Neutrino Cosmology: Lecture I

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ISAPP School 2024
Neutrinos and Dark Matter
in the lab and in the Universe
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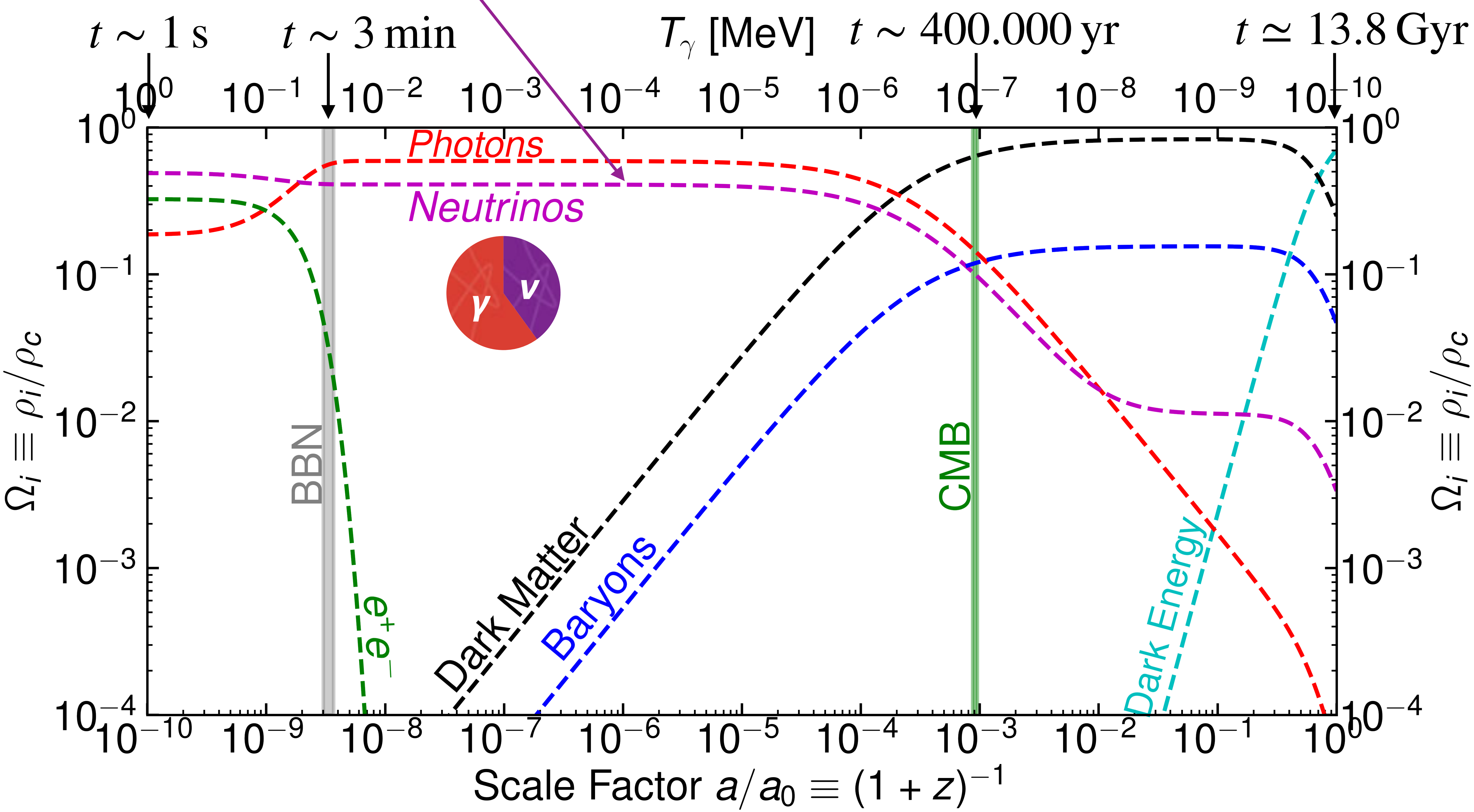
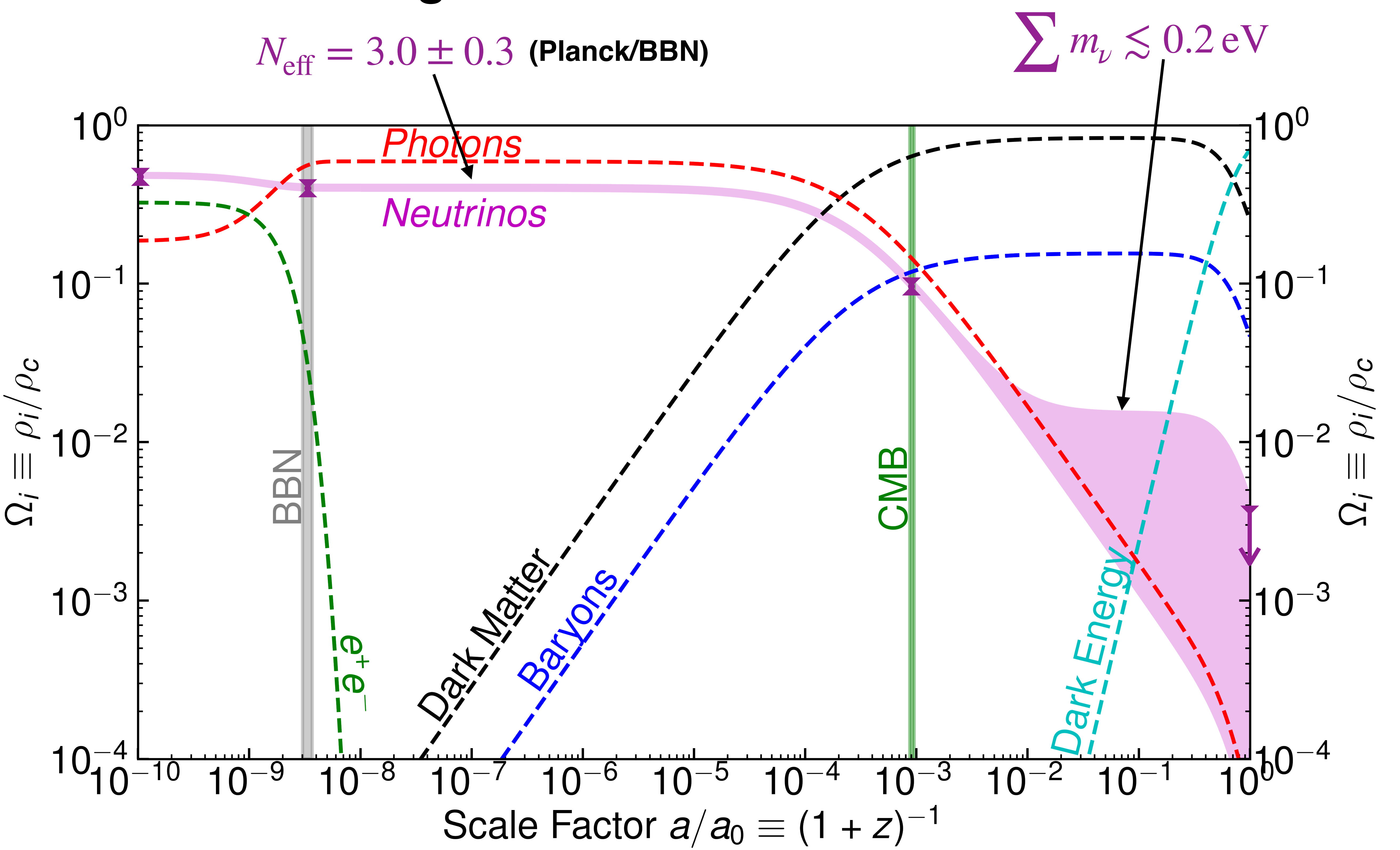
Neutrino Evolution

Neutrinos are always a relevant species in the Universe's evolution



Global Perspective

Current knowledge:

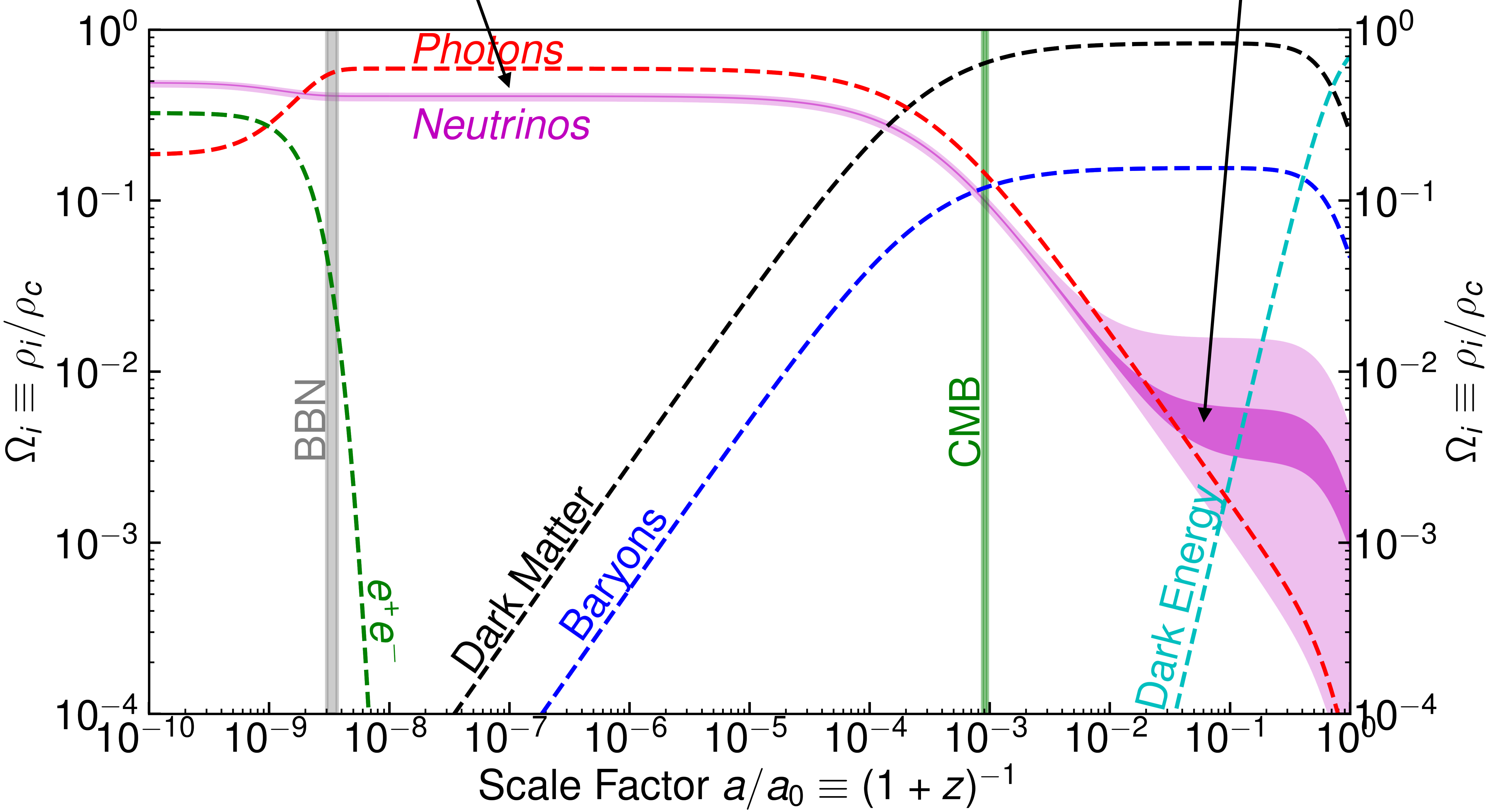


Global Perspective

In the next 5-6 years: **(DESI/Euclid + Planck)**

$N_{\rm eff} = 3.043 \pm 0.06$ **(Simons Observatory)**

$\sum m_\nu = 0.06 \pm 0.02\,{\rm eV}$



Goals

1) Understand what is the role played by neutrinos in Cosmology

2) Understand the evidence that we have for the Cosmic Neutrino Background and have a flavor of the types of BSM physics that can be tested with neutrinos in cosmology

3) Understand why can one derive neutrino mass bounds using cosmological data and what are the assumptions behind these constraints

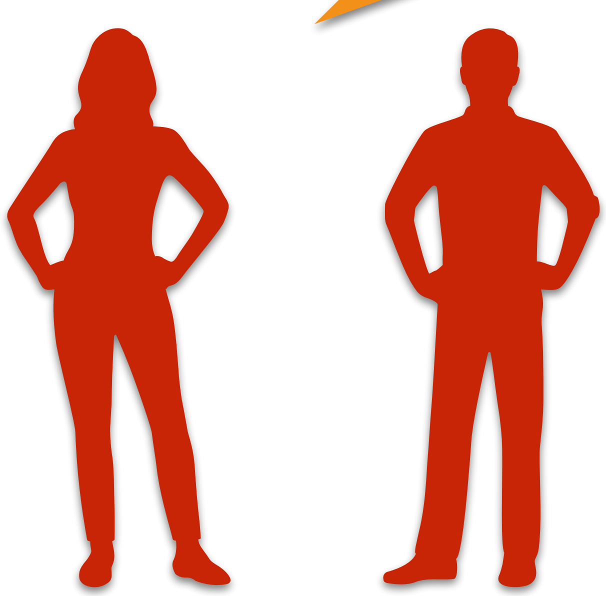
4) What are we going to learn in the upcoming years?

Set Up

Unlike neutrinos, I do like to interact 😃

The plan is to learn and therefore:

Questions and Comments are most welcome, at any time!!!!



Outline

Lecture I

Crash course on early Universe cosmology

Neutrino decoupling in the Standard Model

Evidence for the Cosmic Neutrino Background

BSM constraints: Sterile Neutrinos and Thermal Dark Matter

Lecture II

Neutrino Masses in Cosmology

Lecture III

The Hubble tension and neutrinos

Can we directly detect the Cosmic Neutrino Background?

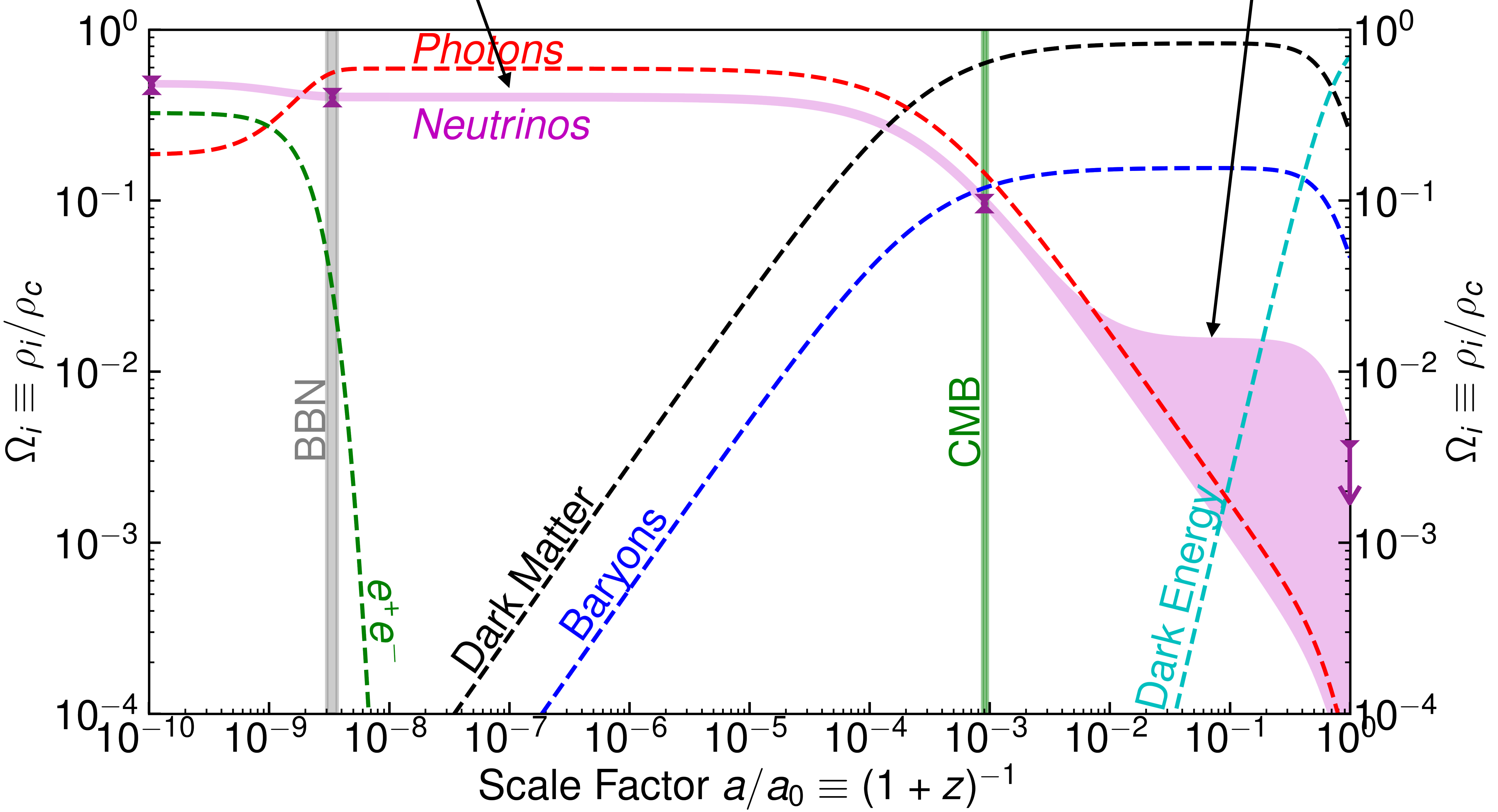
Outline

Current knowledge:

$N_{\text{eff}} = 3.0 \pm 0.3$ **(Planck/BBN)**

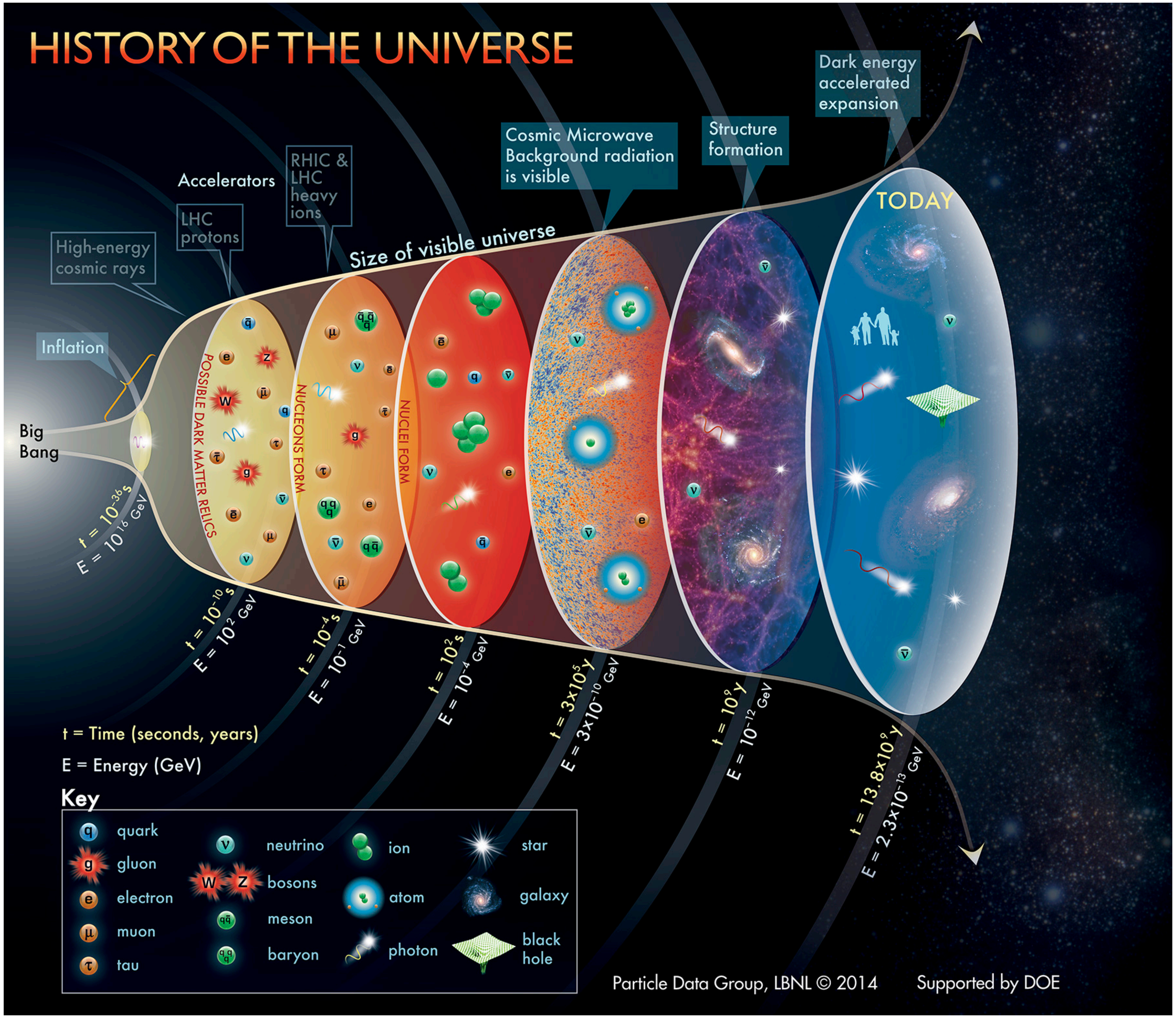
(Planck+BAO)

$\sum m_\nu \lesssim 0.2\,\text{eV}$

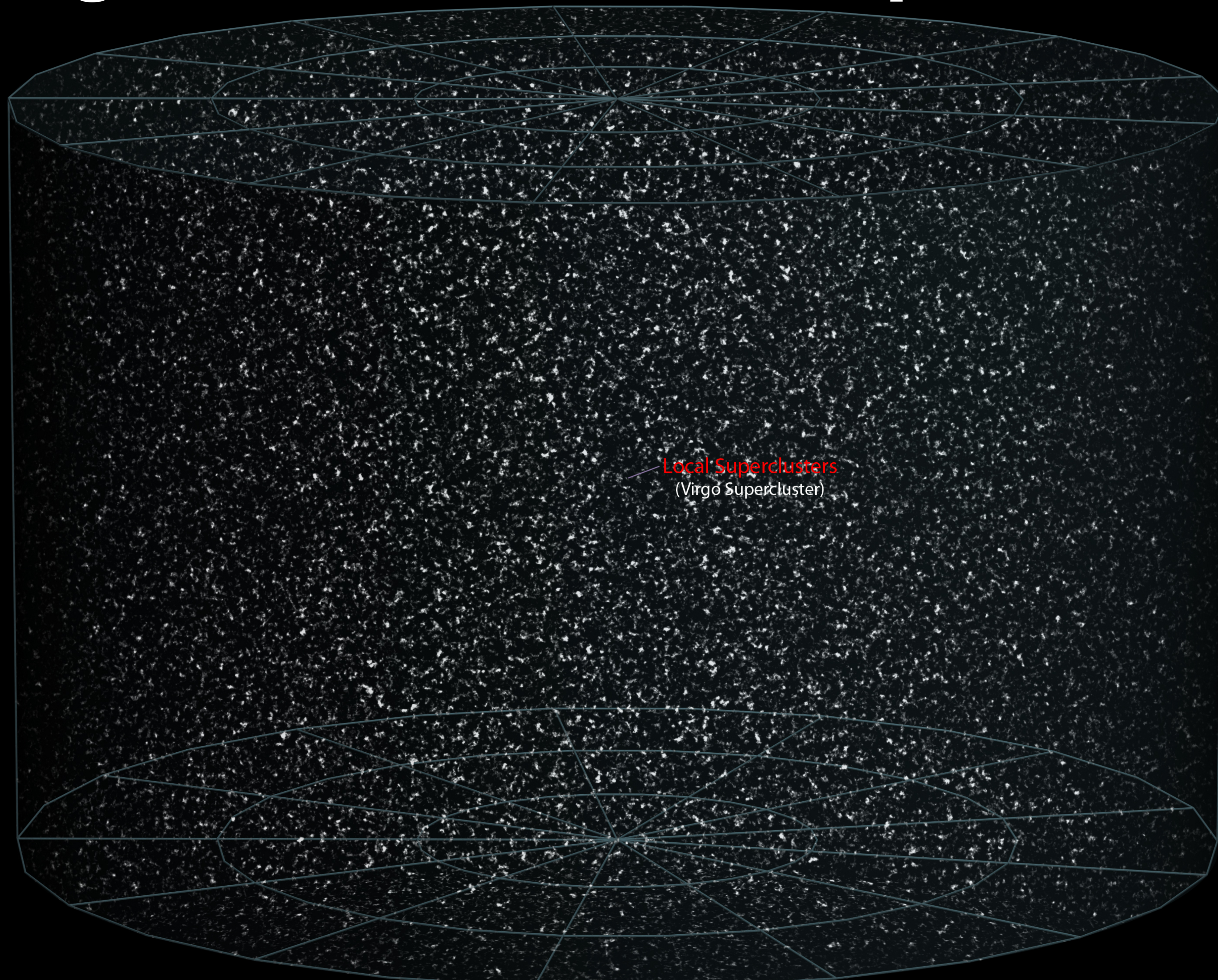


A Crash Course on Cosmology

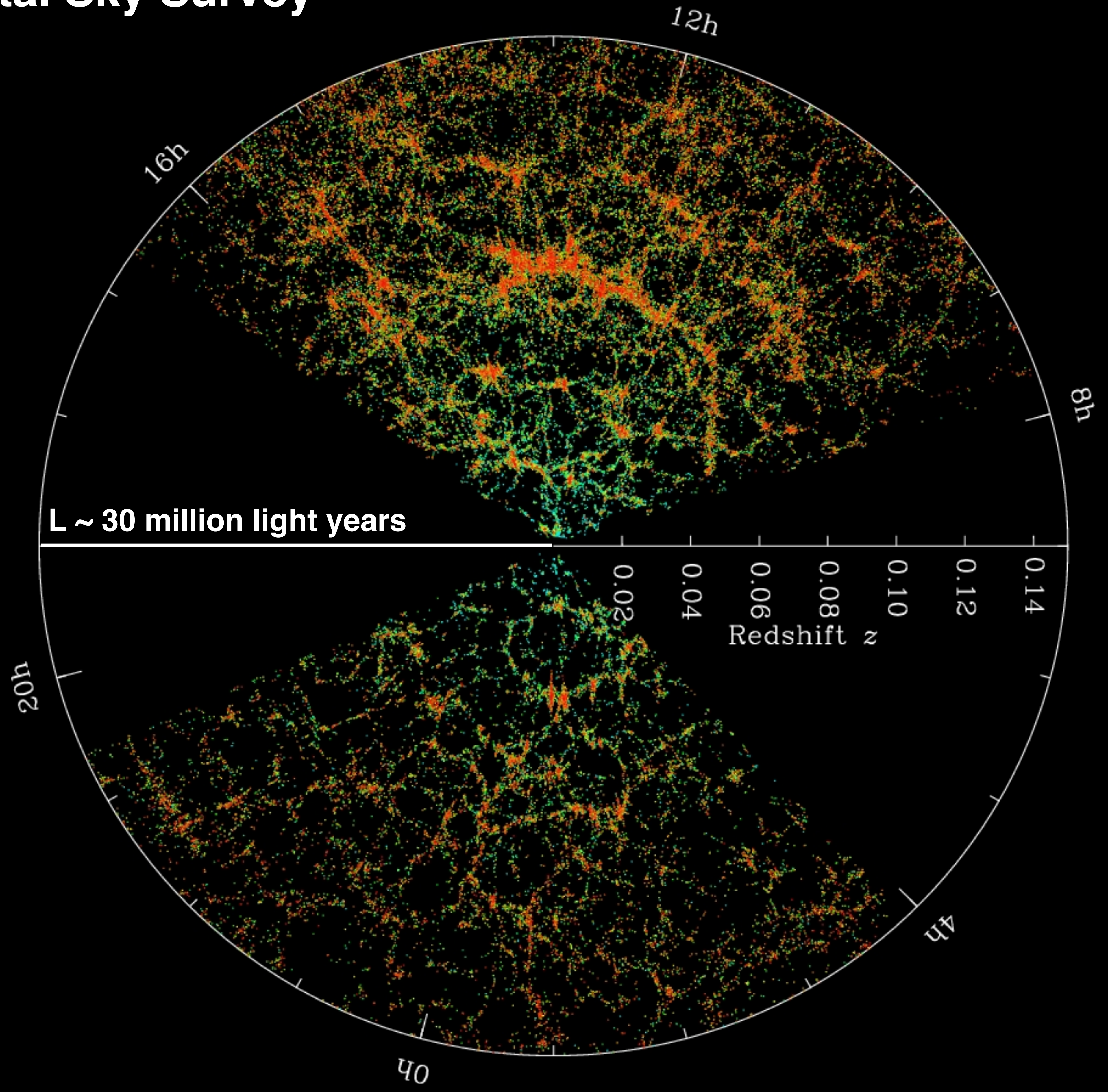
In 10 slides! 💪

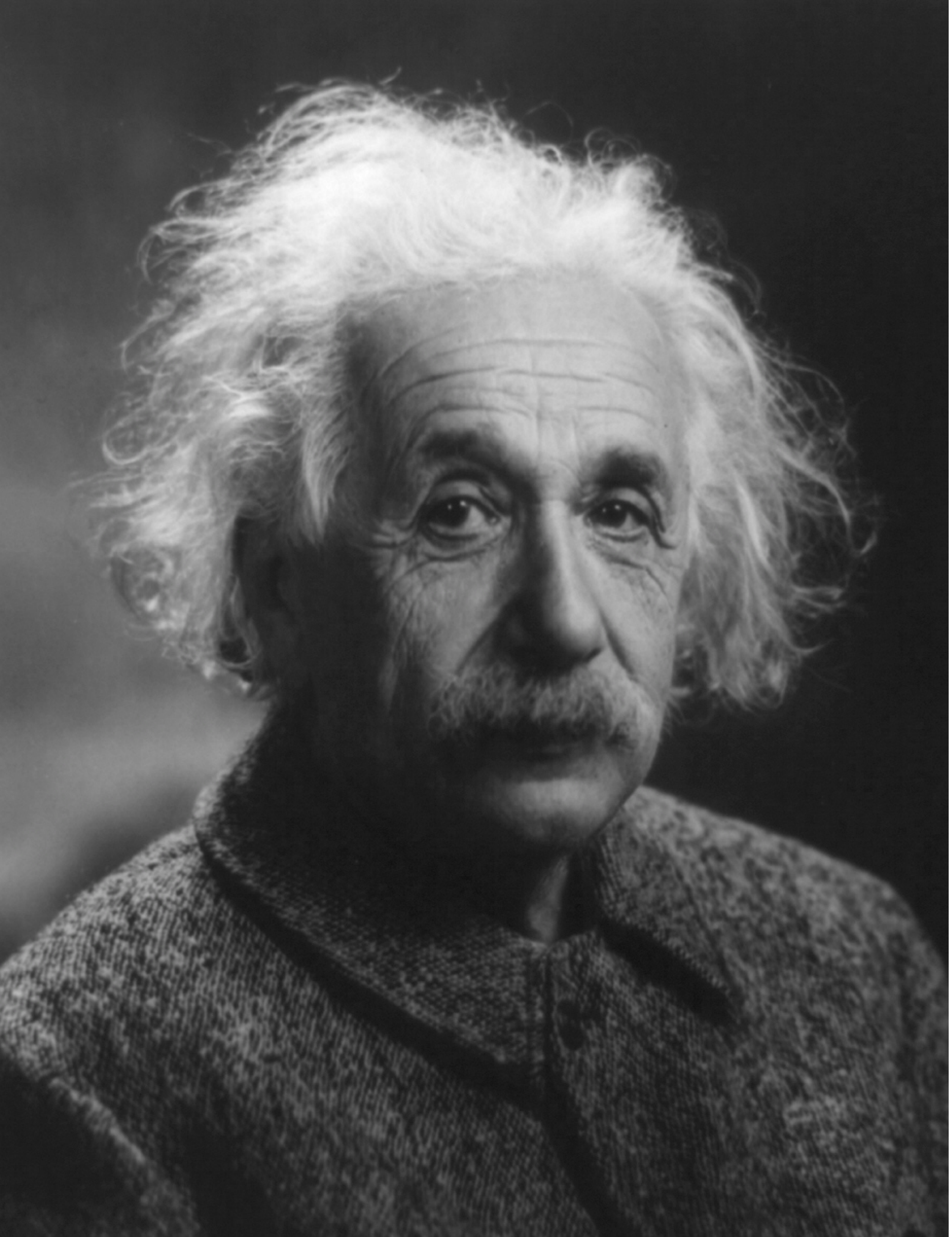


Homogeneous and Isotropic Universe



Sloan Digital Sky Survey





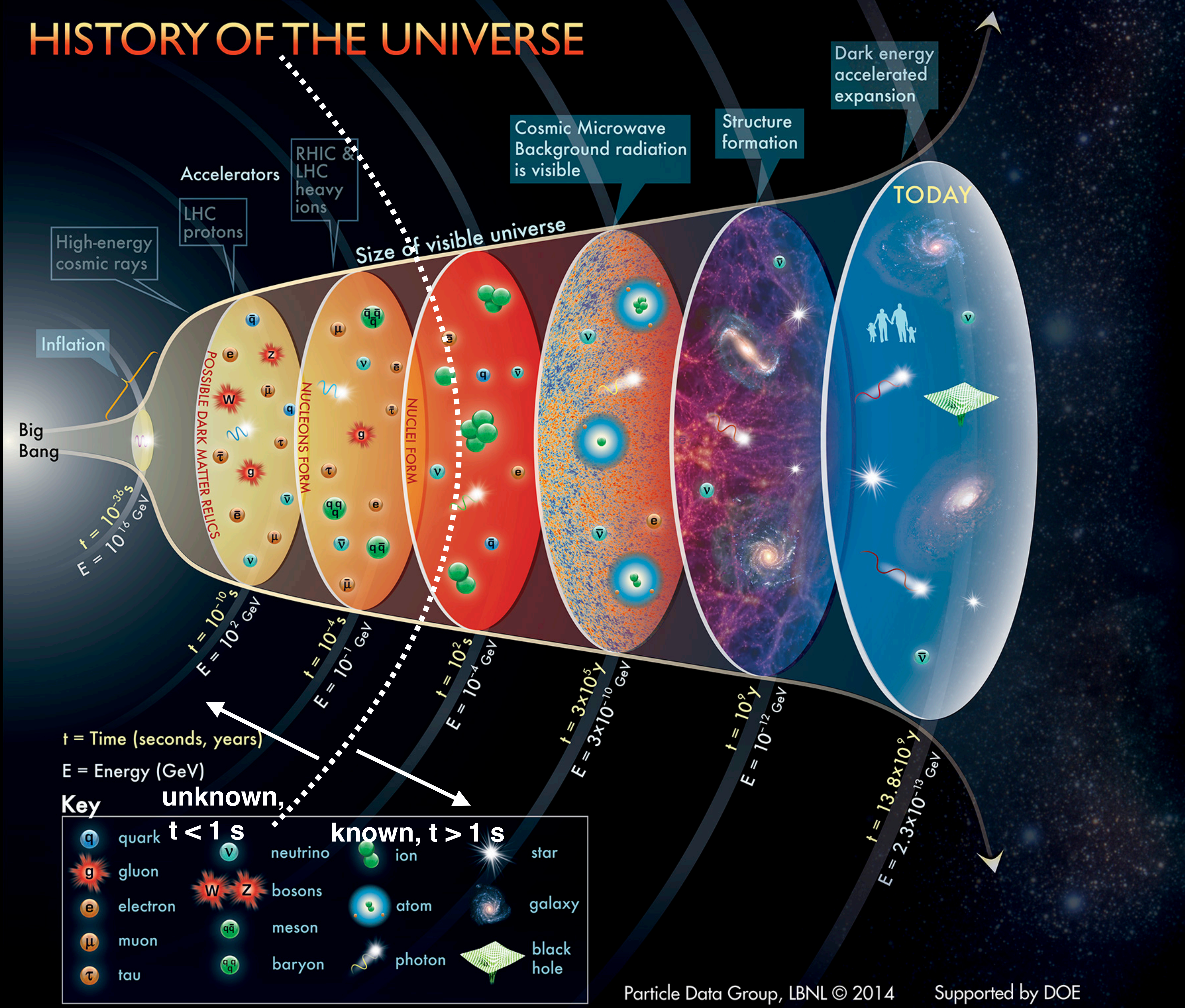
Einstein's Equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Matter: $T_{\mu\nu}$

Space-Time Geometry: $G_{\mu\nu}$

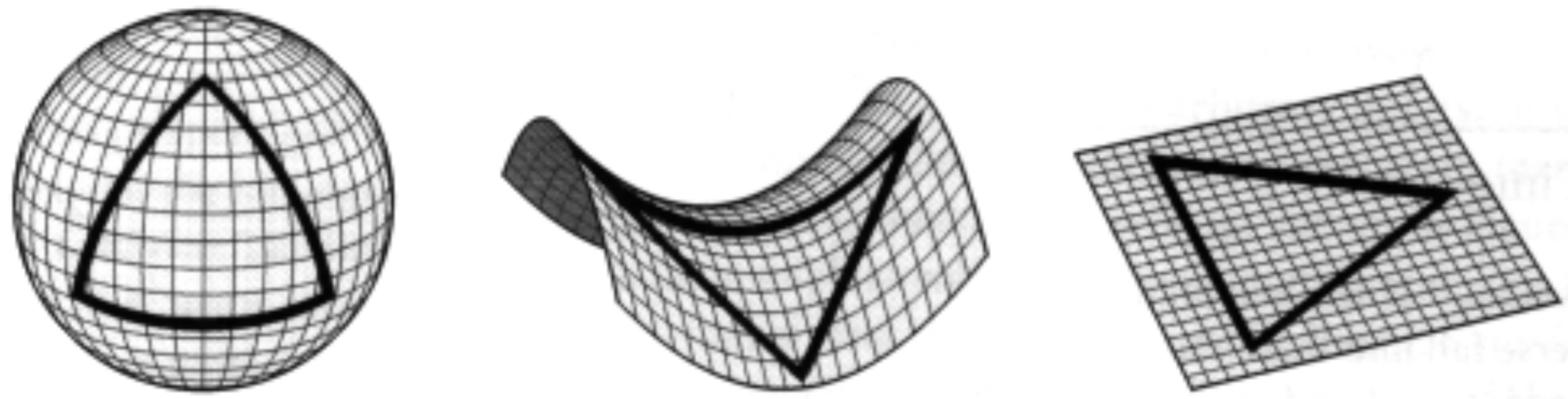
Expansion!



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
unknown, t < 1 s
known, t > 1 s
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
Supported by DOE

Isotropic and Homogeneous Universes

Three possibilities:



From cosmological data we know that even if the Universe is not flat its curvature radius is very large. Which means it will not have an effect on the early Universe! From now on, consider $k = 0$ (as also expected from Inflationary models).

FLRW: Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 \left[dx^2 + dy^2 + dz^2 \right]$$

Only dependent upon a single dynamical variable: The scale factor: $a(t)$

redshift: $a(t) = \dfrac{a_0}{1+z}$ **temperature:** $T \simeq T_0 (1+z)$ **density:** $n = n_0 (1+z)^3$

Cosmological Dynamics

— **General Relativity relates the expansion rate of the Universe with the energy density in all the species contained on it**

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}$$

↓

Friedmann Equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

H **: Expansion rate (Hubble parameter)**

ρ **: Energy density**

$$\nabla^\mu T_{\mu\nu} = 0$$

↓

Continuity equation:

$$\frac{d\rho}{dt} = -H(\rho + p)$$

p **: pressure**

ρ **: energy density**

Thermodynamics

A given particle species can be fully characterized by its distribution function:

Distribution function: $f \equiv$ **Number of particles in a phase space volume of** $\frac{1}{(2\pi\hbar)^3} d^3x d^3p$

In an homogeneous and isotropic Universe: $f(\vec{x}, \vec{p}) = f(|\vec{p}|) = f(p)$

From the distribution function we can extract all relevant properties of the system:

Number density:

$$n = g_i \int \frac{d^3p}{(2\pi)^3} f(p)$$

Energy density:

$$\rho = g_i \int \frac{d^3p}{(2\pi)^3} E\, f(p)$$

Pressure density:

$$p = g_i \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f(p)$$

$g_i \equiv$ **Number of internal degrees of freedom for the given species**

$g_\gamma = 2$ $\quad g_{e^+ + e^-} = 2 + 2 = 4$ $\quad g_{\nu_e + \bar{\nu}_e} = ?$ $\quad g_{\nu_e + \bar{\nu}_e} = 2$

(only the ν_L and $\bar{\nu}_R$ participate in the weak interactions!)

Equilibrium Thermodynamics

Bosons: $f(E) = \dfrac{1}{-1 + e^{(E-\mu)/T}}$

Fermions: $f(E) = \dfrac{1}{+1 + e^{(E-\mu)/T}}$

Ultrarelativistic regime: $T \gg m \quad \mu \ll T$

$$n = \quad g\frac{\xi(3)}{\pi^2}T^3 \quad \textbf{Bose-Einstein}$$

$$n = \frac{3}{4}g\frac{\xi(3)}{\pi^2}T^3 \quad \textbf{Fermi-Dirac}$$

$$\rho = \quad g\frac{\pi^2}{30}T^4 \quad \textbf{Bose-Einstein}$$

$$\rho = \frac{7}{8}g\frac{\pi^2}{30}T^4 \quad \textbf{Fermi-Dirac}$$

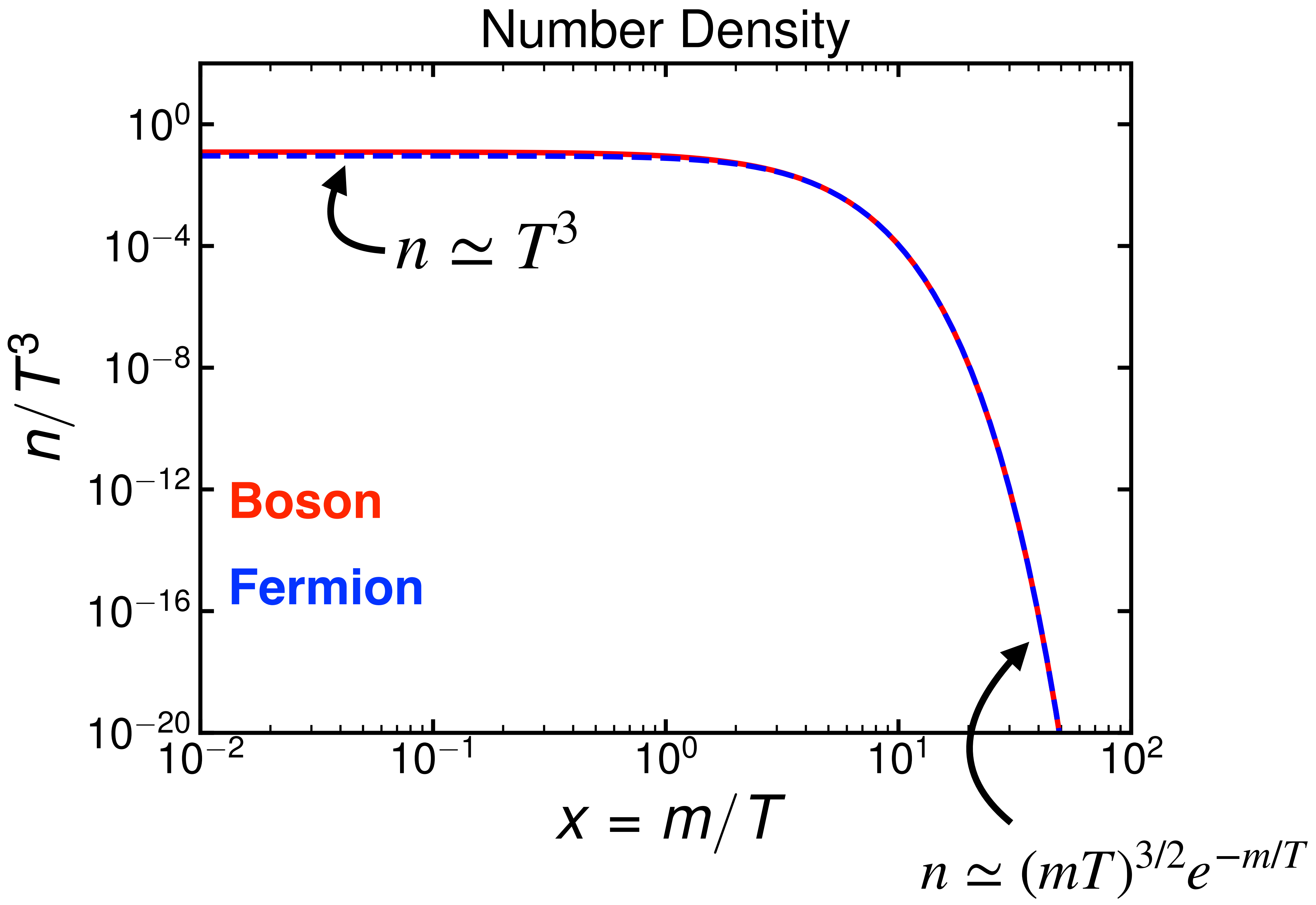
$$p = 1/3\rho$$

non-relativistic regime: $m \ll T \quad \mu \ll T$

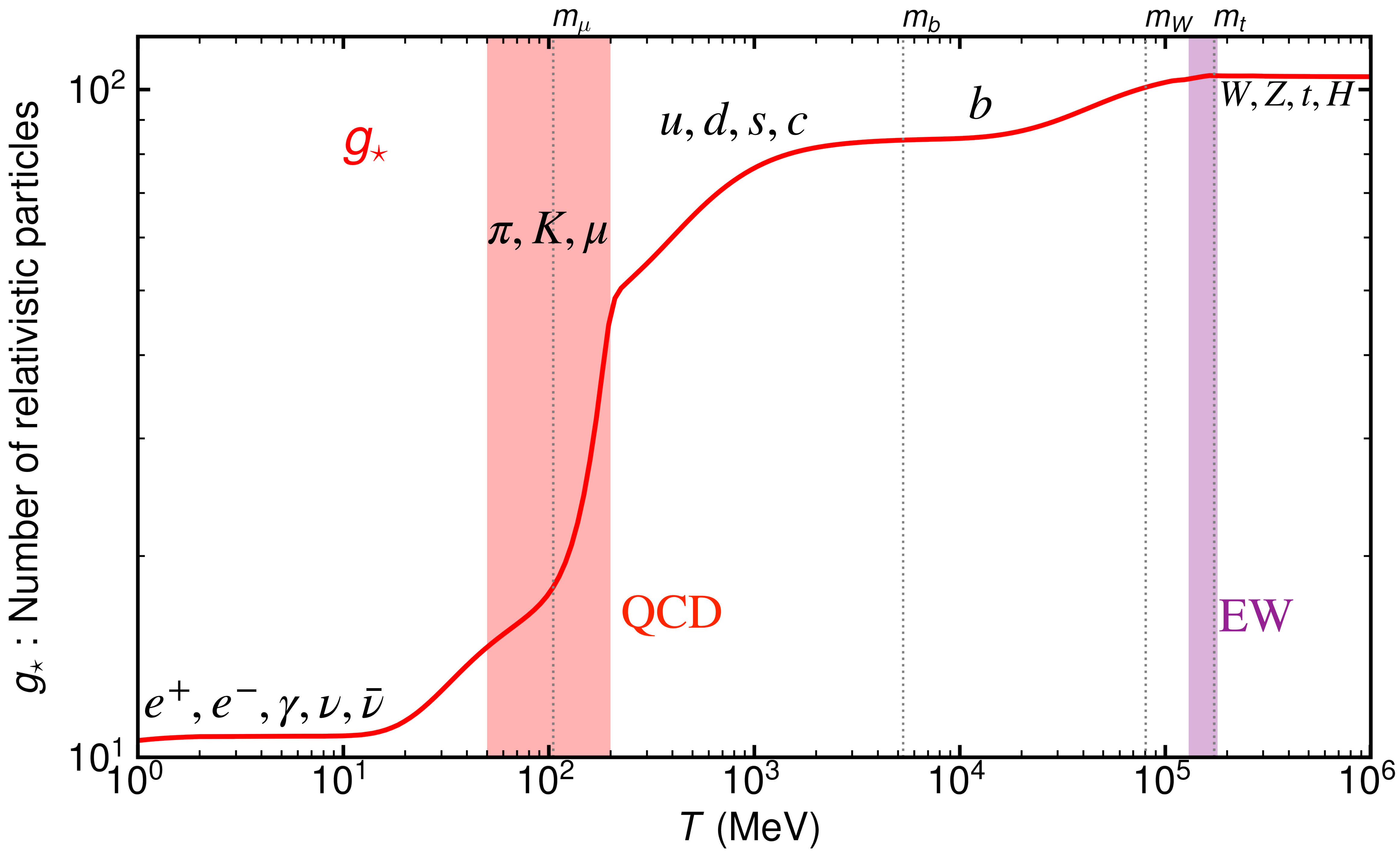
$$n = g(Tm/(2\pi))^{3/2}e^{-m/T}$$

$$\rho = m \times n$$

Equilibrium Thermodynamics



Equilibrium Thermodynamics



Laine & Meyer [1503.04935] http://www.laine.itp.unibe.ch/eos15/

Equilibrium Thermodynamics

Key things to remember:

$$T \gg m$$

$$n \simeq T^3 \quad \langle E \rangle \simeq 3T$$

$$\rho \simeq T^4 \quad p = 1/3\rho$$

$$T \ll m$$

$$n \simeq (Tm)^{3/2} e^{-m/T}$$

$$\rho \simeq mn$$

Main consequence: **Ultra relativistic particles dominate the energy density of the early Universe**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$s = \frac{\rho + p}{T}$$

$$H = 1.66 g_\star^{1/2} \frac{T^2}{M_{\rm Pl}} \quad t = \frac{1}{2H}$$

$$s = \frac{\pi^2}{45} g_{\star s} T^3$$

Departures from Equilibrium

A process will be in equilibrium in the early Universe if:

$$\Gamma \gtrsim H$$ **(equilibrium)**

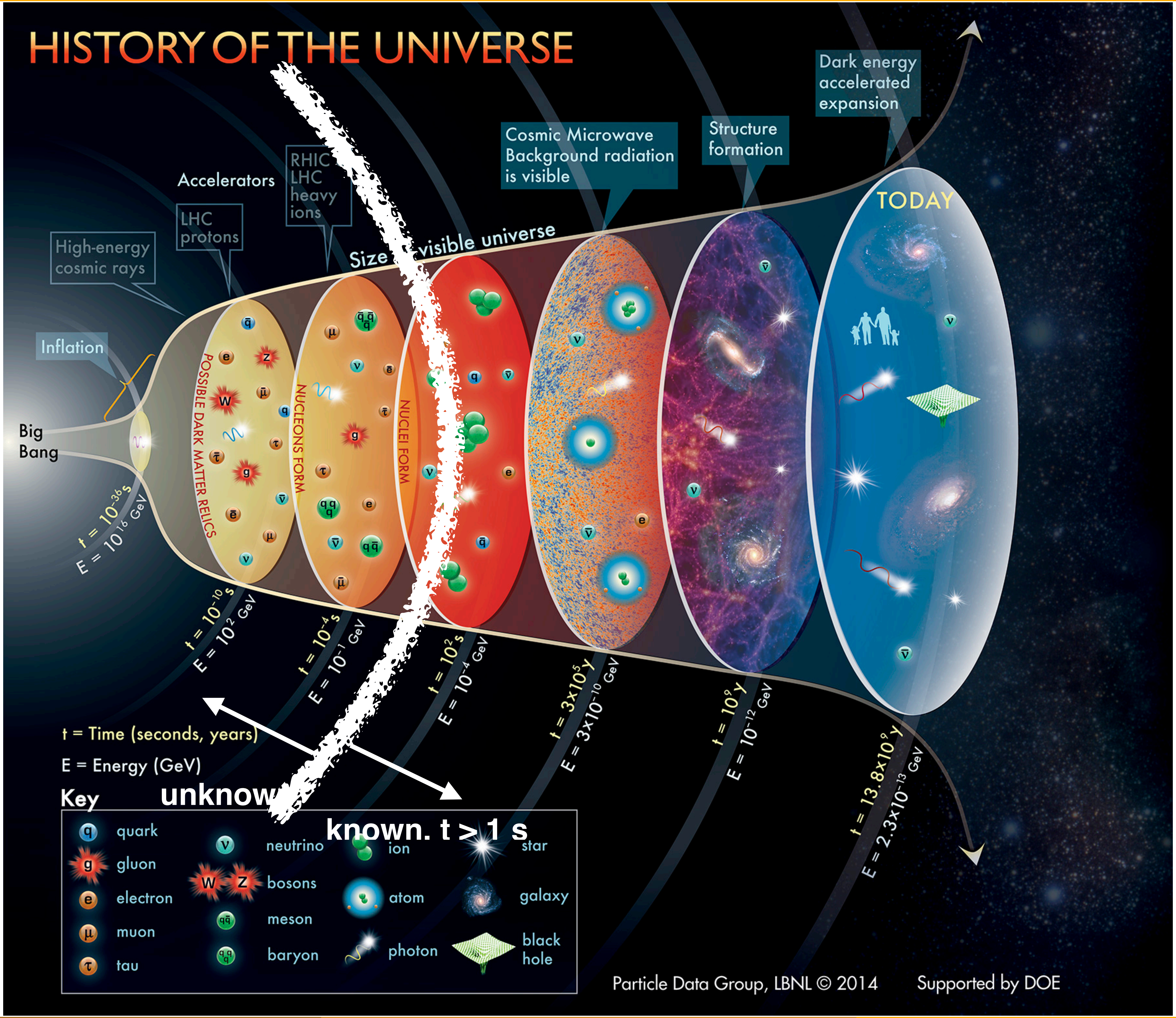
$$\Gamma \lesssim H$$ **(out-of-equilibrium)**

why?

number of interactions over the Universe lifetime will simply be:

$$N \simeq t_U/\tau \simeq \Gamma/H$$

Neutrino Decoupling



Application to Neutrinos

Consider interactions between neutrinos and the Standard Model

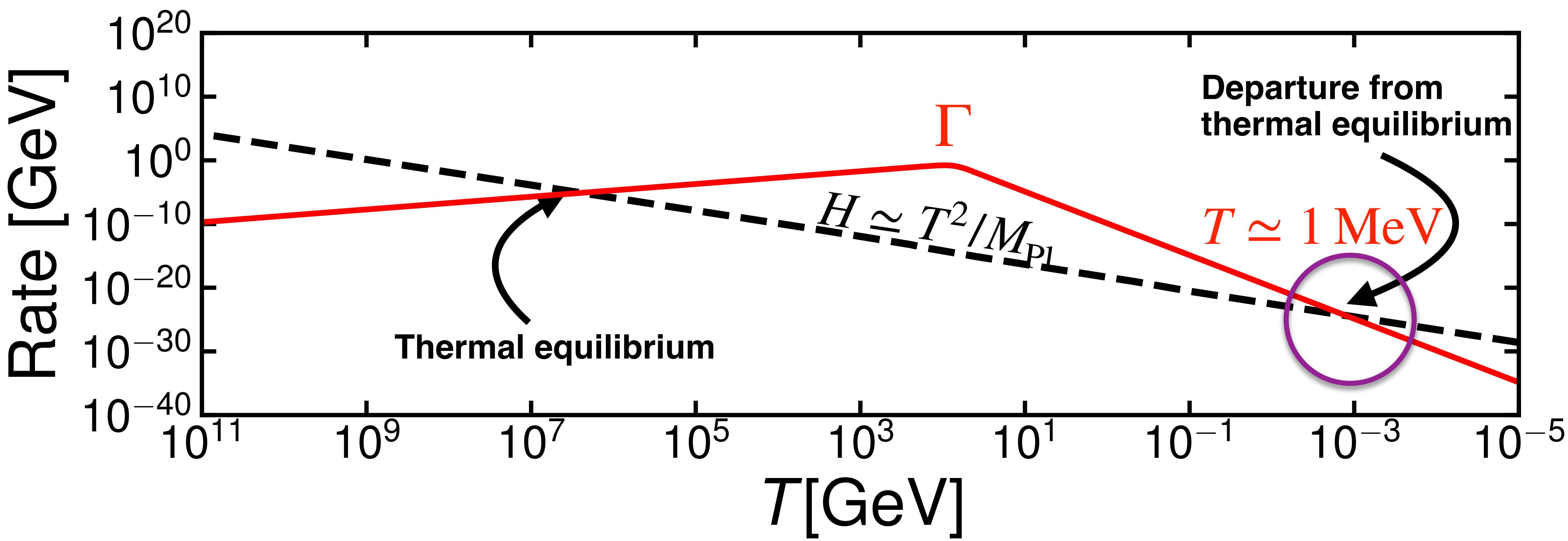
at $T > M_W$ $\quad W^+ \leftrightarrow e^+ + \nu$

$$\Gamma \simeq \frac{m_W}{T}\Gamma(W^+ \to e^+ + \nu) \qquad \Gamma \simeq \frac{m_W}{T}\frac{g^2}{48\pi}m_W$$

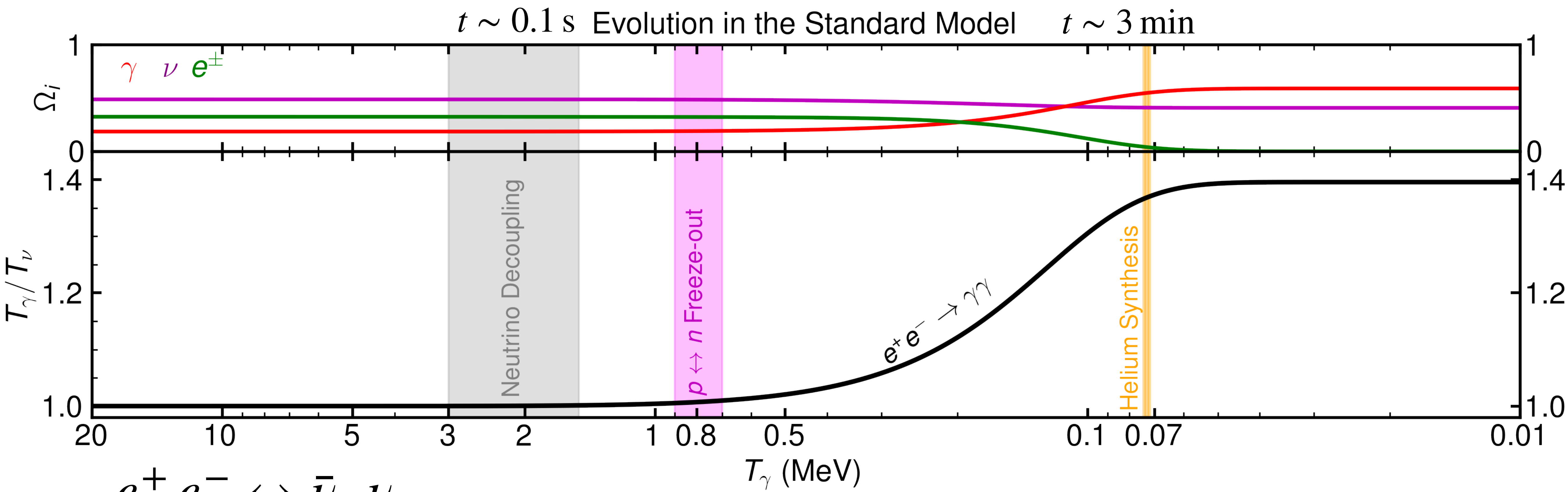
at $T < M_W$ $\quad e^+e^- \leftrightarrow \bar{\nu}\nu$

$$\Gamma = n_e \langle \sigma v \rangle \qquad \sigma \simeq G_F^2 s \qquad v \simeq 1$$

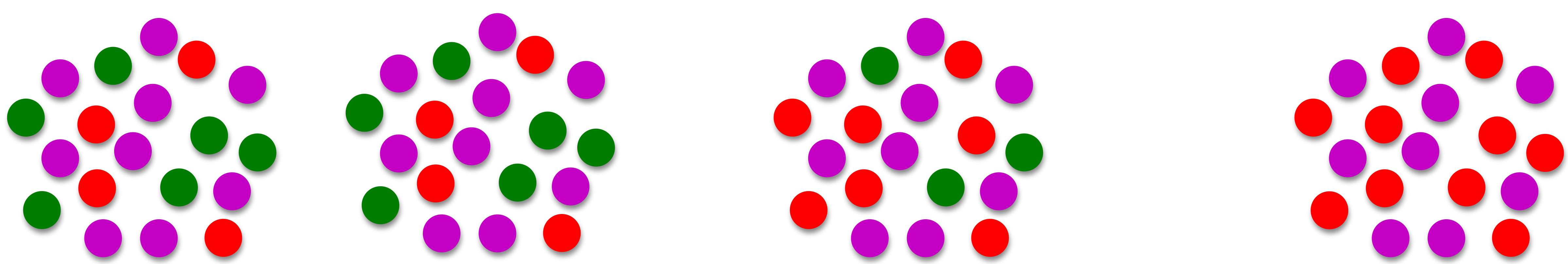
$$\Gamma \simeq T^3 \langle \sigma v \rangle \qquad \langle \sigma v \rangle \simeq G_F^2 T^2 \qquad \Gamma \simeq G_F^2 T^5$$



Neutrino Decoupling

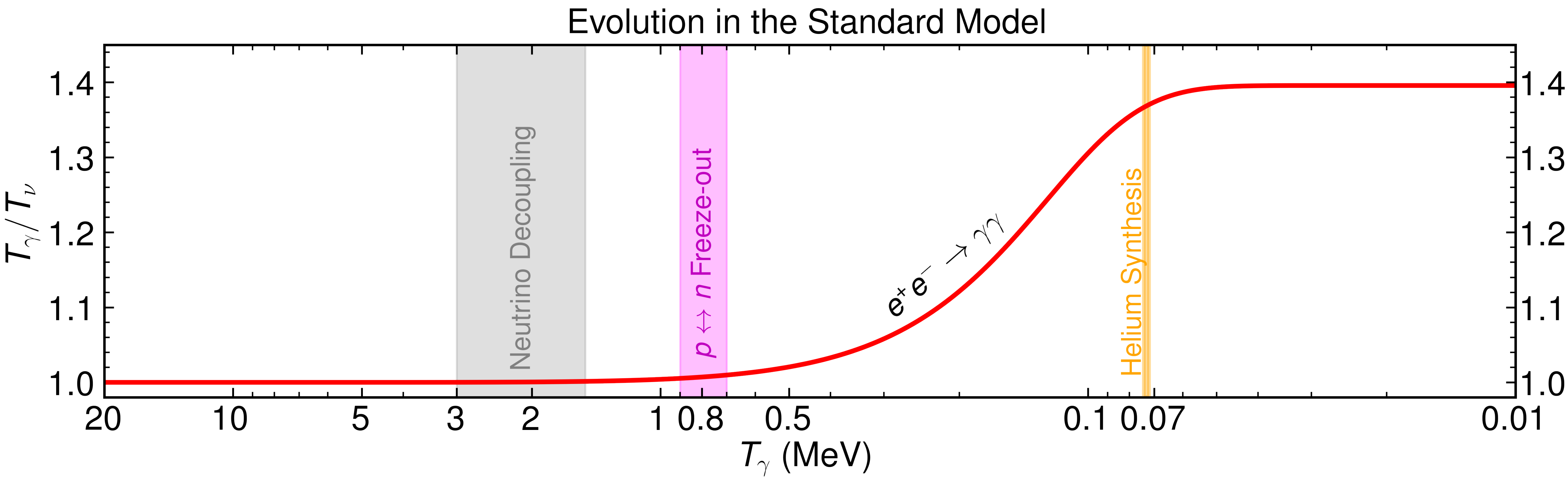


$$e^+ e^- \leftrightarrow \bar{\nu}_\alpha \nu_\alpha$$
$$e^{\pm} \nu_\alpha \leftrightarrow e^{\pm} \nu_\alpha$$
$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$



Neutrinos **Electrons** **Photons**

Neutrino Decoupling



- **How do we measure the energy density in relativistic neutrino species?**
- **The key parameter is:** $$N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad}-\rho_\gamma}{\rho_\gamma}\right)$$
- **when only neutrinos and photons are present:** $$N_{\rm eff} = 3\left(\frac{1.4\,T_\nu}{T_\gamma}\right)^4$$
- **The Standard Model value is:** $$N_{\rm eff}^{\rm SM} = 3.043(1)$$

Bennett, Buldgen, Drewes & Wong 1911.04504
Escudero Abenza 2001.04466
Akita & Yamaguchi 2005.07047
Froustey, Pitrou & Volpe 2008.01074
Gariazzo, de Salas, Pastor et al. 2012.02726
Hansen, Shalgar & Tamborra 2012.03948
Cielo, Escudero, Mangano & Pisanti 2306.05460

Why $N_{\rm eff}^{\rm SM}$ is not exactly 3?

1) Neutrino Decoupling is not instantaneous

$$\sigma \sim G_F^2 E_\nu^2$$

2) Weak Interactions freeze out at T = 2-3 MeV hence, some heating from e^+e^- annihilation

$$n \langle \sigma v \rangle \simeq G_F^2 T^5 \simeq H$$

$$\Delta N_{\rm eff} \simeq +0.03$$

Kolb et al. '82
Dolgov et al. '97

3) Finite Temperature QED corrections

$$\delta m_e^2(T),\ \delta m_\gamma^2(T)$$

$$\Delta N_{\rm eff} \simeq +0.01$$

Heckler '94
Bennet et al. '21

4) Neutrino oscillations are active at T < 10 MeV

$$\Delta N_{\rm eff} \simeq +0.0007$$

Mangano et al. '05
de Salas & Pastor '16

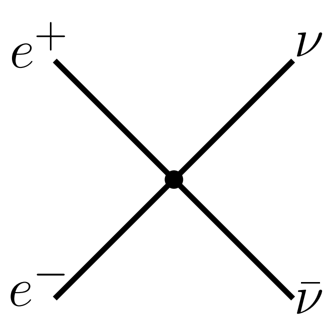
$$t_\nu^{\rm os} \sim \frac{12\,T}{\Delta m^2} \qquad t_{\rm exp} = \frac{1}{2H} \sim \frac{m_{Pl}}{3.44\sqrt{10.75}T^2} \qquad t_\nu^{\rm scat} \sim \frac{1}{G_F^2\, T^5}$$

5) QED corrections to the interaction rates

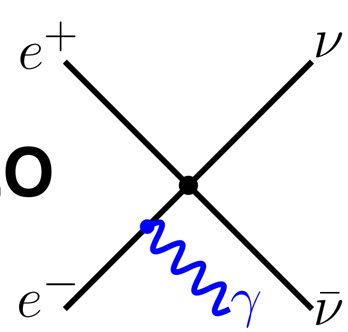
$$|\,\Delta N_{\rm eff}\,| \lesssim 0.0007$$

Cielo et al. '23
Jackson & Laine '23

LO



NLO



CMB-S4

$$\delta N_{\rm eff} \simeq 0.03$$

Neutrino Decoupling in the SM

Why it is worth investigating the process of neutrino decoupling?

1) The ultimate generation of CMB experiments are expected to measure Neff with a precision of 0.03!

That means that small effects cannot be neglected!

2) This will allow us to understand what can happen in scenarios beyond the Standard Model!

Neff in the Standard Model

Methods to solve for neutrino decoupling:

The simplest method:

— Assume neutrinos decouple instantaneously and use entropy conservation to get the neutrino temperature today. Exercise!

Pros: Very easy to do 😃

Con: Does not include dynamics and is not too accurate 😐

The full method:

— Solve the actual Boltzmann equation describing $\nu - e$ and $\nu - \nu$ interactions

Pros: It gives the full result 😃

Con: It is considerably involved as it requires solving a system of *hundreds of stiff integrodifferential equations* ☹️

The intermediate method:

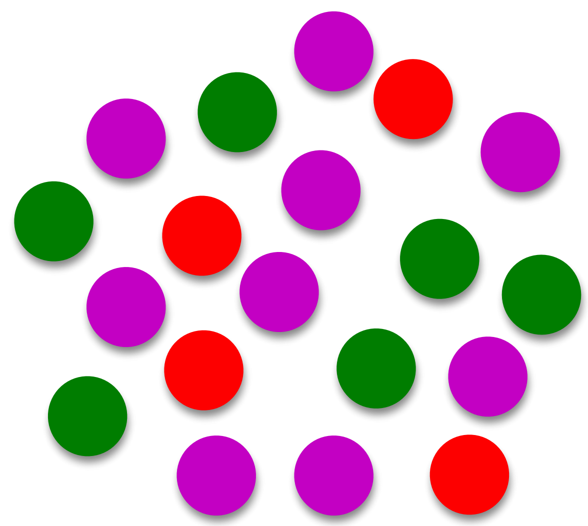
— Track the neutrino energy density of all the species assuming they follow thermal equilibrium distributions

Pros: It is fast, precise and allows one to easily include BSM species in the game! 😃

Neutrino Decoupling Simplified

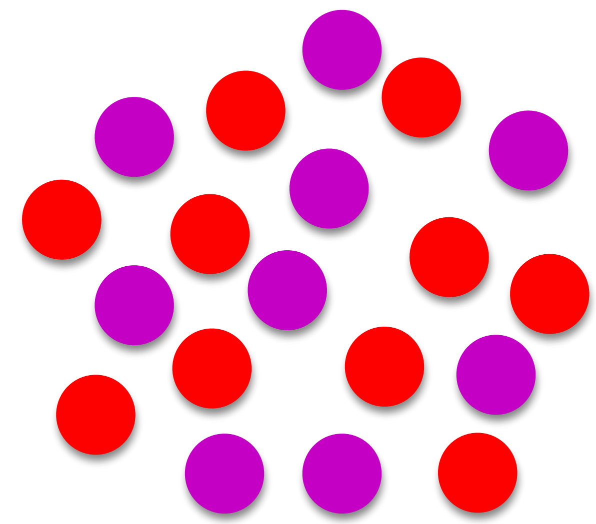
Neutrinos **Electrons** **Photons**

Initial $e^+, e^-, \gamma, \nu, \bar{\nu}$



$t \sim 0.1\,\mathrm{s}$
$T = 2\,\mathrm{MeV}$

Final $\gamma, \nu, \bar{\nu}$



$t \sim 10\,\mathrm{mins}$
$T = 0.01\,\mathrm{MeV}$

$n_{e^-}/n_\gamma \simeq 10^{-9}$
$n_{e^+} = 0$

$$\frac{d\rho}{dt} = -H(\rho + p)$$ **implies entropy conservation:** $S = sa^3 = \text{constant}$

Initial	**Final**

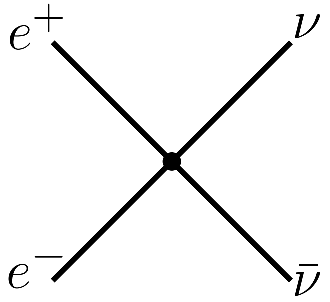
$$a_i^3 s_\nu^i = a_f^3 s_\nu^f$$
$$a_i^3 s_{\gamma+e}^i = a_f^3 s_{\gamma+e}^f$$

$$s \propto g_{\star S} T^3$$
$$T_\nu^i = T_\gamma^i$$

$$\frac{T_\nu^f}{T_\gamma^f} = \left[\frac{g_\gamma}{g_\gamma + g_e}\right]^{1/3} = \left[\frac{2}{2 + 4\frac{7}{8}}\right]^{1/3} = \left[\frac{4}{11}\right]^{1/3} \simeq \frac{1}{1.4}$$

Neutrino Decoupling

- **Interactions between neutrinos and electrons were very efficient for T > 2 MeV. That means that we expect neutrinos to follow a distribution function that roughly resembles an equilibrium one**



- **The full description will be obtained by solving the full Boltzmann equation:**

$$\frac{\partial f}{\partial t} - Hp\frac{\partial f}{\partial p} = C[f]$$

Here, H is the expansion rate of the Universe and $C[f]$ is the collision term that accounts for the interactions of neutrinos with any other species, e.g.: $e^+e^- \leftrightarrow \bar{\nu}\nu$

- **The main issue is that:**

$$C[f] \sim \int_{\mathrm{9D-PhaseSpace}} d\Pi[f_{\nu_\alpha} f_{\nu_\beta} - ..]$$

- **The integral can be simplified to just 2D, but then this equation represents a system of stiff integrodifferential equation that can be rather difficult to solve**

see Mangano et al. astro-ph/0111408 for early calculations and Bennet et al. 2012.02726 for the most recent one

see also the FortEPiaNO code: by Gariazzo, de Salas & Pastor

Neutrino Decoupling

- **A trick to solve it much more easily is to integrate it and make several approximations, see Escudero 1812.05605 and 2001.04466. This is what is typically done in the context of thermal Dark Matter or in Baryogenesis.**

$$\frac{\partial f}{\partial t} - pH\frac{\partial f}{\partial p} = C[f]$$ **integrating this equation by** $\frac{1}{(2\pi)^3} E d^3 p$ **yields:**

$$\frac{d\rho}{dt} + 3(\rho + p)H = \int \frac{Ep^2}{2\pi^2} C[f] dp \equiv \frac{\delta\rho}{\delta t}$$

To actually solve for this we need an ansatz for the distribution function of neutrinos. Lets assume they follow a Fermi-Dirac distribution with a temperature T_ν.

Once this is done one simply needs to solve two ordinary differential equations for T_ν and T_γ:

$$\frac{dT}{dt} = \frac{d\rho}{dt} \Big/ \frac{\partial\rho}{\partial T} = \left[-3H(\rho + p) + \frac{\delta\rho}{\delta t}\right] \Big/ \frac{\partial\rho}{\partial T}$$

The energy transfer rates are analytical expressions if one neglects the electron mass and assumes Maxwell-Boltzmann statistics for the distributions:

as a result of a 12D integral!:

$$\left.\frac{\delta\rho_\nu}{\delta t}\right|_{\rm SM} = 4\frac{G_F^2}{\pi^5}\left(g_L^2 + g_R^2\right)\left[32\left(T_\gamma^9 - T_\nu^9\right) + 56\, T_\gamma^4 T_\nu^4 \left(T_\gamma - T_\nu\right)\right]$$

Neutrino Decoupling in the SM

Solutions after electron-positron annihilation:

Neutrino Decoupling in the SM	$T_{\nu_e} = T_{\nu_{\mu,\tau}}$		$T_{\nu_e} \neq T_{\nu_{\mu,\tau}}$		
Scenario	T_γ/T_ν	$N_{\rm eff}$	T_γ/T_{ν_e}	T_γ/T_{ν_μ}	$N_{\rm eff}$
Instantaneous decoupling	1.4010	3.000	1.4010	1.4010	3.000
Instantaneous decoupling + QED.	1.3998	3.010	1.3998	1.3998	3.010
FD+m_e collision term	1.3969	3.036	1.3957	1.3976	3.035
FD+m_e collision term + NLO-QED	1.39578	**3.045**	1.3946	1.3965	**3.044**

From these results we can draw some conclusions:

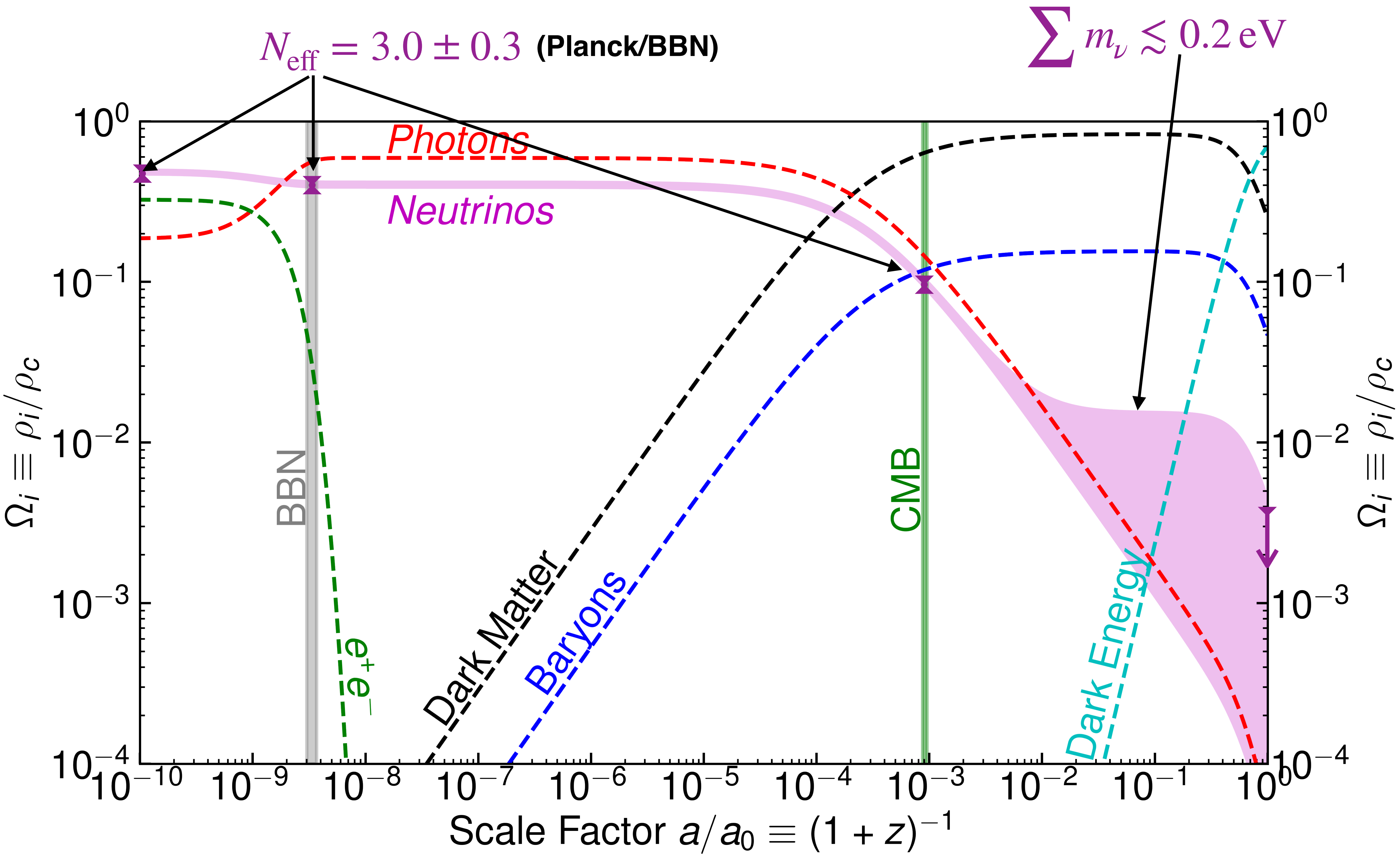
1) The main contributions to $\Delta N_{\rm eff}^{\rm SM}$ come from residual electron-positron annihilations into neutrinos $\Delta N_{\rm eff}^{\rm SM} \simeq 0.036$

2) Finite temperature corrections contribute to $\Delta N_{\rm eff}^{\rm SM} \simeq 0.009$

3) Neutrino oscillations contribute to $\Delta N_{\rm eff}^{\rm SM} \simeq 0.001$

Global Perspective

Current knowledge:



Mid Lecture Pause

Key things to remember:

- **In the Standard Model, neutrinos are a relevant component of the Universe across its entire history**

- **When neutrinos are relativistic, their energy density is measured by $N_{\rm eff}$ which in the Standard Model is 3.043(1)**

- **Neutrinos decouple at a temperature of $T \simeq 2\,{\rm MeV}$. From then onwards, they do not interact with anything.**

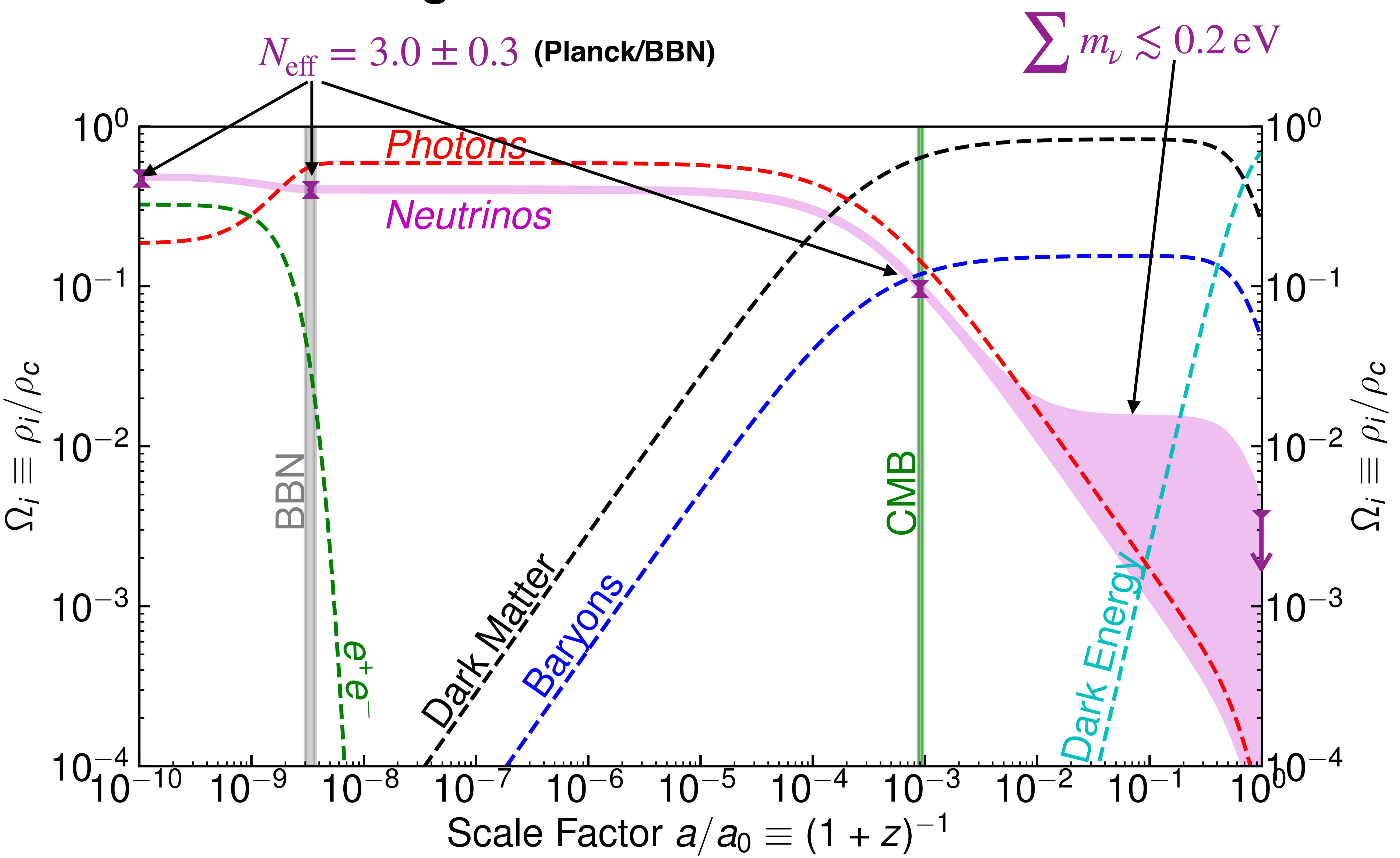
- **After e^+e^- have annihilated, neutrinos have a temperature of $T_\nu \simeq T_\gamma/1.4$**

- **There should be $n_\nu \simeq T_\nu^3 \simeq 300\,{\rm cm}^{-3}$ in every point in the Universe**

Time for questions!

Global Perspective

Current knowledge:



Evidence for Cosmic Neutrinos

Big Bang Nucleosynthesis

Current measurements are broadly consistent with the SM picture

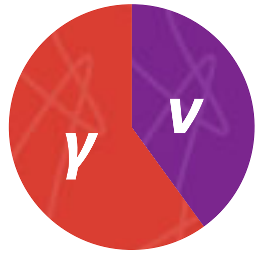
H ~ 75% **^{4}He ~ 25%** **D ~ 0.005%**

This implies that neutrinos should have been present:

1) It is impossible to have successful BBN without neutrinos.
They participate in $p \leftrightarrow n$ conversions up to $T \gtrsim 0.7\,\mathrm{MeV}$

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$
$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$
$$n + \nu_e \leftrightarrow p + e^-$$

2) Neutrinos contribute to the expansion rate $H \propto \sqrt{\rho}$



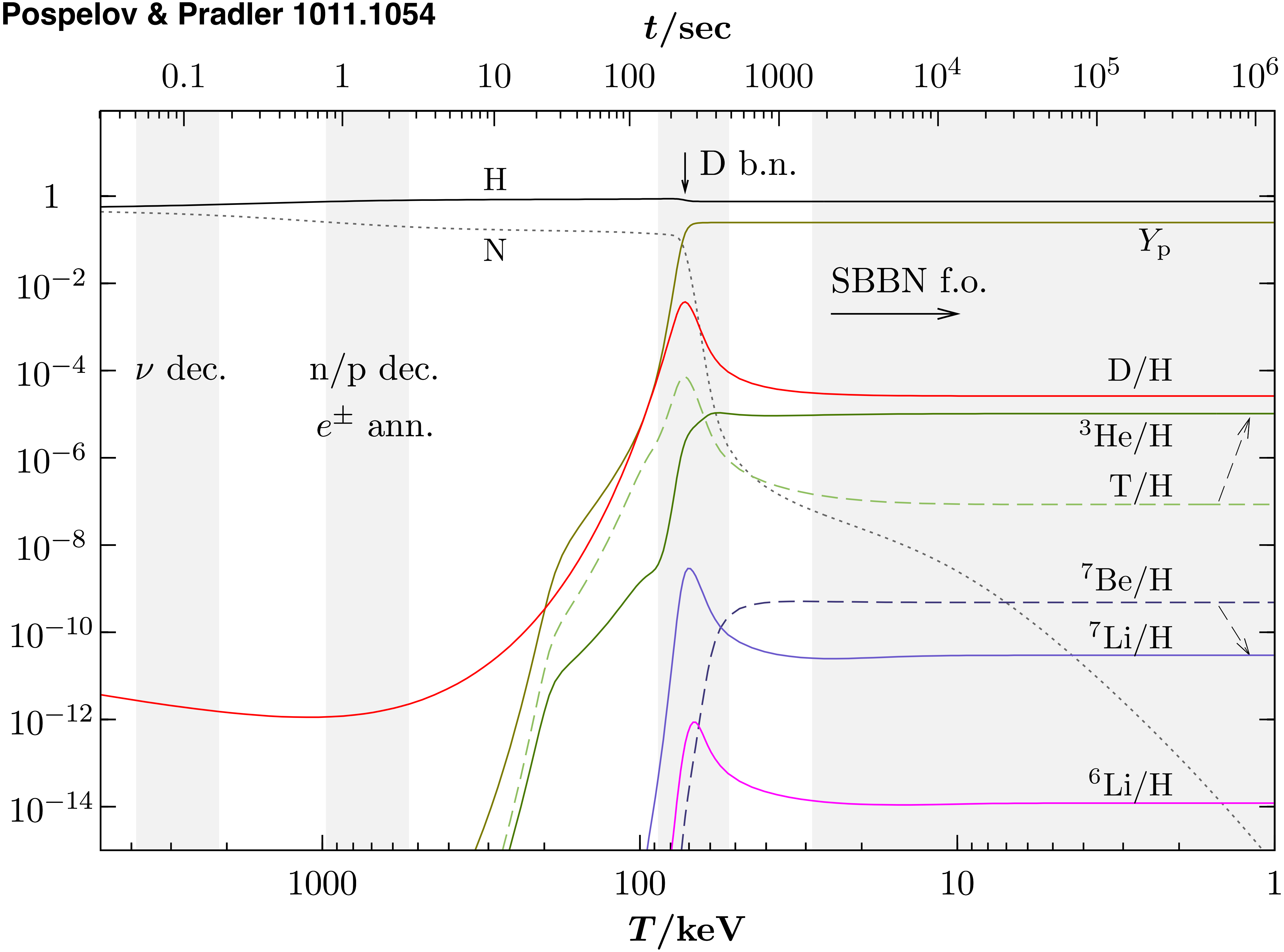
By comparing predictions against observations, we know:

$$N_{\mathrm{eff}}^{\mathrm{BBN}} = 2.86 \pm 0.28$$

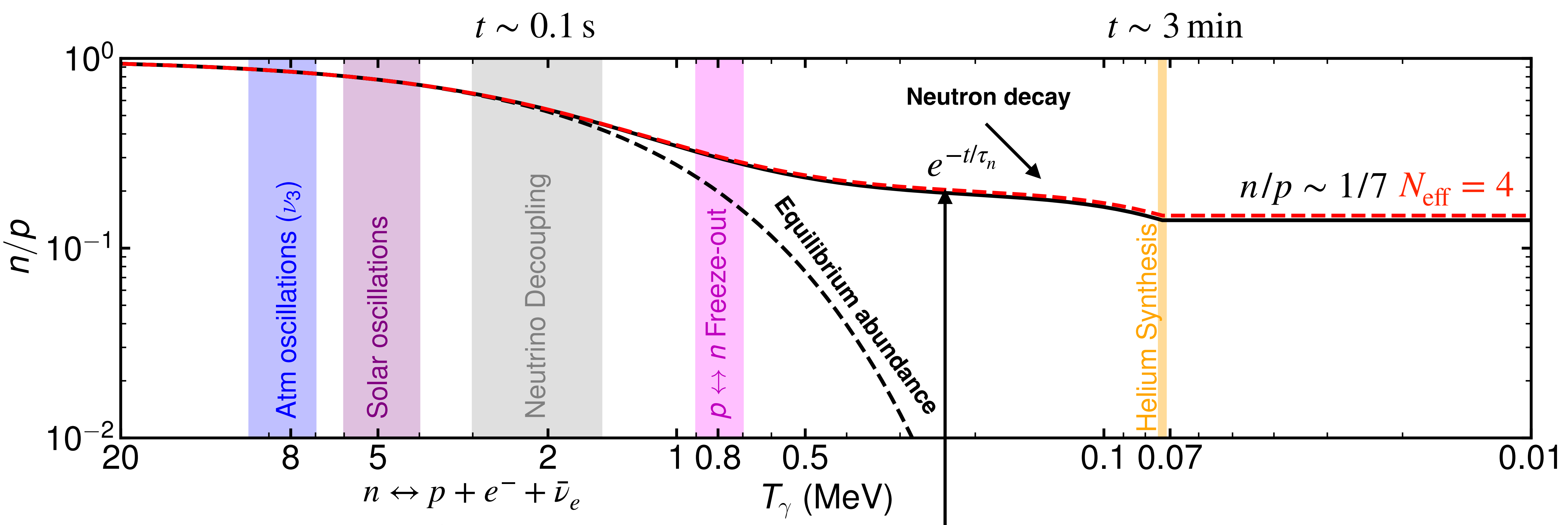
see e.g. Pisanti et al. 2011.11537

Big Bang Nucleosynthesis and Neff

Pospelov & Pradler 1011.1054



Big Bang Nucleosynthesis and Neff



$n \leftrightarrow p + e^- + \bar{\nu}_e$

$n + e^+ \leftrightarrow p + \bar{\nu}_e$

$n + \nu_e \leftrightarrow p + e^-$

Exponential sensitivity to Neff!

Theory: **Fields, Olive, Yeh & Young [1912.01132]**

Observations: **PDG 2024** **1% errors**

$$Y_p = 0.24696 \times \left[\frac{\eta_{10}}{6.129}\right]^{0.039} \times \left[\frac{N_{\rm eff}}{3.044}\right]^{0.163}$$

$$Y_P = 0.245 \pm 0.003$$ **sys dom**

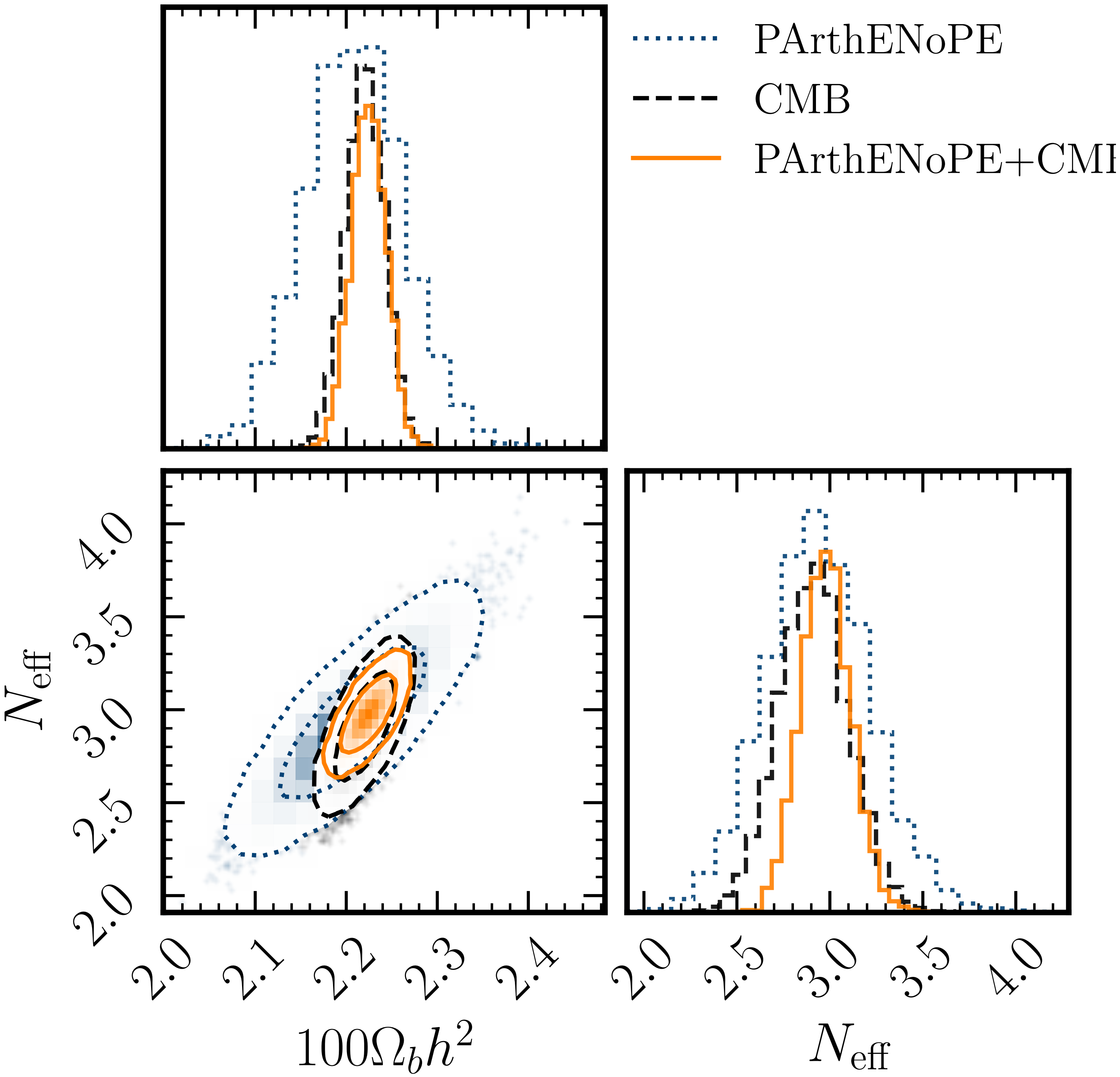
$$\mathrm{D/H} = (2.60 \pm 0.13) \times 10^{-5} \times \left[\frac{\eta_{10}}{6.129}\right]^{-1.597} \times \left[\frac{N_{\rm eff}}{3.044}\right]^{0.396}$$

$$\mathrm{D/H} = (2.55 \pm 0.03) \times 10^{-5}$$

stat dom

Big Bang Nucleosynthesis and Neff

2408.14531 Giovanetti et al.



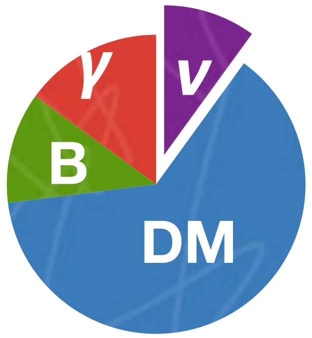
$$N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28$$

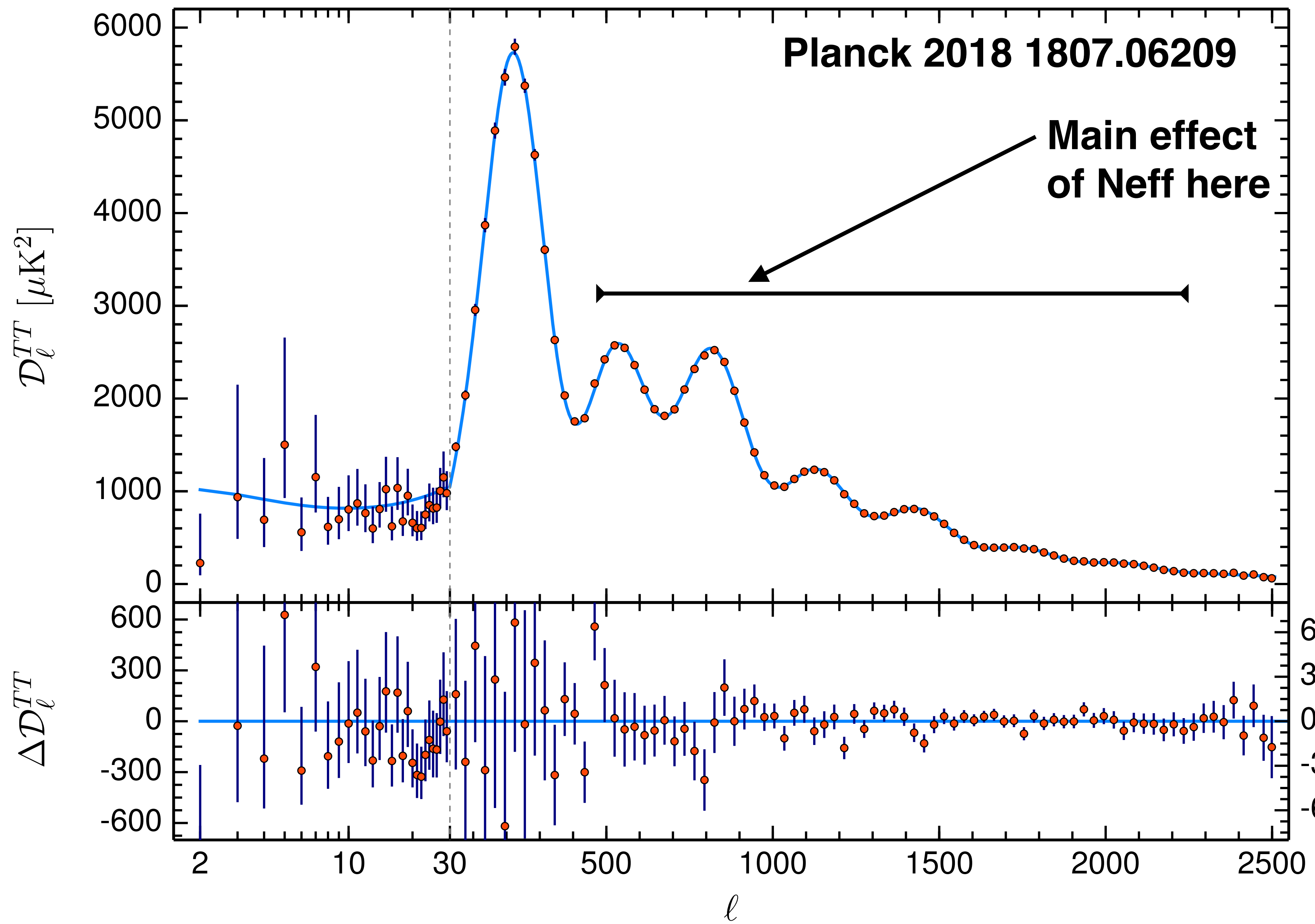
Pisanti et al. 2011.11537
Yeh et al. 2207.13133
Giovanetti et al. 2408.14531

Evidence for Cosmic Neutrinos

Cosmic Microwave Background

Why? **Ultra-relativistic neutrinos represent a large fraction of the energy density of the Universe,** $H \propto \sqrt{\rho}$





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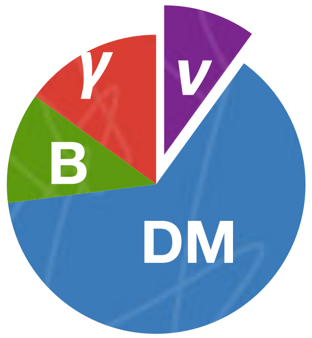
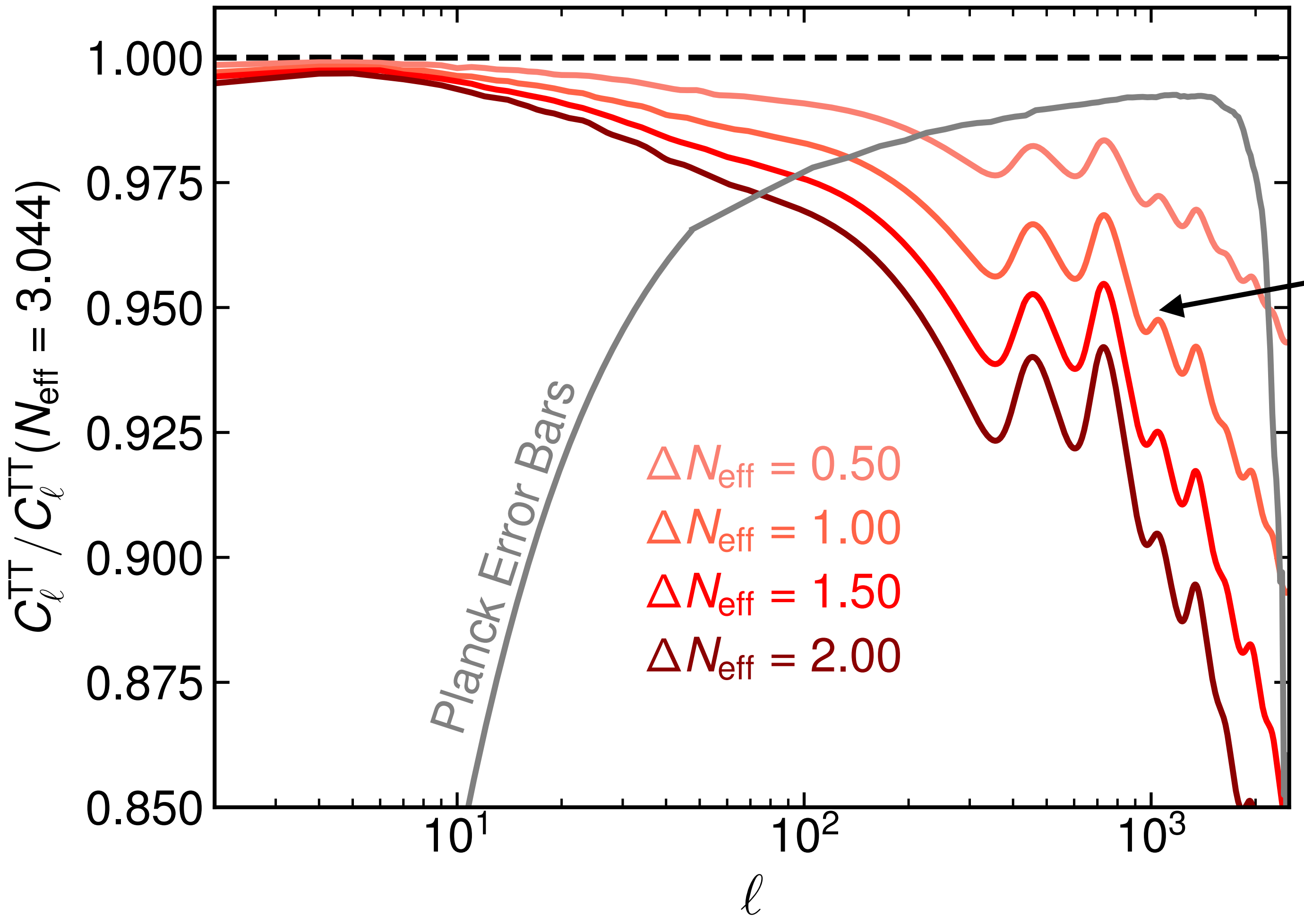


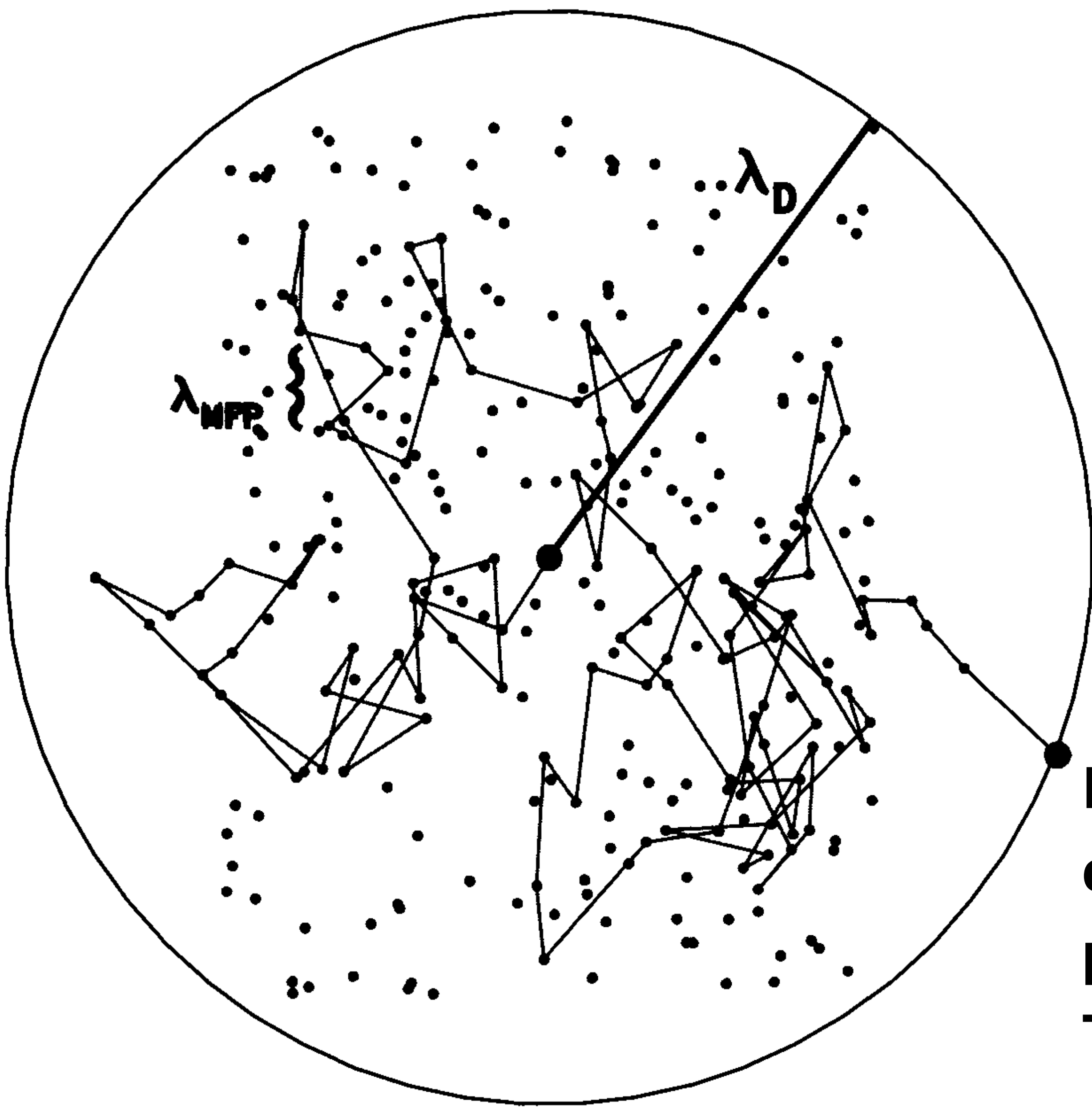


N_{eff} **is constrained by the high-**ℓ **multipoles, i.e. Silk damping**

$$N_{eff}^{CMB+BAO} = 2.99 \pm 0.17$$

Planck 2018 1807.06209

The physics of diffusion damping



Perturbations on scales are $\lambda < \lambda_D$ are erased:

$$\lambda < \lambda_D = \lambda_{\text{Mean-Free-Path}}\sqrt{N_{\text{steps}}} = (n_e\sigma_T)^{-1}\sqrt{n_e\sigma_T H^{-1}}$$

$$\lambda < \lambda_D = \frac{1}{\sqrt{n_e\sigma_T H}}$$

Effectively, the energy density of neutrinos controls the physical length scale over which photons diffuse.
The larger Neff the smaller this distance is.

Evidence for Cosmic Neutrinos

- **Current constraints**

BBN	$N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28$	**Pisanti et al. 2011.11537**
Planck+BAO	$N_{\rm eff}^{\rm CMB} = 2.99 \pm 0.17$	**Planck 2018, 1807.06209**

- **Standard Model prediction:** $N_{\rm eff}^{\rm SM} = 3.043(1)$
- **Data is in excellent agreement with the Standard Model prediction**
- **This provides strong (albeit indirect) evidence for the Cosmic Neutrino Background.**

Today! **Tomorrow!**

Implications:

1) Stringent constraint on many BSM settings

2) We can use cosmological data to test neutrino properties

Constraints from N_{eff}

N_{eff} measurements constrain very light particles that decoupled while relativistic after the Big Bang (very much like neutrinos). Their energy density is parametrized by

$$\Delta N_{\rm eff} = N_{\rm eff} - 3.043$$

We have thousands of BSM models where $\Delta N_{\rm eff} > 0$

Some examples:

- **Sterile Neutrino** $m_N \sim \text{eV}$ $\Delta N_{\rm eff} = 1$ **(e.g. Gariazzo, de Salas & Pastor 1905.11290)**
- **Goldstone Bosons**

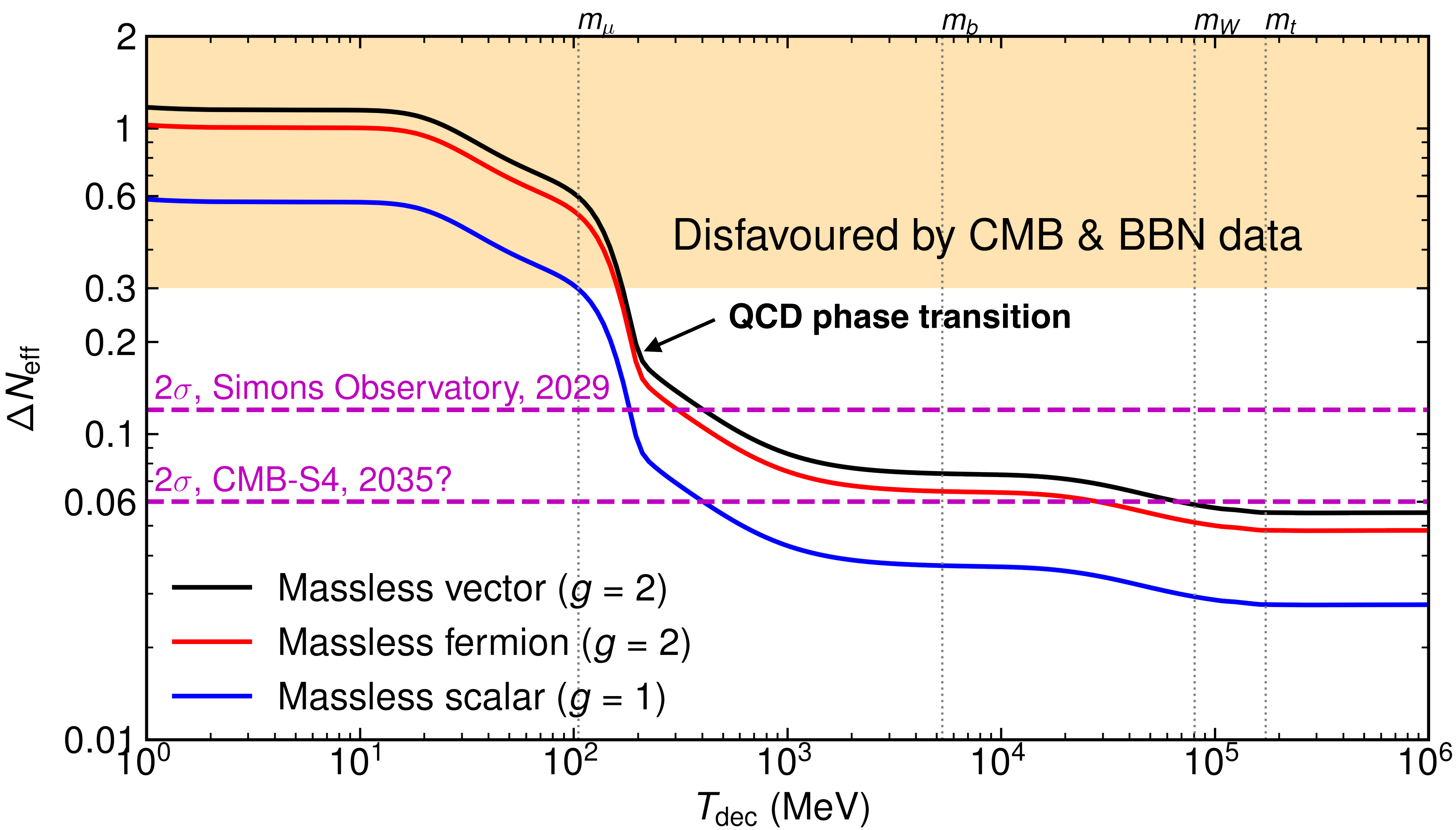
  Editors' Suggestion

  Goldstone Bosons as Fractional Cosmic Neutrinos

  Steven Weinberg
  Phys. Rev. Lett. **110**, 241301 – Published 10 June 2013
- **Other sterile long-lived particles** **Gravitino, hidden sector particles ...**

Constraints from N_{eff}

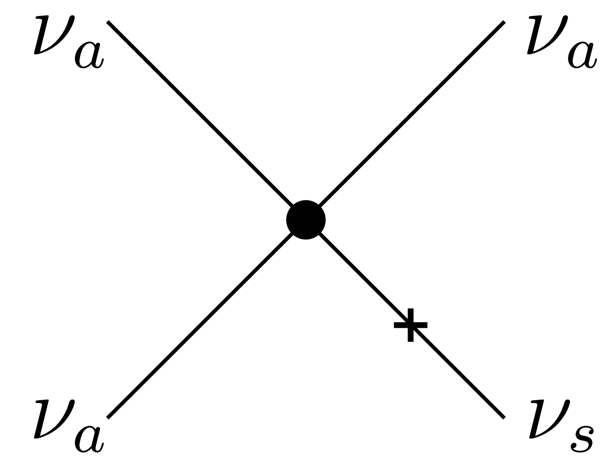
Contribution to $\Delta N_{\rm eff}$ from a massless particles that decoupled at $T_{\rm dec}$



Take Away: **Any new massless state in thermal equilibrium with the SM plasma should have decoupled at $T_{\rm dec} \gtrsim 100\,{\rm MeV}$**

Sterile Neutrinos and Neff

Production of sterile neutrinos in the early Universe proceeds via collisions/neutrino oscillations



Typical production rate: $\Gamma \simeq \Gamma_\nu \frac{1}{1+(100\Gamma_\nu/\Gamma_{\rm osc})^2} \times \sin^2(2\theta_s)$

Abazajian astro-ph/0511630

$$\Gamma_{\rm osc} = \Delta m^2/T$$
$$\Gamma_\nu \simeq G_F^2 T^5$$

Production can be amplified in the presence of large lepton asymmetries

Shi & Fuller [astro-ph/9810076] Abazajian [astro-ph/0511630]

Production can be suppressed in the presence of self-interactions

Dasgupta & Kopp [1310.6337], Chu, Dasgupta & Kopp [1505.02795], Hannestad. Hansen & Tram [1310.5926]

Production can be suppressed in the presence of a low-reheating temperature see e.g. Hasegawa et al. [2003.13302]

Sterile Neutrino Constraints

Standard case for sterile neutrinos

From Gariazzo, de Salas & Pastor 1905.11290

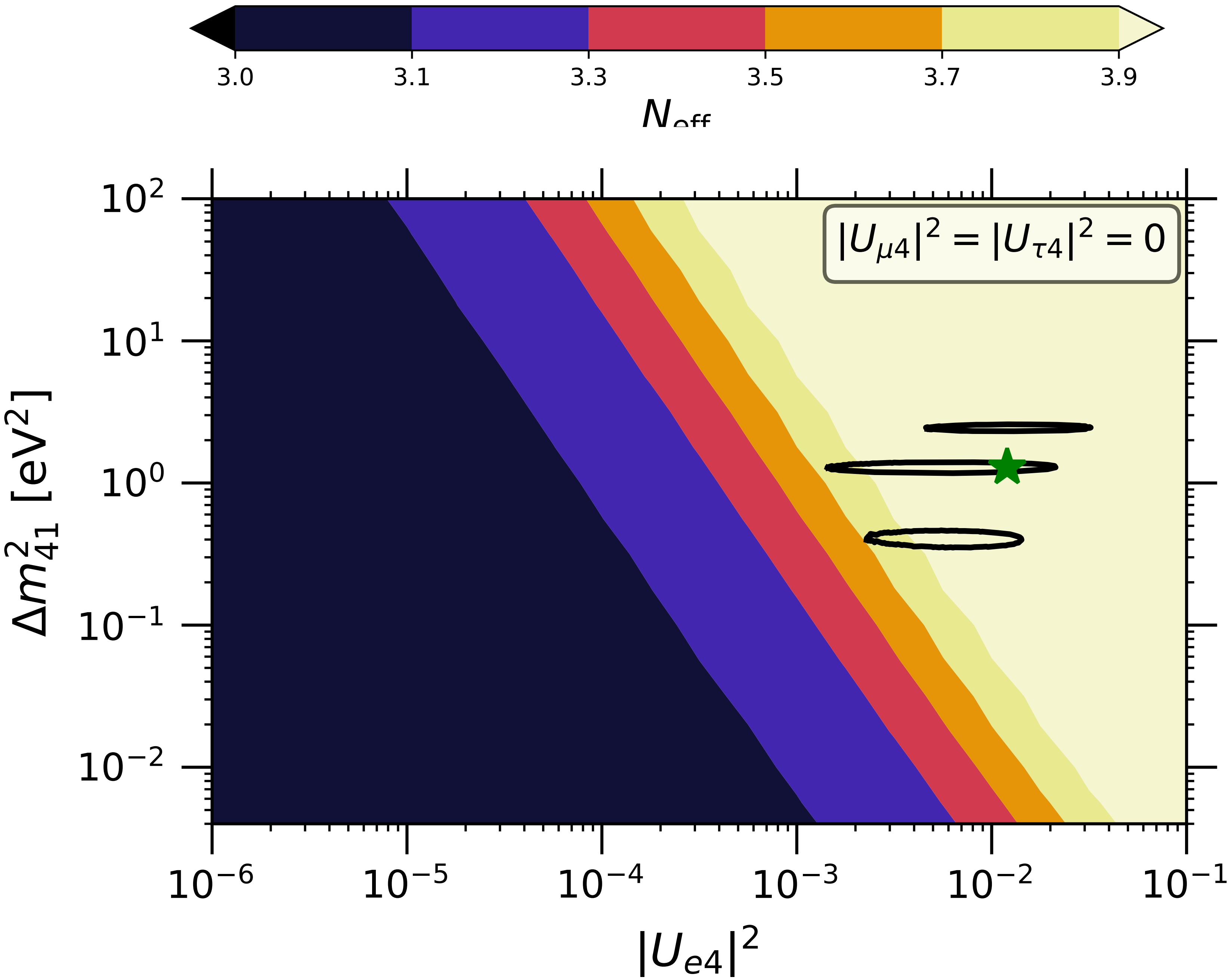


Figure 9. Final $N_{\rm eff}$ in the 3+1 case for different values of Δm^2_{41} and $|U_{e4}|^2$ when considering normal ordering for the active neutrinos. The other two active-sterile components of the mixing matrix take the values as labelled. The black closed contours represent the 3σ preferred regions and the green star the best-fit point from [44].

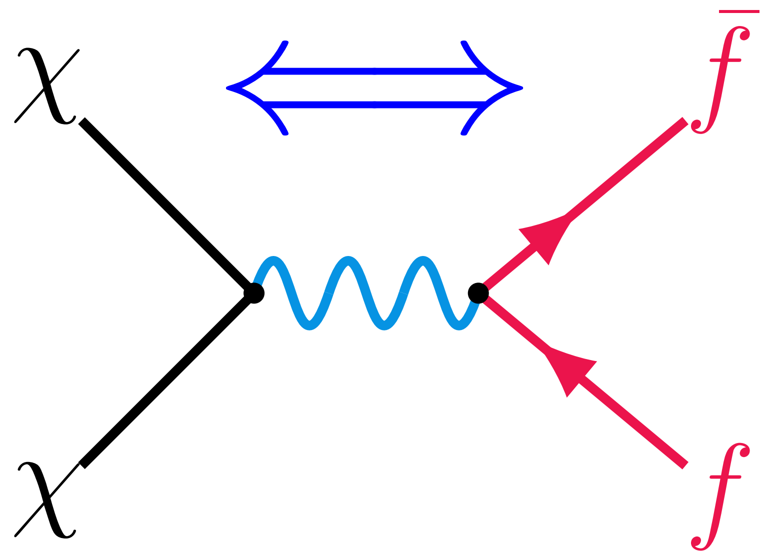
Constraints from N_{eff}

Constraints are relevant in many other BSM settings:

WIMPs	$m_{\rm WIMP} > (4-10)\,{\rm MeV}$	Sabti et al. 1910.01649 Boehm et al. 1303.6270
GeV-Sterile Neutrinos	$\tau_N \lesssim 0.05\,{\rm s}$	Sabti et al. 2006.07387 Dolgov et al. hep-ph/0008138
Vector Bosons	$g \lesssim 10^{-10} \quad m \lesssim 10\,{\rm MeV}$	Escudero et al. 1901.02010 Kamada & Yu 1504.00711
Axions		Raffelt et al. 1011.3694 Blum et al. 1401.6460
Low Reheating	$T_{\rm RH} > (2-5)\,{\rm MeV}$	de Salas et al. 1511.00672 Hasegawa et al. 1908.10189
Variations of G_N	$G_{\rm BBN}/G_0 = 0.98 \pm 0.03$	Alvey et al. 1910.10730 Copi et al. astro-ph/0311334
PBHs	$6\times 10^8\,{\rm g} < M_{\rm PBH} < 2\times 10^{13}\,{\rm g}$	Carr et al. 0912.5297 Keith et al. 2006.03608
Stochastic GW backgrounds	$\Omega_{\rm GW} h^2 < 3\times 10^{-6}$	Caprini & Figueroa 1801.04268

Constraints from $N_{\rm eff}$: WIMPs

WIMPs are in thermal equilibrium until $T \sim m_\chi/20$

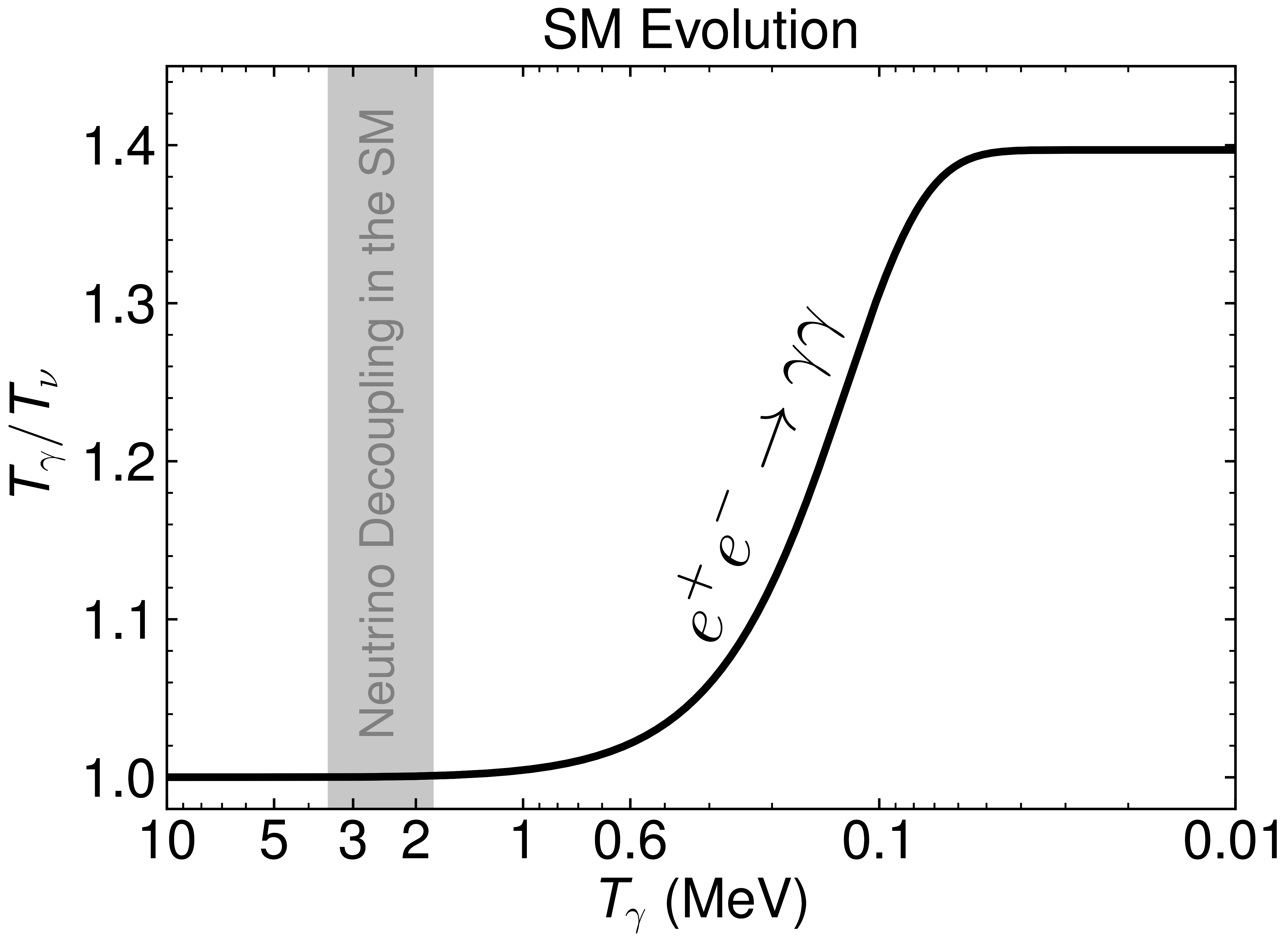


That means that WIMPs with $m_\chi \lesssim 20\,\mathrm{MeV}$ **can affect neutrino decoupling, and therefore** $N_{\rm eff}$

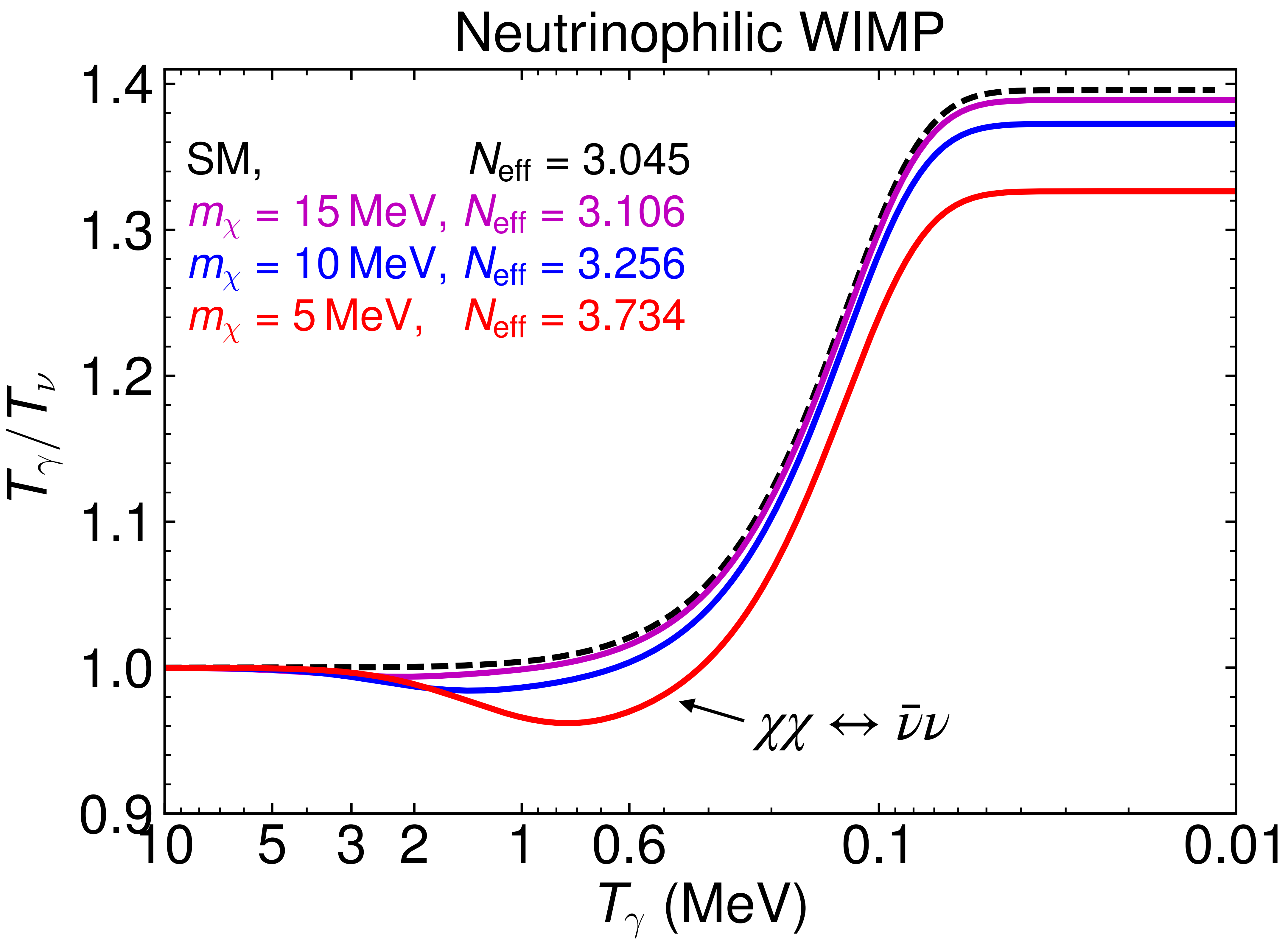
— **They can release entropy into the SM sectors**

— **Could delay the process of neutrino decoupling**

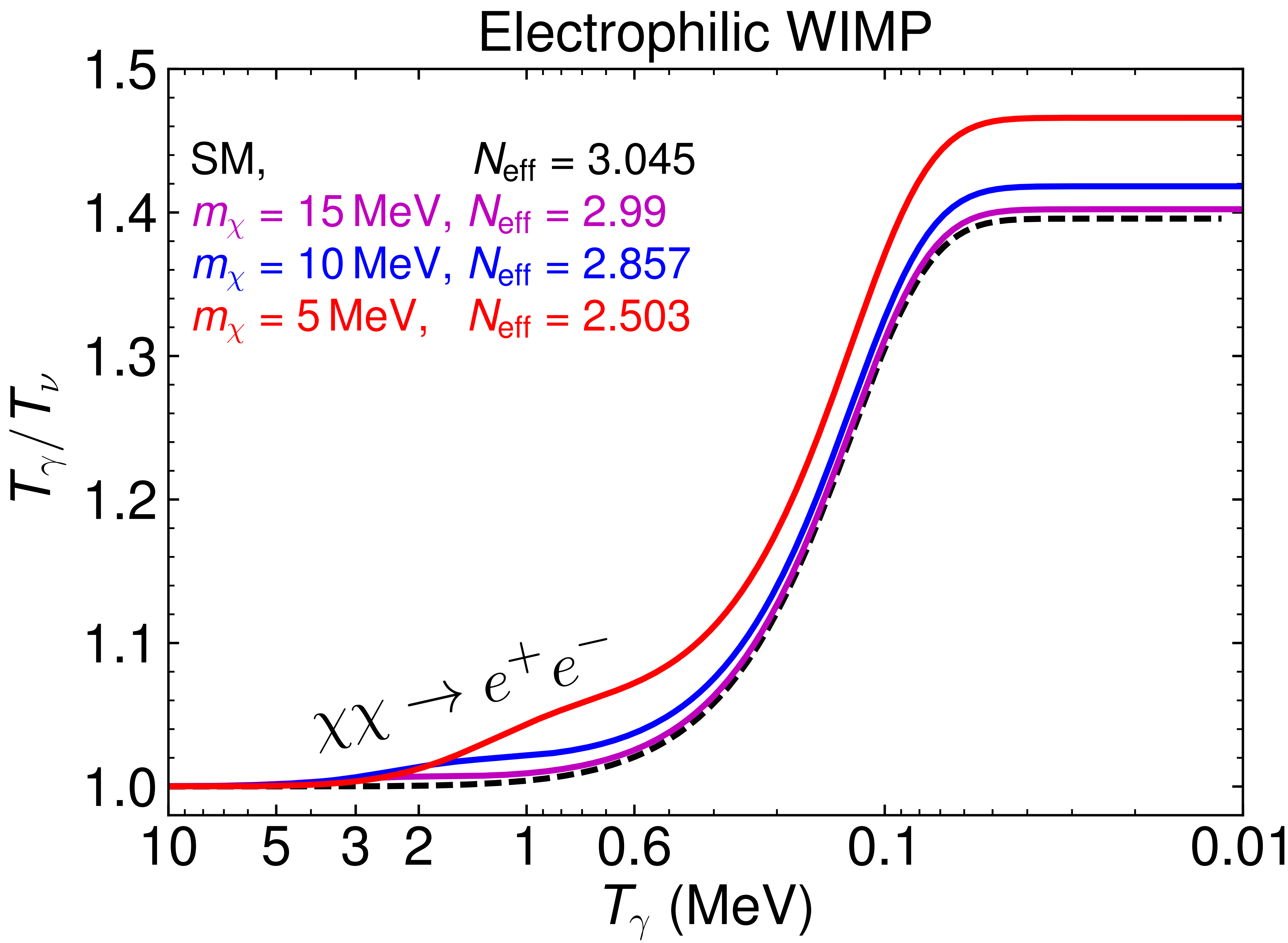
Impact of Thermal Dark Matter



Neutrinophilic WIMP: $N_{eff} > 3.044$



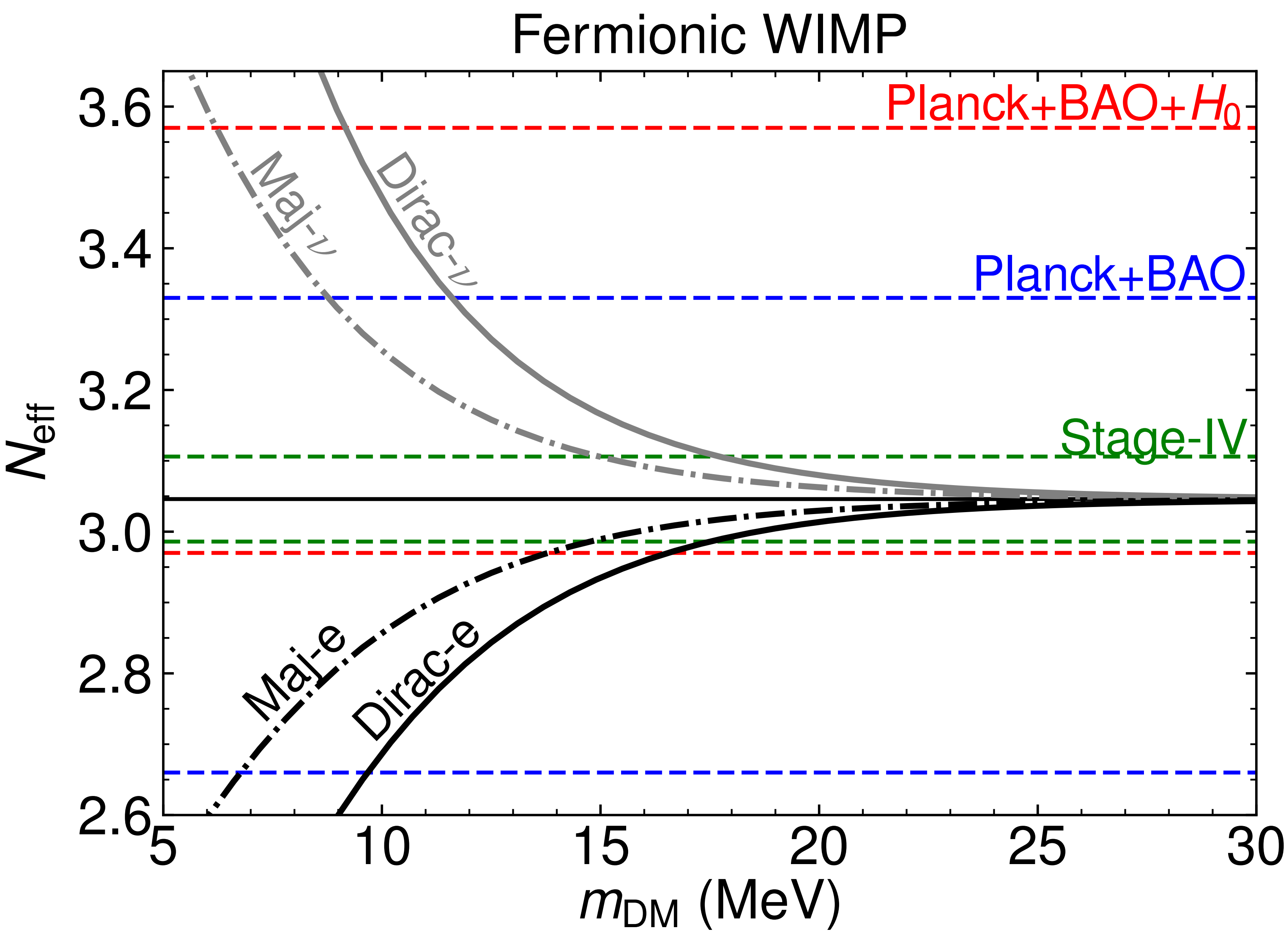
Electrophilic WIMP: $N_{\mathrm{eff}} < 3.044$



This is one of the very few scenarios where $N_{\mathrm{eff}} < 3.044$!

Lower bound on the DM mass

Comparing prediction vs. observations:



$$m_{\rm DM} > 4\,{\rm MeV}$$

at 95% CL

Sabti et al. 1910.01649
Boehm et al. 1303.6270

In addition, we could test WIMPs of $m_{\rm DM} \lesssim 15\,{\rm MeV}$ with CMB Stage-IV experiments

Summary

Number of effective neutrino species

$$N_{\rm eff}^{\rm BBN} = 2.86 \pm 0.28 \qquad N_{\rm eff}^{\rm CMB} = 2.99 \pm 0.17$$

$$N_{\rm eff}^{\rm SM} = 3.044(1)$$

Agreement between measurements of $N_{\rm eff}$ and the SM prediction implies:

Strong evidence that the CNB should be there as expected in the SM

This represents an important constraint on many BSM settings

e.g.: $\theta_s^2 \lesssim 10^{-3}\,{\rm eV}/\sqrt{(m_s^2 - m_\nu^2)}$

e.g.: $m_{\rm WIMP} > 4\,{\rm MeV}$

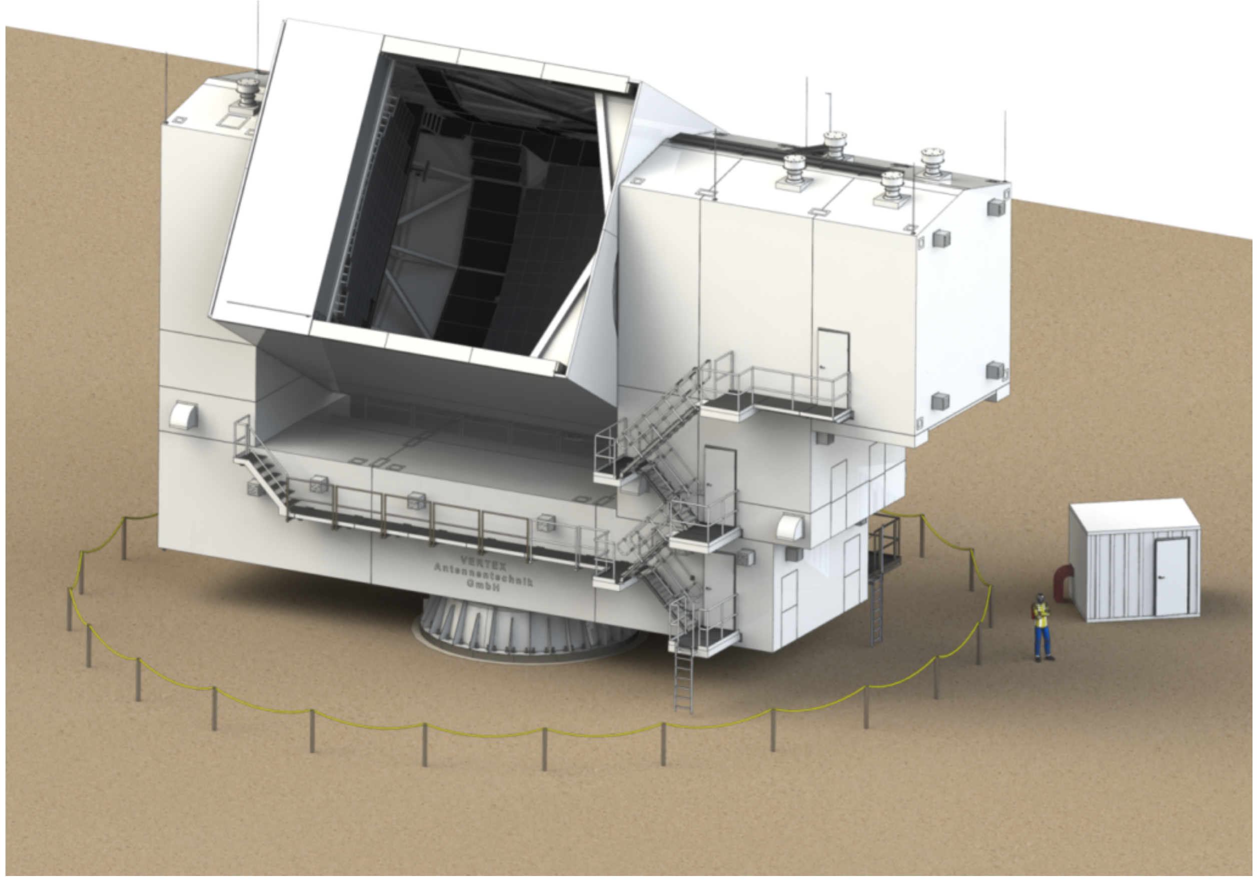
Outlook: Number of Neutrinos

The next generation of CMB experiments are expected to significantly improve the sensitivity on Neff.

Simons Observatory

$$\sigma(N_{\rm eff}) = 0.06$$ **~2029**

CMB-S4



$$\sigma(N_{\rm eff}) = 0.03$$ **~2035?**

These measurements will represent an important test to BSM physics and perhaps may yield a BSM signal!

Take Home Messages

1) In the Standard Model, neutrinos are always a relevant component of the Universe across its entire history

2) When neutrinos are relativistic, their energy density is measured by $N_{\rm eff}$ which in the Standard Model is 3.044(1)

3) The agreement between measurements of $N_{\rm eff}$ and its prediction represents an important constraint for many BSM settings, including sterile neutrinos and WIMPs

4) Cosmological bounds are cosmological model dependent, but given a cosmological model in some scenarios very strong constraints can be drawn

Key facts/numbers to remember

- **Neutrinos decouple at a temperature of $T \simeq 2\,\mathrm{MeV}$. From then onwards, they do not interact with anything.**

- **After e^+e^- have annihilated, neutrinos have a temperature of** $T_\nu \simeq T_\gamma/1.4$

- **There should be $n_\nu \simeq T_\nu^3 \simeq 300\,\mathrm{cm}^{-3}$ in every point in the Universe**

- **Neutrinos become non-relativistic when $T_\nu \lesssim m_\nu/3$.**

  This corresponds to $z_{\rm nr} \simeq 200\, m_\nu/(0.1\,\mathrm{eV})$

- **We have measured the mass squared differences between neutrinos which means that at least two of them should be non-relativistic today! Exercise: explicitly check when!**

Recommended References

Introductory:

Modern Cosmology
Scott Dodelson & Fabian Schmidt, Academic Press, 2020

General:

The Early Universe
Edward Kolb & Michael Turner, Front. Phys. 69, 1990

Introduction to the Theory of the Early Universe
Valery Rubakov & Dmitry Gorbunov, World Scientific, 2017

Advanced/Neutrinophilic:

Neutrino Cosmology
Lesgourgues, Mangano, Miele & Pastor, Cambridge University Press, 2013

Neutrinos in Cosmology
Alexander Dolgov, Physics Reports 370 (2002) 333–535

Kinetic Theory in the Expanding Universe
Jeremy Berstein, Cambridge University Press, 1988

Tomorrow's plan

Lecture II

Neutrino Masses in Cosmology

Lecture III

The Hubble tension and neutrinos
Can we directly detect the Cosmic Neutrino Background?

Exercises

I have written down some exercises for the lectures

They are on indico

I will stay until Friday afternoon and would be happy to go over them with you 😃

Neutrino Cosmology Exercises

Lecturer: Miguel Escudero Abenza

Strategy: Doing exercises 1-4 should not be too time consuming and will give you a good idea of key numbers and calculations relevant for the cosmology of neutrinos. Exercises 5-8 are a bit more advanced, but can be done and will give you a flavor of how cosmological bounds are derived on sterile neutrinos as well as other types of light particles from cosmological observations.

1. **Number and energy density in the Cosmic Neutrino Background today**
   Estimated time: 15 mins, *difficulty*: 3/10, *result*: $n_\nu^{\rm today} = 346{\rm cm}^{-3}$, $\Omega_\nu h^2|^{\rm rel} = 1.75 \times 10^{-5}$, $\Omega_\nu h^2|^{\rm non-rel} = \sum m_\nu/(91.4\,{\rm eV})$.
   Taking the CNB temperature today to be $T_\nu = 2.75/1.4\,{\rm K}$, obtain: 1) the number density of neutrinos+antineutrinos today. 2) The fraction of the energy density of the Universe they represent. Note that today's critical energy density is $\rho_c = 1.054 \times 10^4 h^2 {\rm eV/cm}^3$. Do the calculation for neutrinos which are relativistic today, as well as for those which are non-relativistic today.

2. **Time of neutrinos becoming non-relativistic**
   Estimated time: 10 mins, *difficulty*: 3/10, *result*: $z_\nu \simeq 200 m_\nu/(0.1{\rm eV})$, at least 2.
   Taking $T_\nu = 2.75\,{\rm K}/1.4$, obtain the redshift at which neutrinos would become non-relativistic. Note that this happens when $T_\nu \lesssim m_\nu/3$. Given the observed mass squared differences in neutrino oscillation experiments, how many neutrinos are non-relativistic today?

3. **Calculation of the temperature of the Cosmic Neutrino Background**
   Estimated time: 10 mins, *difficulty*: 3/10, *result*: $T_\nu \simeq T_\gamma/1.4$.
   Using entropy conservation, calculate the temperature of the cosmic neutrino background after electrons and positrons have annihilated from the plasma. Take into account that neutrinos decoupled at a temperature of around 2 MeV, at which point electrons and positrons were still relativistic.

4. **Neutrino Oscillations in the Early Universe**
   Estimated time: 20 mins, *difficulty*: 4/10, *result*: $T_\nu^{\rm osc} \simeq \mathcal{O}(10)\,{\rm MeV}$.
   Using the 2-neutrino oscillation formula $P(\nu_\alpha \to \nu_\beta, t) = \sin^2(2\theta)\sin^2(\Delta m^2 t/(4E_\nu))$ estimate the time at which neutrino oscillations will take place. For the numerics, use $\theta = 0.1$, $\Delta m^2 = 10^{-3}\,{\rm eV}^2$, and $t = 1/(2H)$ with $H = 1.66\sqrt{g_\star}T^2/M_{\rm Pl}$, with $g_\star \simeq 10.75$ and $M_{\rm Pl} = 1.22 \times 10^{19}\,{\rm GeV}$.

5. **Calculation of the contribution to $\Delta N_{\rm eff}$ from a thermal Axion**
   Estimated time: 15 mins, *difficulty*: 6/10, *result*: $\Delta N_{\rm eff} \simeq 0.03$.
   Using entropy conservation, calculate the contribution to $\Delta N_{\rm eff}$ of an axion (scalar particle with one internal degree of freedom) which was present in thermal abundances in the early Universe but decoupled at a temperature of $T = 1\,{\rm TeV}$ at which point all the SM particles were present and relativistic. In particular, at the time $g_\star = 106.75$. *Remember the definition of $N_{\rm eff} \equiv 8/7(11/4)^{4/3}[\rho_{\rm rad} - \rho_\gamma]/\rho_\gamma$ at $T \ll m_e$.

Time for Questions and Comments

End of Lecture I

Thank you for your attention!

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