



Collaborative Research Center TRR 257  
Particle Physics Phenomenology after the Higgs Discovery

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# Four top final states with NLO accuracy in perturbative QCD: 4 lepton decay channel

Nikolaos Dimitrakopoulos, RWTH Aachen University

*Preliminary results* in collaboration with **Małgorzata Worek**, RWTH Aachen University

Young Scientists Meeting of the CRC TRR 257, 17/10/23

# Why top quark physics?

- Top quark is the heaviest SM particle  $m_t \approx 172.5 \text{ GeV}$
- Substantial Yukawa coupling  $\rightarrow y_t = \sqrt{2} m_t/v \approx 1$
- Top quarks are produced copiously at the LHC
  - $\rightarrow$  Their properties can be studied in high precision
- Top quark is extremely *short-lived*
  - $\rightarrow$  Decays before bound states can form
  - $\rightarrow$  Unique opportunity to study a "bare" quark
  - $\rightarrow$  Study of top-quark properties through its decay products



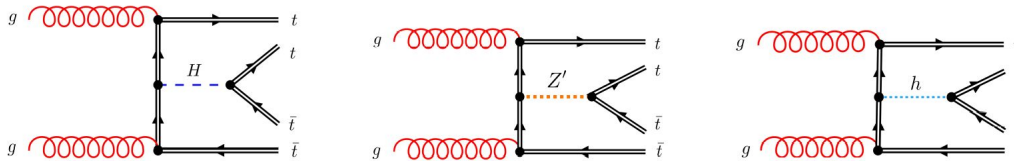
# Why four tops?

- Four top production is an extremely rare process with an estimated cross section

$$\sigma_{tt\bar{t}\bar{t}}^{NLO(QCD+EW)+NLL'} = 13.4_{-1.8}^{+1.0} \text{ fb} \quad \text{at} \quad \sqrt{s} = 13 \text{ TeV}$$

*van Beekveld, Kulesza, Valero '22*

- Direct way to measure the top Yukawa coupling complementary to  $t\bar{t}H$  production
- Very sensitive to many New Physics (NP) models
  - Study modifications in the Higgs sector e.g. two-Higgs-doublet models
  - Top philic models → new BSM heavy resonances decaying to top quark pairs



Diagrams were created  
via FeynGame:  
*Harlander, Klein, Lipp '20*

**Highly accurate SM calculations are essential alongside BSM modeling**

# $t\bar{t}t\bar{t}$ state of the art

- **First NLO QCD predictions for 4 stable tops:** General idea about the size of the NLO QCD calculations. Top decays are not considered.  
*Bevilacqua, Worek '12 / Maltoni, Pagani, Tsirikos '16*
- **Complete-NLO predictions for 4 stable tops with sub-leading effects:** All the non-vanishing contributions of  $\mathcal{O}(\alpha_s^i \alpha^j)$  with  $i + j = 4, 5$  are taken into account without any approximation. Top quark decays are omitted.  
*Frederix, Pagani, Zaro '18*
- **NLO QCD matched to parton shower (NLO+PS):** Besides NLO QCD corrections, the inclusion of subleading EW production channels at LO accuracy was also considered. LO spin correlated effects in top quark decays were also studied for the first time.  
*Ježo, Kraus '22*
- **Threshold resummation for the production of four top quarks:** Results for the total cross section for 4-top production at next-to-leading logarithmic (NLO + NLL') accuracy. Top quark decays are not included either.  
*van Beekveld, Kulesza, Valero '22*

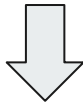
# Observation of four top production

Discovery of four top production  
in 2023

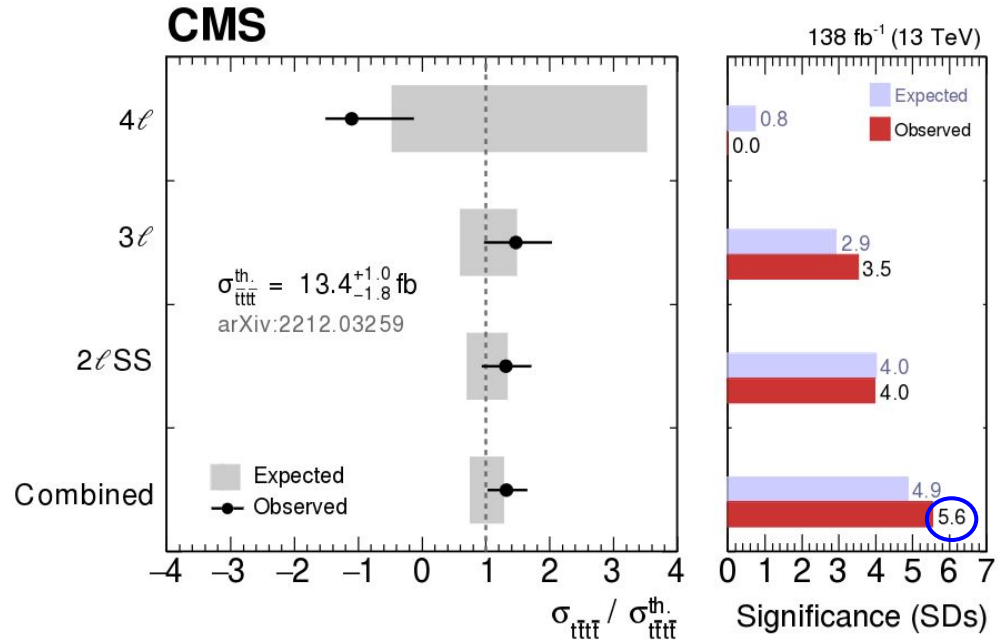
CMS: [arXiv:\[2305.13439\]](https://arxiv.org/abs/2305.13439)

ATLAS: [arXiv:\[2303.15061\]](https://arxiv.org/abs/2303.15061)

- Three different Signal Regions (SR) were taken into account: 4-lepton channel, 3-lepton channel and 2lSS channel

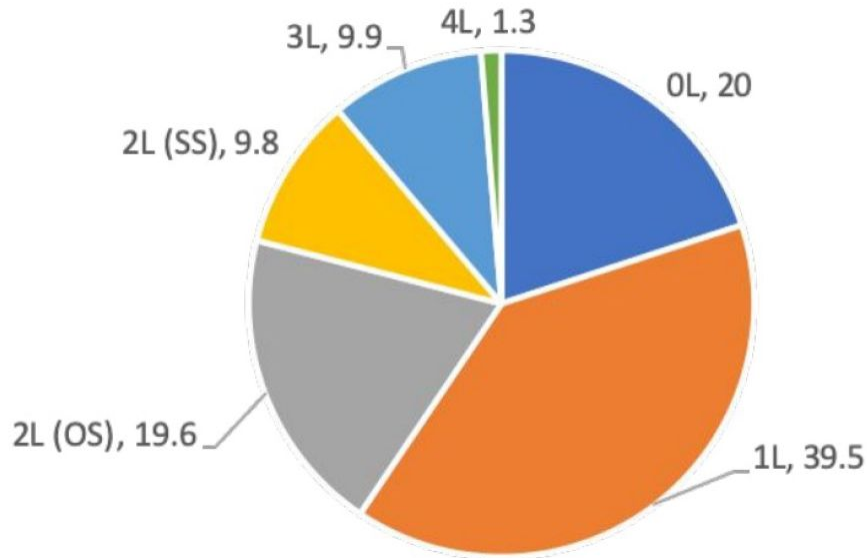


Observed significance of **5.6 $\sigma$**  for CMS



CMS: [arXiv:\[2305.13439\]](https://arxiv.org/abs/2305.13439)

# Branching ratios



Manganelli, Quinnan '22

- $t \rightarrow Wb$ : Top quark decays almost entirely through weak interaction to a  $W$  boson and a bottom quark with a branching ratio of  $\sim 100\%$
- $W$  boson decays to either a pair of lepton with its corresponding neutrino or two quarks.

$$Br(W \rightarrow lv_l) \approx 10.8\%$$

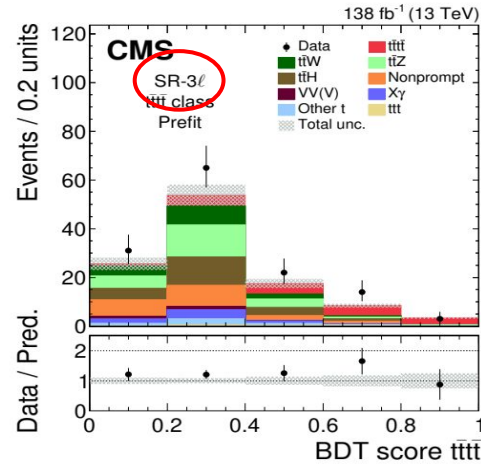
$$\sum_{qq'} Br(W \rightarrow q\bar{q}') \approx 67.6\%$$

# Project description / Motivation

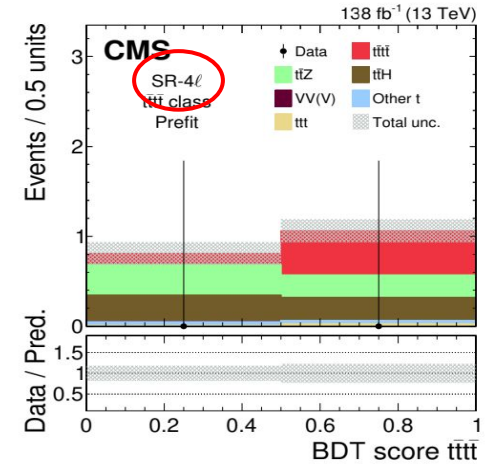
- The 3-lepton and 2-lepton channels are currently more promising due to bigger cross sections!
- However, the **4-lepton** channel is a stepping stone to studying the 3-lepton and 2-lepton channels!

Goal of my project

- Theoretical predictions for NLO QCD calculations for the fully leptonic channel using the NWA
- Compare the full NLO QCD calculations to NLO with LO decays



CMS: [arXiv:\[2305.13439\]](https://arxiv.org/abs/2305.13439)



- Investigate the effects of expanding the decay rate of the top quark in the calculation

# Process description

LO

$$pp \rightarrow t\bar{t}t\bar{t} \rightarrow W^+W^-W^+W^-b\bar{b}b\bar{b} \rightarrow \ell^+\nu_\ell \ell^-\bar{\nu}_\ell \ell^+\nu_\ell \ell^-\bar{\nu}_\ell b\bar{b}b\bar{b}.$$

- We treat top and  $W$  in the NWA  $\rightarrow$  valid for inclusive observables since  $\Gamma_t$  and  $\Gamma_W$  are much smaller compared to  $m_t$  and  $m_W$  respectively

$$\begin{aligned} \Gamma_t/m_t &\approx 0.008 \\ \Gamma_W/m_W &\approx 0.026 \end{aligned}$$

$$\lim_{\Gamma/m \rightarrow 0} \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} = \frac{\pi}{m\Gamma} \delta(p^2 - m^2)$$

NLO

Emission of an extra parton either at the **production** stage or during a top quark **decay**



Virtual corrections both at the production and the decays

- ★ Only the combination of real emission and virtual corrections is IR safe!

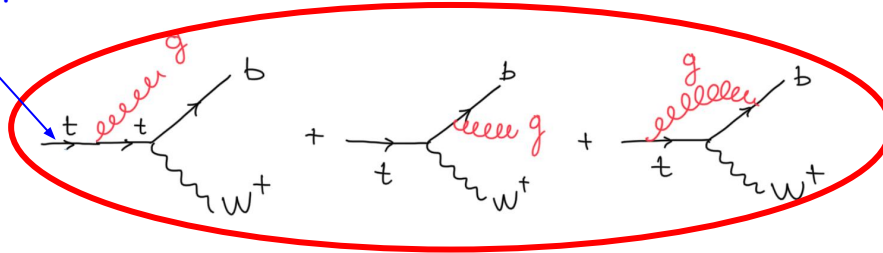


# Process description

$$d\sigma_{\text{NWA}_{\text{full}}}^{\text{NLO}} = d\sigma_{t\bar{t}\bar{t}\bar{t}}^{\text{NLO}} \times \left( \frac{d\Gamma_{t \rightarrow W^+b}^{(0)} d\Gamma_{\bar{t} \rightarrow W^- \bar{b}}^{(0)}}{(\Gamma_t^{\text{NLO}})^4} \right)$$

$$d\sigma_{t\bar{t}\bar{t}\bar{t}}^{\text{LO}} \times \left( 2 \frac{d\Gamma_{t \rightarrow W^+b}^{(1)} d\Gamma_{t \rightarrow W^+b}^{(0)} d\Gamma_{\bar{t} \rightarrow W^- \bar{b}}^{(0)} d\Gamma_{\bar{t} \rightarrow W^- \bar{b}}^{(0)}}{(\Gamma_t^{\text{NLO}})^4} + 2 \frac{d\Gamma_{t \rightarrow W^+b}^{(0)} d\Gamma_{t \rightarrow W^+b}^{(0)} d\Gamma_{\bar{t} \rightarrow W^- \bar{b}}^{(1)} d\Gamma_{\bar{t} \rightarrow W^- \bar{b}}^{(0)}}{(\Gamma_t^{\text{NLO}})^4} \right)$$

on-shell



- NWA<sub>LOdecays</sub>**: No QCD corrections at the decays stage → Only the first term contributes with the replacement  $\Gamma_{t,\text{NWA}}^{\text{NLO}} \rightarrow \Gamma_{t,\text{NWA}}^{\text{LO}}$

# Process description

- **NWA<sub>exp</sub>**: Expansion of the decay rate of the top quark is taken place. To all orders of perturbation theory it holds that:

$$d\sigma = d\sigma_{t\bar{t}\bar{t}} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_t}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} \times \frac{d\Gamma_{\bar{t}}}{\Gamma_t} \qquad d\sigma_{t\bar{t}\bar{t}} = d\sigma_{t\bar{t}\bar{t}}^{(0)} + \alpha_s d\sigma_{t\bar{t}\bar{t}}^{(1)} + \mathcal{O}(\alpha_s^2)$$
$$d\Gamma_t = d\Gamma_t^{(0)} + \alpha_s d\Gamma_t^{(1)} + \mathcal{O}(\alpha_s^2)$$

- By expanding the above formula and keeping terms up to  $\mathcal{O}(\alpha_s)$  we end up with

$$d\sigma_{\text{NWA}_{\text{exp}}} = d\sigma_{\text{NWA}_{\text{full}}}^{\text{NLO}} \times \left( \frac{\Gamma_t^{\text{NLO}}}{\Gamma_t^{\text{LO}}} \right)^4 - d\sigma_{\text{NWA}}^{\text{LO}} \times \frac{4(\Gamma_t^{\text{NLO}} - \Gamma_t^{\text{LO}})}{\Gamma_t^{\text{LO}}}$$

where  $\Gamma_t^{\text{NLO}} = \Gamma_t^{(0)} + \alpha_s \Gamma_t^{(1)}$

- Advantage of not including higher order effects corresponding to simultaneous QCD corrections both at the production and the decay stages

# Integrated cross sections at LO, NLO

PDF	$\sigma^{\text{LO}}$ [ab]	$\delta_{\text{scale}}$	$\sigma^{\text{NLO}}$ [ab]	$\delta_{\text{scale}}$	$\delta_{\text{PDF}}$	$\mathcal{K} = \sigma^{\text{NLO}}/\sigma^{\text{LO}}$
$\mu_R = \mu_F = \mu_0 = 2m_t$						
MSHT20	4.387(3)	+3.224 (73%) -1.733 (40%)	5.448(3)	+0.147 (3%) -0.848 (16%)	+0.233 (4%) -0.173 (3%)	1.24
NNPDF31	3.739(2)	+2.681 (72%) -1.455 (39%)	5.398(3)	+0.161 (3%) -0.853 (16%)	+0.117 (2%) -0.117 (2%)	1.44
CT18	4.676(3)	+3.375 (72%) -1.831 (39%)	5.406(3)	+0.149 (3%) -0.840 (16%)	+0.319 (6%) -0.261 (5%)	1.16
$\mu_R = \mu_F = \mu_0 = E_T/4$						
MSHT20	4.748(3)	+3.516 (74%) -1.885 (40%)	5.718(3)	+0.138 (2%) -0.875 (15%)	+0.243 (4%) -0.181 (3%)	1.20
NNPDF31	4.093(3)	+2.979 (73%) -1.606 (39%)	5.670(4)	+0.138 (2%) -0.876 (15%)	+0.123 (2%) -0.123 (2%)	1.39
CT18	5.000(3)	+3.615 (72%) -1.962 (39%)	5.671(4)	+0.137 (2%) -0.867 (15%)	+0.332 (6%) -0.273 (5%)	1.13

Different values of  $\alpha_s$  have been used for the PDFs at LO and NLO



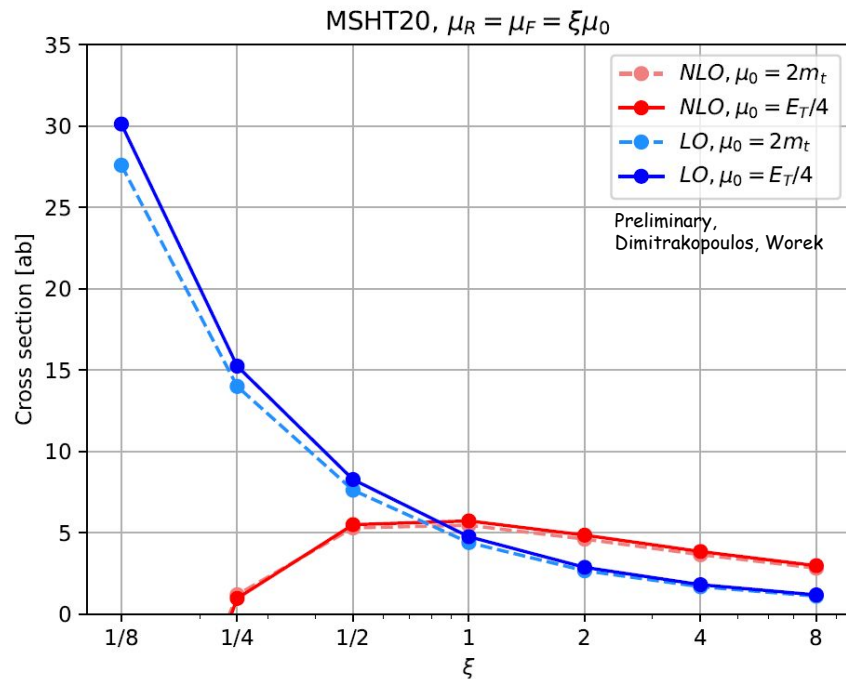
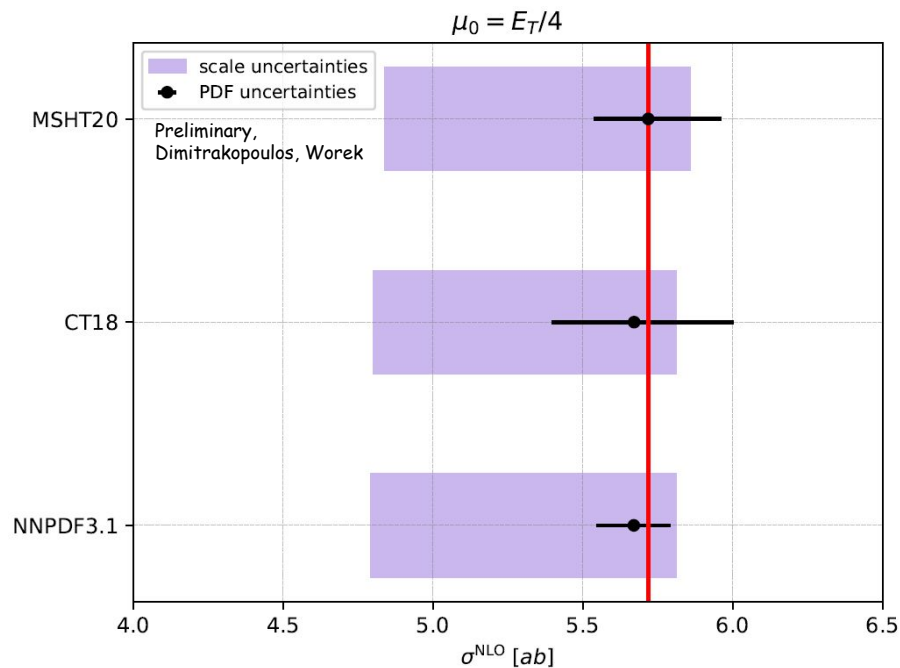
K factors are very sensitive to that choice due to the large powers of  $\alpha_s$  appearing in the cross section:

$$\sigma_{\text{LO}} \sim \alpha_s^4, \quad \sigma_{\text{NLO}} \sim \alpha_s^5$$

LO results are only sufficient for an order-of-magnitude estimation: No reliable conclusions can be drawn

However, results for different PDF sets are stabilized at NLO

# Scale and PDF uncertainties



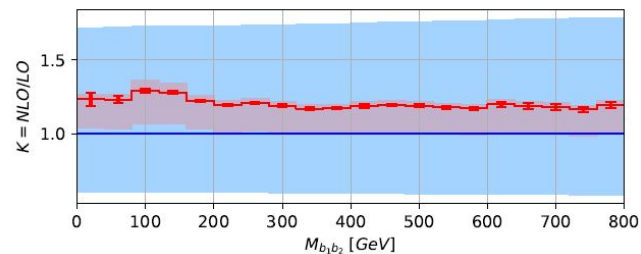
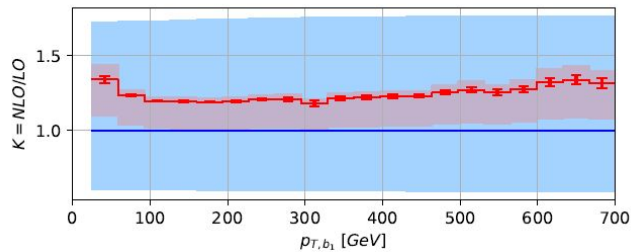
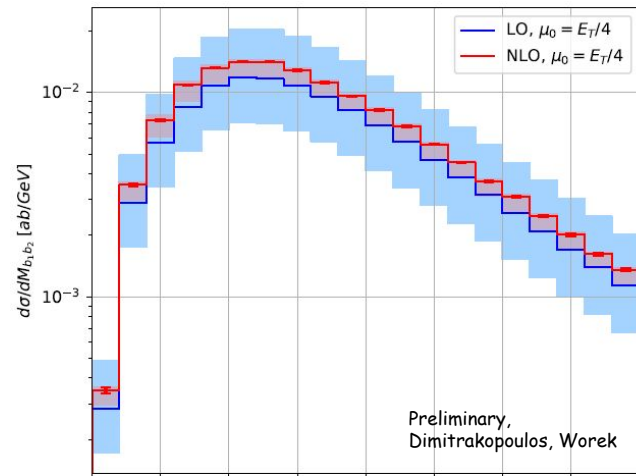
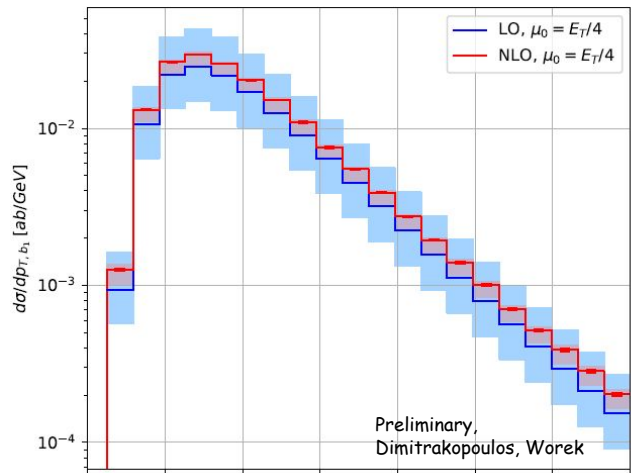
# NWA<sub>full</sub> vs NWA<sub>LOdec</sub> vs NWA<sub>exp</sub>

Preliminary results,  
Dimitrakopoulos, Worek

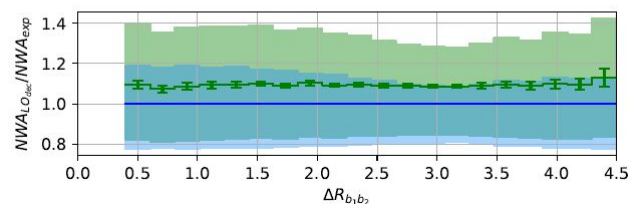
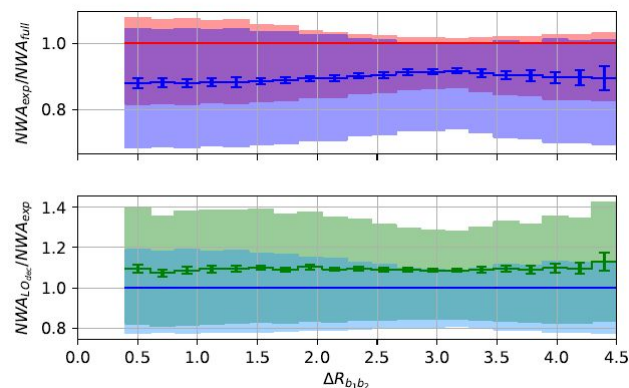
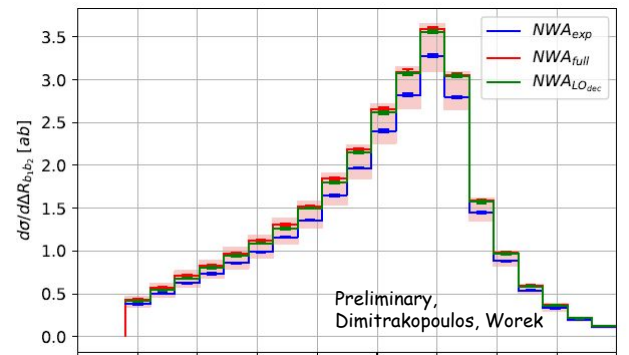
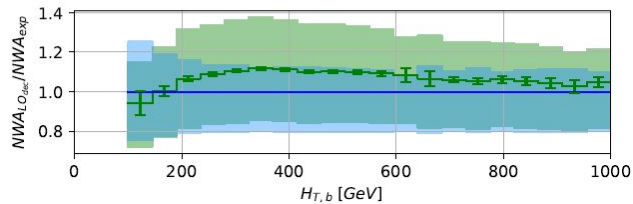
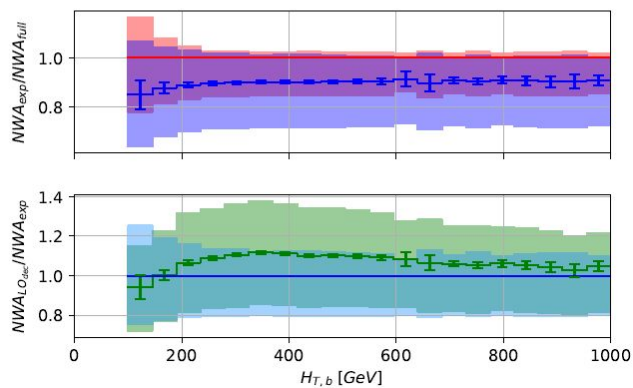
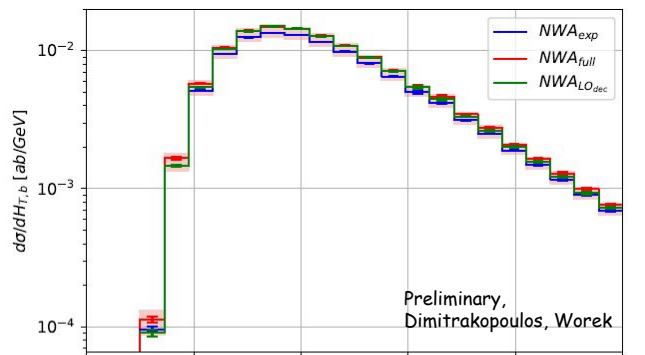
Decay treatment	$\sigma_i^{\text{NLO}}$ [ab]	$+\delta_{\text{scale}}$ [ab]	$-\delta_{\text{scale}}$ [ab]	$\sigma_i^{\text{NLO}}/\sigma_{\text{NWA}_{\text{full}}} - 1$	$\sigma_i^{\text{NLO}}/\sigma_{\text{NWA}_{\text{exp}}} - 1$
$\mu_R = \mu_F = \mu_0 = 2m_t$					
NWA <sub>full</sub>	5.448(3)	+0.147 (3%)	-0.848 (16%)	-	+11.5%
NWA <sub>LOdec</sub>	5.280(3)	+1.114 (21%)	-1.219 (23%)	-3.1%	+8.1%
NWA <sub>exp</sub>	4.885(2)	+0.618 (13%)	-0.998 (20%)	-10.3%	-
$\mu_R = \mu_F = \mu_0 = E_T/4$					
NWA <sub>full</sub>	5.718(3)	+0.138 (2%)	-0.875 (15%)	-	+10.9%
NWA <sub>LOdec</sub>	5.628(3)	+1.213 (22%)	-1.310 (23%)	-1.6%	+9.1%
NWA <sub>exp</sub>	5.158(3)	+0.630 (12%)	-1.051 (20%)	-9.8%	-

Significant reduction of the scale uncertainties when QCD corrections are applied to the decays as well: **23% → 15%**

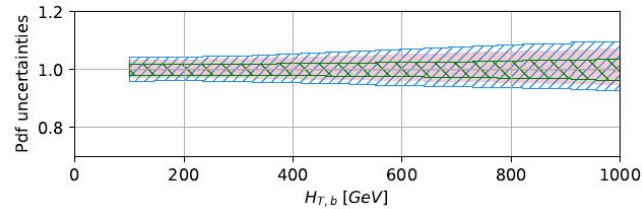
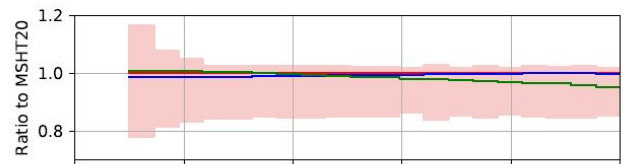
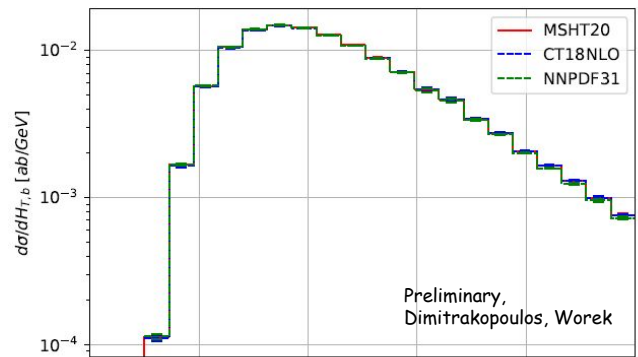
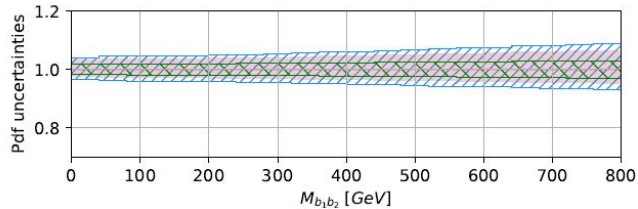
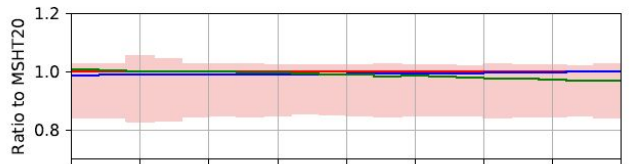
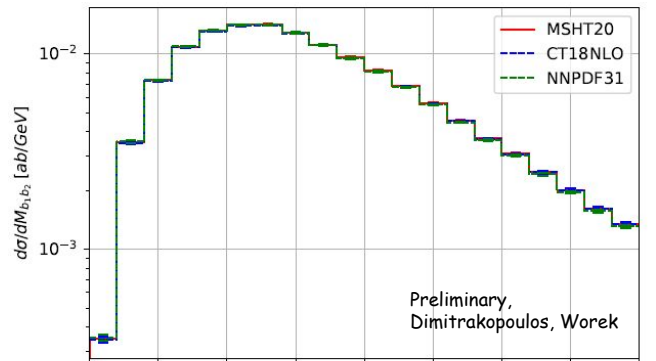
# Differential distributions



# Differential distributions



# Differential agreement among different PDF sets





# Summary & Outlook



- The recent observation of 4-top production makes high precision measurements more important than ever → need for at least full NLO QCD calculations at 4-top production and decays
- Results are stable between different PDF sets at NLO QCD where the scale uncertainties are of the order of  $\sim 15\%$
- Full NWA calculations significantly reduce the scale uncertainties compared to the  $NWA_{LOdecays}$ : reduction of approximately **30-40%**
- Incorporation of higher order effects of the order of  $\sim 10\%$  in  $NWA_{full}$  where  $\Gamma_t$  is not expanded in terms of the strong coupling
- Future goal:
  - 1) NLO calculations in perturbative QCD for the 3-lepton channel
  - 2) Comparisons to NLO QCD calculations matched to Parton Shower (PS)



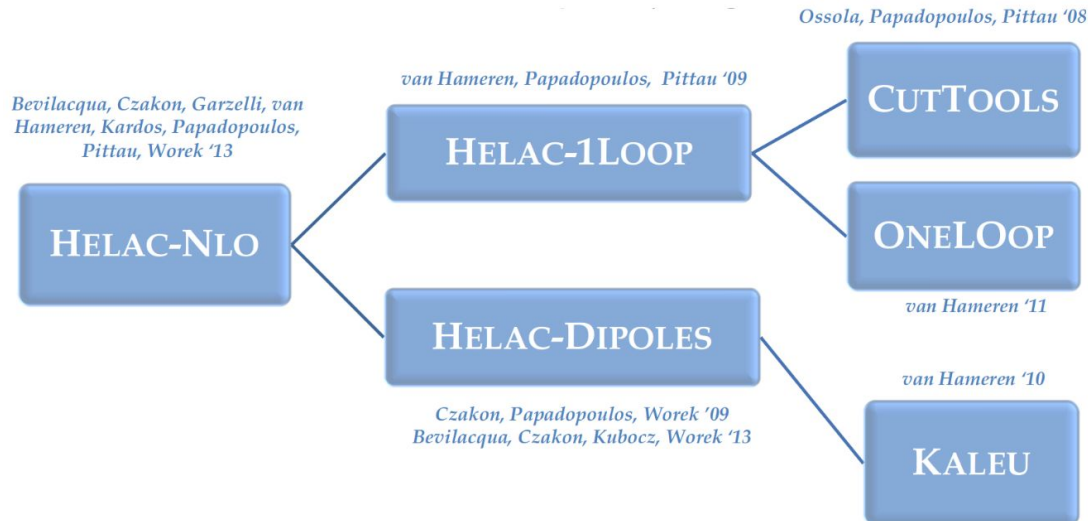
*Thanks for your attention!*



# Backup

# Backup slides

## HELAC-NLO



- The output is saved in **Les Houches & Root files, and Ntuple files**  
<https://arxiv.org/abs/hep-ph/0609017>, <https://arxiv.org/abs/1310.7439>
- It can be further analysed by adding new cuts, changing the renormalization and factorization scales, using different PDF set

# Backup slides

## Setup for the calculation

- We perform our calculations with a center of mass energy  $\sqrt{s} = 13.6 \text{ TeV}$
- We try to be as inclusive as possible in the fiducial phase-space:  
$$p_{T,\ell} > 25 \text{ GeV}, \quad |y_\ell| < 2.5, \quad \Delta R_{\ell\ell} > 0.4,$$
$$p_{T,b} > 25 \text{ GeV}, \quad |y_b| < 2.5, \quad \Delta R_{bb} > 0.4$$
- In our fixed order calculation we use both a fixed and a dynamical scale  $\mu_0 = 2m_t$ ,  $\mu_0 = E_T/4$

$$E_T = \sum_{i=1,2} \sqrt{m_i^2 + p_T^2(t_i)} + \sum_{i=1,2} \sqrt{m_i^2 + p_T^2(\bar{t}_i)}$$

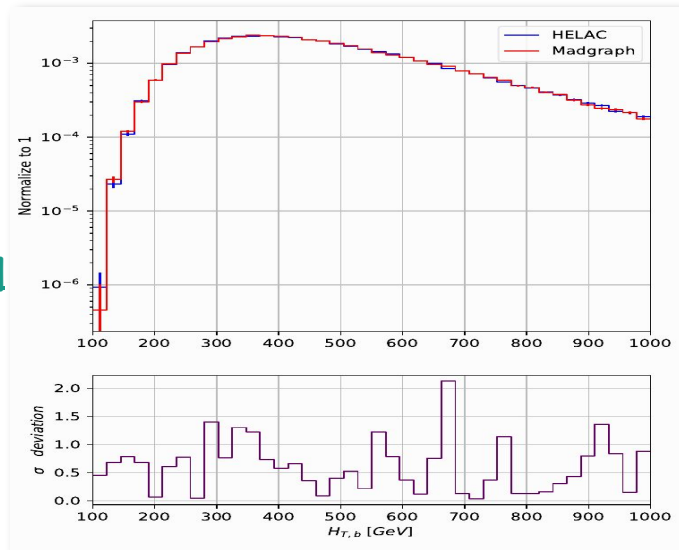
- Scale variations are calculated by varying both  $\mu_R$  and  $\mu_F$

$$\left( \frac{\mu_R}{\mu_0}, \frac{\mu_F}{\mu_0} \right) = \left\{ (2, 1), (0.5, 1), (1, 2), (1, 1), (1, 0.5), (2, 2), (0.5, 0.5) \right\}.$$

# Backup slides

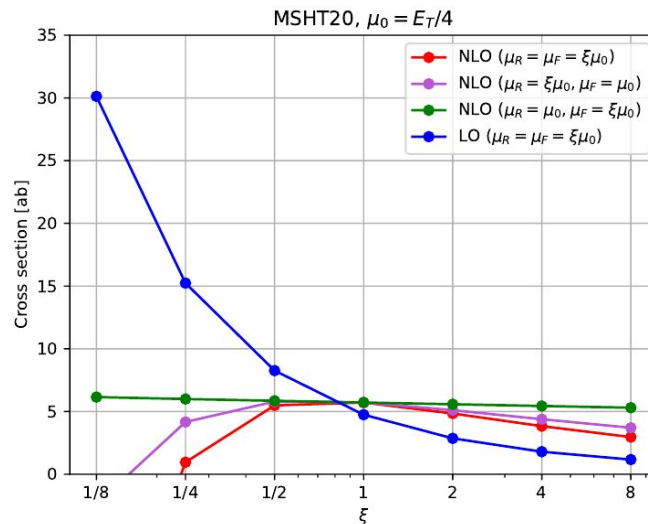
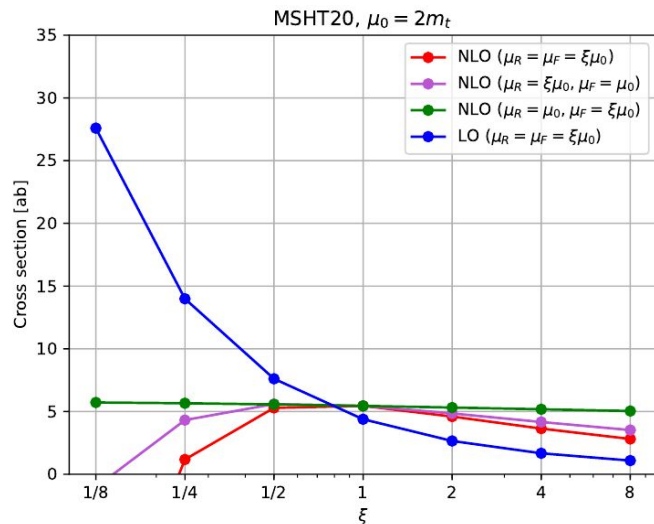
## Cross-checks

- LO calculations have been cross-checked both at the integrated and differential level with the help of MadSpin in MadGraph5\_aMC@NLO [arxiv:\[1405.0301\]](#)
- For subtracting the IR divergences we employed two different subtraction schemes to cross-check our results, namely the **Catani-Seymour** and **Nagy Soper** subtraction schemes
- Cancellation of the  $1/\epsilon$  and  $1/\epsilon^2$  poles between the virtual corrections and the real emission has also been confirmed for multiple phase space points
- The finite value for the virtual amplitude has also been cross checked with RECOLA [arxiv:\[1605.01090\]](#)



# Backup slides

## Scale variations



The primary source of scale uncertainties originates from variations in  $\mu_R$