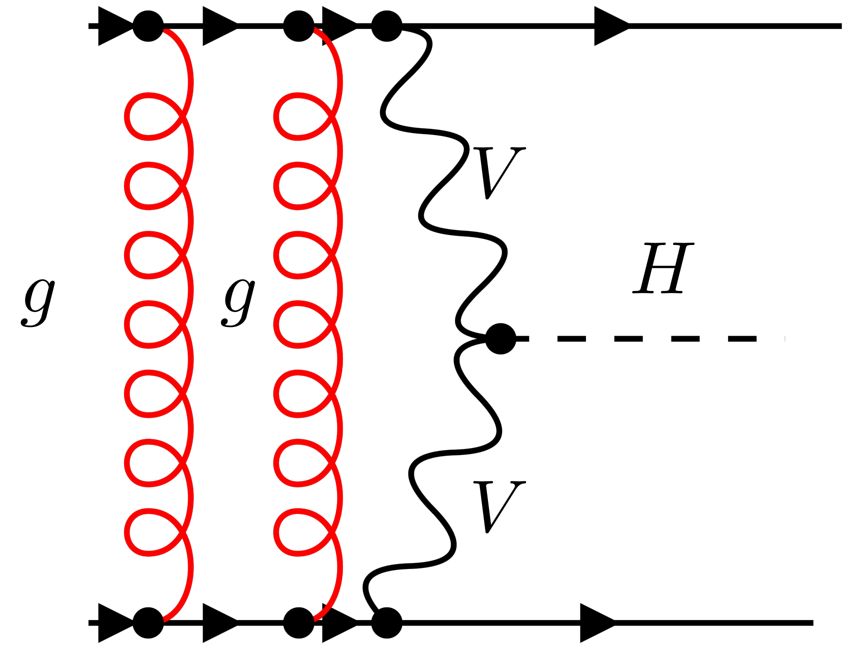
Karlsruher Institut für Technologie

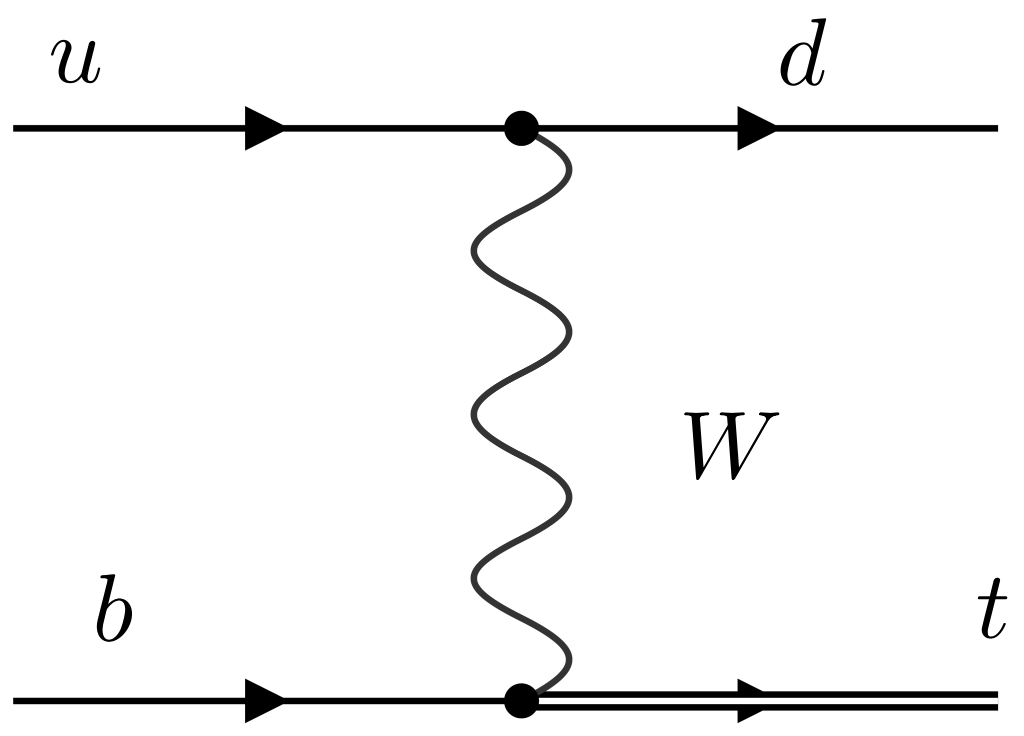
Particle Physics Phenomenology
after the Higgs Discovery

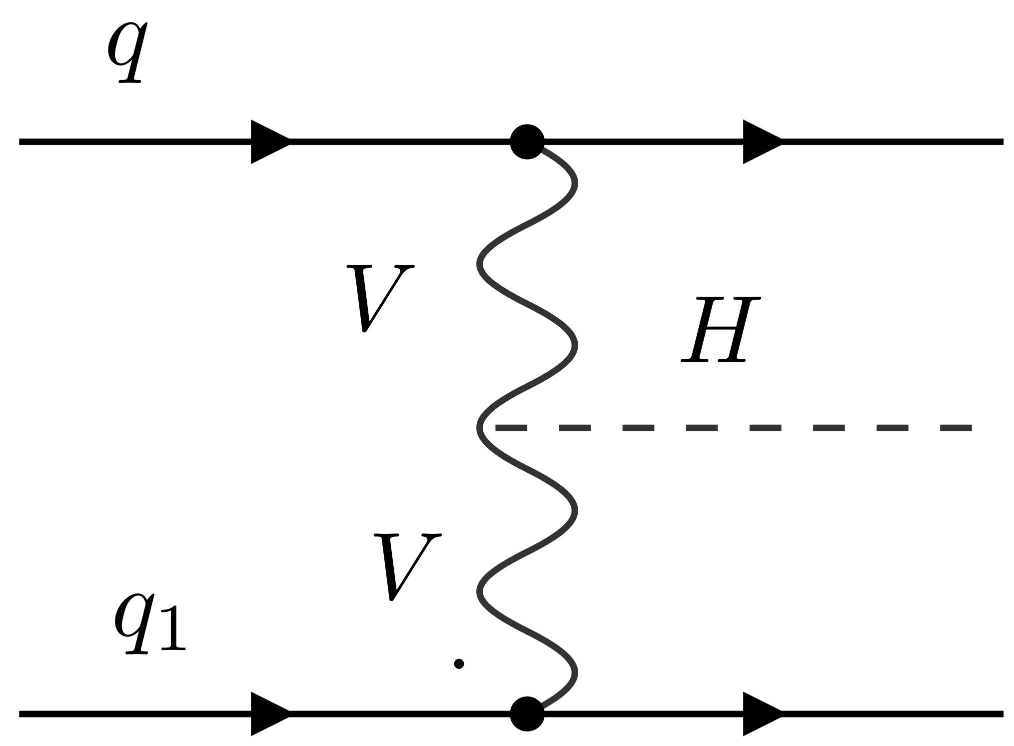
# Non-factorizable QCD corrections in hadron collider processes

**Kirill Melnikov**

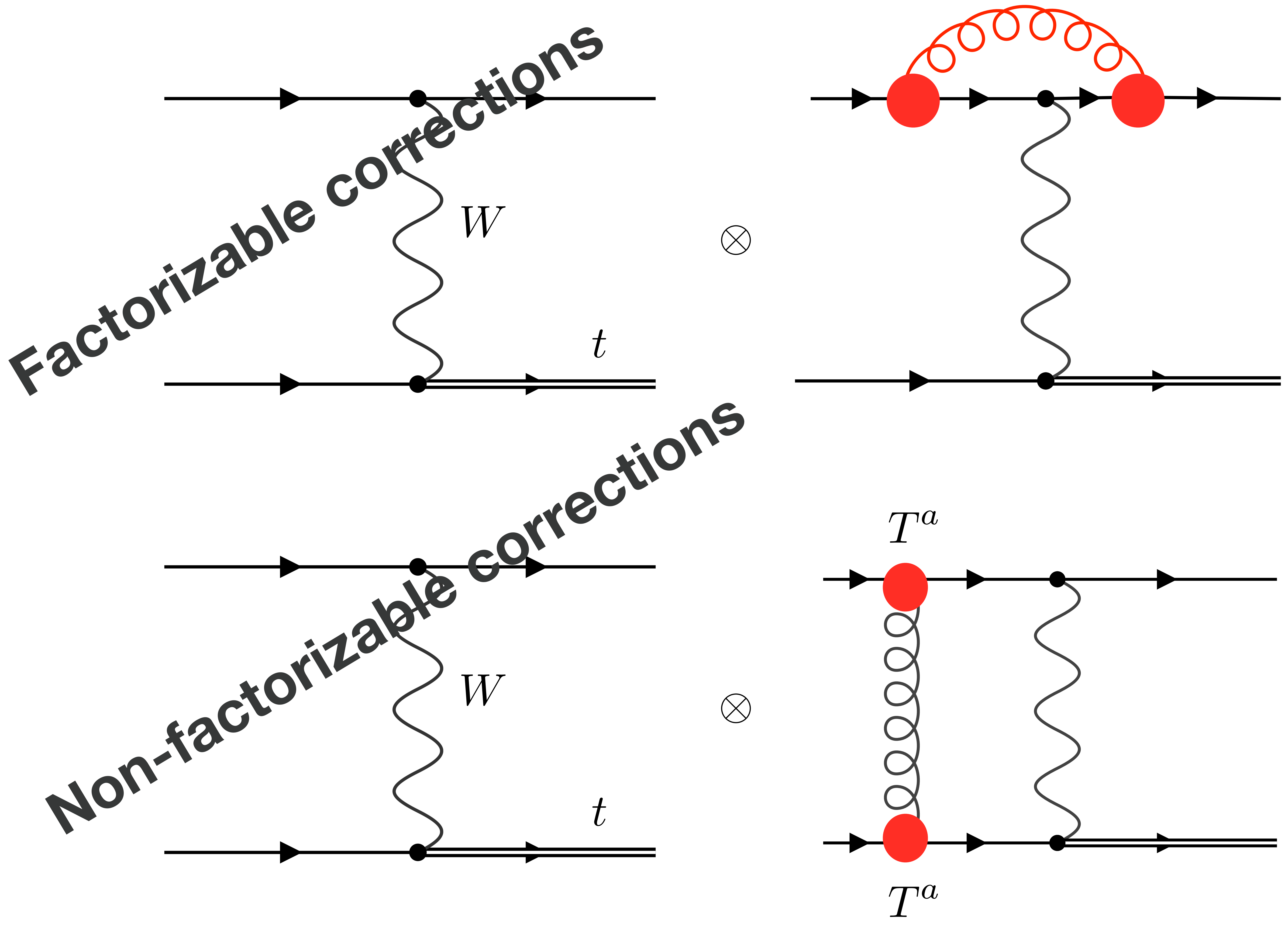


Non-factorizable corrections appear in QCD processes which, at tree level, are mediated by exchanges of colorless particles.  Two most famous examples are single top production and Higgs production in weak boson fusion.   These processes are used by the LHC collaborations to study e.g. tbW and HVV couplings, as well as other physical quantities related to top and Higgs physics.



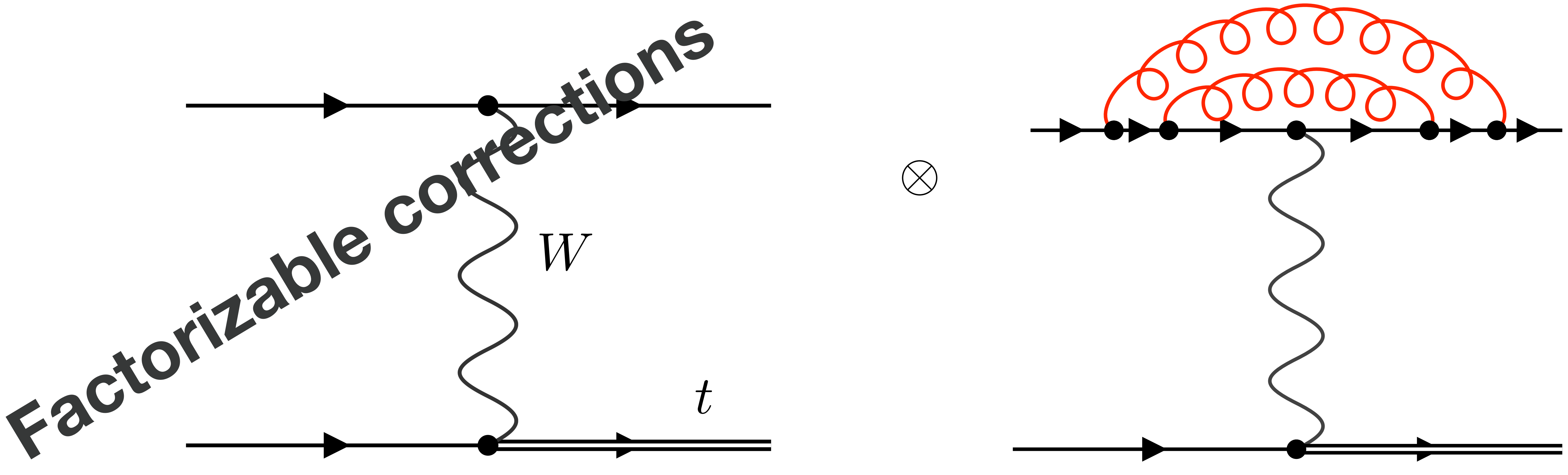
At NLO QCD, two classes of contributions appear. However, the non-factorizable contributions vanish at this order because of color conservation.



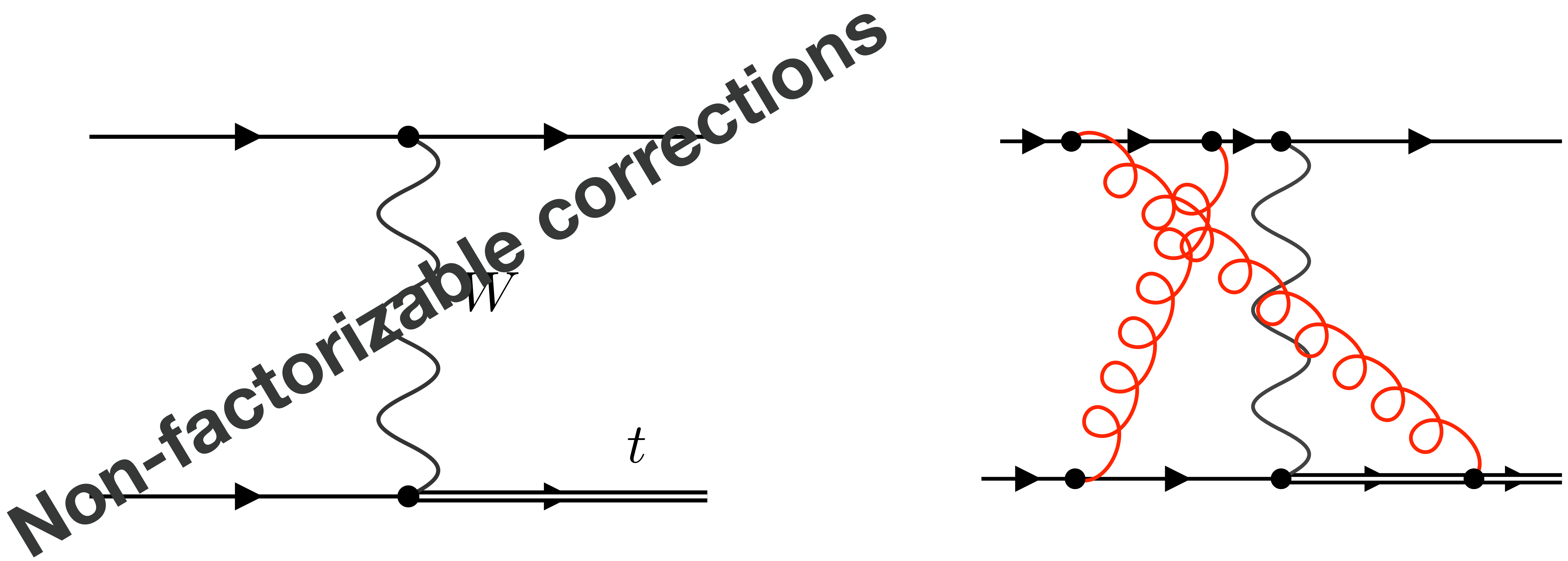

$$T^a T^a = C_F$$

$$= 0 \quad \Leftrightarrow \quad \mathrm{Tr}[T^a] = 0$$

At NNLO QCD, the situation changes: non-factorizable contributions do not vanish anymore. However, they are color-suppressed relative to the factorizable ones. As the result, the NNLO QCD corrections to both single top production and to Higgs boson production in WBF were originally computed neglecting the non-factorizable contributions.




$$\frac{\text{nfact}}{\text{fact}} \sim N_c^{-2} \sim 10^{-1}$$



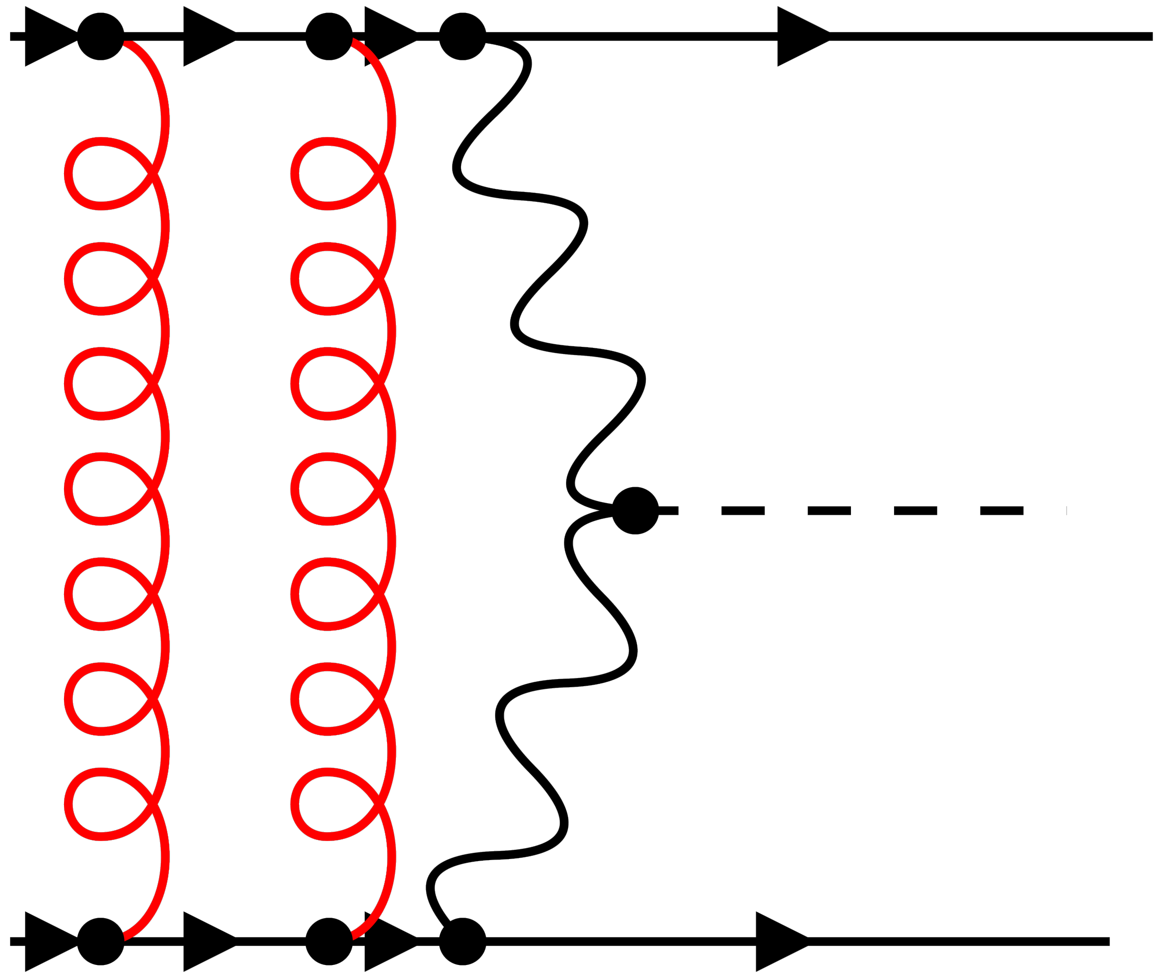

**Non-factorizable corrections at NNLO are effectively abelian. This reduces the number of diagrams one needs to consider and leads to other simplifications.**

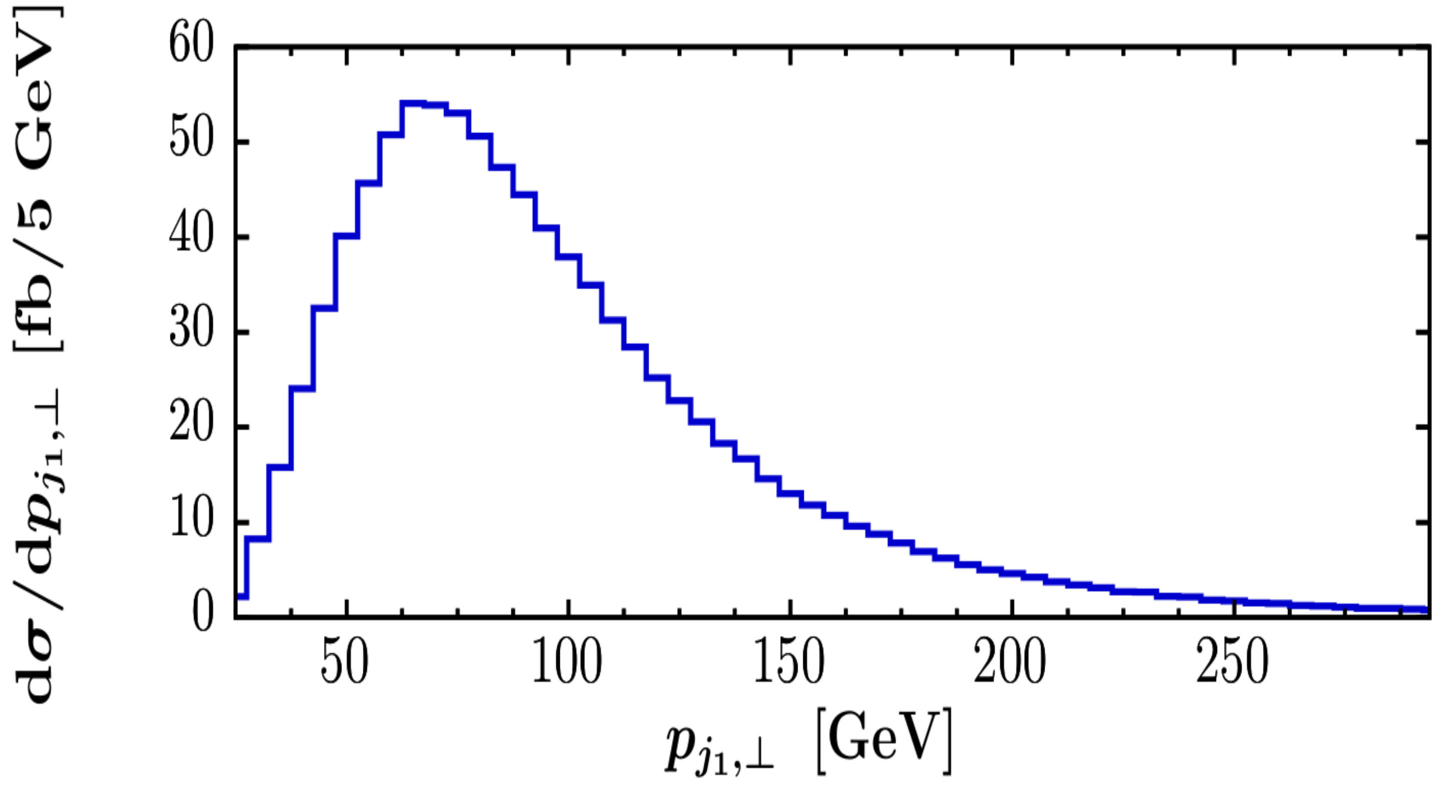
$$2T^aT^b = \{T^a, T^b\} + [T^a, T^b]$$

The color suppression argument does not appear to hold upon a more careful analysis because there is a peculiar dynamical enhancement of the non-factorizable corrections. This enhancement was discovered when trying to understand if it is possible to compute the required two-loop amplitude, if even approximately.

Liu, Melnikov, Penin

To understand how to construct an expansion of the amplitude, we employ the kinematics of weak boson fusion where forward (tagging) jets with large invariant mass and large rapidity separation are selected.






$$p_\perp^{j_1,j_2} > 25~\mathrm{GeV}, \quad |y_{j_1,j)2}| < 4.5$$
$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600~\mathrm{GeV}$$

Expansion around the forward limit of tagging jets leads to the eikonal approximation for the loop integrand.

$$p_{\perp,3} \sim p_{\perp,4} \sim m_V \sim m_H \ll \sqrt{s}$$

The main idea behind the eikonal approximation is that high-energy quarks follow straight lines and do not recoil because the exchanged gluons are soft. Rules for constructing the amplitude are:

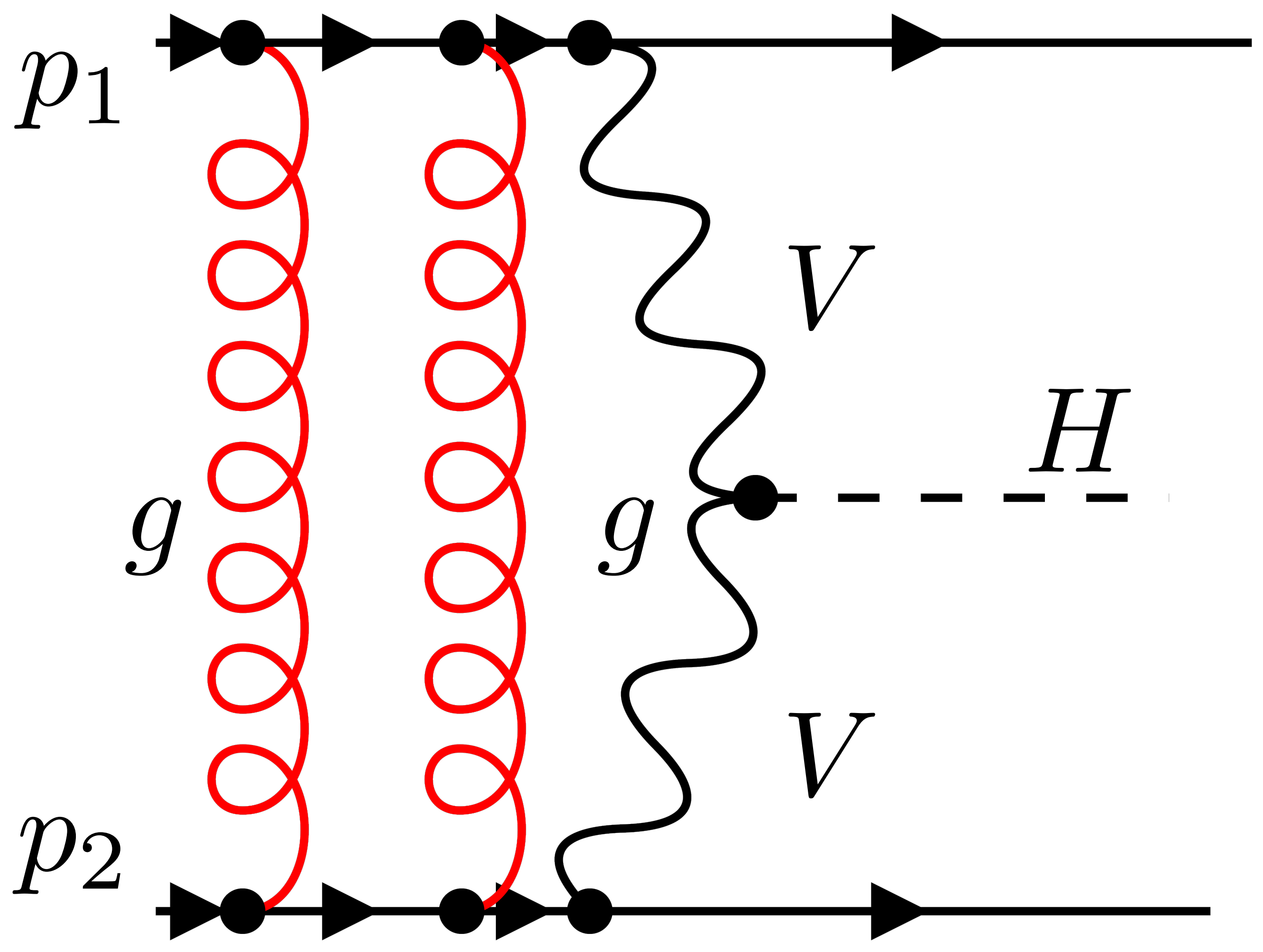
1) eikonal propagators for quark lines: $(2pk + i0)^{-1}$

2) eikonal couplings of quarks to gluons: $-2ie\, p_\mu$

3) no longitudinal loop momenta components in gluon and vector boson propagators: $k^{-2} \to -k_\perp^{-2}$

$$k = \alpha p_1 + \beta p_2 + k_\perp$$

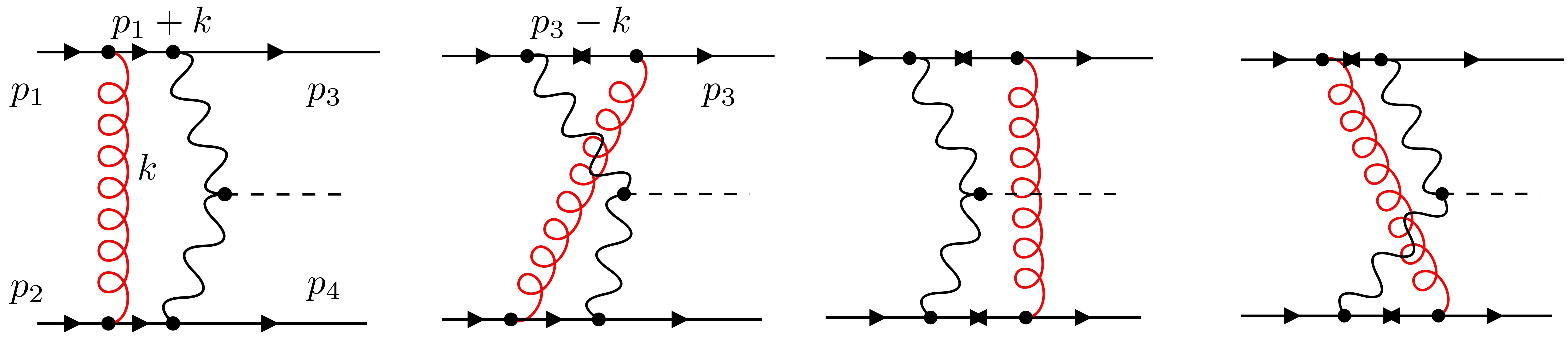
**Sudakov, Lipatov, Gribov, Cheng, Wu, Chang, Ma**




Typical WBF cuts

$$p_\perp^{j_1,j_2} > 25 \text{ GeV}, \quad |y_{j_1,j)2}| < 4.5$$
$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600 \text{ GeV}$$

Dramatic simplifications occur if the eikonal approximation for integrands are used and if various diagrams, that contribute to the amplitude, are combined before attempting to integrate them over the loop momentum.



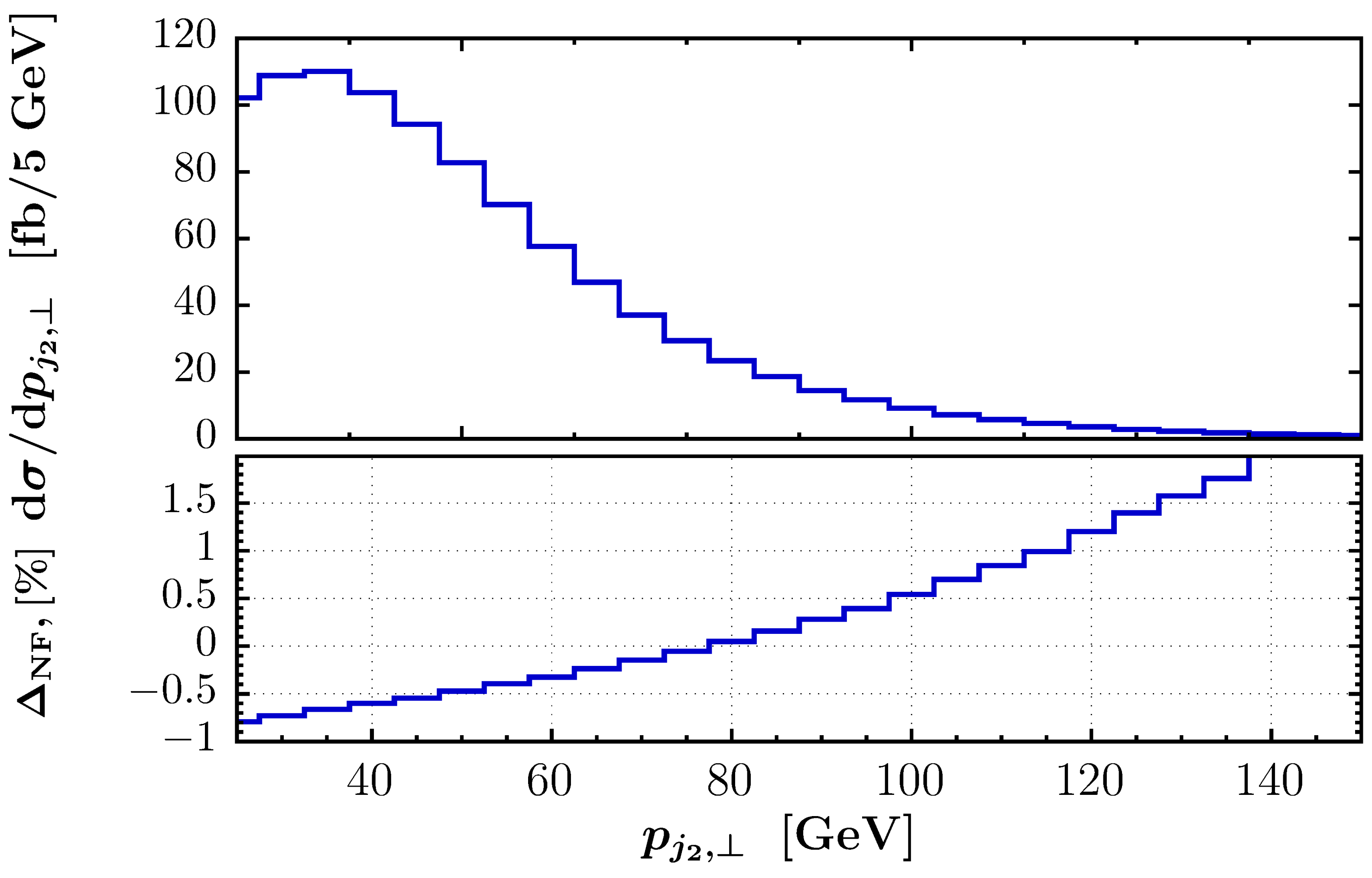

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k_\perp^2} \frac{1}{(k_\perp - q_{3,\perp})^2 + M_V^2} \frac{1}{(k_\perp + q_{4,\perp})^2 + M_V^2} \left[\frac{1}{2p_1k + i0} + \frac{1}{-2p_3k + i0}\right]\left[\frac{1}{-2p_2k + i0} + \frac{1}{2p_4k + i0}\right]$$

$$\lim_{p_3 \to p_1} \left[\frac{1}{2p_1k + i0} - \frac{1}{-2p_3k + i0}\right] = -\frac{2i\pi}{s}\delta(\beta) \qquad \lim_{p_4 \to p_2} \left[\frac{1}{-2p_2k + i0} + \frac{1}{2p_4k + i0}\right] = -\frac{2i\pi}{s}\delta(\alpha)$$

$$k = \alpha p_1 + \beta p_2 + k_\perp \qquad d^4k = \frac{s}{2}\, d\alpha\, d\beta\, d^2k_\perp$$

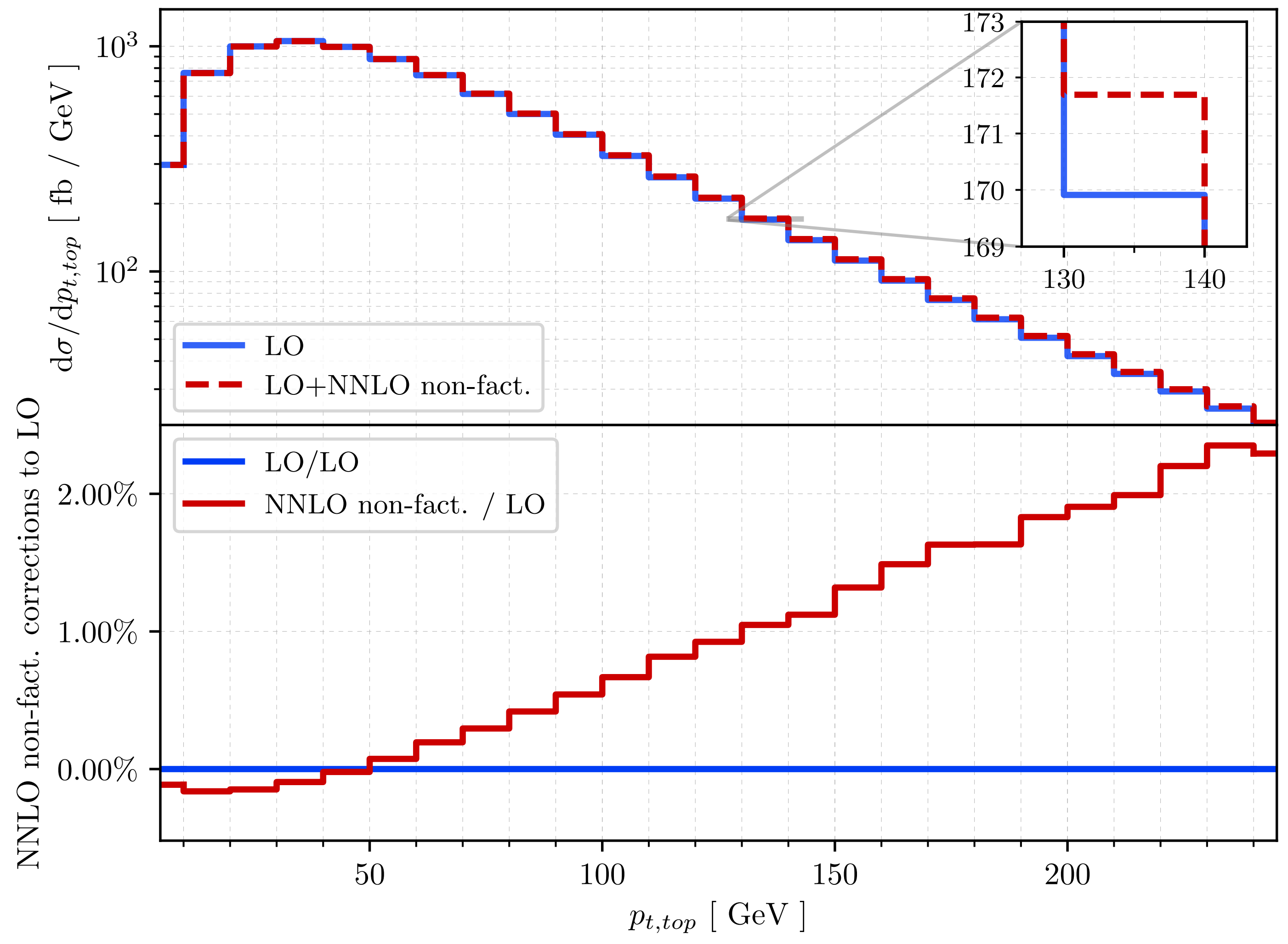
We find non-factorizable corrections to Higgs production in WBF to be between 0.5 and 1 percent, depending on a kinematic distribution. This result is not significantly smaller than NNLO QCD factorizable corrections and is more important than the N3LO QCD ones.

For the single top production, the exact calculation can be done using semi-numerical methods and very similar results are obtained.




Higgs production in WBF

Liu, Melnikov, Penin



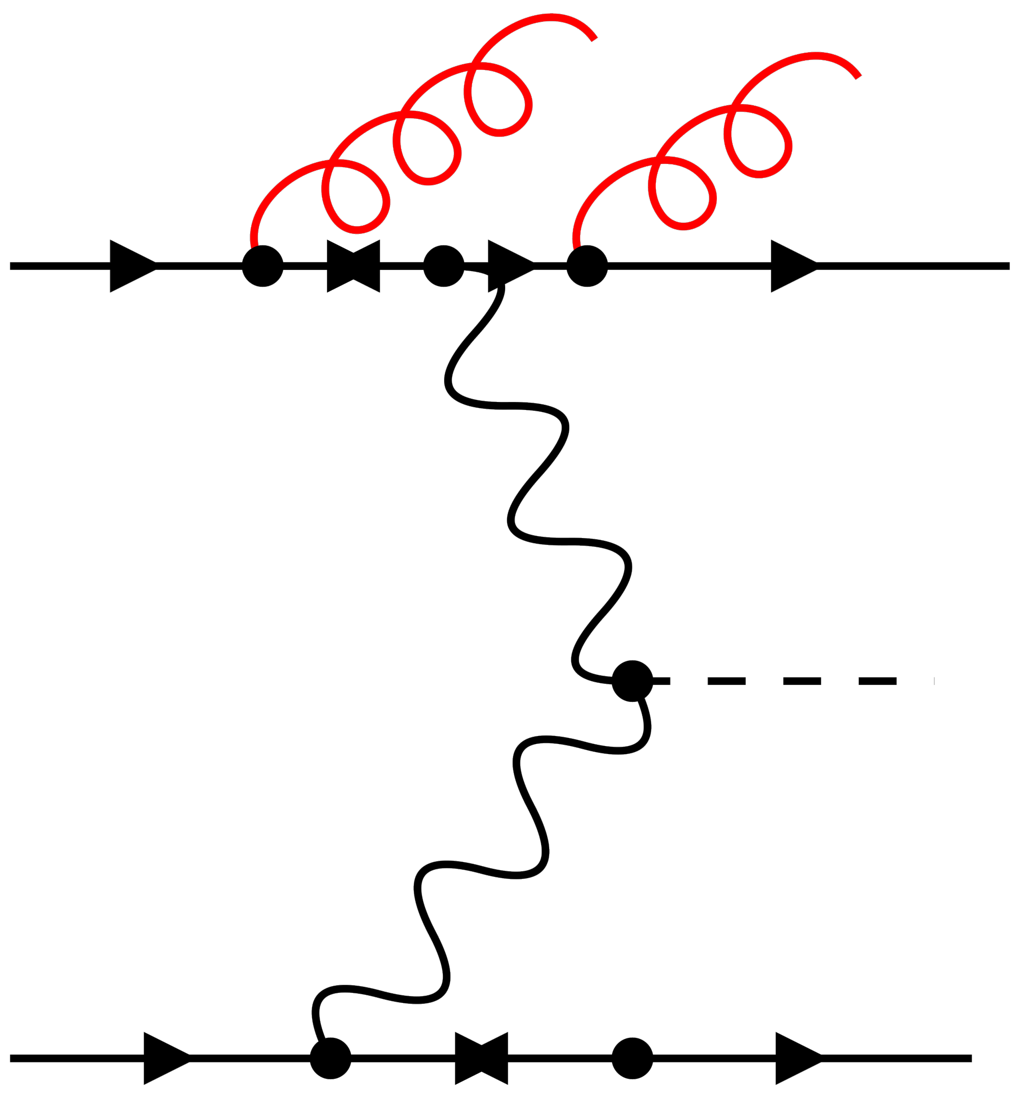

Single top production

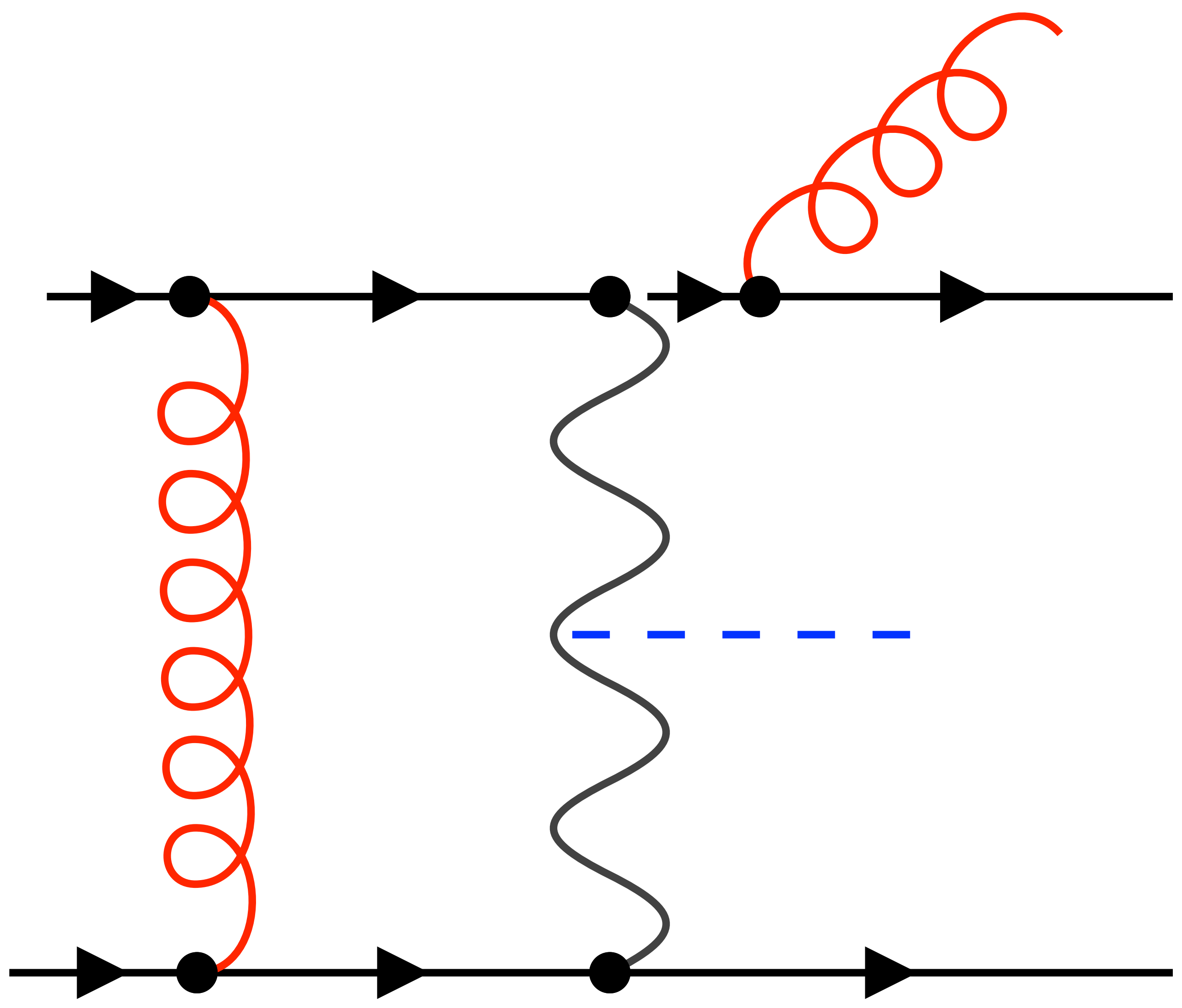
Quarroz, Bronnum-Hansen, Melnikov, Wang

Since non-factorizable contributions may be somewhat relevant, two questions arise:

1) how reliable is the leading eikonal approximation for Higgs boson production in WBF?

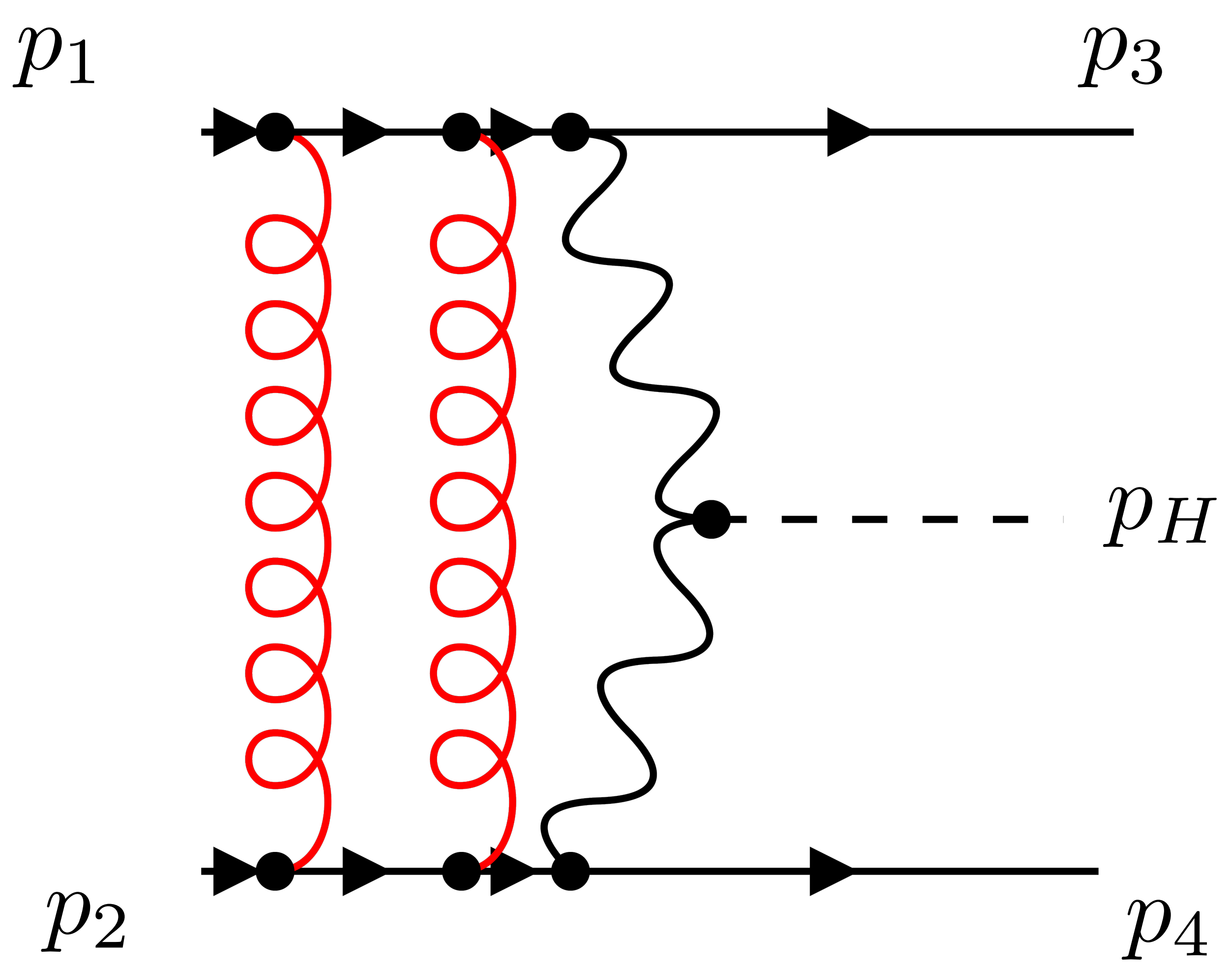
2) how important are all other contributions to NNLO corrections (the emission of two gluons or a one-loop correction to the single gluon emission) in the non-factorizable case? This question applies to both single top production and to Higgs production in WBF.





To go beyond the leading eikonal approximation, we need to parametrise the external momenta in a way that makes it clear how the forward scattering limit is approached.

It follows from the Higgs boson on-shell condition that changes in the "longitudinal" momenta fractions are proportional to the absolute value of the transverse momentum, not the transverse momentum squared.




$$q(p_1) + q(p_2) \to q(p_3) + q(p_4) + H(p_H)$$

$$p_3 = \alpha_3 p_1 + \beta_3 p_2 + p_{3,\perp} \qquad p_4 = \alpha_4 p_1 + \beta_4 p_2 + p_{4,\perp}$$

$$\beta_3 = \frac{\mathbf{p}_{3,\perp}^2}{s\alpha_3}, \quad \alpha_4 = \frac{\mathbf{p}_{4,\perp}^2}{s\beta_4}$$

$$\delta_3 = 1 - \alpha_3 \qquad \delta_4 = 1 - \beta_4$$

$$\boxed{\delta_3 \delta_4 s = m_H^2 + \frac{\mathbf{p}_{3,\perp}^2}{\alpha_3} + \frac{\mathbf{p}_{4,\perp}^2}{\beta_4} + 2\mathbf{p}_{3,\perp} \cdot \mathbf{p}_{4,\perp} - \frac{\mathbf{p}_{3,\perp}^2 \mathbf{p}_{4,\perp}^2}{\alpha_3 \beta_4 s}}$$

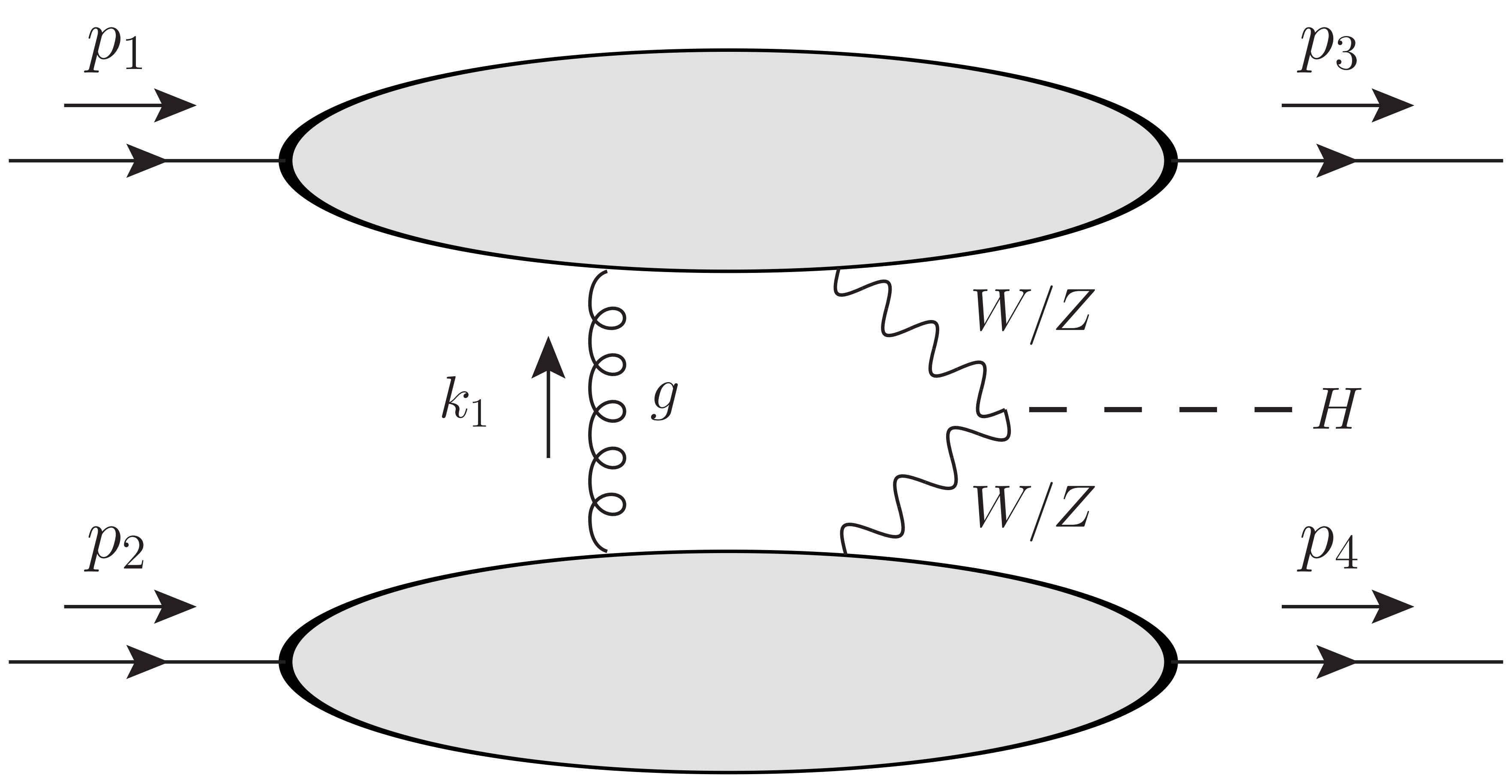
$$\delta_3 \delta_4 \sim \frac{m_V^2}{s} \sim \frac{m_H^2}{s} \sim \frac{\mathbf{p}_{3,\perp}^2}{s} \sim \frac{\mathbf{p}_{4,\perp}^2}{s} \sim \lambda \ll 1$$

$$\boxed{\delta_3 \sim \delta_4 \sim \sqrt{\lambda}}$$

To understand how to expand the virtual amplitude for Higgs boson production in WBF around the eikonal limit, we study the dependence of the integrand on the small parameter and identify various "integration regions". A power counting applied to individual diagrams indicates that, already at leading power, a large number of various regions contributes.

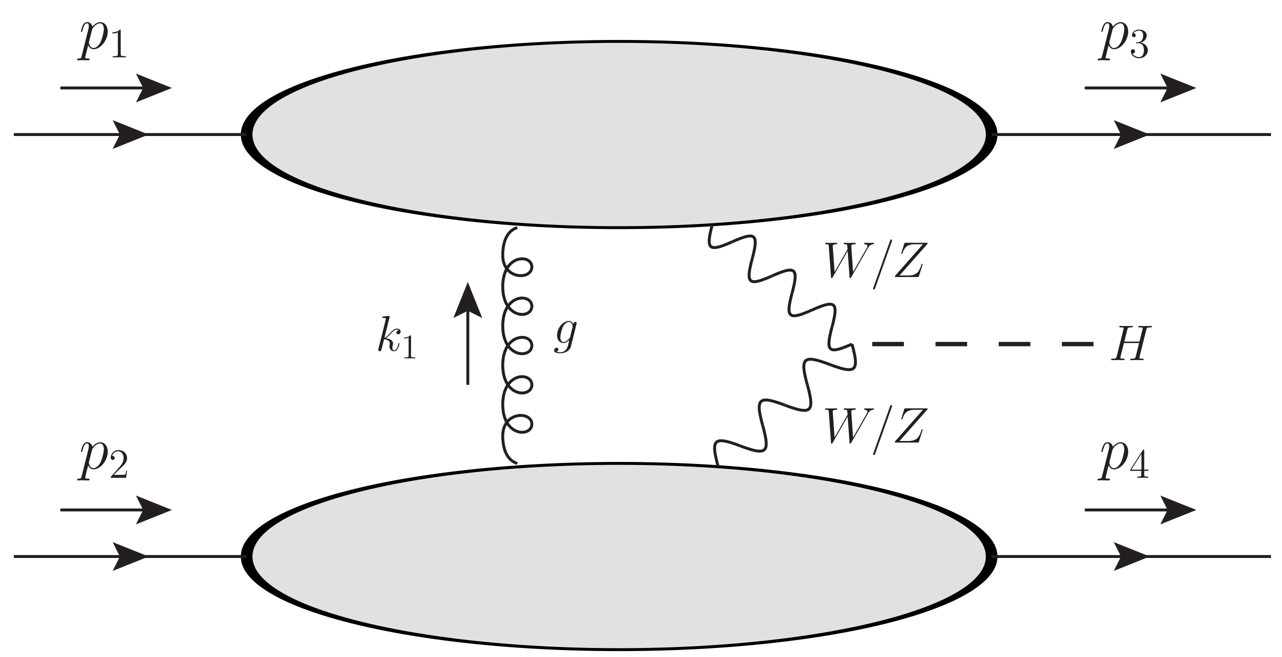
$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}$$

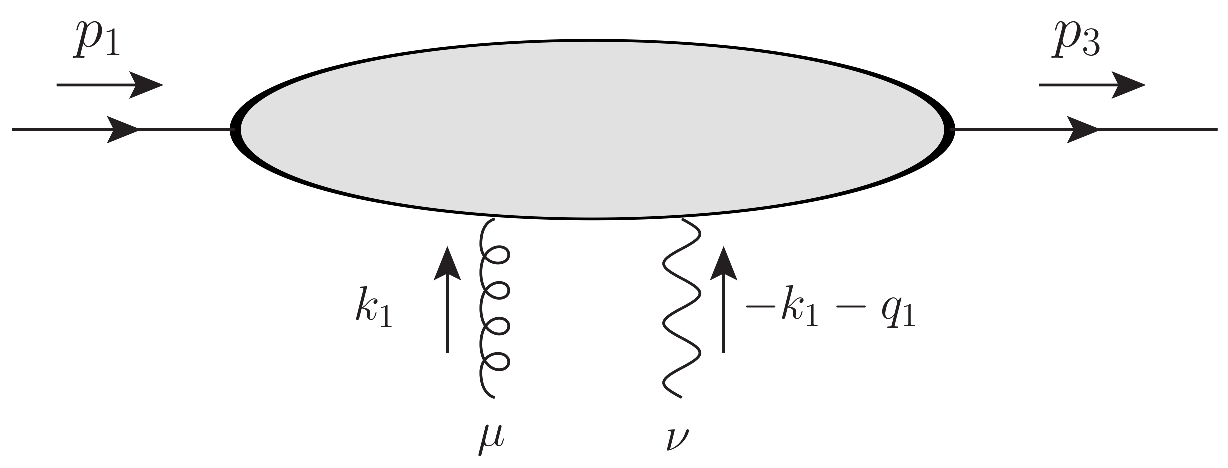
| Region | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ |
|---|---|---|---|
| a | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ |
| b | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| c | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| d | 1 | $\lambda$ | $\sqrt{\lambda}$ |
| e | 1 | 1 | 1 |




$$\mathcal{M}^{(a)} \sim \lambda^{-2}, \quad \mathcal{M}^{(b)} \sim \lambda^{-2}, \quad \mathcal{M}^{(c)} \sim \lambda^{-2}, \quad \mathcal{M}^{(d)} \sim \lambda^{-3/2}, \quad \mathcal{M}^{(e)} \sim 1$$

We now proceed in a standard way. Starting from the general expression for the integrand, we apply scaling of the external momenta and the loop momentum in a particular region to simplify the various propagators and, eventually, the integrand.




$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}$$

| Region | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ |
|---|---|---|---|
| a | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ |
| b | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| c | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| d | 1 | $\lambda$ | $\sqrt{\lambda}$ |
| e | 1 | 1 | 1 |

$$\mathcal{A}_1 = \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{1}{d_1 d_3 d_4} J_{\mu\nu}(k_1, -k_1 - q_1)\, \tilde{J}^{\mu\nu}(-k_1, k_1 - q_2)$$

$$d_1 = k_1^2 + i0, \quad d_3 = (k_1 + q_1)^2 - m_V^2 + i0, \quad d_4 = (k_1 - q_2)^2 - m_V^2 + i0$$

$$\rho_i(k) = \frac{1}{(p_i + k)^2 + i0} \qquad i = 1, 2, 3, 4$$

$$J^{\mu\nu}(k_1, -k_1 - q_1) = \langle 3| \left[ \frac{\gamma^\nu(\hat{p}_1 + \hat{k}_1)\gamma^\mu}{\rho_1(k_1)} + \frac{\gamma^\mu(\hat{p}_3 - \hat{k}_1)\gamma^\nu}{\rho_3(-k_1)} \right] |1]$$

$$\tilde{J}^{\mu\nu}(-k_1, k_1 - q_2) = \langle 4| \left[ \frac{\gamma^\nu(\hat{p}_2 + \hat{k}_1)\gamma^\mu}{\rho_2(k_1)} + \frac{\gamma^\nu(\hat{p}_4 - \hat{k}_1)\gamma_\mu}{\rho_4(-k_1)} \right] |2]$$

Simplifications in region "a"

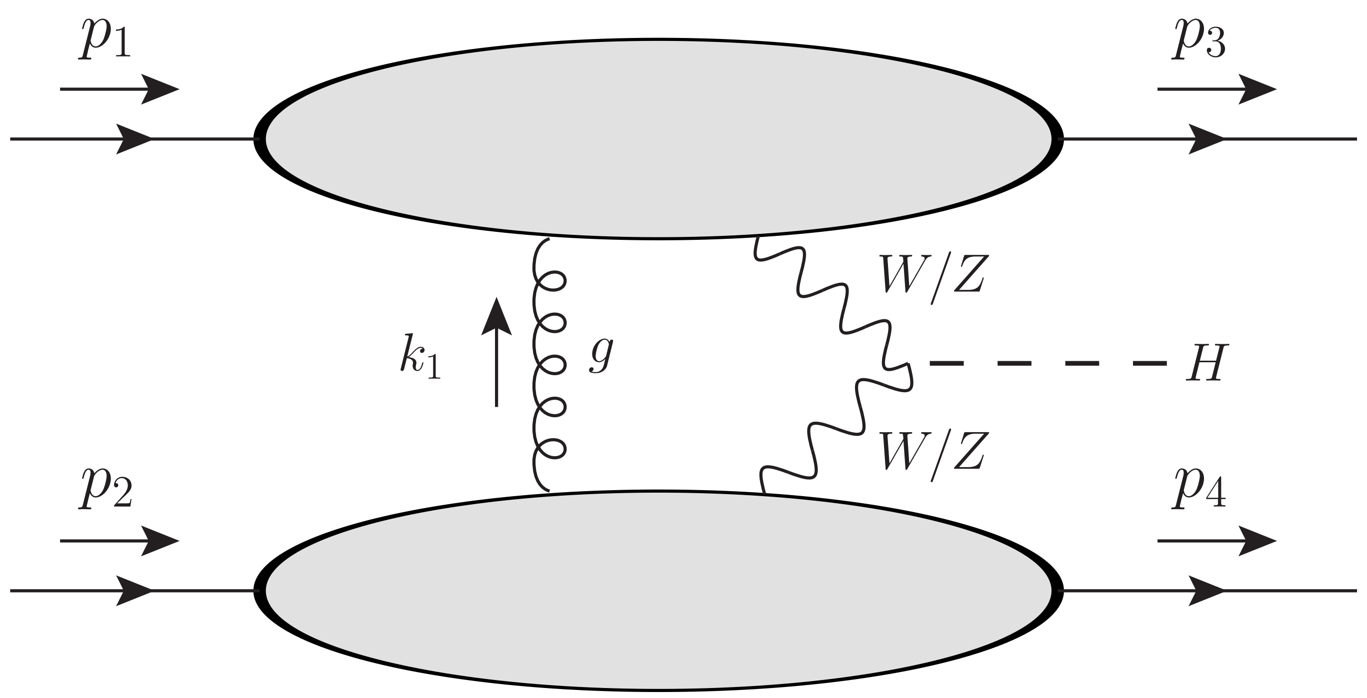
$$d_1 = \alpha_1 \beta_1 - \mathbf{k}_{1,\perp}^2 \to -\mathbf{k}_{1,\perp}^2$$

$$\rho_1(k_1) = k_1^2 + 2p_1 k_1 \to s\beta_1 - \mathbf{k}_{1,\perp}^2$$

$$\rho_2(-k_1) = k_1^2 - 2p_2 k_1 \to -s\alpha_1 - \mathbf{k}_{1,\perp}^2.$$

The possibility  to compute both leading and first subleading correction to the non-factorizable amplitude in WBF with a relative ease is the consequence of the fact that integrations over two longitudinal components of the loop momentum factorize and that gauge cancellations ensure that contributions of many regions are suppressed relative to expectations based on “naive” power counting.

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}$$




$$\mathcal{A}_1^{(a)} = -\frac{s}{2} \int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}} \frac{1}{\Delta_1 \Delta_{3,1} \Delta_{4,1}} \Phi^{\mu\nu} \, \tilde{\Phi}_{\mu\nu}$$

$$\Delta_1 = -\mathbf{k}_{1,\perp}^2$$

$$\Delta_{3,1} = -(\mathbf{k}_{1,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2$$

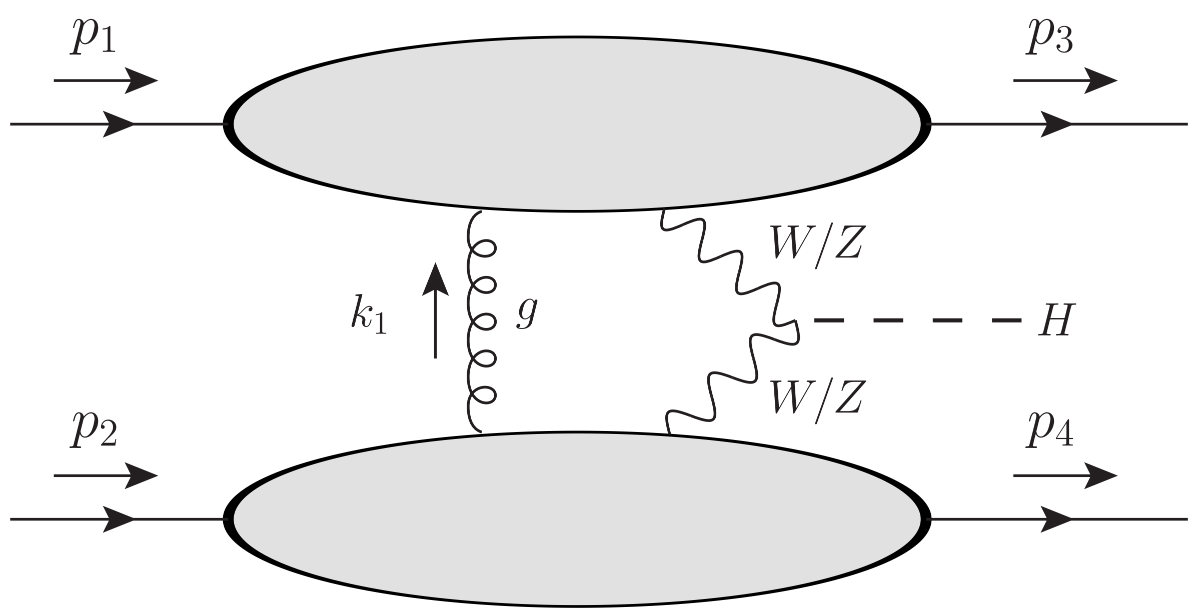
$$\Delta_{4,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2$$

$$\Phi^{\mu\nu} = \int\limits_{-\sigma}^{\sigma} \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\Delta_{3,1}}{s\delta_3(\beta_1 - \beta_3) + \Delta_{3,1} + i0} \langle 3| \left[ \frac{\gamma^\nu (\hat{p}_1 + \hat{k}_{1,\perp})\gamma^\mu}{s\beta_1 + \Delta_1 + i0} + \frac{\gamma^\mu (\hat{p}_3 - \hat{k}_{1,\perp})\gamma^\nu}{-s\alpha_3\beta_1 + \Theta_{3,1} + i0} \right] |1]$$

$$\tilde{\Phi}^{\mu\nu} = \int\limits_{-\sigma}^{\sigma} \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\Delta_{4,1}}{-s\delta_4(\alpha_1 + \alpha_4) + \Delta_{4,1} + i0} \langle 4| \left[ \frac{\gamma^\nu (p_2 + \hat{k}_{1,\perp})\gamma^\mu}{-s\alpha_1 + \Delta_1 + i0} + \frac{\gamma^\nu (p_4 - \hat{k}_{1,\perp})\gamma^\mu}{s\beta_4\alpha_1 + \Theta_{4,1} + i0} \right] |2]$$

In region "b", gauge cancellations ensure additional suppression of one of the currents. Similar cancellations allow us to completely discard regions "c" and "d" at next-to-leading power.

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}$$




$$\Delta_1 = -\mathbf{k}_{1,\perp}^2$$
$$\Delta_{4,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2$$
$$\Delta_{3,1} = -(\mathbf{k}_{1,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2$$
$$\Theta_{3,1} = -\left(\mathbf{k}_{1,\perp}^2 - 2\mathbf{k}_{1,\perp}\cdot\mathbf{p}_{3,\perp}\right)$$
$$\Theta_{4,1} = -\left(\mathbf{k}_{1,\perp}^2 + 2\mathbf{k}_{1,\perp}\cdot\mathbf{p}_{4,\perp}\right)$$

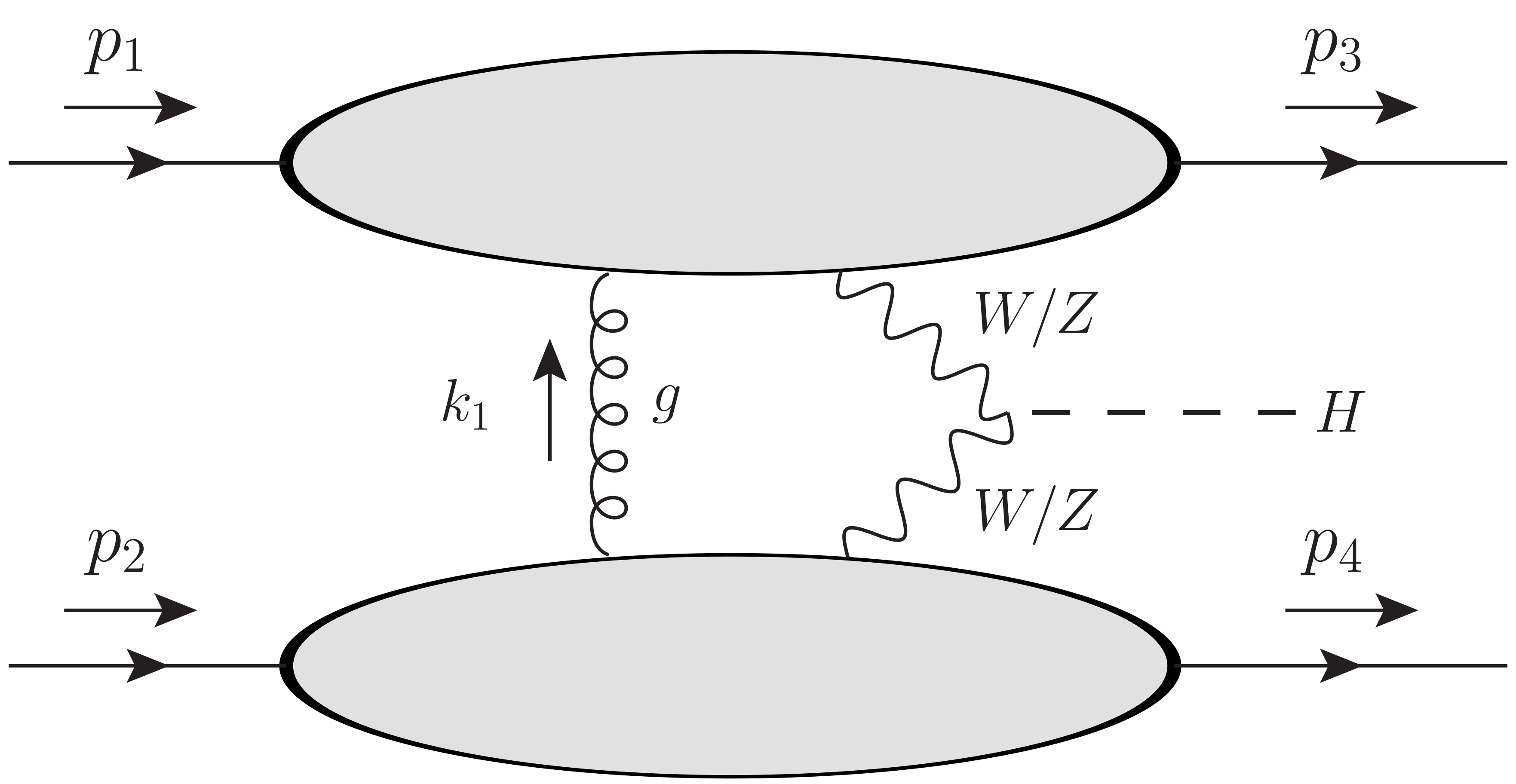
| Region | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ |
|---|---|---|---|
| a | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ |
| b | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| c | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| d | 1 | $\lambda$ | $\sqrt{\lambda}$ |
| e | 1 | 1 | 1 |

$$J^{\mu\nu}(k_1, -q_1 - k_1) \approx p_1^\mu p_1^\nu \left(\frac{1}{s\beta_1 + \Delta_1 + i0} + \frac{\alpha_3}{-s\alpha_3\beta_1 + \Theta_{3,1} + i0}\right) \approx -\frac{p_1^\mu p_1^\nu}{s\beta_1^2}(\Delta_1 + \Theta_{3,1})$$

$$\mathcal{A}_1^{(b)} = -\langle 3|\gamma_\mu|1]\langle 4|\gamma^\mu|2] \int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}} \frac{1}{\Delta_1\Delta_{3,1}\Delta_{4,1}} \Delta\Phi\,\tilde{\Phi}$$

$$\Delta\Phi = \left(-\frac{\Delta_1}{s} - \frac{\Theta_{3,1}}{s}\right)\int\limits_{-\infty}^{\infty} \frac{\mathrm{d}\beta_1}{2\pi i} \frac{(\theta(\beta_1 - \sigma) + \theta(-\sigma - \beta_1))\Delta_{3,1}}{(s\delta_3\beta_1 + \Delta_{3,1} + i0)\ \beta_1^2} \qquad \tilde{\Phi} = (-1)\left[1 + \frac{\delta_4}{2\Delta_{4,1}}(2s\alpha_4 + \Delta_1 - \Theta_{4,1})\right] \to -1$$

The final result for the one-loop non-factorisable amplitude with next-to-eikonal accuracy is given by a simple integral over transversal components of the loop momentum.




| Region | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ |
|---|---|---|---|
| a | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ |
| b | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| c | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ |
| d | 1 | $\lambda$ | $\sqrt{\lambda}$ |
| e | 1 | 1 | 1 |

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}$$

$$\Delta_1 = -\mathbf{k}_{1,\perp}^2$$

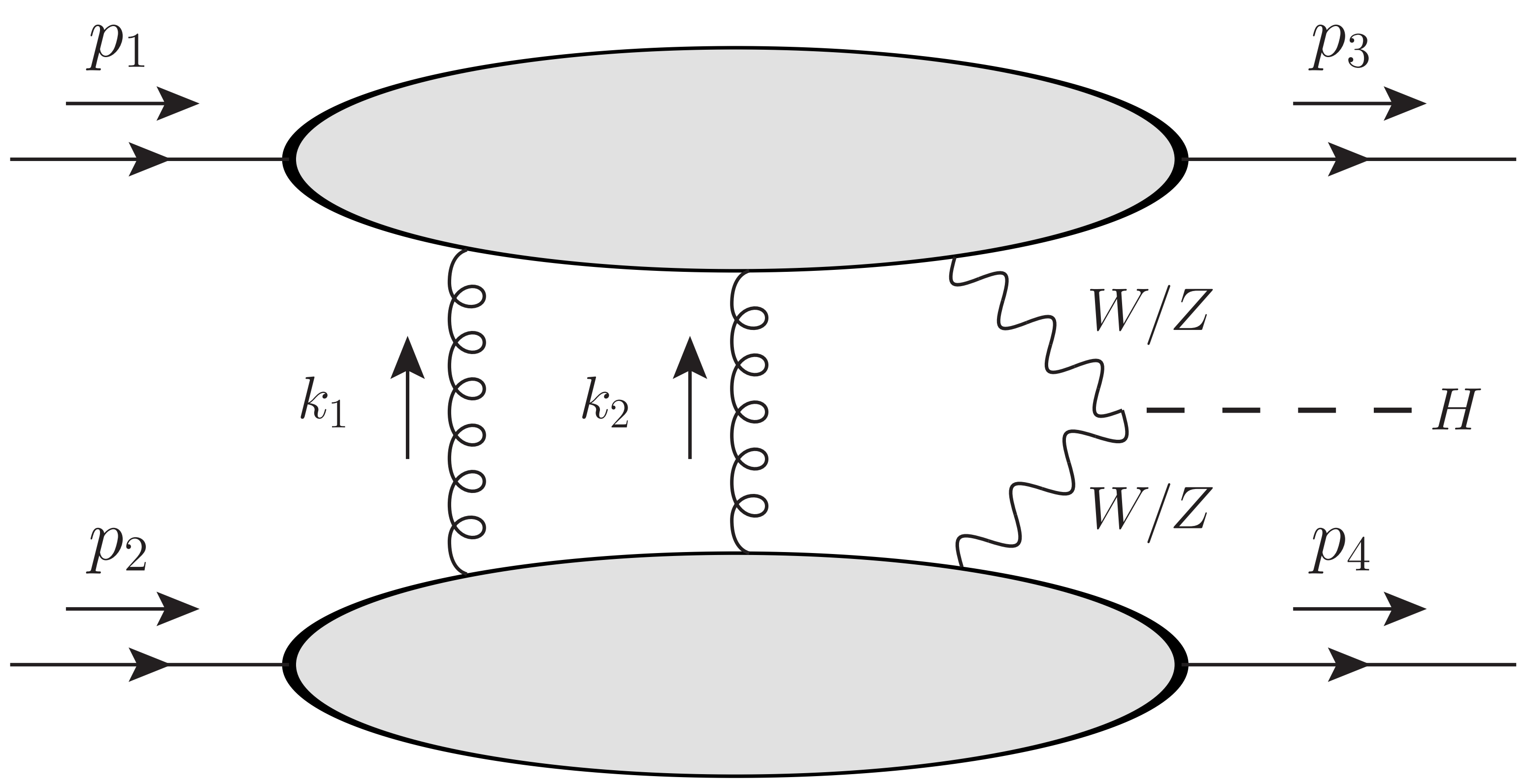
$$\Delta_{3,1} = -(\mathbf{k}_{1,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2$$

$$\Delta_{4,1} = -(\mathbf{k}_{1,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2$$

$$\mathcal{M}_1 = i\frac{g_s^2}{4\pi}\, T^a_{i_3 i_1} T^a_{i_4 i_2}\, \mathcal{M}_0\, \mathcal{C}_1$$

$$\mathcal{C}_1 = 2\int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}} \frac{(\mathbf{p}_{3,\perp}^2 + m_V^2)(\mathbf{p}_{4,\perp}^2 + m_V^2)}{\Delta_1 \Delta_{3,1} \Delta_{4,1}} \left[1 - \delta_3\left(\frac{m_V^2}{\mathbf{p}_{3,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{3,1}}\right) - \delta_4\left(\frac{m_V^2}{\mathbf{p}_{4,\perp}^2 + m_V^2} + \frac{m_V^2}{\Delta_{4,1}}\right)\right]$$

A similar analysis of the two-loop amplitude leads to the conclusion that contributions of momenta regions which are identical to the one-loop case need to be considered. Importantly, factorization works in a very similar manner, so that the "upper-line" and the "lower-line" currents factorize and can be treated separately.




$$k_{1,2} = \alpha_{1,2} p_1 + \beta_{1,2} p_2 + k^{\mu}_{\perp,1,2}$$

$$\Delta_i = -\mathbf{k}_{i,\perp}^2$$

$$\Delta_{3,i} = -(\mathbf{k}_{i,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2$$

$$\Delta_{4,i} = -(\mathbf{k}_{i,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2$$

$$\mathcal{M}_2 = -\frac{1}{2}\frac{g_s^4}{(4\pi)^2}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_3 i_1}\left(\frac{1}{2}\{T^a,T^b\}\right)_{i_4 i_2}\mathcal{M}_0\,\mathcal{C}_2$$

$$\mathcal{C}_2 = 4\int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}}\frac{\mathrm{d}^{d-2}\mathbf{k}_{2,\perp}}{\pi(2\pi)^{1-2\epsilon}}\frac{(\mathbf{p}_{3,\perp}^2+m_V^2)(\mathbf{p}_{4,\perp}^2+m_V^2)}{\Delta_1\Delta_2\Delta_{3,12}\Delta_{4,12}}\left[1-\delta_3\left(\frac{m_V^2}{\mathbf{p}_{3,\perp}^2+m_V^2}+\frac{m_V^2}{\Delta_{3,12}}\right)-\delta_4\left(\frac{m_V^2}{\mathbf{p}_{4,\perp}^2+m_V^2}+\frac{m_V^2}{\Delta_{4,12}}\right)\right]$$

The cross section is obtained by computing the sum of the square of the one-loop amplitude and the interference of the two-loop amplitude with the Born amplitude. In this combination, the infra-red divergences cancel out and the finite remainder is obtained. The finite remainder can be computed analytically, by applying standard methods of multi-loop computations, albeit at d = 2.

$$\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{\mathrm{nf}} = \frac{N_c^2-1}{4N_c^2}\,\alpha_s^2\,\mathcal{C}_{\mathrm{nf}}\,\mathrm{d}\hat{\sigma}^{\mathrm{LO}}$$

$$\mathcal{C}_1 = -\frac{1}{\epsilon} + \mathcal{C}_{1,0} + \epsilon\,\mathcal{C}_{1,1} \qquad \mathcal{C}_2 = \frac{1}{\epsilon^2} - \frac{2}{\epsilon}\mathcal{C}_{1,0} + \mathcal{C}_{2,0}$$

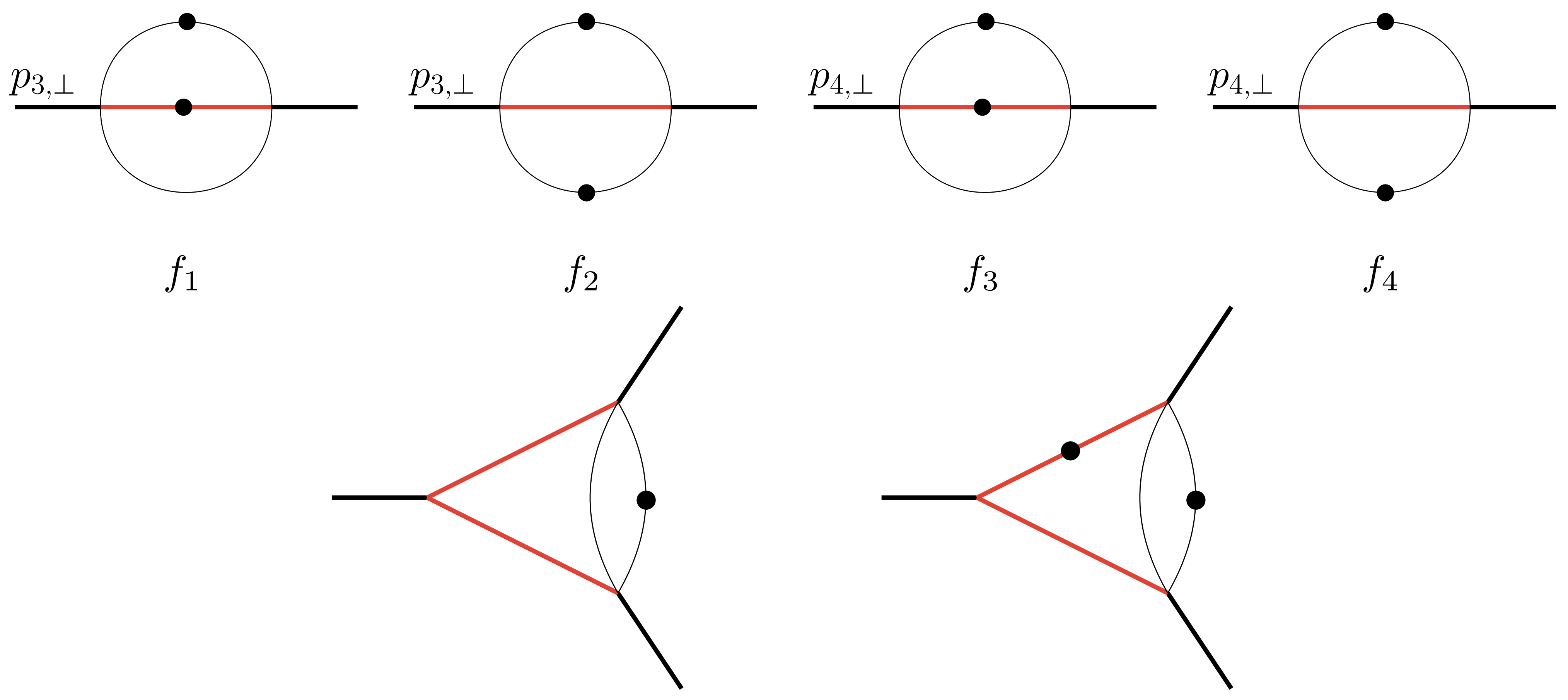
$$\mathcal{C}_{\mathrm{nf}} = \mathcal{C}_{1,0}^2 - 2\,\mathcal{C}_{1,1} - \mathcal{C}_{2,0}$$

$$j[a_1,a_2,a_3,a_4] = \frac{(m_V^2)^{2\epsilon}}{\pi^{d-2}\Gamma(1+\epsilon)^2}\int \frac{\mathrm{d}\mathbf{k}_{1,\perp}^{d-2}\mathrm{d}\mathbf{k}_{2,\perp}^{d-2}}{\Delta_1^{a_1}\Delta_2^{a_2}\Delta_{3,12}^{a_3}\Delta_{4,12}^{a_4}}$$

$$\Delta_i = -\mathbf{k}_{i,\perp}^2$$

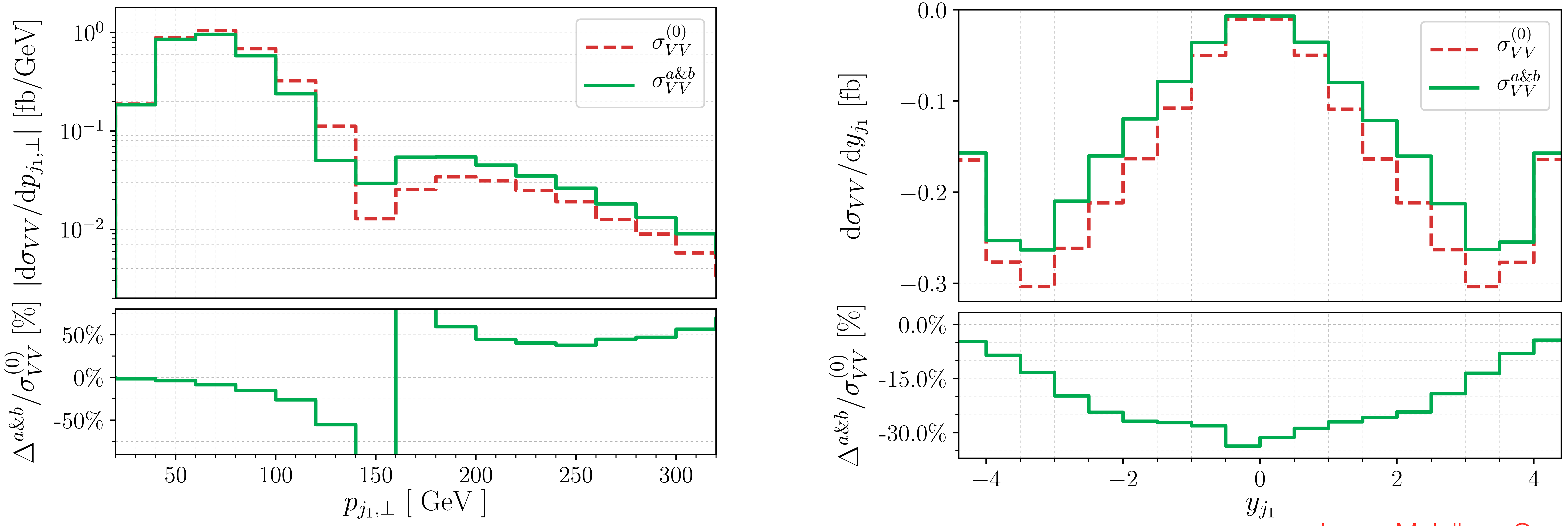
$$\Delta_{3,i} = -(\mathbf{k}_{i,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2$$

$$\Delta_{4,i} = -(\mathbf{k}_{i,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2$$




$$x = \frac{\mathbf{p}_{3,\perp}^2}{m_V^2}, \quad y = \frac{\mathbf{p}_{4,\perp}^2}{m_V^2}, \quad z = \frac{\mathbf{p}_{H,\perp}^2}{m_V^2}$$

The next-to-leading terms change the leading eikonal contribution to the non-factorizable corrections by thirty percent, depending on the observable. Hence, the eikonal expansion is not perfect but, most likely, it provides a reasonable order-of-magnitude estimate for non-factorizable corrections.




Long, Melnikov, Quarroz

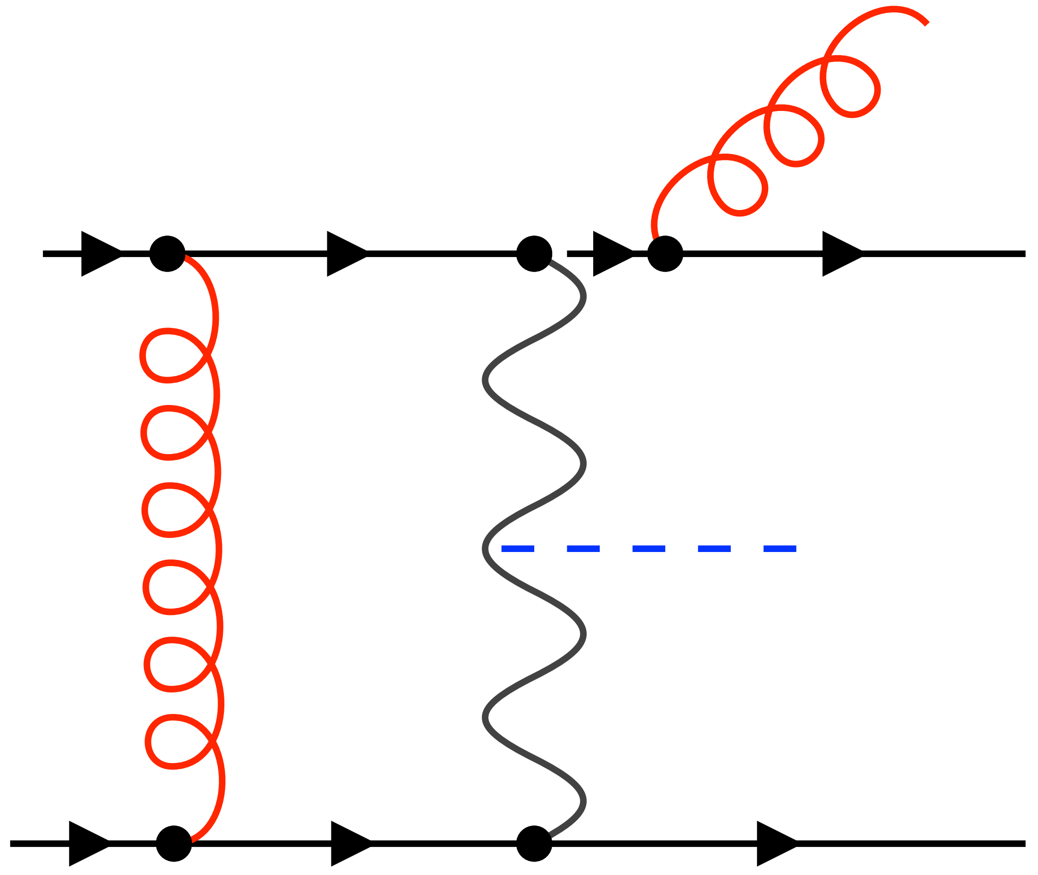
$$\mathrm{d}\hat{\sigma}_{\mathrm{nf}}^{\mathrm{NNLO}} = \frac{N_c^2-1}{4N_c^2}\,\alpha_s^2\,\mathcal{C}_{\mathrm{nf}}\,\mathrm{d}\hat{\sigma}^{\mathrm{LO}}$$

$$p_\perp^{j_1,j_2} > 25\ \mathrm{GeV}, \quad |y_{j_1,j)2}| < 4.5$$
$$|y_{j_1} - y_{j_2}| > 4.5, \quad m_{j_1 j_2} > 600\ \mathrm{GeV}$$

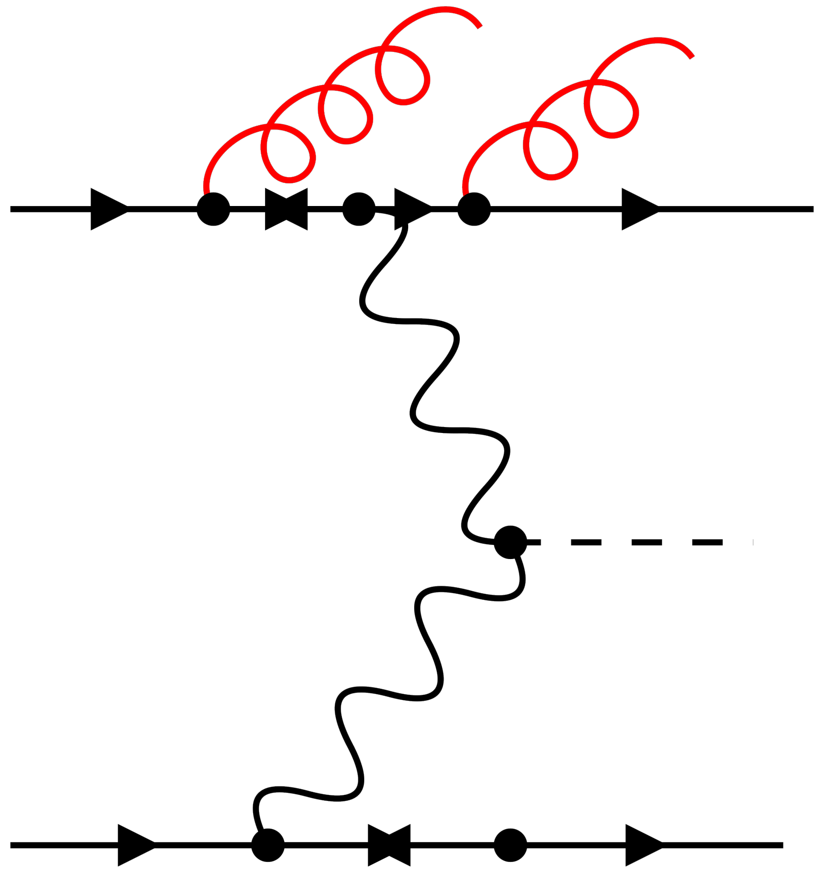
$$\sigma_{\mathrm{nf}}^{\mathrm{NNLO}} = (-3.1 + 0.53)\ \mathrm{fb}$$

Other contributions that need to be considered for computing physical quantities (such as the cross section) are the double-real emission contribution and the real-virtual contribution. An important problem that one faces, when dealing with these terms, is to extract and remove singularities that arise upon integration over energies and angles of the emitted gluon (soft and collinear singularities).

A general solution to this problem requires the development of intricate infra-red subtraction schemes. However, in case of non-factorizable corrections, this problem simplifies because 1) there are no collinear singularities (emissions and absorptions must occur on different lines) and 2) the non-factorizable corrections are, effectively, abelian.



Real-virtual contribution to NNLO corrections

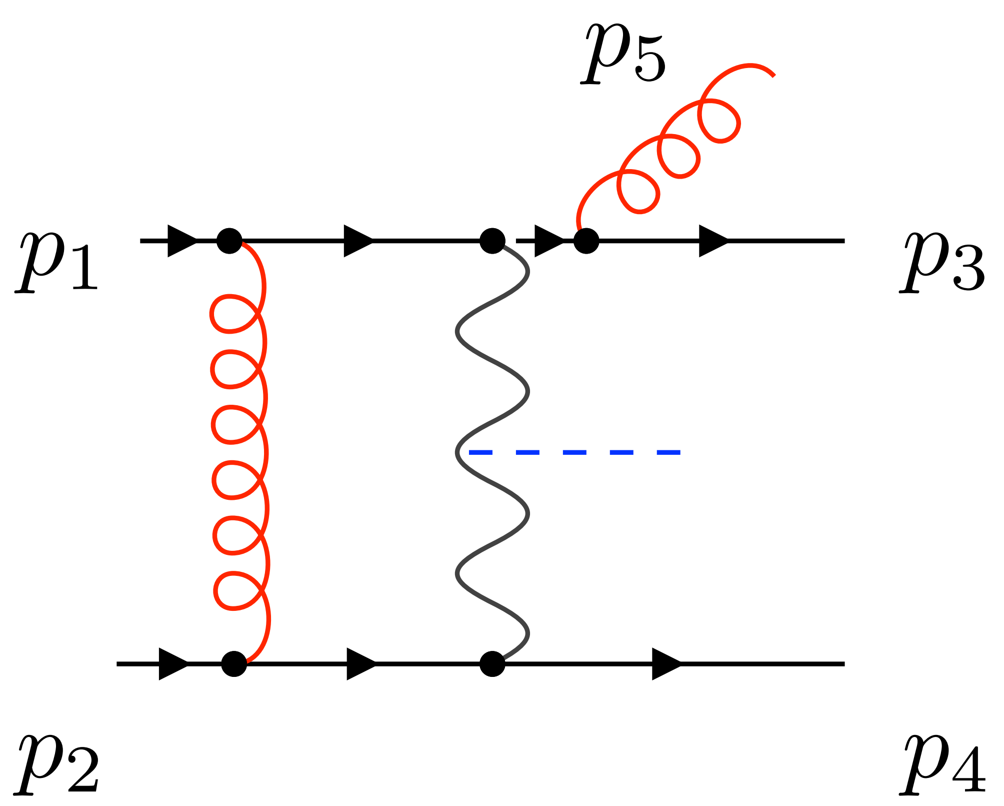


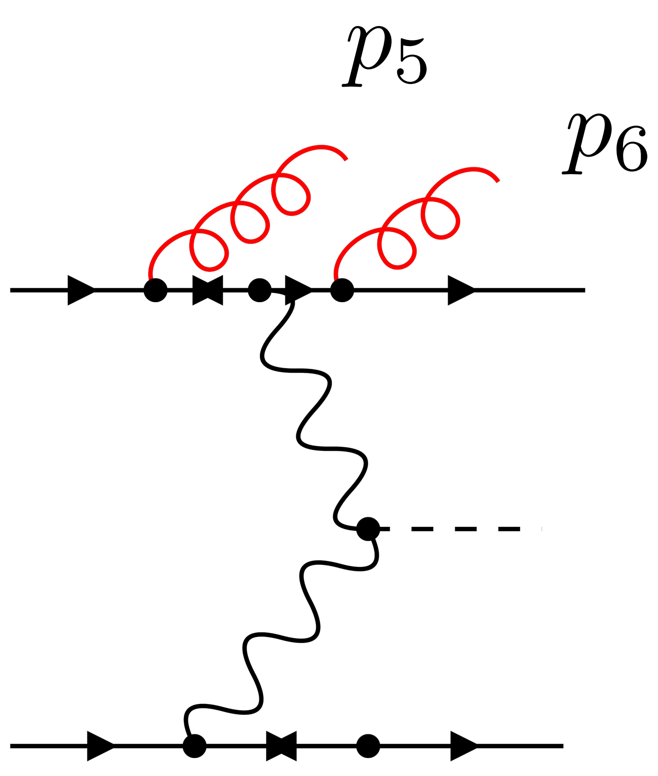
Double-real contribution to NNLO corrections

Iterative subtraction of soft singularities from the double-real and real-virtual contributions as well as the use of Catani's formula for divergences of double-virtual corrections leads to the following compact result for complete NNLO contribution to differential cross section.

$$
\begin{aligned}
d\sigma^{\rm nf}_{\rm nnlo} &= \frac{T_R^2(N_c^2-1)}{2s}\Big[\big\langle F^{\rm nf}_{\rm LM}(1,2,3,4\,|\,5,6)\big\rangle \\
&\quad + \big\langle F^{\rm nf}_{\rm LV}(1,2,3,4\,|\,5)\big\rangle + \big\langle F^{\rm nf}_{\rm LVV}(1,2,3,4)\big\rangle\Big] \\
&= \frac{T_R^2(N_c^2-1)}{2s}\Big[\big\langle [I-S_6]\,F^{\rm nf}_{\rm LM}(1,2,3,4\,|\,5,6)\big\rangle \\
&\quad - 2\,\frac{\tilde\alpha_s}{2\pi}\big\langle [I-S_5]\,\mathcal{W}(E_5;1,..,4)\,F^{\rm nf}_{\rm LM}(1,2,3,4\,|\,5)\big\rangle \\
&\quad + 2\left(\frac{\tilde\alpha_s}{2\pi}\right)^2\big\langle \mathcal{W}(E_{\rm max};1,..,4)^2 F^{\rm nf}_{\rm LM}(1,2,3,4)\big\rangle \\
&\quad + \big\langle [I-S_5]\,F^{\rm nf}_{\rm LV,fin}(1,2,3,4\,|\,5)\big\rangle \\
&\quad - 2\,\frac{\tilde\alpha_s}{2\pi}\big\langle \mathcal{W}(E_{\rm max};1,..,4)\,F^{\rm nf}_{\rm LV,fin}(1,2,3,4)\big\rangle \\
&\quad + \big\langle F^{\rm nf}_{\rm LVV,fin}(1,2,3,4)\big\rangle\Big]\,.
\end{aligned}
$$

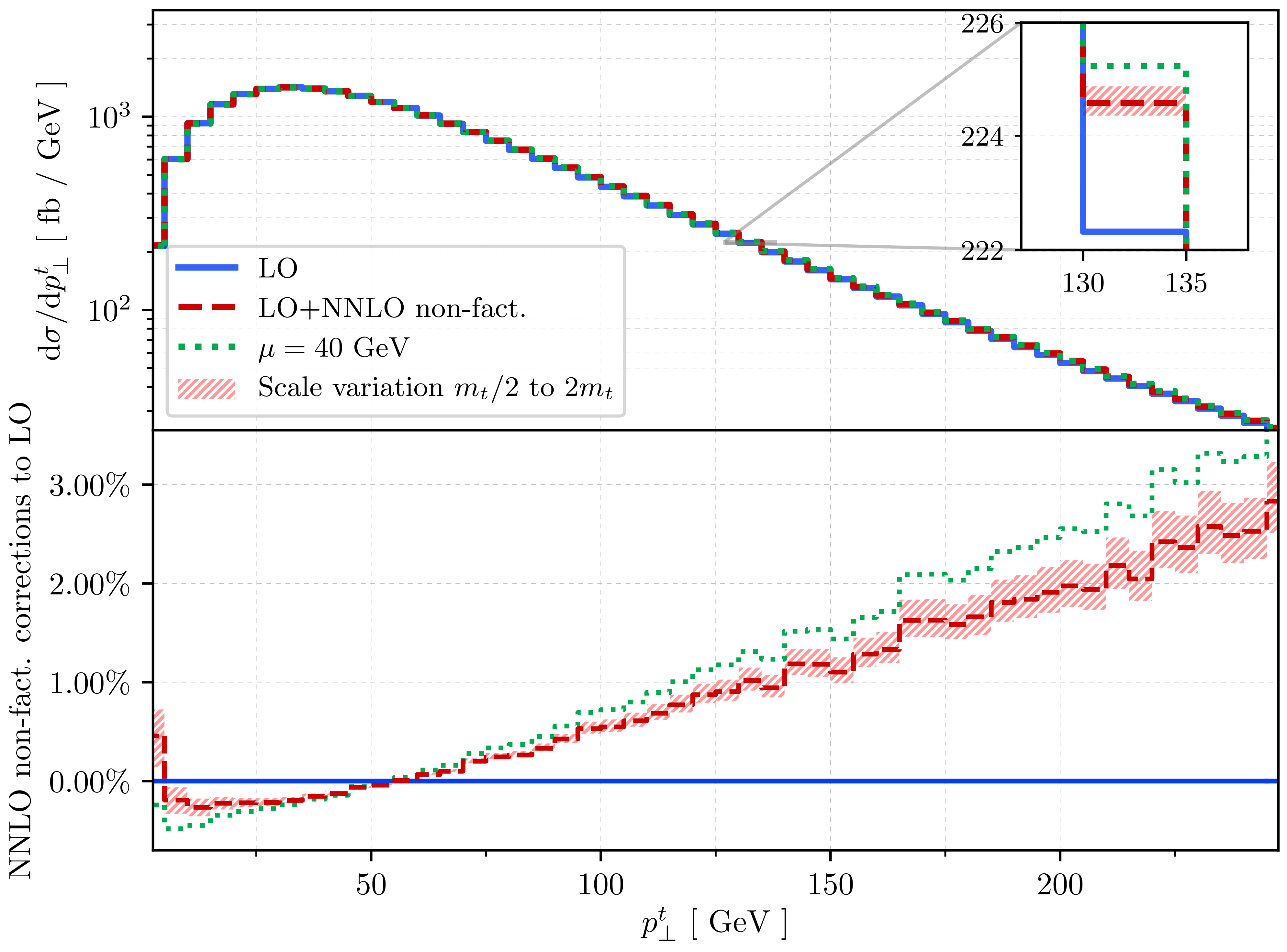
$$
\begin{aligned}
\mathcal{W}(E;1,2,3,4) &\equiv \kappa_{qQ}\left[\left(\frac{2E}{\mu}\right)^{-2\epsilon} K_{\rm nf}(\epsilon) - {\rm I}_1(\epsilon)\right] \\
&= \kappa_{qQ}\Bigg[-2\ln\left(\frac{2E}{\mu}\right)\ln\left(\frac{p_1\cdot p_4\; p_3\cdot p_2}{p_1\cdot p_2\; p_3\cdot p_4}\right) \\
&\quad + \sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}} \lambda_{ij}\left(\frac{1}{2}\ln^2(\eta_{ij}) + {\rm Li}_2(1-\eta_{ij})\right)\Bigg] + \mathcal{O}(\epsilon)
\end{aligned}
$$



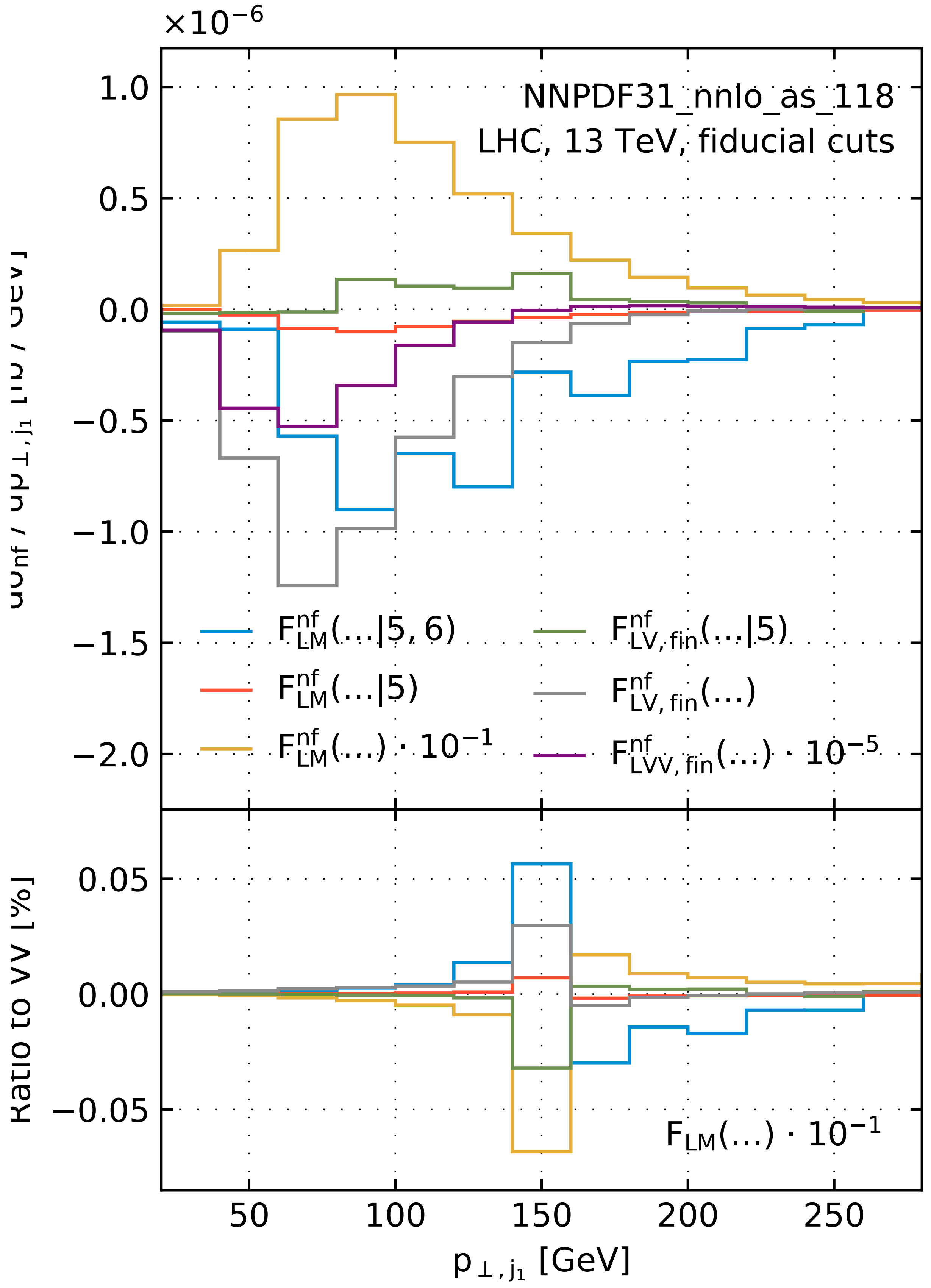




Campanario, Figi, Plätzer

The non-factorizable contributions are dominated by the virtual corrections. This is true for both the single top and the Higgs production in WBF, but this feature becomes extreme (a factor of a $10^5$ difference) in the latter case (the consequence of how WBF events are selected).




[Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile,Wang 2022]

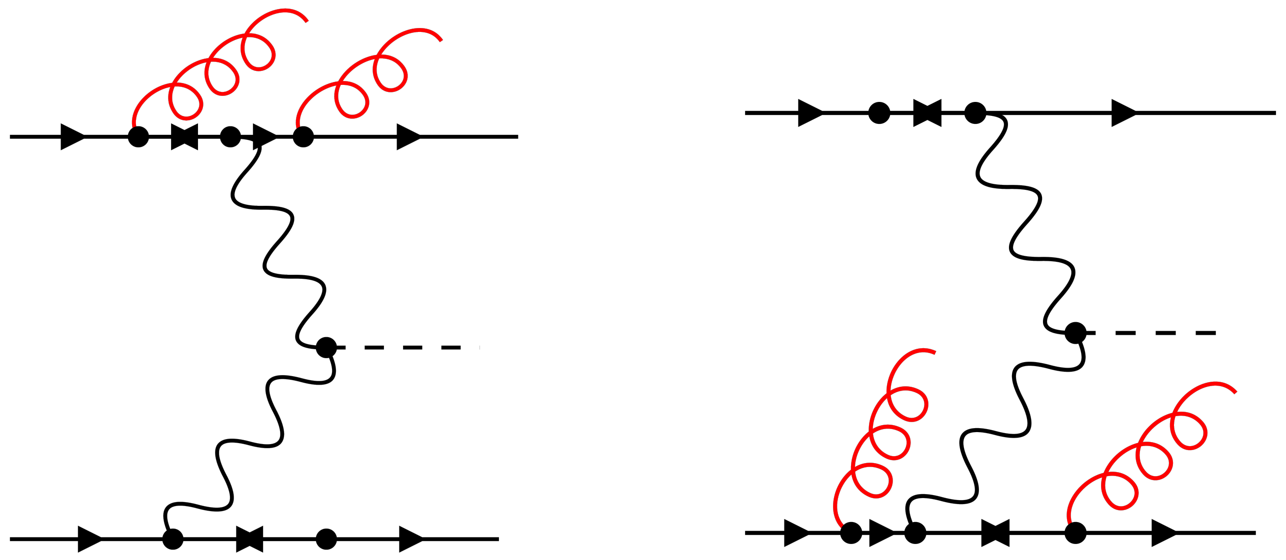



[Asteriadis, Brønnum-Hansen, Melnikov 2305.08016]

To understand this suppression, consider emission of soft gluon, compare the change of the cross section caused by the emission of two soft gluons with the double-virtual corrections.

$$\lim_{p_5,p_6\to 0} |\mathcal{M}(1_q,2_Q,3_q,4_Q)|^2_{\rm nf} = (N_c^2-1){\rm Eik}_{\rm nf}(p_5){\rm Eik}_{\rm nf}(p_6)\ \mathcal{A}_0^2(1,2,3,4)$$

$$\mathcal{W}(E;1,2,3,4) \equiv \kappa_{qQ}\left[\left(\frac{2E}{\mu}\right)^{-2\epsilon} K_{\rm nf}(\epsilon) - {\rm I}_1(\epsilon)\right]$$
$$= \kappa_{qQ}\Bigg[-2\ln\left(\frac{2E}{\mu}\right)\ln\left(\frac{p_1\cdot p_4\ p_3\cdot p_2}{p_1\cdot p_2\ p_3\cdot p_4}\right)$$
$$+\sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}}\lambda_{ij}\left(\frac{1}{2}\ln^2(\eta_{ij})+{\rm Li}_2(1-\eta_{ij})\right)\Bigg]+\mathcal{O}(\epsilon)$$



$${\rm Eik}(p) = \sum_{i\in[1,3];j\in[2,4]} \lambda_{ij}\frac{p_i p_j}{(p_i p)(p_j p)}$$

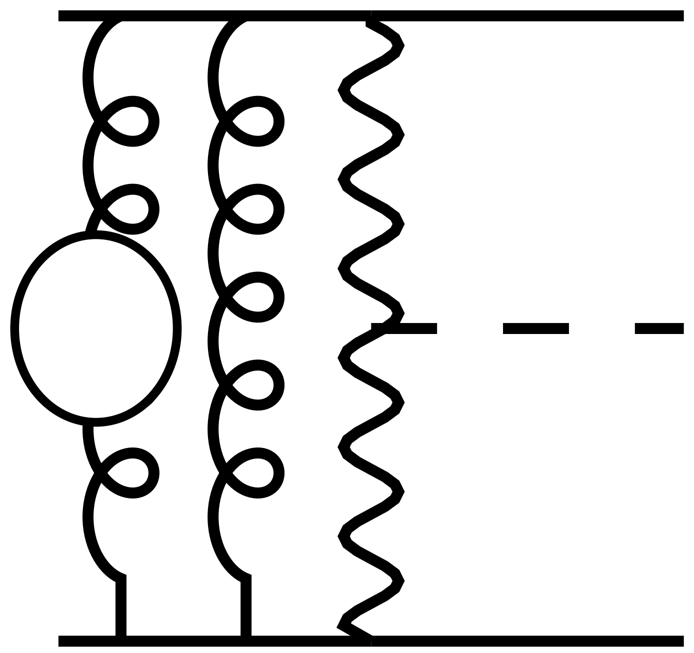
$$L(1,2,3,4) = \ln\left(\frac{p_1\cdot p_4\ p_3\cdot p_2}{p_1\cdot p_2\ p_3\cdot p_4}\right) \quad \approx \frac{2\vec{p}_{3,\perp}\cdot\vec{p}_{4,\perp}}{s} \quad \sim 10^{-2}$$

$$\sigma_{RR}\sim\left(\frac{\tilde\alpha_s}{2\pi}\right)^2 N_c^2\langle L^2(1,2,3,4)F_{\rm LM}^{\rm nf}(1_q,2_q,3_q,4_q)\rangle \quad \sim \left(\frac{\tilde\alpha_s}{2\pi}\right)^2 10^{-4}\ \sigma_{\rm LO}$$

$$\sigma_{VV}\sim\left(\frac{\tilde\alpha_s}{2\pi}\right)^2 N_c^2\langle\ \chi_{\rm nf}(1,2,3,4)\ F_{\rm LM}^{\rm nf}(1,2,3,4)\rangle \quad \approx \left(\frac{\tilde\alpha_s}{2\pi}\right)^2 10\,\sigma_{\rm LO}$$

$$\Bigg\} \rightarrow \quad \frac{\sigma_{RR}}{\sigma_{VV}} \sim 10^{-5}$$

The dependence of not-factorizable corrections on the renormalization scale is quite strong; one can try to accommodate effects of the running coupling constant into the calculation by employing Brodsky-Lepage-Mackenzie philosophy.



Bronum-Hansen, Long, Melnikov

$$C_{\rm nf} = 4\int \frac{{\rm d}^2\mathbf{k}_1}{(2\pi)}\frac{{\rm d}^2\mathbf{k}_2}{(2\pi)}\frac{\Delta_3\Delta_4}{\tilde\Delta_1\tilde\Delta_2}\left(\frac{\Delta_3\Delta_4}{\Delta_{3,1}\Delta_{4,1}\Delta_{3,2}\Delta_{4,2}} - \frac{1}{\Delta_{3,12}\Delta_{4,12}}\right)$$

$$\Delta_i = \mathbf{k}_i^2,\quad \Delta_{3,i} = (\mathbf{k}_i - \mathbf{p}_3)^2 + m_V^2,\quad \Delta_{4,i} = (\mathbf{k}_i + \mathbf{p}_4)^2 + m_V^2,\quad i = 1,2,12$$

$$\tilde\Delta_i = \Delta_i\left(1 + \frac{\beta_0\alpha_s}{2\pi}\ln\frac{\mathbf{k}_i^2}{\mu^2 e^{5/3}}\right)$$

$$C_1(\nu) = -2\int\frac{{\rm d}^2\mathbf{k}_1}{2\pi}\frac{\Delta_3\,\Delta_4\,m_V^{2\nu}}{\Delta_1^{1+\nu}\Delta_{3,1}\Delta_{4,1}}$$

$$C_2(\nu_1,\nu_2) = 4\int\frac{{\rm d}^2\mathbf{k}_1}{2\pi}\frac{{\rm d}^2\mathbf{k}_2}{2\pi}\frac{\Delta_3\,\Delta_4\,m_V^{2(\nu_1+\nu_2)}}{\Delta_1^{1+\nu_1}\Delta_2^{1+\nu_2}\Delta_{3,12}\Delta_{4,12}}$$

$$C_2(\nu_1,\nu_2) = \frac{\nu_{12}}{\nu_1\nu_2}\frac{\Gamma(1+\nu_{12})}{\Gamma(1+\nu_1)\Gamma(1+\nu_2)}\frac{\Gamma(1-\nu_1)\Gamma(1-\nu_2)}{\Gamma(1-\nu_{12})}C_1(\nu_{12})$$

$$C_1(\nu) = \frac{1}{\nu} + \sum_{i=1} C_1^{(i)}\nu^i$$

$$C_{\rm nf}^{(0)} = \left(C_1^{(0)}\right)^2 - 2C_1^{(1)}$$

$$C_{\rm nf} = C_{\rm nf}^{(0)} + \frac{\alpha_s\beta_0}{\pi}\left(C_{\rm nf}^{(0)}\ln\left(\frac{\mu^2 e^{5/3}}{m_V^2}\right) + C_{\rm nf}^{(1)}\right) + \mathcal{O}(\alpha_s^2\beta_0^2)$$

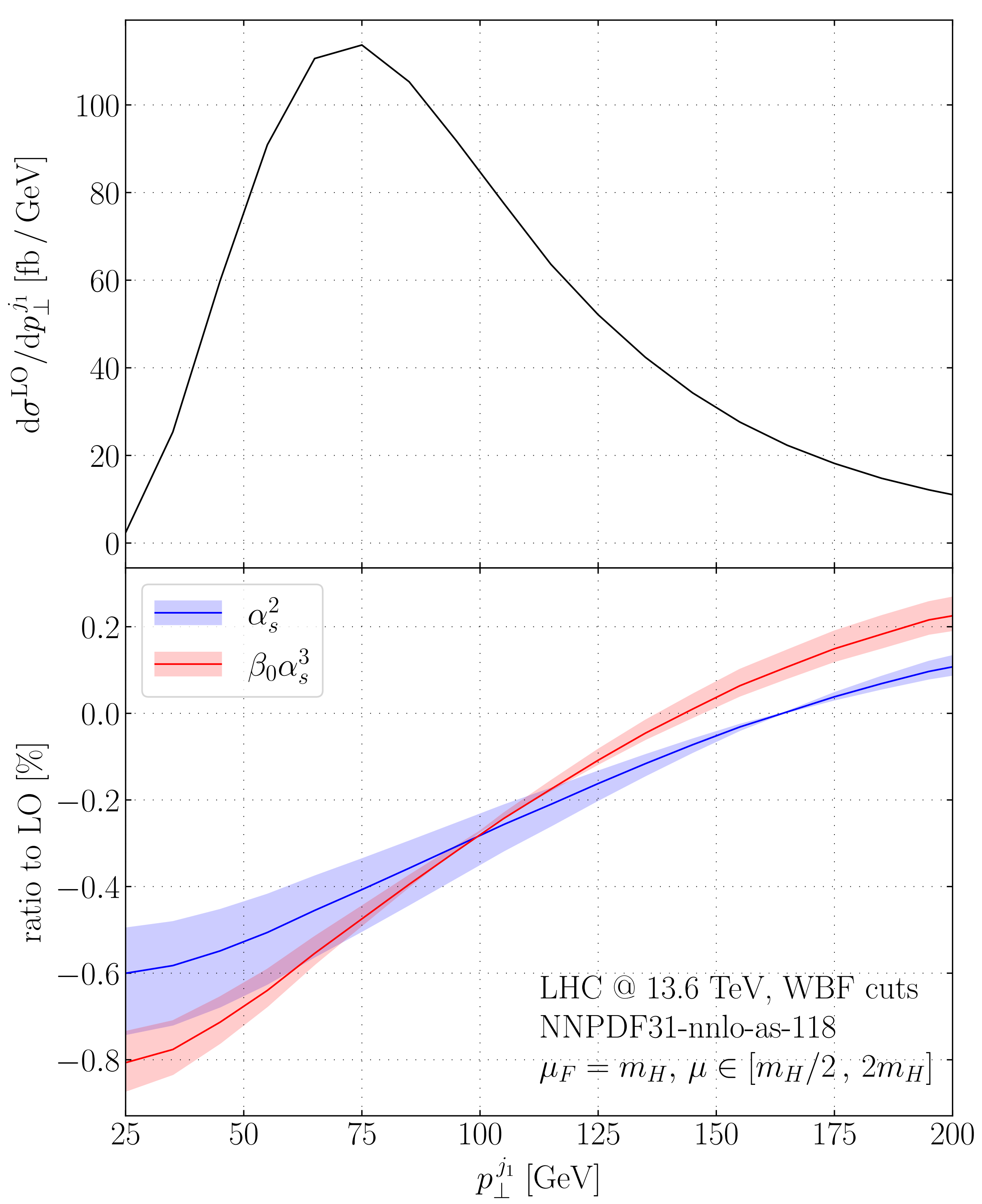
$$C_{\rm nf}^{(1)} = C_1^{(0)}C_1^{(1)} - 3C_1^{(2)} + 2\zeta_3$$

BLM corrections can be computed analytically; the scale dependence stabilizes.

$$C_1^{(0)} = \int_0^1 \mathrm{d}t \, \frac{\Delta_x \Delta_y}{r_{12}^2} \left[ \ln r_2 - 2\ln r_{12} + \frac{r_2 - r_1}{r_2} \right],$$

$$C_1^{(1)} = \int_0^1 \mathrm{d}t \, \frac{\Delta_x \Delta_4}{r_{12}^2} \left[ \frac{1}{2}\ln^2 r_{12} - \ln r_{12} \left( \frac{r_2 - r_1}{r_2} + \ln \frac{r_2}{r_{12}} \right) + 2\ln\frac{r_2}{r_{12}} + \frac{\pi^2}{6} - \mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) \right],$$

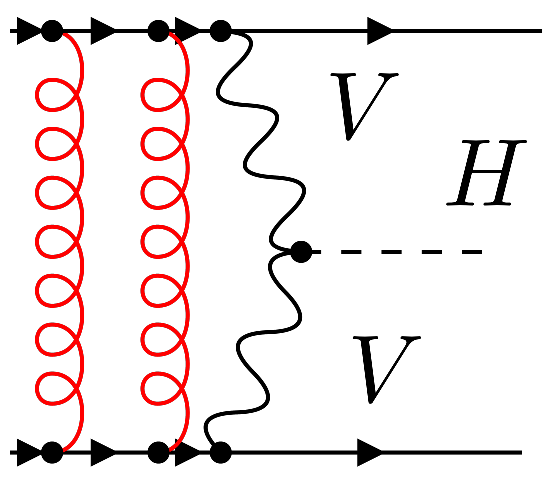
$$C_1^{(2)} = \int_0^1 \mathrm{d}t \, \frac{\Delta_x \Delta_4}{r_{12}^2} \Bigg[ -\frac{1}{6}\ln^3 r_{12} + \frac{1}{2}\ln^2 r_{12} \left( \frac{r_2 - r_1}{r_2} + \ln\frac{r_2}{r_{12}} \right) + \frac{\pi^2}{6}\frac{r_2 - r_1}{r_2} + \ln^2\left(\frac{r_2}{r_{12}}\right) \ln\frac{r_1}{r_{12}} - \ln r_{12} \left( \frac{\pi^2}{6} + 2\ln\frac{r_2}{r_{12}} - \mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) \right) - \frac{r_2 - r_1}{r_2}\mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) - \ln\frac{r_2}{r_{12}} \left( \frac{\pi^2}{6} - \mathrm{Li}_2\left(\frac{r_1}{r_{12}}\right) \right) + 2\mathrm{Li}_3\left(\frac{r_2}{r_{12}}\right) - 2\zeta_3 \Bigg]$$




$$r_1 = \frac{\mathbf{p}_3^2}{m_V^2} + \frac{\mathbf{p}_4^2}{m_V^2} + \frac{\mathbf{p}_H^2}{m_V^2} t(1-t) \qquad r_2 = 1 - \frac{\mathbf{p}_H^2}{m_V^2} t(1-t) \qquad r_{12} = r_1 + r_2$$

$$\sigma_{\mathrm{nf}}^{\mathrm{LO}} = -2.97^{-0.69}_{+0.52}\ \mathrm{fb}, \qquad \sigma_{\mathrm{nf},\beta_0}^{\mathrm{NLO}} = -3.20^{-0.01}_{+0.14}\ \mathrm{fb}$$

We discussed the non-factorizable corrections to process of the weak boson fusion type (Higgs production in WBF, single top production). Our interest to understand these effect is related to an impressive progress in computing factorizable corrections, e.g. where N3LO QCD corrections to inclusive WBF have been computed.




Non-factorizable corrections have the following properties:

1) they start contributing at next-to-next-to-leading order for the first time;

2) they are colour-suppressed but dynamically enhanced; the enhancement is related to the Coulomb (Glauber) phase;

3) they can reach a percent level in kinematic distributions and they are strongly kinematic-dependent; they can be studied independently of real-emission contributions since they are infra-red finite.

4) for Higgs production in WBF, the non-factorizable corrections can be studied using expansion around the forward limit (eikonal expansion), that can also be extended to provide the next-to-leading power accuracy;

5) the non-factorizable corrections are (very) strongly dominated by the double-virtual contributions; the real-emission contributions are very much suppressed.

6) the scale-dependence of non-factorizable contributions can be strongly reduced by computing $O(n_f)$ three-loop corrections and treating them in the spirit of BLM scale-setting procedure.