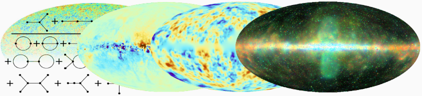

Information Field Theory

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für Bildung
und Forschung

Teaser: Radio Aperture Synthesis

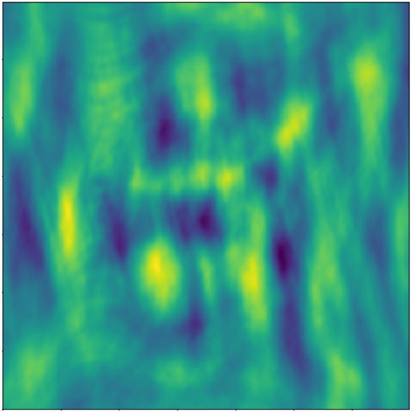


Figure 1: Dirty image

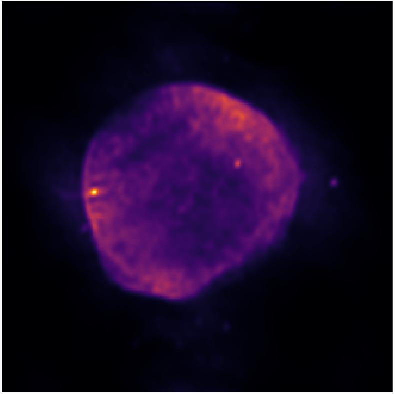


Figure 2: Reconstructed sky model

Supernova remnant SN1006, public data VLA (24/01/2003), 12.5 MHz bandwidth, single-channel mode, one spectral windows at 1.37 GHz. Reconstructed with RESOLVE, only Stokes I.

Acknowledgements

- Torsten Enßlin, my **supervisor**.
- Henrik Junklewitz, **inventor** of RESOLVE.
- Philipp Frank, Sebastian Hutschenreuter, Jakob Knollmüller, Reimar Leike, Natalia Porqueres, Daniel Pumpe, Julian Rüstig and many more, **fellow sufferers**.
- Martin Reinecke, **numerical genius**.
- NIFTy, **solves all problems** (https://gitlab.mpcdf.mpg.de/ift/NIFTy).

Inverse Problems

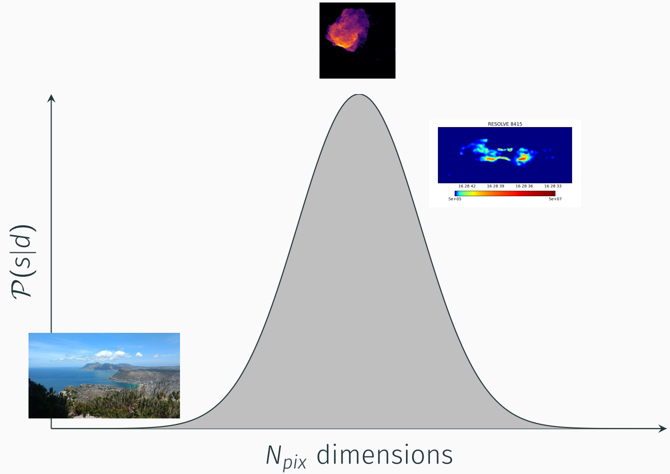
True sky s ($N_{\text{dof}} = \infty$)
$\longrightarrow$ Fields (e.g. $\rho : \mathbb{R}^3 \to \mathbb{R}_{\geq 0}$)

Data d ($N_{\text{dof}} < \infty$)
$\longrightarrow$ Data arrays
(e.g. `vis.shape = (4132094,)`)

$$\mathcal{P}(s|d)$$

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\mathcal{P}(s)}{\mathcal{P}(d)}$$

Probability Distributions Over All Possible Fields

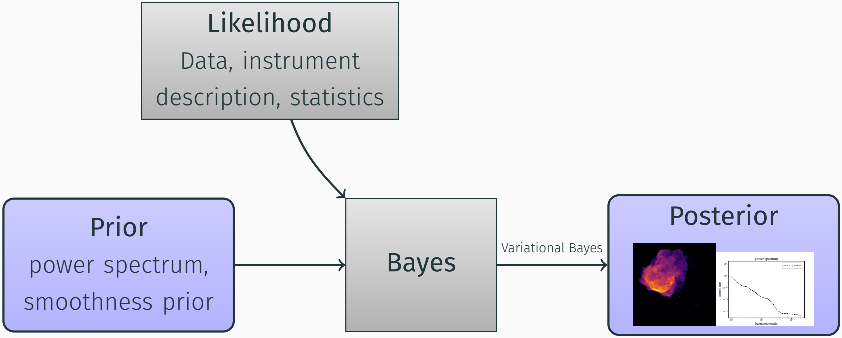


This is a lot!

Theoretically, infinite-dimensional.

Practically, e.g. $N_{pix} = 1024^2 \rightarrow 1$ Mio. dimensions.

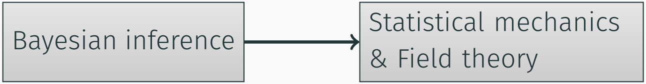
Bayesian Reconstruction Algorithms



Information Field Theory

Information Field Theory

- Information Field Theory := Information theory with fields.
- Provides dictionary:



- Define $\mathcal{H}(s, d) := -\log \mathcal{P}(s, d)$. Then:

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(s, d)}{\mathcal{P}(d)} = \frac{e^{-\mathcal{H}(s,d)}}{\int \mathcal{D}s\, e^{-\mathcal{H}(s,d)}}.$$

- Exploits continuity and correlation.
- Variational Bayes corresponds to minimizing the Gibbs free energy.

Example: Log-normal Poisson

Log-normal Poisson Reconstruction

- Imagine a light curve.
- Data: Point-wise drawn from Poisson distribution:

$$d_i \curvearrowleft P_{\lambda_i}(d_i).$$

  λ: expectation value of poisson process.
- Measurement equation:

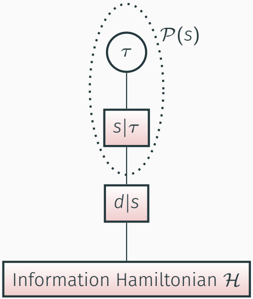
$$\lambda_i = R(\phi_t) = R(e^{s_t}),$$

  ϕ: continuous photon flux field.
- Prior knowledge: s_t is a Gaussian process whose kernel is known.

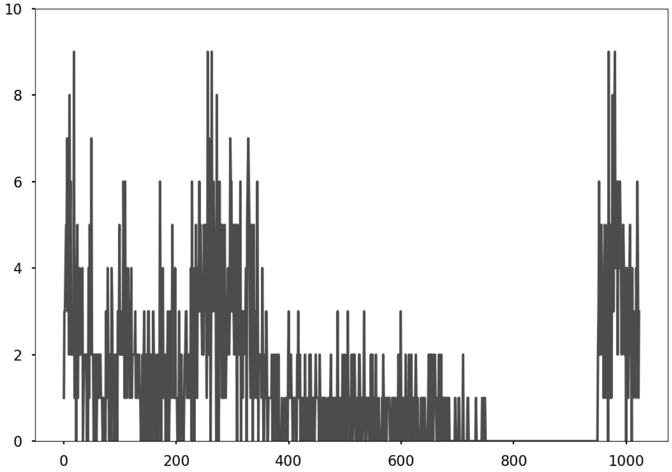
- Bayes theorem:

$$\mathcal{P}(s|d) \propto \mathcal{P}(d|s)\mathcal{P}(s).$$

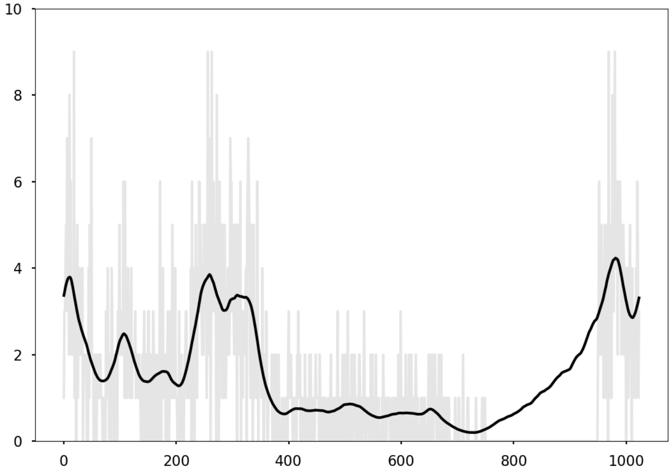
- Bayesian hierarchical model:



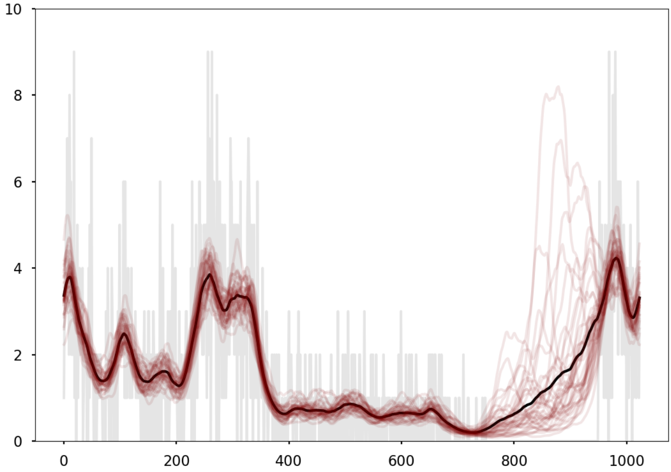
Log-normal Poisson Reconstruction



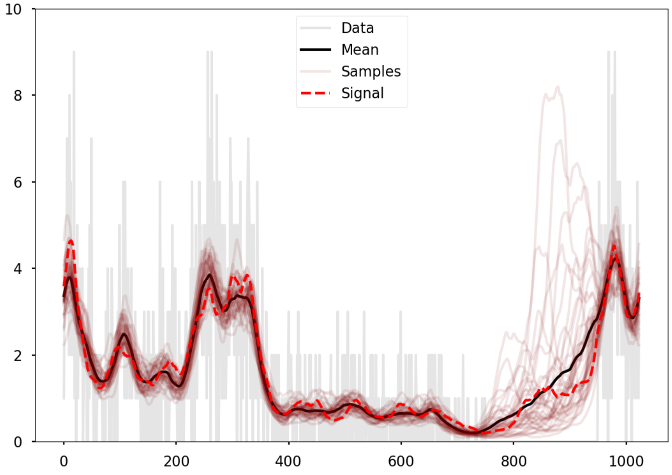
Log-normal Poisson Reconstruction



Log-normal Poisson Reconstruction



Log-normal Poisson Reconstruction



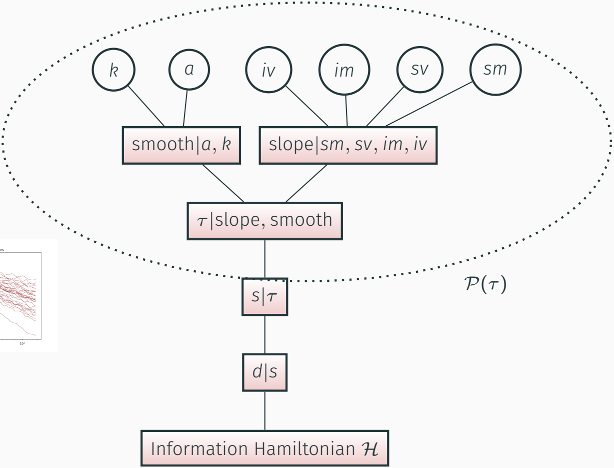
Example: Estimating Power Spectra

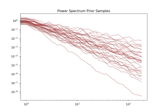
Critical Log-normal Poisson Reconstruction

- Infer power spectrum as well:

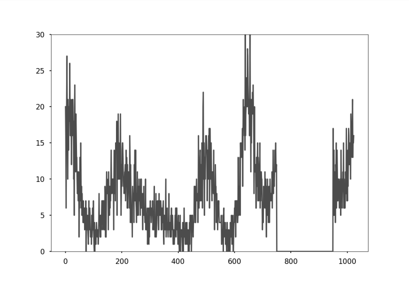
$$\mathcal{P}(s|d) \propto \mathcal{P}(d|s)\mathcal{P}(s|\tau)\mathcal{P}(\tau).$$

- Bayesian hierarchical model:

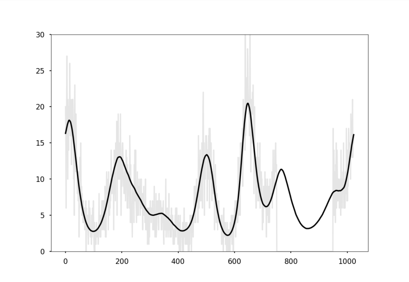


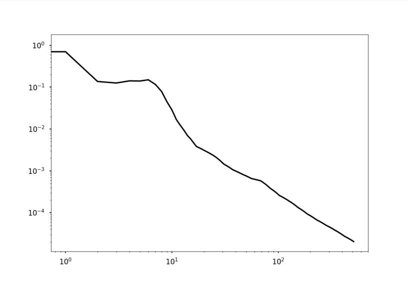


Critical Log-normal Poisson Reconstruction

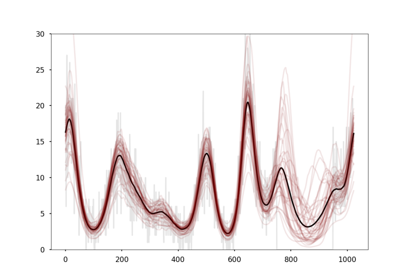


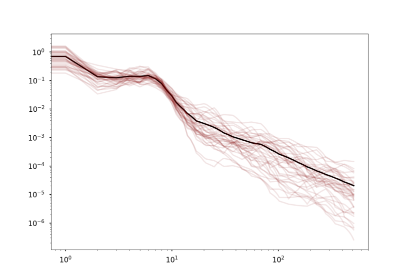
Critical Log-normal Poisson Reconstruction



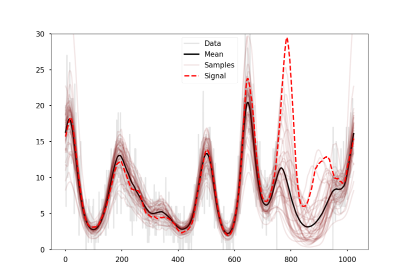


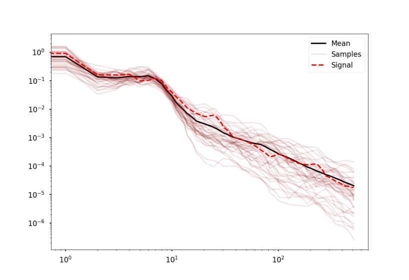
Critical Log-normal Poisson Reconstruction





Critical Log-normal Poisson Reconstruction





Example: Multi-Messenger

Multi-Messenger with IFT

- Observe a physical field s with two different measurement devices:

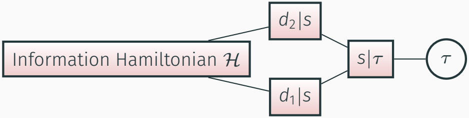
$$d_1 = R_1 s + n_1$$
$$d_2 = R_2 s + n_2$$

- Reconstruct s from $d = (d_1, d_2)$:

$$\mathcal{P}(s|d_1, d_2) \propto \mathcal{P}(d_1, d_2|s)\mathcal{P}(s) = \mathcal{P}(d_1|s)\mathcal{P}(d_2|s)\mathcal{P}(s)$$

$\longrightarrow$ One only needs to multiply likelihoods.

- Bayesian hierarchical model:



Multi-Messenger with IFT

Information source aka dirty image: $j = R^\dagger N^{-1} d$.

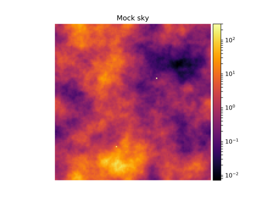


Figure 3: Ground truth

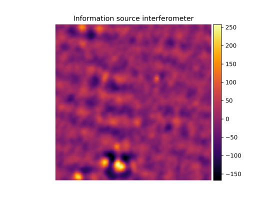


Figure 4: Information source interferometer

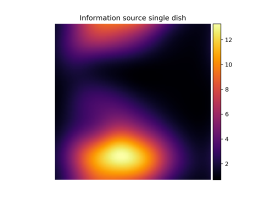
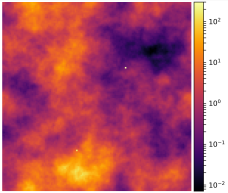
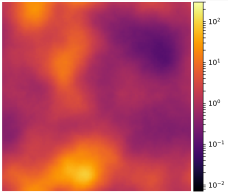


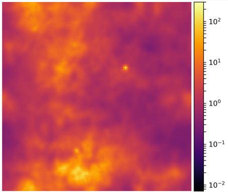
Figure 5: Information source single dish



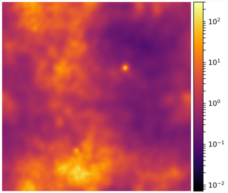
Ground Truth



Single Dish



Interferometer



Combined

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