C1c: Lattice and Gradient Flow

Oliver Witzel
Matthew Black, Robert Harlander, Fabian Lange, Antonio Rago, Andrea Shindler

Universität Siegen / TP1 Theoretical Particle Physics / CPPS Center for Particle Physics Siegen

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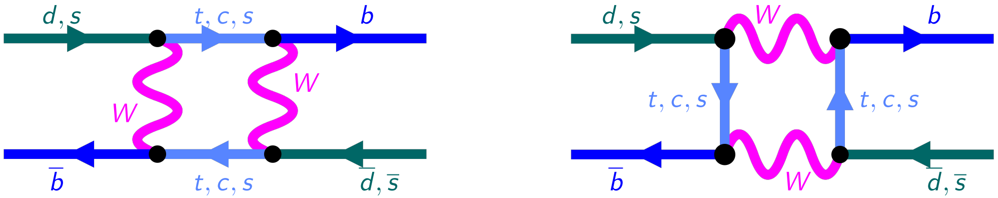
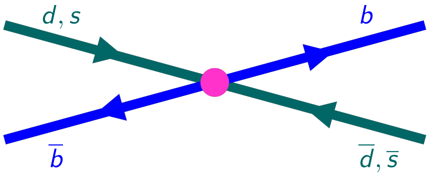
Lattice calculation of neutral $B^0_{(s)}$ meson mixing

- Standard model process described by box diagrams



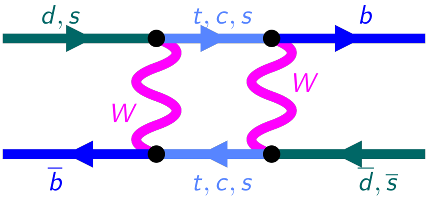
- Top quark contribution dominates ⇒ short-distance process

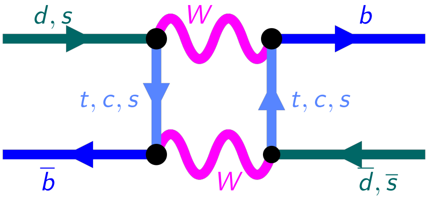
  → Describe by point-like 4-quark operators



Lattice calculation of neutral $B^0_{(s)}$ meson mixing

- Standard model process described by box diagrams



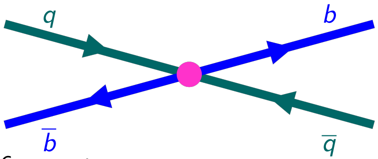


- Top quark contribution dominates ⇒ short-distance process
  - → Describe by point-like 4-quark operators
  - → Parameterize experimentally measured oscillation frequencies Δm_q by

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 \hat{B}_{B_q} \left| V_{tq}^* V_{tb} \right|^2, \qquad q = d, s$$

  - → Nonperturbative contribution decay constant $f_{B_q}^2$ times bag parameter $\hat{B}_{B_q}$

$\Delta B = 2$ mixing operators



- Standard model process described by

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta$$

$\rightarrow$ General BSM considerations give rise to four additional dim-6 operators

- Calculate matrix element

$$\langle \mathcal{O}_1^q \rangle = \langle \overline{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q} B_{B_q}$$

- Convert "lattice" bag parameter B_{B_q} to RGI bag parameter $\hat{B}_{B_q}$

$\rightarrow$ Renormalization/matching procedures used in the literature

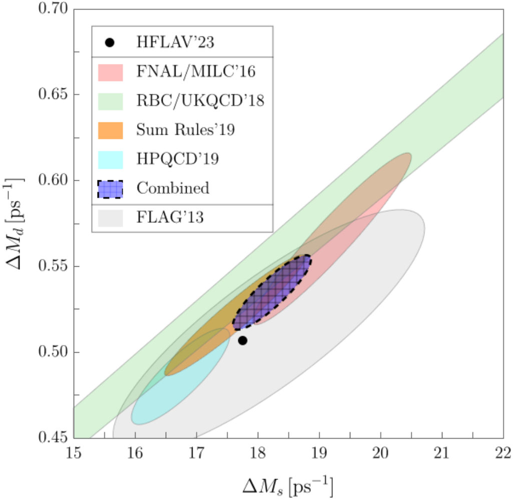
Perturbative scheme: Fermilab/MILC, HPQCD

Nonperturbative scheme: ETMC, RBC-UKQCD

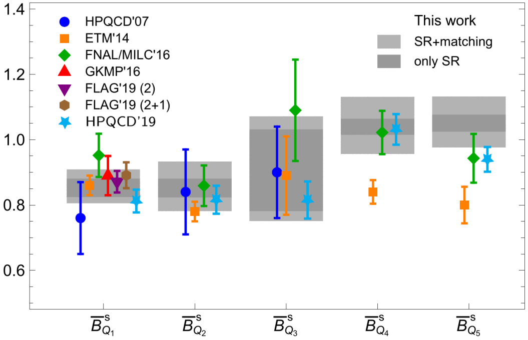
- Operator mixing occurs for non-chiral lattice fermions

$\Delta B = 2$ mixing operators (literature)

[Albrecht et al. 2402.04224]



[King Thesis 2022]



- Ongoing work by RBC-UKQCD+JLQCD
  [Boyle et al. PoS Lattice 2021 224] [Tsang Lattice 2023]
- Dim-7 operators pioneered by HPQCD
  [HPQCD PRL 124 (2020) 082001]

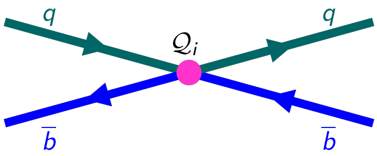
Heavy meson lifetimes ($\Delta B = 0$ operators)

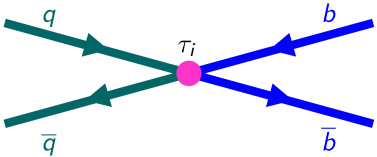
- Using heavy quark expansion (HQE), lifetimes of heavy mesons are described by 4-quark operators with $\Delta B = 0$

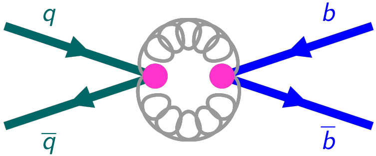
- Operators $\mathcal{Q}_1$, $\mathcal{Q}_2$, τ_1, τ_2, contribute

- $\Delta B = 0$ operators mix under renormalization
  → To date no complete LQCD determination (only exploratory work 20+ years ago)

- Quark-line disconnected contributions
  → Notoriously noisy, hard to calculate on the lattice

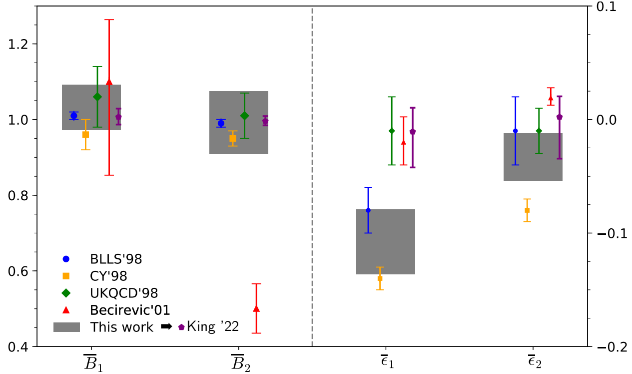






Heavy meson lifetimes (literature)

[Kirk, Lenz, Rauh JHEP 12 (2017) 068]



- Sum rule results taken in HQET limit
- Lattice results are exploratory; no full calculation with error budget

How can we calculate heavy meson lifetimes on the lattice?

- Need a new way to handle the operator mixing

- Use Gradient flow (GF) in combination with short flow-time expansion (SFTX)
  → GF effectively acts as renormalization
  → SFTX allows to directly match GF renormalized results to $\overline{\text{MS}}$

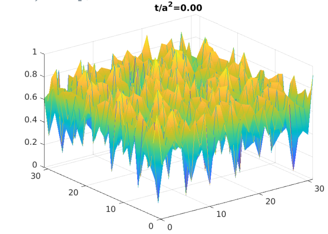
- Operators do not mix under the GF on the lattice
  → Mixing is pushed to the perturbative part of the SFTX where we can manage it

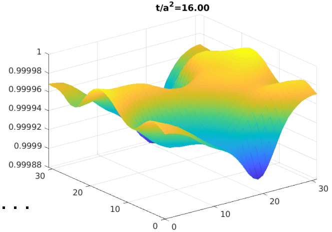
Gradient flow (GF)

- By now standard tool for calculating scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
  [Narayanan, Neuberger JHEP 03 (2006) 064] [Lüscher JHEP 08 (2010) 071][JHEP 04 (2013) 123], ...
- Introduce auxiliary dimension, flow time τ to regularize UV
  → Well-defined smearing of gauge and fermion fields
  → Smoothening UV fluctuations
- First order differential equation

$$\partial_t B_\mu(\tau,x) = \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau,x), \quad B_\mu(0,x) = A_\mu(x)$$
$$\partial_t \chi(\tau,x) = \mathcal{D}^2(\tau)\chi(\tau,x), \quad \chi(0,x) = q(x)$$

- Two concepts for GF renormalization
  → GF as an RG transformation [Carosso et al. PRL 121 (2018) 201601]
  [Hasenfratz et al. PoS Lattice 2021 155]
  → Short flow-time expansion (SFTX)
  [Lüscher, Weisz JHEP 02 (2011) 051] [Makino, Suzuki PTEP (2014) 063B02]
  [Monahan, Orginos PRD 91 (2015) 074513] [Rizik et al. PRD 102 (2020) 034509], ...





Short flow-time expansion (SFTX)

- Re-express effective Hamiltonian in terms of 'flowed' operators

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau)$$

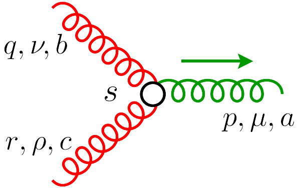
- Relate to regular operators in SFTX

ME of flowed operator (lattice)

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

PT calculated matching matrix

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$



new Feynman diagrams

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in $\overline{\text{MS}}$ found in $\tau \to 0$ limit $\Rightarrow$ 'window' problem
  - $\to$ Large systematic effects at very small flow times
  - $\to$ Large flow time dominated by operators $\propto O(\tau)$

Exploratory setup

- RBC/UKQCD's 2+1 flavor DWF + Iwasaki gauge action ensembles, 3 lattice spacings
  [PRD 78 (2008) 114509] [PRD 83 (2011) 074508] [PRD 93 (2016) 074505] [JHEP 12 (2017) 008]
- Setup of the lattice calculation follows [Boyle et al. 1812.08791]
  - → Z2 wall sources for all quark propagators [Boyle et al. JHEP 08 (2008) 086]
  - → Gaussian source smearing for strange quarks [Allton et al. PRD 47 (1993) 5128]
  - → Multiple source separations $\Delta T \in \{10, 30\}$
- Fully-relativistic, chiral action for all quarks
  - → Shamir domain-wall fermions for light and strange quarks
    [Kaplan PLB 288 (1992) 342] [Shamir NPB 406 (1993) 90] [Furman, Shamir NPB 439 (1995) 54]
  - → Stout-smeared Möbius domain-wall fermions for heavy quarks
    [Morningstar, Peardon PRD 69 (2004) 054501] [Brower, Neff, Orginos CPC 220 (2017) 1]
- Simulate "neutral" charm-strange mesons
  - → Easy to tune to physical strange and charm quarks
  - → Avoid more expensive chiral or heavy quark extrapolation

Steps of the calculation

- Implement matrix elements of $\Delta Q = 2$ and $\Delta Q = 0$ operators for charm-strange mesons
- Carry out measurements for different ensembles (lattice spacing, sea quark masses)
  - → Each ensemble requires measurements on many configurations
  - → Each gauge field and fermion propagator needs to be evolved along the GF time τ
- Extract bag parameters from 3-pt and 2-pt functions
  - → For each operator to be done for many flow times and on each ensemble
- Take continuum limit of lattice data, again for many flow times
- Combine with PT calculation to extract renormalized quantities in the $\overline{\text{MS}}$ scheme
- First results for $\Delta Q = 2$, $\mathcal{O}_1$ operator [PoS Lattice2023 263]

PROCEEDINGS OF SCIENCE

Using Gradient Flow to Renormalise Matrix Elements for Meson Mixing and Lifetimes

Matthew Black,[a,*] **Robert Harlander,**[b] **Fabian Lange,**[c,d,e,f] **Antonio Rago,**[g] **Andrea Shindler**[b] **and Oliver Witzel**[a]

[a] *Center for Particle Physics Siegen, Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*

[b] *Institute for Theoretical Particle Physics and Cosmology, RWTH Aachen University, 52056 Aachen, Germany*

[c] *Physik-Institut, Universität Zürich, Winterthurerstrasse 190, 8057 Zürich, Switzerland*

[d] *Paul Scherrer Institut, 5232 Villigen PSI, Switzerland*

[e] *Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology, Wolfgang-Gaede-Straße 1, 76128 Karlsruhe, Germany*

[f] *Institut für Astroteilchenphysik, Karlsruhe Institute of Technology, Hermann-von-Helmholtz-Platz 1, 76344 Eggenstein-Leopoldshafen, Germany*

[g] *IMADA and Quantum Theory Center, University of Southern Denmark, Odense, Denmark*

E-mail: matthew.black@uni-siegen.de

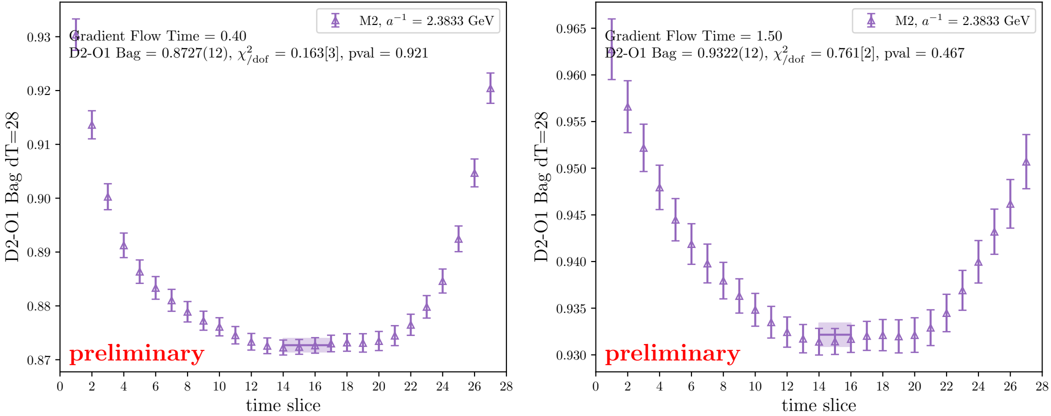
Neutral meson mixing and meson lifetimes are theory-side parametrised in terms four-quark operators which can be determined by calculating weak decay matrix elements using lattice Quantum Chromodynamics. While calculations of meson mixing matrix elements are standard, determinations of lifetimes typically suffer from complications in renormalisation procedures because dimension-6 four-quark operators can mix with operators of lower mass dimension and, moreover, quark-line disconnected diagrams contribute.

We present work detailing the idea to use fermionic gradient flow to non-perturbatively renormalise matrix elements describing meson mixing or lifetimes, and combining it with a perturbative calculation to match to the $\overline{\text{MS}}$ scheme using the short-flow-time expansion.

PoS(LATTICE2023)263

Extract bag parameter

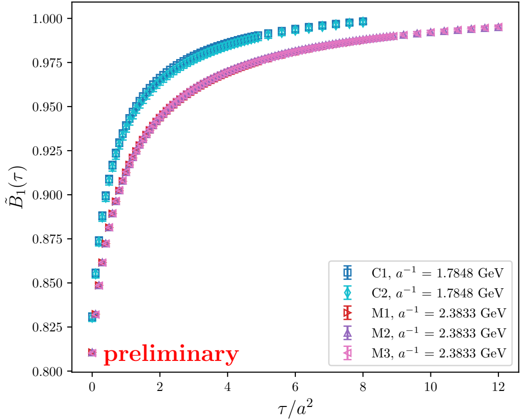
- For each ensemble extract bag parameter for different flow times



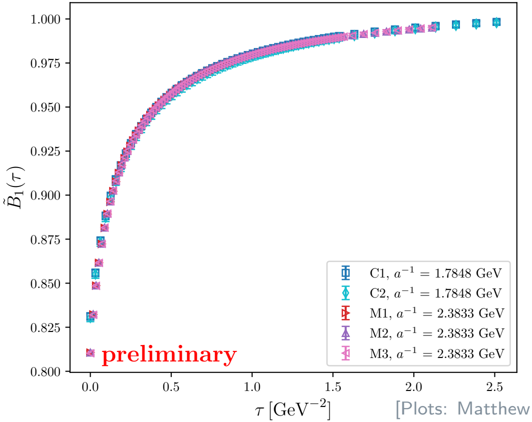
[Plots: Matthew Black]

$\mathcal{O}_1$ mixing operator vs. GF time

- Operator is renormalized in GF scheme as it evolves along flow time τ
- No light sea quark effects



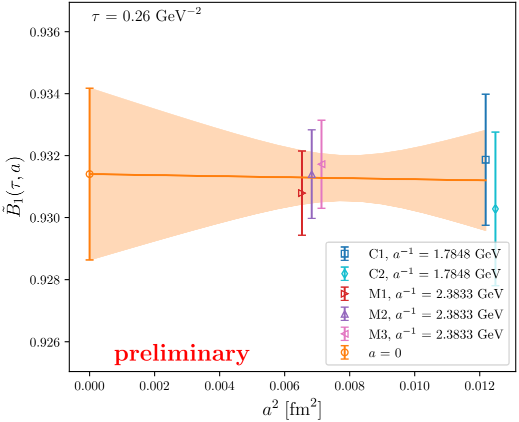
- Convert to "physical" flow time
  → Mild continuum limit

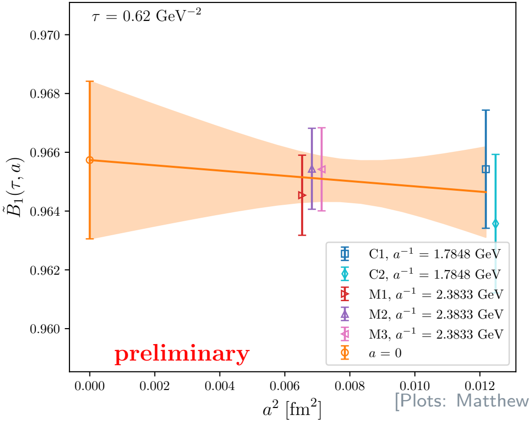


[Plots: Matthew Black]

Continuum limit

- Very mild continuum limit for positive flow times





[Plots: Matthew Black]

Perturbative matching to $\overline{\text{MS}}$ scheme

- Relate to regular operators in SFTX

ME of flowed operator (lattice)

$$\tilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

PT calculated matching matrix

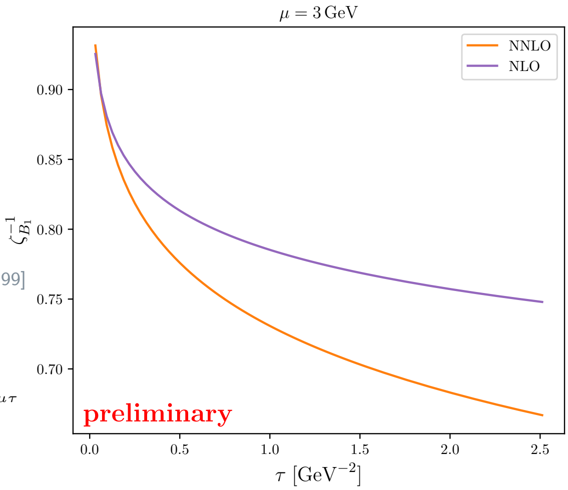
$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\tilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$

- Calculated at two-loop for $\mathcal{B}_1$ at $\mu = 3\,\text{GeV}$

[Harlander, Lange PRD 105 (2022) L071504] [Borgulat et al. 2311.16799]

$$\zeta^{-1}_{\mathcal{B}_1}(\mu,\tau) = 1 + \frac{a_s}{4}\left(-\frac{11}{3} - 2L_{\mu\tau}\right)$$
$$+ \frac{a_s^2}{43200}\Big[-2376 - 79650L_{\mu\tau} - 24300L_{\mu\tau}^2 + 8250n_f + 6000\,n_f\,L_{\mu\tau}$$
$$+ 1800\,n_f\,L_{\mu\tau}^2 - 2775\pi^2 + 300\,n_f\,\pi^2 - 241800\log 2$$
$$+ 202500\log 3 - 110700\,\mathrm{Li}_2\left(\frac{1}{4}\right)\Big]$$

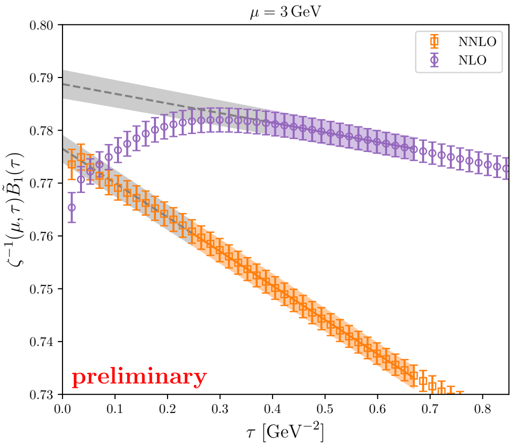
$$L_{\mu\tau} = \log(2\mu^2\tau) + \gamma_E$$



[Plot: Matthew Black]

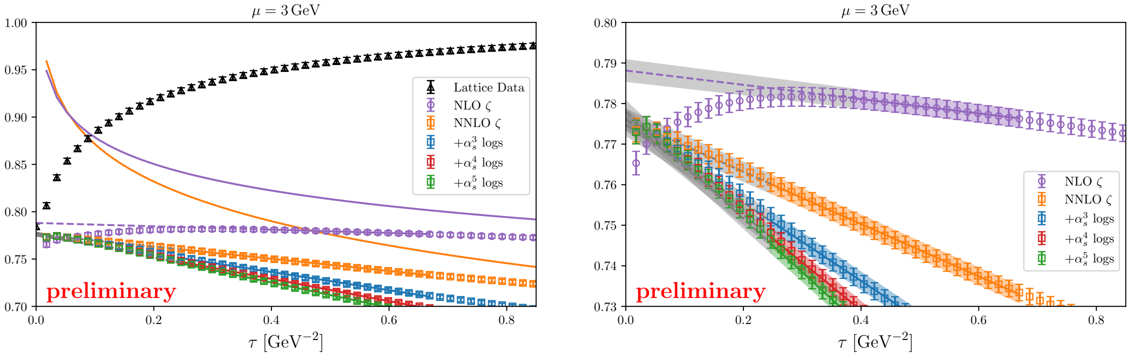
Matched result for $\mathcal{O}_1$ mixing operator

- NNLO improves over NLO by extending linear region to smaller flow time
- Statistical errors only!
- NNLO and NLO in similar ballpark
  $\rightarrow$ Systematic errors needed for comparison
- Reasonable value compared to short distance D^0 meson mixing
  [ETMC 2015] 0.757(27)
  [Fermilab/MILC] 0.795(56)



[Plot: Matthew Black]

Checking effects of higher order logarithms



[Plots: Matthew Black]

Summary and outlook

- First results for $\Delta Q = 2$ operator $\mathcal{O}_1$ look qualitatively extremely promising
  $\rightarrow$ Any quantitative statement warrants a more careful analysis and studies of systemtatic effects
- Work for first $\Delta Q = 0$ operators is in progress!
- Measurements on last ensemble at third lattice spacing currently running on LumiG
- Complete proof of principle
- Set-up full scale calculation including physical light quarks and multiple heavy quarks to target $B_{(s)}$ meson mixing and lifetimes

Acknowledgment

- Grid [Peter Boyle et al.]
- Hadrons [Antonin Portelli et al.]
- Feyngame [Harlander et al.]
- OMNI, Universität Siegen
- HAWK, HLR Stuttgart
- LumiG, DEIC

$\Delta B = 2$ operators

- Full operator basis:

$$\mathcal{O}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \ \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta, \quad \langle \mathcal{O}_1^q \rangle = \langle \bar{B}_q | \mathcal{O}_1^q | B_q \rangle = \frac{8}{3} f_{B_q}^2 M_{B_q}^2 B_1^q$$

$$\mathcal{O}_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \ \bar{b}^\beta (1-\gamma_5) q^\beta, \quad \langle \mathcal{O}_2^q \rangle = \langle \bar{B}_q | \mathcal{O}_2^q | B_q \rangle = \frac{-5 M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_2^q,$$

$$\mathcal{O}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \ \bar{b}^\beta (1-\gamma_5) q^\alpha, \quad \langle \mathcal{O}_3^q \rangle = \langle \bar{B}_q | \mathcal{O}_3^q | B_q \rangle = \frac{M_{B_q}^2}{3(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 B_3^q,$$

$$\mathcal{O}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \ \bar{b}^\beta (1+\gamma_5) q^\beta, \quad \langle \mathcal{O}_4^q \rangle = \langle \bar{B}_q | \mathcal{O}_4^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{(m_b+m_q)^2} + \frac{1}{3} \right] f_{B_q}^2 M_{B_q}^2 B_4^q,$$

$$\mathcal{O}_5^q = \bar{b}^\alpha (1-\gamma_5) q^\beta \ \bar{b}^\beta (1+\gamma_5) q^\alpha, \quad \langle \mathcal{O}_5^q \rangle = \langle \bar{B}_q | \mathcal{O}_5^q | B_q \rangle = \left[\frac{2 M_{B_q}^2}{3(m_b+m_q)^2} + 1 \right] f_{B_q}^2 M_{B_q}^2 B_5^q.$$

$\Delta B = 2$ operators

- Transformed basis (color singlets only)

$$\mathcal{Q}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta,$$
$$\mathcal{Q}_2^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1+\gamma_5) q^\beta,$$
$$\mathcal{Q}_3^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1+\gamma_5) q^\beta,$$
$$\mathcal{Q}_4^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{b}^\beta (1-\gamma_5) q^\beta,$$
$$\mathcal{Q}_5^q = \frac{1}{4} \bar{b}^\alpha \sigma_{\mu\nu} (1-\gamma_5) q^\alpha \; \bar{b}^\beta \sigma_{\mu\nu} (1-\gamma_5) q^\beta$$

$$\begin{pmatrix} \mathcal{O}_1^+ \\ \mathcal{O}_2^+ \\ \mathcal{O}_3^+ \\ \mathcal{O}_4^+ \\ \mathcal{O}_5^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \mathcal{Q}_3^+ \\ \mathcal{Q}_4^+ \\ \mathcal{Q}_5^+ \end{pmatrix}$$

- Advantages for both lattice calculation and the NPR procedure
- We are only concerned with parity-even components which then can be transformed back

$\Delta B = 0$ operators

- For lifetimes, the dimension-6 $\Delta B = 0$ operators are

$$Q_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta, \qquad \langle Q_1^q \rangle = \langle B_q | Q_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \mathcal{B}_1^q,$$

$$Q_2^q = \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1-\gamma_5) b^\beta, \qquad \langle Q_2^q \rangle = \langle B_q | Q_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \mathcal{B}_2^q,$$

$$T_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma \gamma_\mu (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \quad \langle T_1^q \rangle = \langle B_q | T_1^q | B_q \rangle = f_{B_q}^2 M_{B_q}^2 \epsilon_1^q,$$

$$T_2^q = \bar{b}^\alpha (1-\gamma_5) (T^a)^{\alpha\beta} q^\beta \; \bar{q}^\gamma (1-\gamma_5) (T^a)^{\gamma\delta} b^\delta, \qquad \langle T_2^q \rangle = \langle B_q | T_2^q | B_q \rangle = \frac{M_{B_q}^2}{(m_b+m_q)^2} f_{B_q}^2 M_{B_q}^2 \epsilon_2^q.$$

- For simplicity of computation, we want these to be color-singlet operators

$$\begin{aligned}
\mathcal{Q}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) q^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) b^\beta \\
\mathcal{Q}_2 &= \bar{b}^\alpha (1-\gamma_5) q^\alpha \; \bar{q}^\beta (1+\gamma_5) b^\beta) \\
\mathcal{T}_1 &= \bar{b}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta \\
\mathcal{T}_2 &= \bar{b}^\alpha \gamma_\mu (1+\gamma_5) b^\alpha \; \bar{q}^\beta \gamma_\mu (1-\gamma_5) q^\beta
\end{aligned}
\qquad
\begin{pmatrix} Q_1^+ \\ Q_2^+ \\ T_1^+ \\ T_2^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2N_c} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2N_c} & 0 & \frac{1}{4} \end{pmatrix} \begin{pmatrix} \mathcal{Q}_1^+ \\ \mathcal{Q}_2^+ \\ \mathcal{T}_1^+ \\ \mathcal{T}_2^+ \end{pmatrix}$$