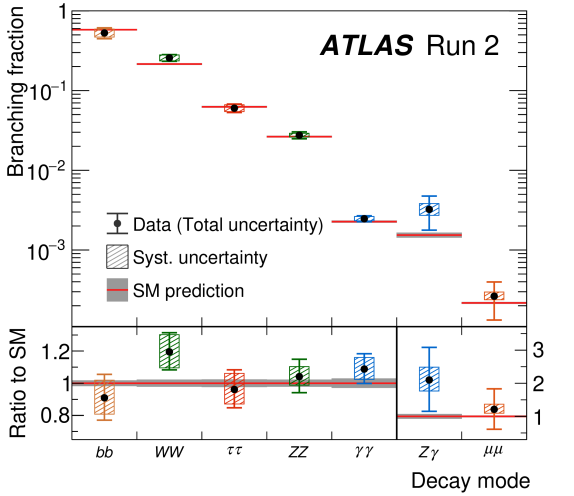
Does the discovered Higgs boson behave as the SM predicts?

What we know after Run2 $(139\,\mathrm{fb}^{-1})$

- CP-even scalar
- Mass measured with **permille** precision
- Production and decay channels all compatible with SM predictions
- Experimental uncertainties in the 10-20% range

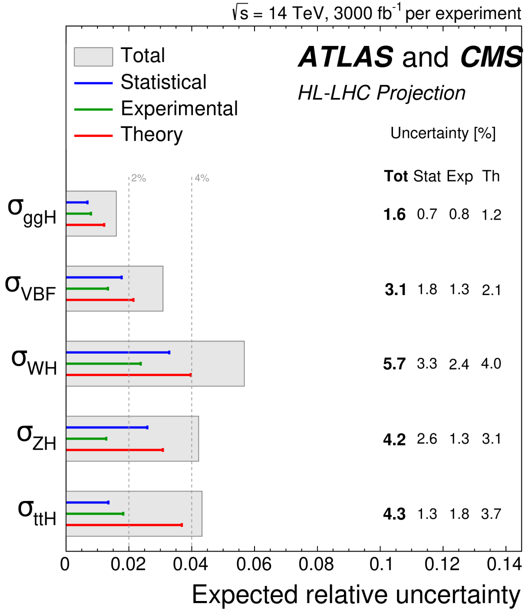
[ATLAS-2207.00092]

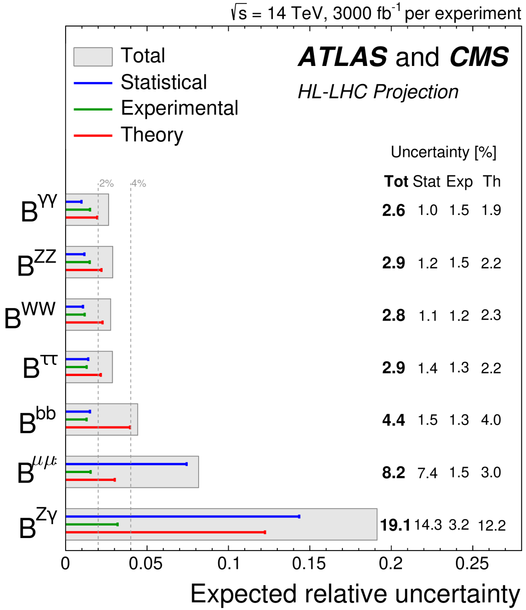




What next? Projections for High-Luminosity LHC

- Systematic uncertainties will play important role





[Cepeda et al. - 1902.00134]

Theory uncertainties need to be reduced

GOAL : **percent accuracy**

[THIS TALK]

Missing higher orders in perturbative calculations

(multi-)loop Feynman diagrams

Other theory uncertainties

- Parametric uncertainties
- PDF determination
- Matching with parton showers

Where to look for improvements?

- Les Houches **precision wishlist** [Huss et al. - 2207.02122]

Table 1. Precision wish list: Higgs boson final states. $\mathrm{N}^x\mathrm{LO}^{(\mathrm{VBF}*)}_{\mathrm{QCD}}$ means a calculation using the structure function approximation. $V = W, Z$.

Process	Known	Desired
$pp \to H$	$\mathrm{N^3LO_{HTL}}$ $\mathrm{NNLO}^{(t)}_{\mathrm{QCD}}$ $\mathrm{N^{(1,1)}LO^{(HTL)}_{QCD\otimes EW}}$ $\mathrm{NLO_{QCD}}$	$\mathrm{N^4LO_{HTL}}$ (incl.) $\mathrm{NNLO}^{(b,c)}_{\mathrm{QCD}}$
$pp \to H+j$	$\mathrm{NNLO_{HTL}}$ $\mathrm{NLO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to H+2j$	$\mathrm{NLO_{HTL}} \otimes \mathrm{LO_{QCD}}$ $\mathrm{N^3LO^{(VBF*)}_{QCD}}$ (incl.) $\mathrm{NNLO^{(VBF*)}_{QCD}}$ $\mathrm{NLO^{(VBF)}_{EW}}$	$\mathrm{NNLO_{HTL}} \otimes \mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{N^3LO^{(VBF*)}_{QCD}}$ $\mathrm{NNLO^{(VBF)}_{QCD}}$
$pp \to H+3j$	$\mathrm{NLO_{HTL}}$ $\mathrm{NLO^{(VBF)}_{QCD}}$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to VH$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $\mathrm{NLO}^{(t,b)}_{gg\to HZ}$	
$pp \to VH+j$	$\mathrm{NNLO_{QCD}}$ $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$
$pp \to HH$	$\mathrm{N^3LO_{HTL}} \otimes \mathrm{NLO_{QCD}}$	$\mathrm{NLO_{EW}}$

Table 3. Precision wish list: vector boson final states. $V = W, Z$ and $V', V'' = W, Z, \gamma$. Full leptonic decays are understood if not stated otherwise.

Process	Known	Desired
$pp \to V$	$\mathrm{N^3LO_{QCD}}$ $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^3LO_{QCD}} + \mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$ $\mathrm{N^2LO_{EW}}$
$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $+ \mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
$pp \to V+j$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$	hadronic decays
$pp \to V+2j$	$\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (QCD component) $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ (EW component)	$\mathrm{NNLO_{QCD}}$

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$pp \to VV'$	$\mathrm{NNLO_{QCD}} + \mathrm{NLO_{EW}}$ $+ \mathrm{NLO_{QCD}}$ (gg channel)	$\mathrm{NLO_{QCD}}$ (gg channel, w/ massive loops) $\mathrm{N^{(1,1)}LO_{QCD\otimes EW}}$
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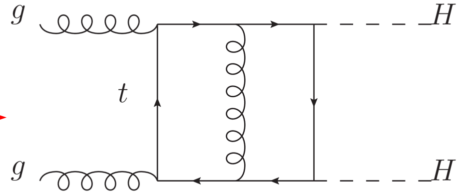

Process	Known	Desired
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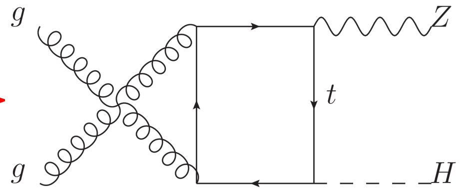
	$\mathrm{NLO}^{(t,b)}_{gg \to HZ}$	



Process	Known	Desired
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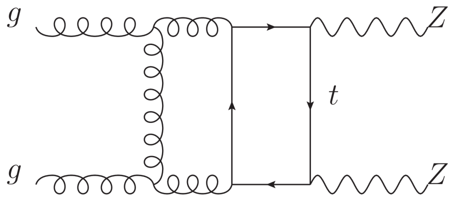
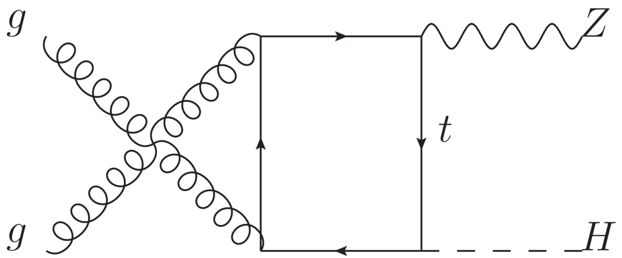


Gluon-initiated 2 → 2 processes
Two-loop diagrams with **massive** internal lines

Main problem in the NLO calculation
Multi-scale (m_Z, m_H, m_t, s, t)
two-loop integrals
No full analytic results

$gg \to ZH$



Solutions

Numerical Evaluation [Chen, Heinrich, Jones, Kerner, Klappert, Schlenk - 2011.12325]

- Exact results
- Demanding in terms of computing resources and time
- Issues with flexibility

Analytic Approximations: exploit **hierarchies** of masses/kinematic invariants

- Reduce the number of scales in Feynman integrals
- Proliferation of integrals
- Restricted to specific phase-space regions

- Limit $m_t \to \infty$
  [Altenkamp, Dittmaier, Harlander, Rzehak, Zirke - 1211.50]
- Large mass expansion: add finite top-mass effects
  [Hasselhuhn, Luthe, Steinhauser - 1611.05881]
- High-energy expansion: $m_Z^2, m_H^2 \ll m_t^2 \ll \hat{s}, \hat{t}$
  [Davies, Mishima, Steinhauser - 2011.12314]
- Small-mass expansion: $m_Z, m_H \to 0$
  [Wang, Xu, Xu, Yang - 2107.08206]
- pT expansion: $m_Z^2, m_H^2, p_T^2 \ll m_t^2, \hat{s}$
  [Alasfar, Degrassi, Giardino Groeber, MV – 2103.06225]

Solutions

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pT Expansion - Calculation Overview

1. Generation of Feynman diagrams - O(100 diags) (FeynArts [Hahn - 0012260])

2. Lorentz decomposition of the amplitude: **projectors** and **scalar form factors** (FeynCalc [Mertig et al. ('91) ; Shtabovenko et al. - 1601.01167]): contractions, Dirac traces...

$$\mathcal{A}_{\mu\nu\rho} = \sum_{i=1}^{6} \mathcal{P}^{(i)}_{\mu\nu\rho} F^{(i)} \qquad F^{(i)} = \sum_{i=1}^{n} C^{(i)}\ I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2)$$

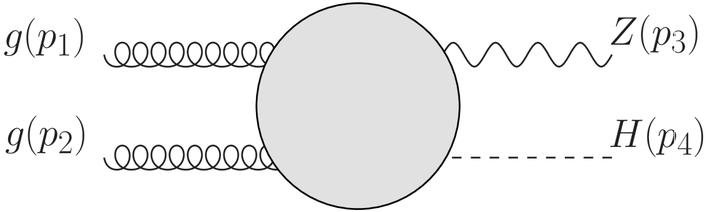
3. Expansion of the form factors in the limit of small pT

4. Decomposition of scalar integrals using integration-by-parts (IBP) identities (LiteRed [Lee - 1310.1145])

5. Evaluation of master integrals

Steps implemented in **Mathematica** code on a **desktop machine**

pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1 + p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\frac{m_H^2}{\hat{s}}, \frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1 \qquad \frac{p_T^2}{4m_t^2} \ll 1$$

Expansion at integrand level

- Now scalar loop integrals depend on fewer scales

$$I(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2) \to I'(\hat{s}, \hat{t}, m_t^2)$$

- The new scalar integrals are decomposed in MIs using IBP relations
- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **only one scale**

$$I(\hat{s}, \hat{t}, m_Z^2, m_H^2, m_t^2) \to I'(\hat{s}, \hat{t}, m_t^2) \to \mathrm{MI}(\hat{s}/m_t^2)$$

- 52 MIs already known in the literature
  SAME MIs FOR $gg \to HH$, $gg \to ZH$, $gg \to ZZ$

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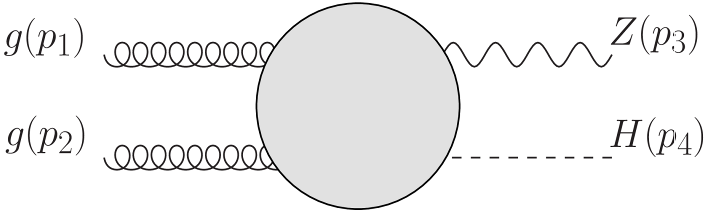

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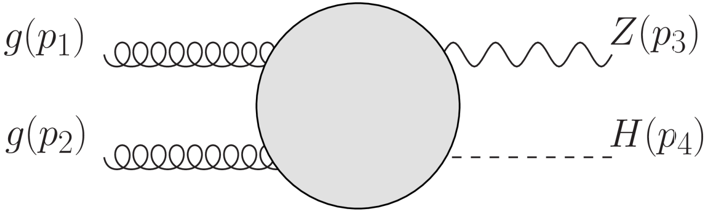

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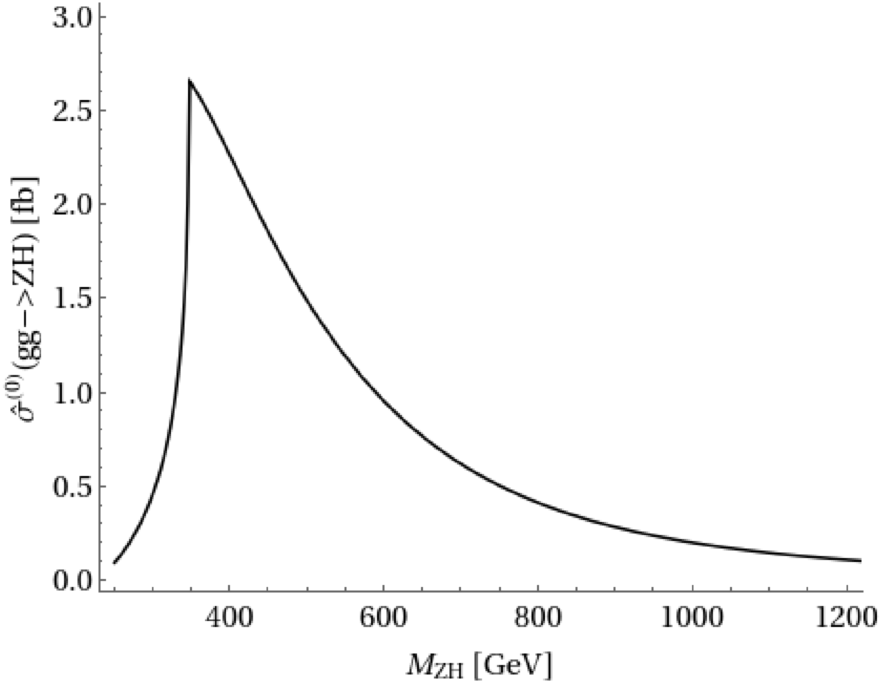
Comparing Validity Ranges



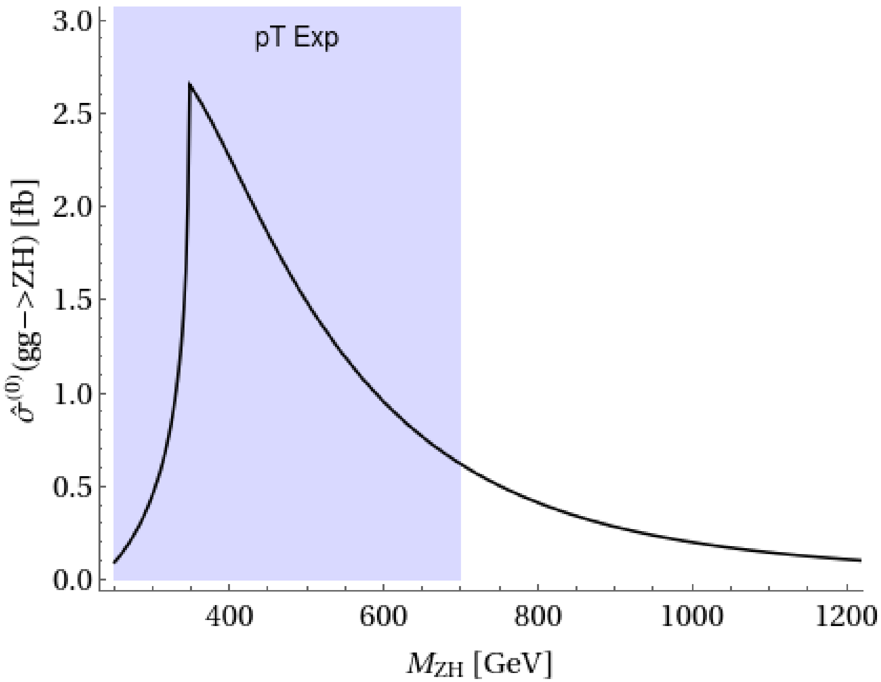
Comparing Validity Ranges

- p_T exp: valid for

$$p_T^2 \lesssim 4m_t^2$$

or

$$\hat{t} \lesssim 4m_t^2$$



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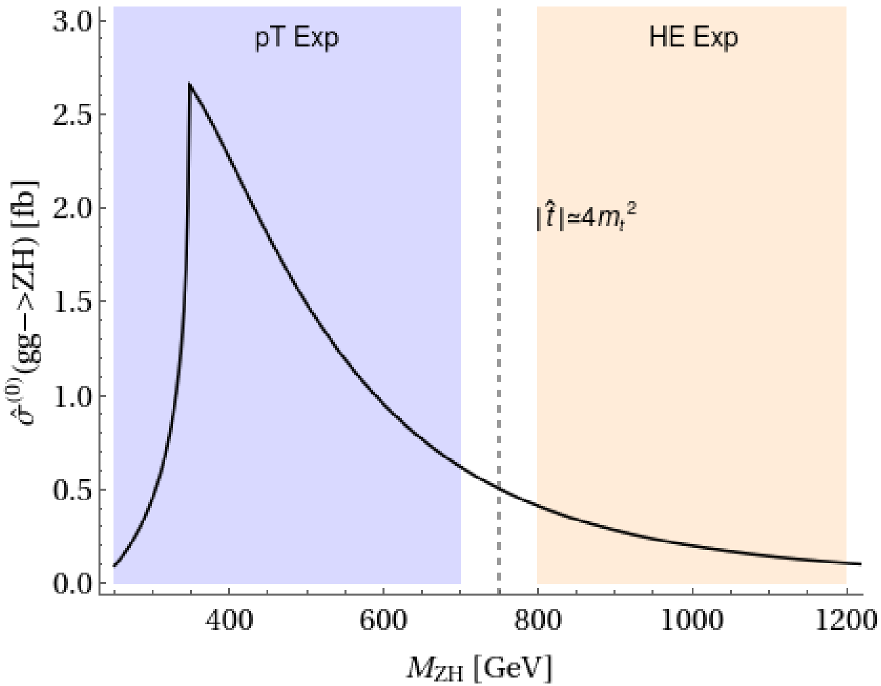

[Davies, Mishima, Steinhauser - 2011.12314]

- High-Energy exp:

$$\hat{t} \gtrsim 4m_t^2$$

The two expansions can be combined!

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

Merging pT and HE expansions at NLO

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Improve the convergence of a series expansion by matching the coefficients of the **Pade approximant** [m/n] [e.g. Fleisher, Tarasov ('94)]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \quad (q = m+n)$$

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

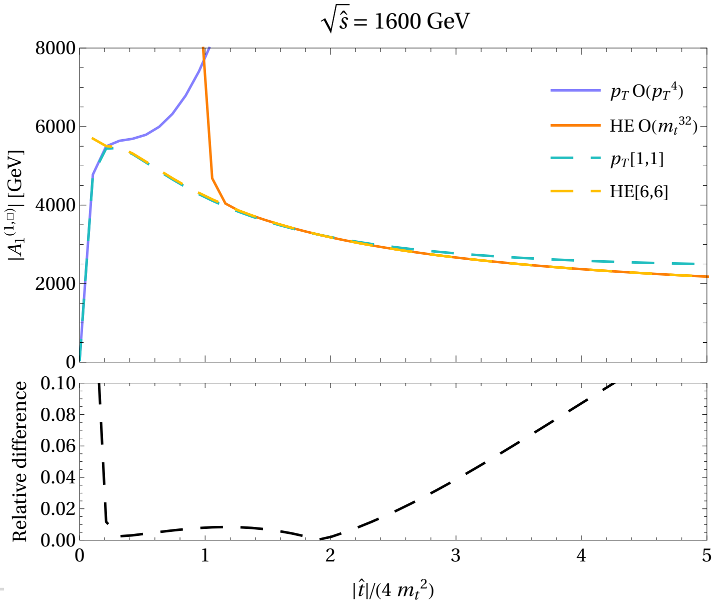
- For each FF we merged the following results

  - pT exp improved by [1/1] Padé

  - HE exp improved by [6/6] Padé

- Padé results are stable and comparable in the region $|\hat{t}| \sim 4\, m_t^2$ → can switch without loss of accuracy (% level or below)

- Evaluation time for a phase-space point below 0.1 $s \Rightarrow$ suitable for Monte Carlo

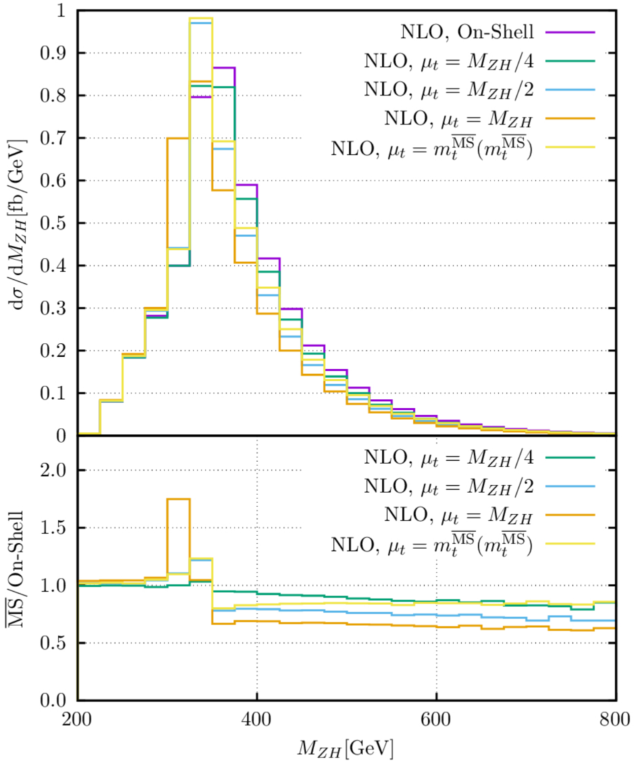


Full NLO QCD Results

Inclusive cross section

Top-mass scheme	LO [fb]	$\sigma_{LO}/\sigma_{LO}^{OS}$	NLO [fb]	$\sigma_{NLO}/\sigma_{NLO}^{OS}$	$K = \sigma_{NLO}/\sigma_{LO}$
On-Shell	$64.01^{+27.2\%}_{-20.3\%}$	—	$118.6^{+16.7\%}_{-14.1\%}$	—	1.85
$\overline{\rm MS}, \mu_t = M_{ZH}/4$	$59.40^{+27.1\%}_{-20.2\%}$	0.928	$113.3^{+17.4\%}_{-14.5\%}$	0.955	1.91
$\overline{\rm MS}, \mu_t = m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}})$	$57.95^{+26.9\%}_{-20.1\%}$	0.905	$111.7^{+17.7\%}_{-14.6\%}$	0.942	1.93
$\overline{\rm MS}, \mu_t = M_{ZH}/2$	$54.22^{+26.8\%}_{-20.0\%}$	0.847	$107.9^{+18.4\%}_{-15.0\%}$	0.910	1.99
$\overline{\rm MS}, \mu_t = M_{ZH}$	$49.23^{+26.6\%}_{-19.9\%}$	0.769	$103.3^{+19.6\%}_{-15.6\%}$	0.871	2.10

- NLO corrections are the same size as LO (K~2)
- Scale uncertainties reduced by 2/3 wrt LO
- Agreement with independent calculations
  [Wang et al. - 2107.08206] [Chen et al. - 2204.05225]



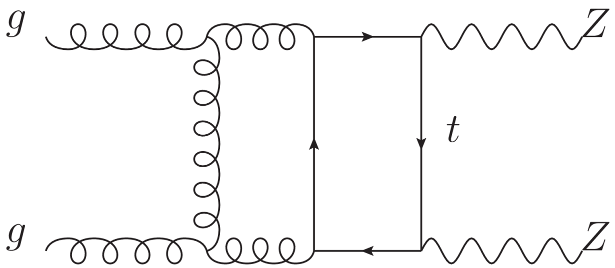
[Degrassi, Gröber, MV, Zhao - 2205.02769]

Top mass scheme uncertainty

- Take deviations of $\overline{\rm MS}$ scheme wrt OS result as top mass scheme uncertainty (used for HH production in [Baglio et al. - 1811.05692, 2003.03227])
- Analytic results → change of top mass scheme is straightforward

$$F_i^{NLO,\overline{\rm MS}} = F_i^{NLO,{\rm OS}} - \frac{1}{4}\frac{\partial F_i^{LO}}{\partial m_t^2}\Delta_{m_t^2} \qquad \Delta_{m_t^2} = 2m_t^2 C_F\left[-4 + 3\log\left(\frac{m_t^2}{\mu^2}\right)\right]$$

$gg \rightarrow ZZ$



$gg \to ZZ$ and Higgs Physics

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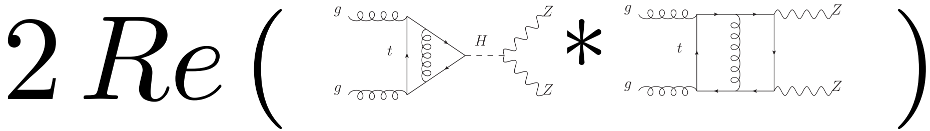
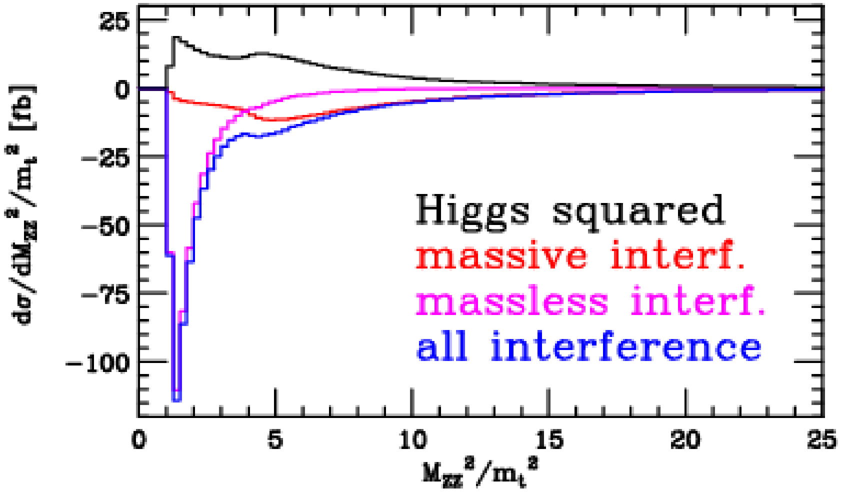


- Destructive interference between
  $gg \to H^* \to ZZ$ and $gg \to ZZ$
  in the off-shell region
- Relevant for indirect measurements of Higgs total width
  [Kauer, Passarino – 1206.4803]
  [Caola, Melnikov – 1307.4935]
  [Campbell. Ellis, Williams - 1311.3589]
- Top loops are dominant in off-shell region
- Use pT expansion for the two-loop box diagrams [Degrassi, Gröber, MV – in preparation]

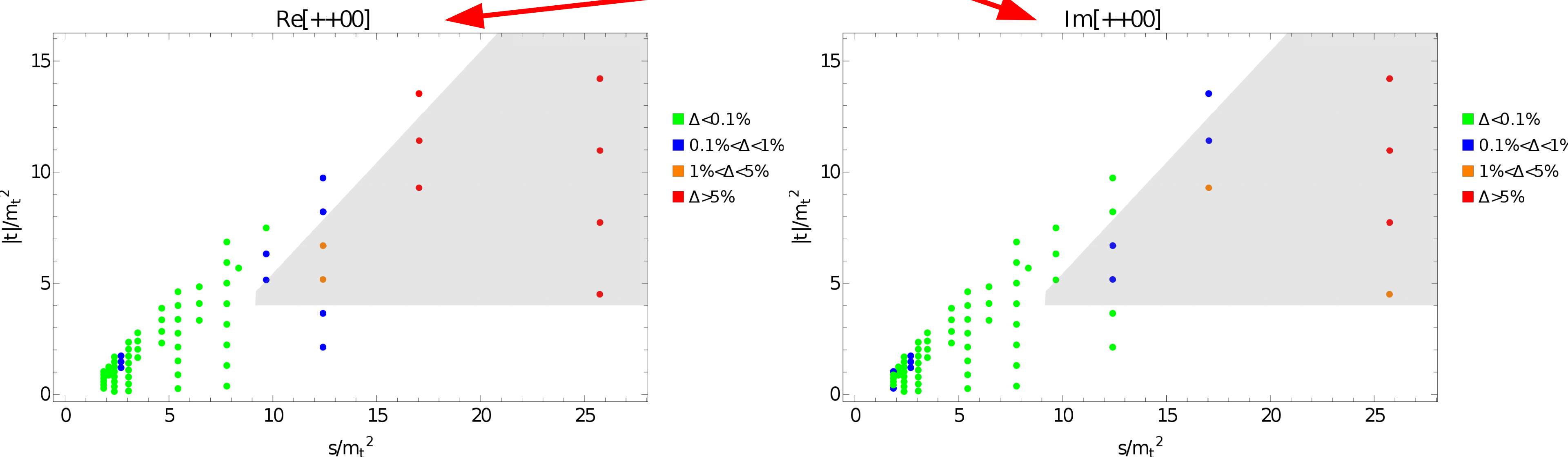


[Campbell et al. - 1605.01380]

Helicity amplitudes at NLO

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \mathcal{O}(\alpha_s^3)$$

[Agarwal, Jones, von Manteuffel - 2011.15113]



[PRELIMINARY]

Next step → combine pT and HE expansions [Davies et al. - 2002.05558]

Conclusions

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- Higgs precision measurements call for improved theoretical predictions
- $2 \to 2$ processes with **massive** loops are hard to compute
- Analytic approximations are useful: flexibility and efficiency
- pT and high-energy expansions can be combined

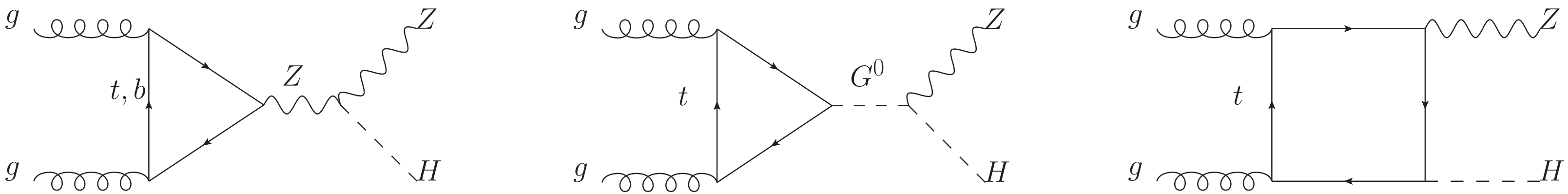
Outlook

- Comparing pT expansion and $t \to 0$ expansion
  [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]
- Application to 3-loop diagrams for NNLO QCD corrections
- EW corrections to $2 \to 2$ processes - New master integrals

Thank you for your attention

Backup

$gg \to ZH$ @ LO



- Third generation gives dominant contribution [Kniehl ('90) - Dicus, Kao ('88)]
- $\mathcal{O}(\alpha_s^2)$ correction to $pp \to ZH$ cross section
- NNLO suppression wrt to $q\bar{q} \to ZH$ but gluon luminosity higher at LHC
- Contributes to about 6% of $\sigma(pp \to ZH)$ for $\sqrt{s} = 14$ TeV
- Only LO included in MC $\to$ scale variation

  leads to **25%** relative uncertainties
- NLO corrections expected to be large in *gg* processes (e.g. *H, HH*)

[Cepeda et al. - 1902.00134]

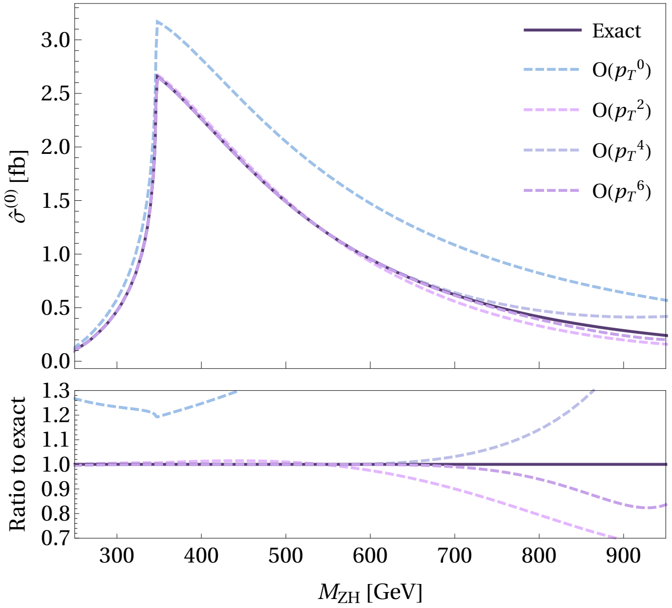
$\sqrt{s}$ [TeV]	$\sigma_{\text{NNLO QCD}\otimes\text{NLO EW}}$ [pb]	Δ_{scale} [%]	$\Delta_{\text{PDF}\oplus\alpha_s}$ [%]
13	0.123	$^{+24.9}_{-18.8}$	4.37
14	0.145	$^{+24.3}_{-19.6}$	7.47
27	0.526	$^{+25.3}_{-18.5}$	5.85

LO Validation

- Three orders sufficient for very good accuracy
- Reliable results for $M_{ZH} \lesssim 700\ \mathrm{GeV}$
- For $M_{ZH} \gtrsim 700\ \mathrm{GeV}$ the assumption

$$p_T^2 \ll 4m_t^2$$

can be violated $\Rightarrow$ the p_T expansion **diverges** (but wait a few slides...)



pT expansion: example

Karlsruhe Institute of Technology

1) Consider a **one-loop** box integral

$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2}(q\cdot p_2)^{n_3}(q\cdot p_3)^{n_4}}{(q^2-m_t^2)[(q+p_2)^2-m_t^2][(q-p_1-p_3)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; explicit the transverse component wrt beam axis

$$\frac{(q\cdot p_3)^{n_4}}{[(q-p_1-p_3)^2-m_t^2]}$$

$$\begin{aligned} p_3^\mu &= \frac{u'}{s'}p_1^\mu + \frac{t'}{s'}p_2^\mu + r_\perp^\mu \\ &= -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + \frac{\Delta_m}{s'}p_1^\mu + r_\perp^\mu \end{aligned}$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2'}(q\cdot p_2)^{n_3'}(q\cdot r_\perp)^{n_4'}}{(q^2-m_t^2)^{l_1}[(q+p_2)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

4) LiteRed searches for MIs with $n'_4=0$ $\rightarrow$ the MIs do not depend on p_T^2

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Master Integrals

- 50 MIs expressed in terms of Generalized Polylogarithms (GPLs)

  [Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

- Two elliptic integrals [von Manteuffel, Tancredi ('17)]

  Semi-analytical evaluation implemented in FORTRAN routine

  [Bonciani, Degrassi, Giardino, Gröber ('18)]