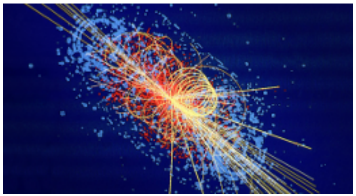
Using Gradient Flow to Renormalise Matrix Elements for Meson Mixing and Lifetimes

Matthew Black

In collaboration with:
R. Harlander, F. Lange, A. Rago, A. Shindler, O. Witzel

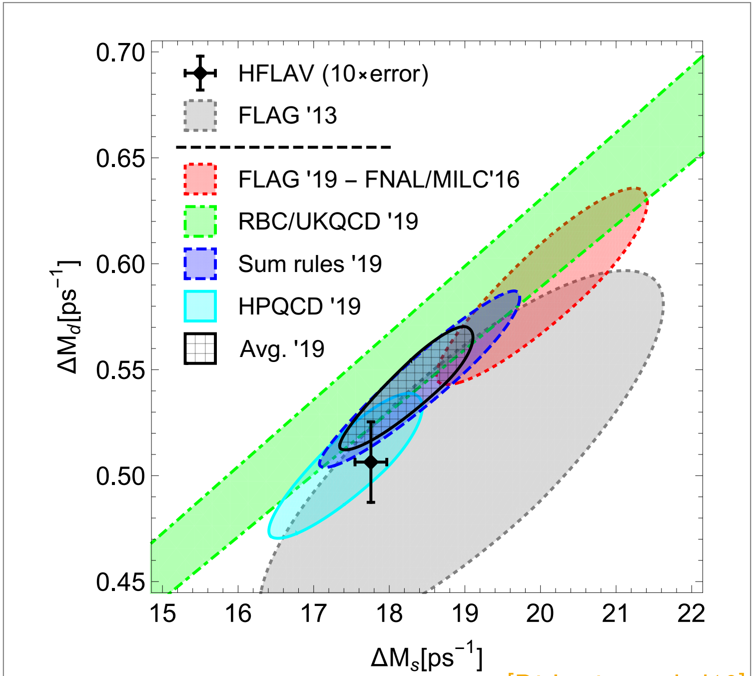
October 16, 2023



Center for Particle Physics Siegen

Universität Siegen

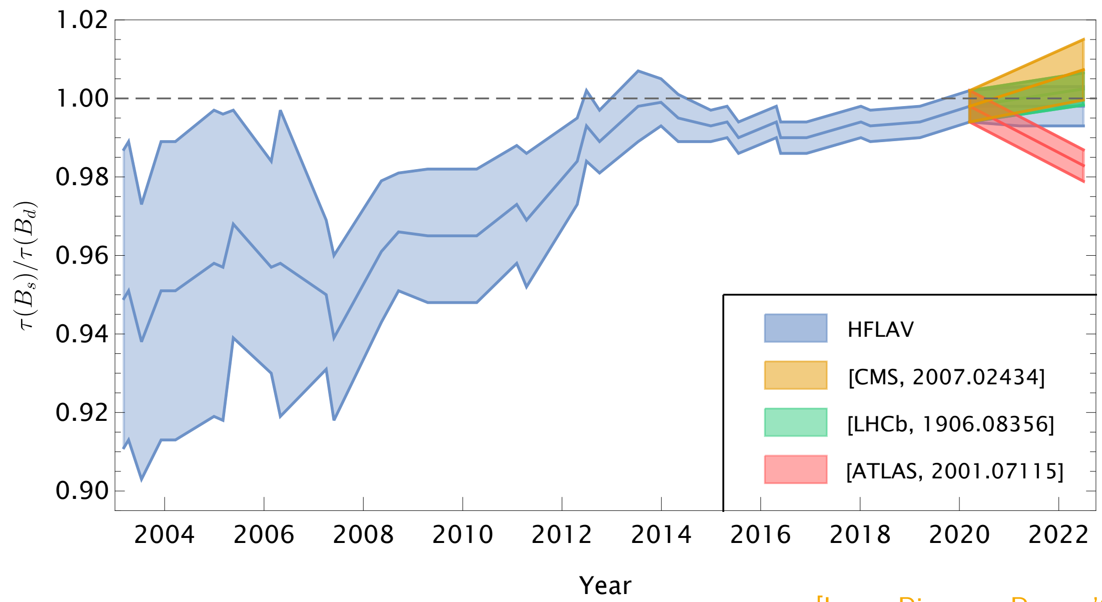
- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**



[Di Luzio et al. '19]

➤ B-meson mixing and lifetimes are measured experimentally to high precision

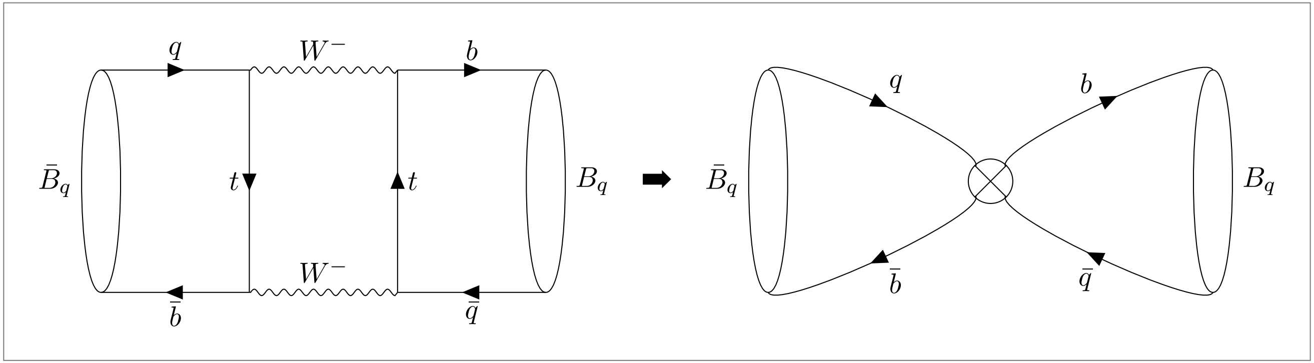
➥ Key observables for probing New Physics ➡ **high precision in theory needed!**



[Lenz, Piscopo, Rusov '22]

- B-meson mixing and lifetimes are measured experimentally to high precision
  - ➥ Key observables for probing New Physics ➡ **high precision in theory needed!**
- For lifetimes and mixing, we use the **Heavy Quark Expansion**

$$\Gamma_{B_q} = \Gamma_3 \langle \mathcal{O}_{D=3} \rangle + \Gamma_5 \frac{\langle \mathcal{O}_{D=5} \rangle}{m_b^2} + \Gamma_6 \frac{\langle \mathcal{O}_{D=6} \rangle}{m_b^3} + \ldots + 16\pi^2 \left[\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_{D=6} \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_{D=7} \rangle}{m_b^4} + \ldots \right]$$



- Factorise observables into ➡ perturbative QCD contributions
  ➡ **Non-Perturbative Matrix Elements**

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing
  - ➥ preliminary $\Delta K = 2$ for Kaon mixing study with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago + new developments for lifetime ratios [Lin, Detmold, Meinel '22]
  - ➥ contributions from gluon disconnected ("eye") diagrams
  - ➥ mixing with lower dimension operators in renormalisation

- Four-quark $\Delta B = 0$ and $\Delta B = 2$ matrix elements can be determined from lattice QCD simulations
- $\Delta B = 2$ well-studied by several groups ➡ precision increasing
  - ➥ preliminary $\Delta K = 2$ for Kaon mixing study with gradient flow [Suzuki et al. '20], [Taniguchi, Lattice '19]
- $\Delta B = 0$ ➡ exploratory studies from ∼20 years ago + new developments for lifetime ratios [Lin, Detmold, Meinel '22]
  - ➥ contributions from gluon disconnected ("eye") diagrams
  - ➥ mixing with lower dimension operators in renormalisation

---

1. Establish gradient flow renormalisation procedure with $\Delta B = 2$ matrix elements
2. Pioneer **connected** $\Delta B = 0$ matrix element calculation
3. Tackle disconnected contributions

- To first test method, only consider $\tilde{\mathcal{O}}_1$ for **mixing**:

$$\tilde{\mathcal{O}}_1 = (\bar{Q}_i\gamma_\mu(1-\gamma_5)q_i)(\bar{Q}_j\gamma_\mu(1-\gamma_5)q_j) \implies \langle\tilde{\mathcal{O}}_1(\mu)\rangle = \frac{8}{3}M_P^2 f_P^2 B_1(\mu)$$

- Hadronic physics encoded in decay constant f_P and **bag parameter** B_1

- To first test method, only consider $\tilde{\mathcal{O}}_1$ for **mixing**:

$$\tilde{\mathcal{O}}_1 = (\bar{Q}_i\gamma_\mu(1-\gamma_5)q_i)(\bar{Q}_j\gamma_\mu(1-\gamma_5)q_j) \implies \langle\tilde{\mathcal{O}}_1(\mu)\rangle = \frac{8}{3}M_P^2 f_P^2 B_1(\mu)$$

- Hadronic physics encoded in decay constant f_P and **bag parameter** B_1

precision $< 1\%$ [FLAG '23]

- Decay constant extracted independently from two-point correlation function:

$$C_{AP}(t) = \sum_n \frac{\langle P_n|A_\mu|0\rangle\langle 0|\gamma_5|P_n\rangle}{2E_{P_n}} e^{-E_{P_n}t} \implies \langle 0|A_\mu|P(0)\rangle = f_P M_P$$

- Bag parameter extracted from ratio of three-point correlator to two-point correlators:

$$R_1 = \frac{C^{3\text{pt}}_{\mathcal{O}_1}(t,\Delta t)}{\frac{8}{3}C^{2\text{pt}}_{AP}(t)\,C^{2\text{pt}}_{PA}(\Delta t - t)} \implies B_1 = \frac{\langle P|\mathcal{O}_1|P\rangle}{\frac{8}{3}\langle P|\gamma_5|0\rangle\langle 0|A_\mu|P\rangle\langle P|A_\mu|0\rangle\langle 0|\gamma_5|P\rangle},$$

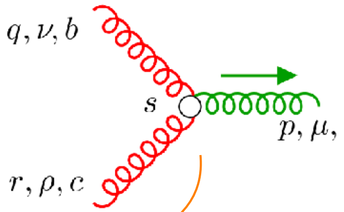
$$C^{3\text{pt}}_{\mathcal{O}_1}(t,\Delta t) = \sum_{n,n'} \frac{P_n P_{n'}}{4M_n M_{n'}}\langle P_n|\mathcal{O}_1|P_{n'}\rangle e^{-(\Delta t - t)M_n} e^{-tM_{n'}}$$

- Formulated by [Lüscher '10], [Lüscher '13] ➡ scale setting, RG β-function, **renormalisation**...
- Introduce auxiliary dimension, **flow time** τ as a way to regularise the UV
- Extend gauge and fermion fields in flow time and express dependence with first-order differential equations:

$$\partial_t B_\mu(\tau,x) = \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau,x), \quad B_\mu(0,x) = A_\mu(x),$$
$$\partial_t \chi(\tau,x) = \mathcal{D}^2(\tau)\chi(\tau,x), \quad \chi(0,x) = q(x).$$

- Re-express effective Hamiltonian in terms of 'flowed' operators:

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau).$$



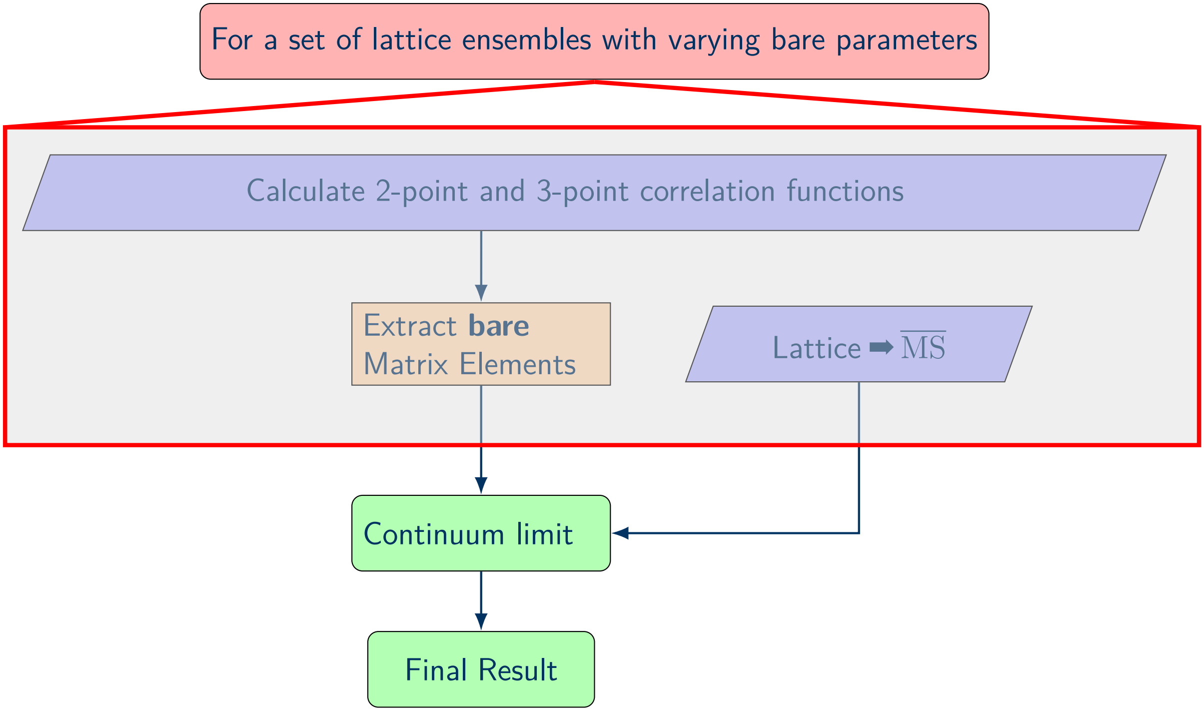
- Relate to regular operators in 'short-flow-time expansion':

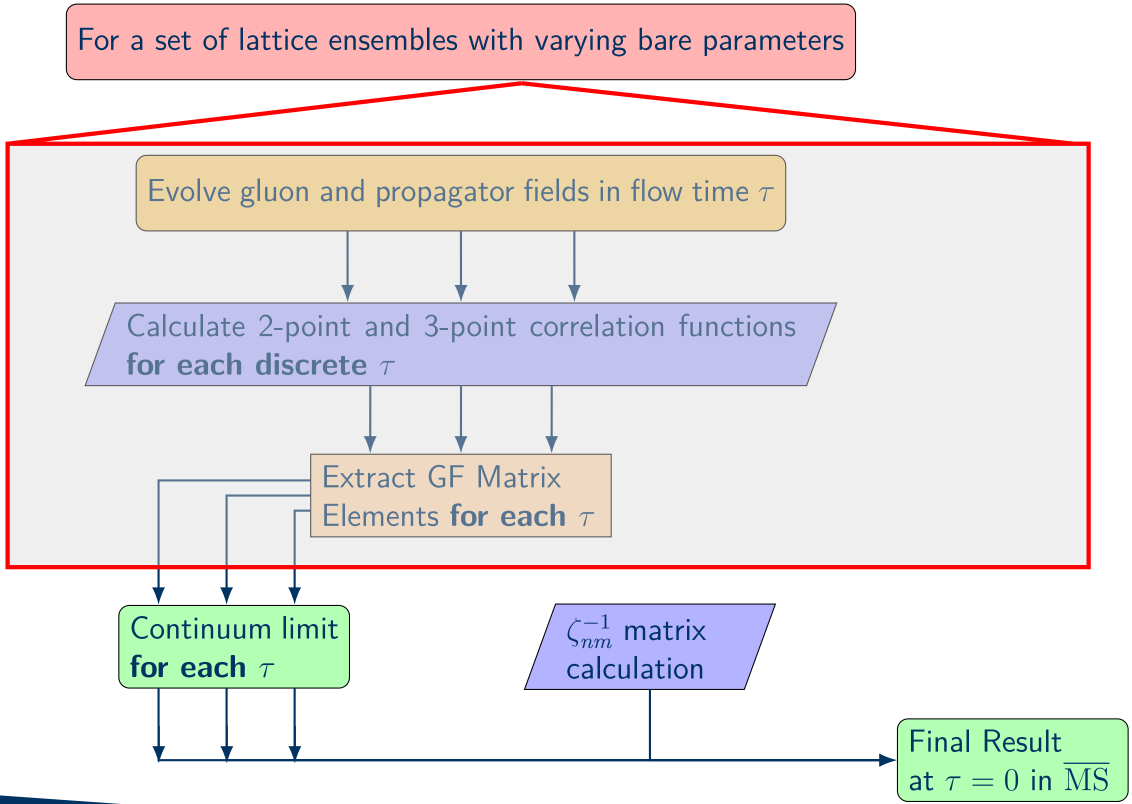
$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice replacing $A_\mu, q \to B_\mu, \chi$

new Feynman diagrams

matching matrix calculated perturbatively


For a set of lattice ensembles with varying bare parameters
Calculate 2-point and 3-point correlation functions
Extract **bare** Matrix Elements
Lattice ➡ $\overline{\mathrm{MS}}$
Continuum limit
Final Result

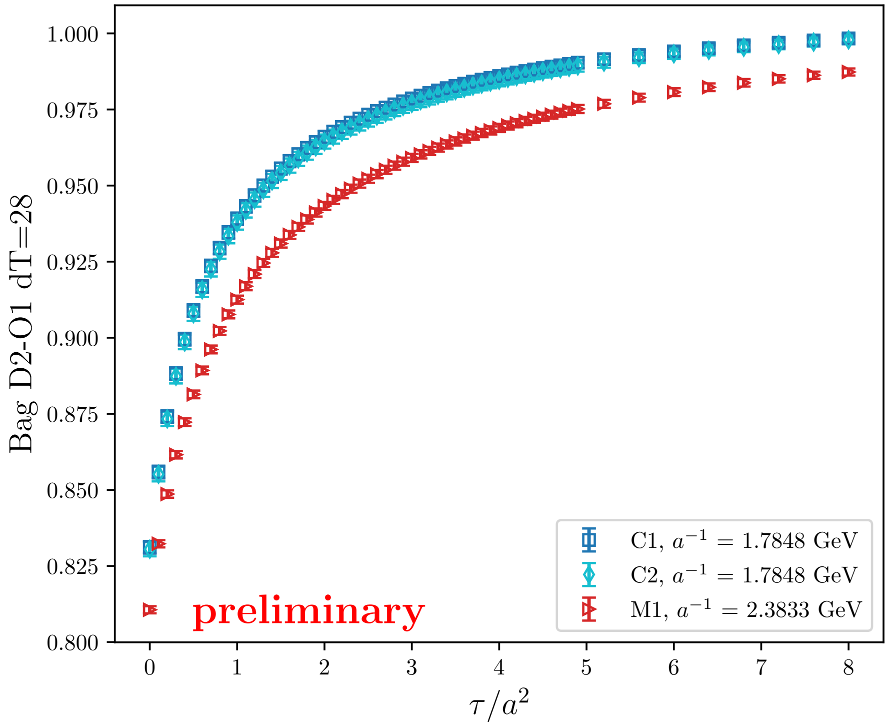

For a set of lattice ensembles with varying bare parameters
Evolve gluon and propagator fields in flow time τ
Calculate 2-point and 3-point correlation functions **for each discrete** τ
Extract GF Matrix Elements **for each** τ
Continuum limit **for each** τ
ζ_{nm}^{-1} matrix calculation
Final Result at $\tau = 0$ in $\overline{\mathrm{MS}}$

- We will consider RBC/UKQCD's 2+1 flavour Shamir DWF + Iwasaki gauge action ensembles
  [Shamir '93] [Iwasaki, Yoshie '84] [Iwasaki '85]

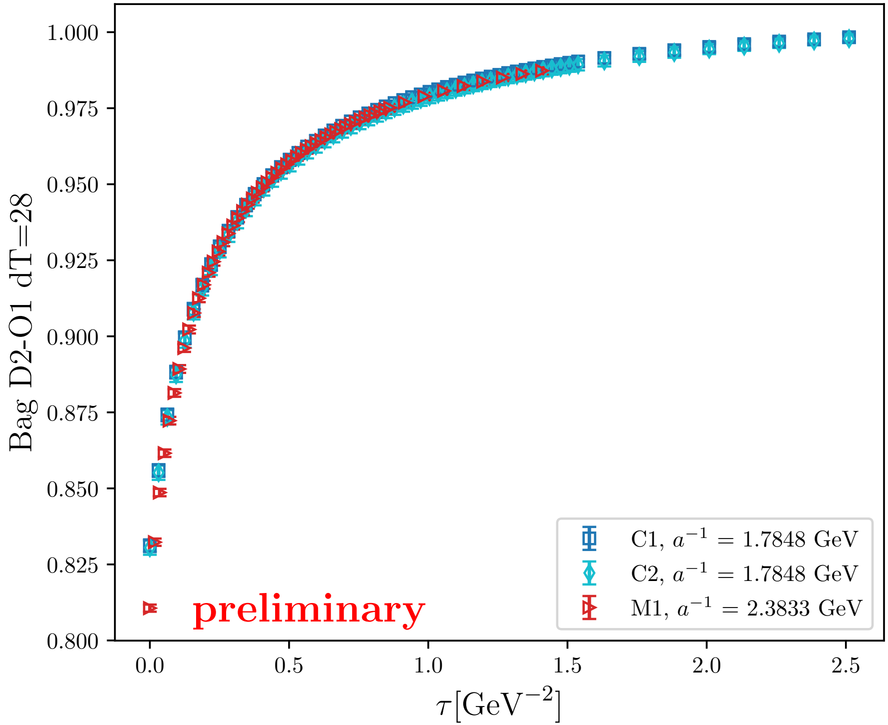
	L	T	a^{-1}/GeV	am_l^{sea}	am_s^{sea}	M_π/MeV	srcs × N_{conf}
C1	24	64	1.7848	0.005	0.040	340	32 × 101
C2	24	64	1.7848	0.010	0.040	433	32 × 101
M1	32	64	2.3833	0.004	0.030	302	32 × 79
M2	32	64	2.3833	0.006	0.030	362	32 × 89
M3	32	64	2.3833	0.008	0.030	411	32 × 68
F1S	48	96	2.785	0.002144	0.02144	267	

[Allton et al. '08]
[Aoki et al. '10]
[Blum et al. '14]
[Boyle et al. '17]

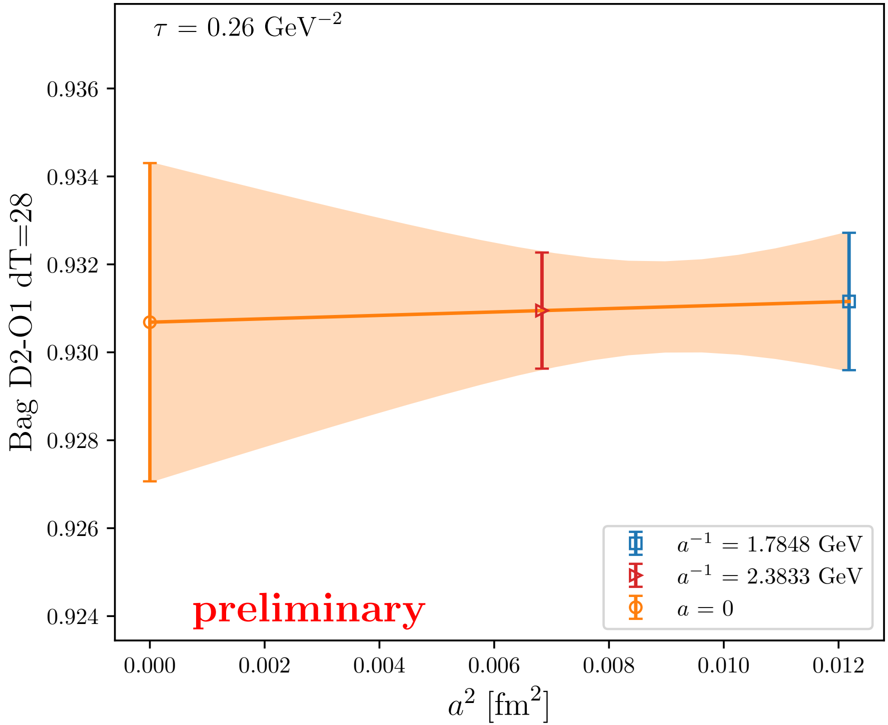
- Exploratory simulations on C1, C2, M1 so far
- To remove additional extrapolations in valence sector, we simulate at physical charm and strange
  - ➥ "neutral D_s" meson mixing



➤ operator is renormalised in 'GF' scheme as it is evolved along flow time



➤ different lattice spacings overlap in physical flow time ➡ mild continuum limit



➤ continuum limit very flat at positive flow time ✔

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice (for $\widetilde{\mathcal{O}}_n(\tau)$)

matching matrix calculated perturbatively (for $\zeta_{nm}(\tau)$)

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

matching matrix
calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Relate to regular operators in 'short-flow-time expansion':

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

'flowed' MEs calculated on lattice

matching matrix calculated perturbatively

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Calculated at two-loop for $\mathcal{B}_1$ based on [Harlander, Lange '22]:

$$\zeta^{-1}_{\mathcal{B}_1}(\mu,\tau) = 1 + \frac{a_s}{4}\left(-\frac{11}{3} - 2L_{\mu\tau}\right) + \frac{a_s^2}{43200}\Big[-2376 - 79650L_{\mu\tau} - 24300L^2_{\mu\tau} + 8250n_f + 6000\,n_f\,L_{\mu\tau}$$
$$+ 1800\,n_f\,L^2_{\mu\tau} - 2775\pi^2 + 300\,n_f\pi^2 - 241800\log 2$$
$$+ 202500\log 3 - 110700\,\mathrm{Li}_2\left(\frac{1}{4}\right)\Big]$$

$$L_{\mu\tau} = \log(2\mu^2\tau) + \gamma_E, \quad \mu = 3\,\mathrm{GeV}$$

- Relate to regul

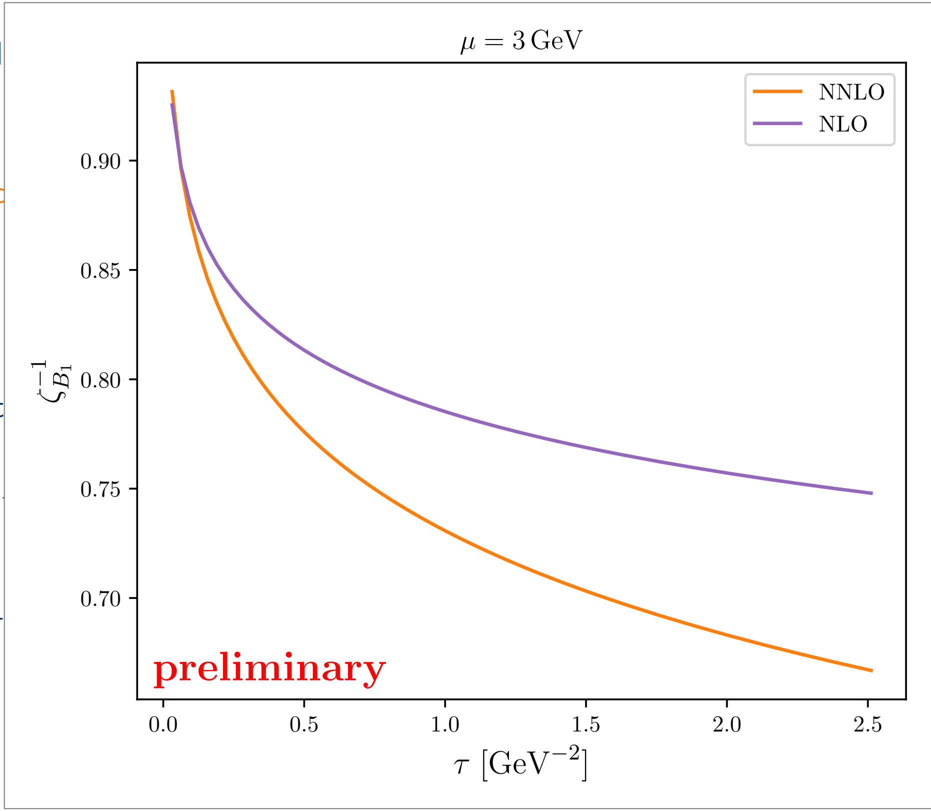
'flowed' MEs c ... k ... rbatively

- Calculated at t

$\zeta_{\mathcal{B}_1}^{-1}(\mu,\tau) = 1$ ... $0 n_f + 6000\, n_f\, L_{\mu\tau}$

... $241800 \log 2$

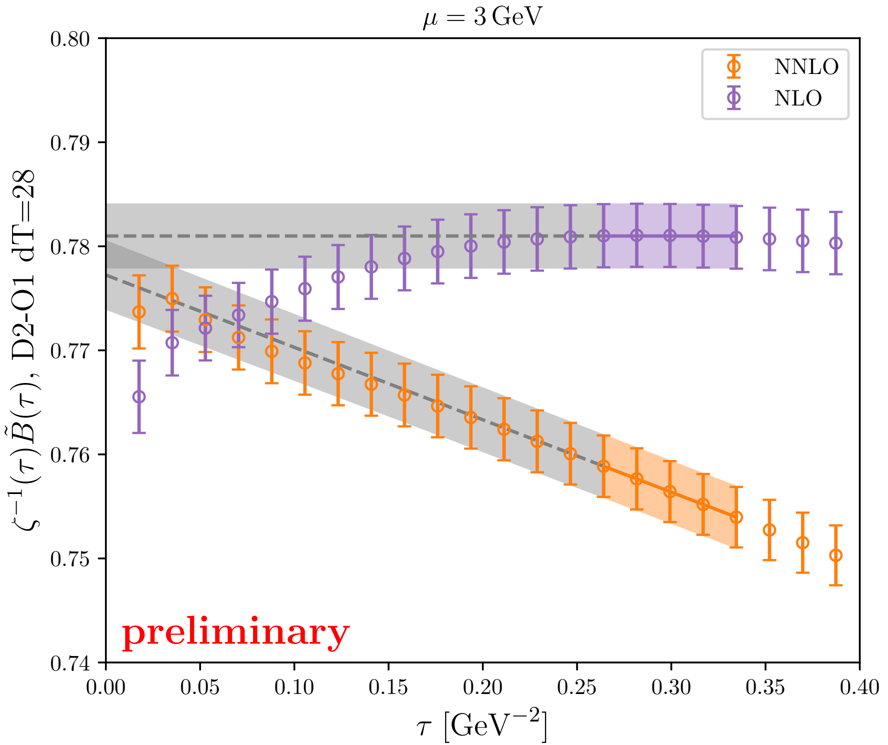
$L_{\mu\tau} = \log(2\mu^2\tau$



- Compare to existing short-distance D^0 mixing results

[ETM '15]	0.757(27)
[FNAL/MILC '17]	0.795(56)

- Promising first signs of agreement
- Different perturbative orders show different behaviour



- $\Delta B = 0$ four-quark matrix elements are strongly-desired quantities
  - Standard renormalisation introduces mixing with operators of lower mass dimension
  - We aim to use the fermionic gradient flow as a non-perturbative renormalisation procedure
- Testing method first with $\Delta Q = 2$ matrix elements
- Shown first simulations for $\Delta C = 2$
- Preliminary results show promising agreement with literature

---

Next Steps:

- Simulate on all ensembles with multiple valence quark masses
- Extrapolate to physical B_s meson mixing
- Repeat analysis for quark-line connected $\Delta B = 0$ matrix elements
- Consider gluon disconnected contributions