INTRODUCTION TO SMEFT

P3H SUMMER SCHOOL

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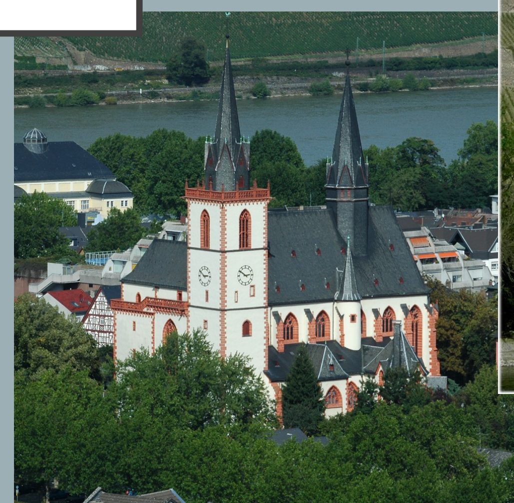

LECTURE 2

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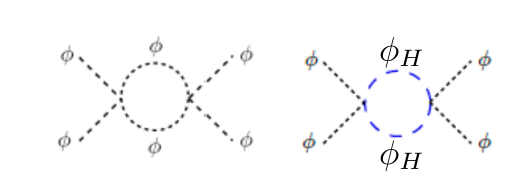
Please find all my typos!

TOY MODEL AT ONE-LOOP, #1

- Light and heavy scalar:

$$L_{UV} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \phi_H\left[-\frac{1}{2}\partial_\mu\partial^\mu - \frac{M_H^2}{2} - \frac{\kappa}{2} \mid \phi \mid^2\right]\phi_H$$

- Calculate $\phi\phi \rightarrow \phi\phi$ in $\overline{MS}$ (ie drop poles). (Also t- and u- channel, plus tree level)



$$A_{UV} = -6\lambda + \frac{27}{8\pi^2}\lambda^2\left[\log\left(\frac{\bar{\mu}^2}{m^2}\right) + \frac{2}{3}\right] + \frac{3}{8\pi^2}\kappa^2 \log\left(\frac{\bar{\mu}^2}{M^2}\right)$$

- No choice of scale eliminates logs

$$\overline{\mu} \equiv \mu^2 \frac{e^{\gamma_E}}{(4\pi)}$$

How does decoupling work?

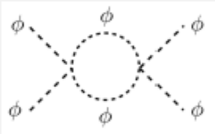
*Integral computed at threshold

TOY MODEL AT ONE- LOOP, #2

- Now compute $\phi\phi \to \phi\phi$ in EFT

$$L_{EFT} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 + \frac{C_{\phi 4}}{4}\phi^4$$

$$A_{EFT} = 6C_{\phi 4} + \frac{27}{8\pi^2}C_{\phi 4}^2\left[\log\left(\frac{\bar{\mu}^2}{m^2}\right) + \frac{2}{3}\right]$$



- Note Lagrangian coefficients can be different in EFT (it's a different theory)
- At matching scale, Λ: $A_{EFT} = A_{UV}$
- At tree level: $C_{\phi 4} = -\lambda$
- At one-loop:

$$\delta C_{\phi 4}(\Lambda) = \frac{\kappa^2}{16\pi^2}\log\left(\frac{\Lambda^2}{M^2}\right)$$

Matching has no logarithmic dependence on low scale, m

Take Λ~M to fix log small

MORE ON SCALES

- Since matching is done at Λ, low energy amplitude is:

$$A_{EFT} = 6C_{\phi 4}(\Lambda) + \frac{27}{8\pi^2} C_{\phi 4}(\Lambda)^2 \left[\log\left(\frac{\Lambda^2}{m^2}\right) + \frac{2}{3}\right]$$

- RGE running of $C_{\phi 4}$ from Λ to μ_L

$$C_{\phi 4}(\mu_L) = C_{\phi 4}(\Lambda) + \frac{9}{16\pi^2} C_{\phi 4}(\Lambda)^2 \log\left(\frac{\mu_L^2}{\Lambda^2}\right)$$

- No large logs in EFT amplitude

$$A_{EFT} = 6C_{\phi 4}(\mu_L) + \frac{27}{8\pi^2} C_{\phi 4}(\mu_L)^2 \left[\log\left(\frac{\mu_L^2}{m^2}\right) + \frac{2}{3}\right]$$

* Can resum logarithms

EVEN MORE ON LOGS

- Keep going with with heavy/light scalar toy model
- Consider a diagram with I heavy and I light propagator:

$$I_{UV} = \mu^{2\epsilon} \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - M^2} \frac{1}{k^2 - m^2}$$
$$= \frac{i}{16\pi^2} \left[\frac{1}{\epsilon} + 1 + \log\left(\frac{\mu^2}{M^2}\right) + \frac{m^2}{M^2 - m^2} \log\left(\frac{m^2}{M^2}\right) \right]$$
$$= \frac{i}{16\pi^2} \left[\frac{1}{\epsilon} + 1 + \log\left(\frac{\mu^2}{M^2}\right) + \left(\frac{m^2}{M^2} + \frac{m^4}{M^4} + \ldots\right) \log\left(\frac{m^2}{M^2}\right) \right]$$

- Order of integration matters
- Non-analytic dependence on m same in both I_{UV} and I_{IR}
- Difference between integrals give matching condition, which is analytic in m

- Now think about an EFT where propagators are expanded first

$$I_{EFT} = \mu^{2\epsilon} \left(-\frac{1}{M^2}\right) \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - m^2} \left[\frac{k^2}{M^2} + \frac{k^4}{M^4} + \ldots \right]$$
$$= \frac{i}{16\pi^2} \left[-\frac{1}{\epsilon} - 1 - \log\left(\frac{\mu^2}{m^2}\right) \right] \left[\frac{m^2}{M^2} + \frac{m^4}{M^4} + \ldots \right]$$

- Matching:

$$I_{UV} - I_{EFT} \sim \frac{i}{16\pi^2} \left[1 + \log\left(\frac{\mu^2}{M^2}\right) \right] \left[1 + \frac{m^2}{M^2} + \ldots \right]$$

TOP DOWN VS BOTTOM UP

- Bottom-up approach, we only consider EFT which describes the low energy physics of any UV model (with no light particles).
  - Can describe experiments using global fits in terms of common Lagrangian
- Top down approach, consider specific UV model and match to EFT (compute coefficients of EFT in terms of parameters of UV model)
  - CONS: lose model independence
  - PROS: fewer parameters
  - Can classify UV models in terms of dictionaries

WARSAW BASIS

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k\sigma^{\mu\nu} u_t)$		

+…..

- The interesting operators are those with derivatives
- Derivative operators introduce new structures into kinematic distributions
- Most of 2499 operators come from flavor permutations
- Systematically eliminate derivative operators using equations of motion and integration by parts

HW: Why do dipole interactions not interfere with the SM for massless fermions?

FIND A BASIS OF OPERATORS

- Start with dimension-6 operators: with no assumptions, 2499 possibilities
- Most popular basis is "WARSAW BASIS"
- Typically work to tree level with one occurrence of dimension-6 operator
- Consider contributions to processes dominated by H/Z/W resonances, and interference with SM only (linear in EFT) (REASONABLE ASSUMPTION)

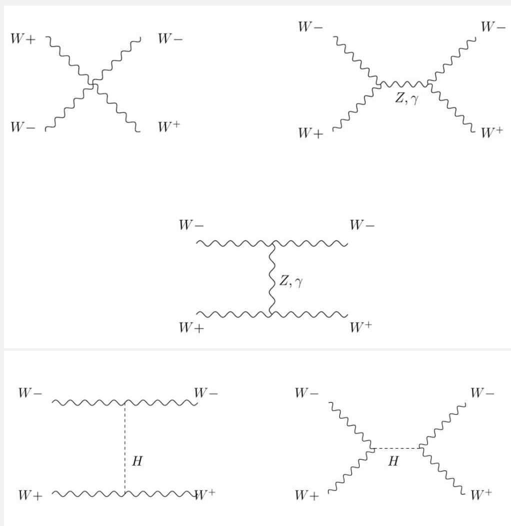
Complete dimension-8 basis known

	Total	Not resonance suppressed
General	2499	46
MFV	108	30
$U(3)^5$	70	24

1709.06492

Useful source of Feynman rules: 1704.03888

SM IS SPECIAL AT HIGH ENERGY



$$A \approx g^2 \frac{E^2}{M_W^2}$$

$$A \approx -g^2 \frac{E^2}{M_W^2}$$

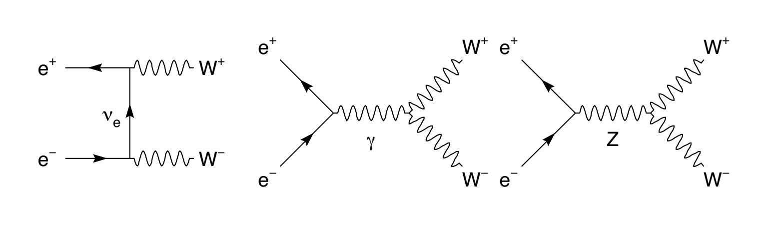
E^4 terms cancel between TGC and QGC

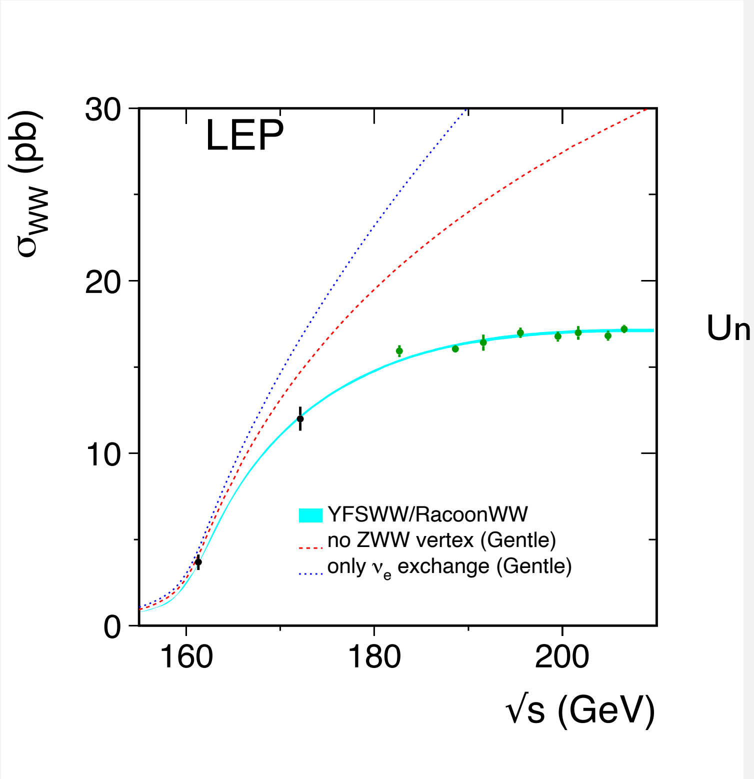
Terms which grow with energy cancel for E >> m_h

SM particles have just the right couplings so amplitudes don't grow with energy

UNITARITY HAS REAL WORLD CONSEQUENCES

- The story started in pre-history with the classic paper:
  - *Probing the Weak Boson Sector in $e^+e^- \rightarrow W^+ W^-$* (Hagiwara, Peccei, Zeppenfeld, Hikasa, 1987)
  - At that time the structure of the 3 gauge boson interactions had not been verified experimentally





Unitarity cancellations

What is a smoking gun signature for SMEFT?

HIGGS MECHANISM IN SMEFT

- Higgs mechanism as usual, but with extra terms

$$
\begin{aligned}
L_h =& (D_\mu\phi)^\dagger(D^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda_h(\phi^\dagger\phi)^2 \\
&+ \frac{C_\phi}{\Lambda^2}(\phi^\dagger\phi)^3 + \frac{C_{\phi\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{C_{\phi D}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi)
\end{aligned}
$$

$$
O_{\phi\Box} = 2(\phi^\dagger\phi)\partial^\mu\phi^\dagger\partial_\mu\phi + (\phi^\dagger\phi)\left[\phi^\dagger\Box\phi + (\Box\phi^\dagger)\phi\right]
$$

- Minimize potential (keeping only terms up to $1/\Lambda^2$):

$$
v = \sqrt{\frac{\mu^2}{\lambda_h} + \frac{3\mu^3}{8\lambda_h^{5/2}}\frac{C_\phi}{\Lambda^2}} \qquad \phi = \begin{pmatrix} \phi_0^+ \\ \frac{1}{\sqrt{2}}(v + h_0 + i\phi_0^0) \end{pmatrix}
$$

This is Warsaw basis

*subscript 0 indicates field before shift of field to get canonical normalization

HIGGS MECHANISM IN SMEFT, #2

- Higgs field is not canonically normalized:

$$L_h \sim \frac{1}{2}\left[1+\frac{v^2}{2\Lambda^2}C_{\phi D}-\frac{2v^2}{\Lambda^2}C_{\phi\Box}\right](\partial_\mu h_0)^2 + \frac{1}{2}\left[\mu^2-3\lambda_h v^2+\frac{15v^4}{4\Lambda^2}C_\phi\right]h_0^2+\text{Goldstones}...$$

- Canonical normalization recovered: $h = Z_h h_0$
- All Higgs interactions shifted

$$Z_h = 1+\frac{v^2}{4\Lambda^2}C_{\phi D}-\frac{v^2}{\Lambda^2}C_{\phi\Box}$$

Other possible purely scalar operators can be eliminated by integration by parts, or by use of the equations of motion

YUKAWAS IN SMEFT

- Consider down quark sector

$$L = -\hat{Y}_d^{ij}\overline{q}_L^i \phi d_R^j + \frac{C_{d\phi}^{ij}}{\Lambda^2}(\phi^\dagger \phi)\overline{q}_L^i \phi d_R^j + hc$$

$$\rightarrow -m_d^{ij}\overline{d}^i d^j - Y_d^{ij} h \overline{d}^i d^j$$

- Diagonalizing mass matrix doesn't simultaneously diagonalize Yukawas

$$m_d^{ij} = \frac{v}{\sqrt{2}}\left[\hat{Y}_d^{ij} - \frac{v^2}{2\Lambda^2}C_{d\phi}^{ij}\right]$$

$$Y_d^{ij} = \frac{m_d^{ij}}{vZ_h} - \frac{v^2}{\sqrt{2}}\frac{C_{d\phi}^{ij}}{\Lambda^2}$$

Possibility for interesting flavor structures

i,j are generation indices

SMEFT GAUGE SECTOR

- Shift fields so that gauge fields have canonical forms
- Find mass eigenstates as usual:

$$M_W = \frac{\overline{g}_2 v}{2}$$
$$M_Z = \frac{v}{2}\sqrt{(\overline{g}_1)^2 + (\overline{g}_2)^2}\left(1 + \frac{\overline{g}_1\overline{g}_2}{(\overline{g}_1)^2 + (\overline{g}_1)^2}\frac{v^2}{\Lambda^2}C_{\phi WB} + \frac{v^2}{4\Lambda^2}C_{\phi D}\right)$$

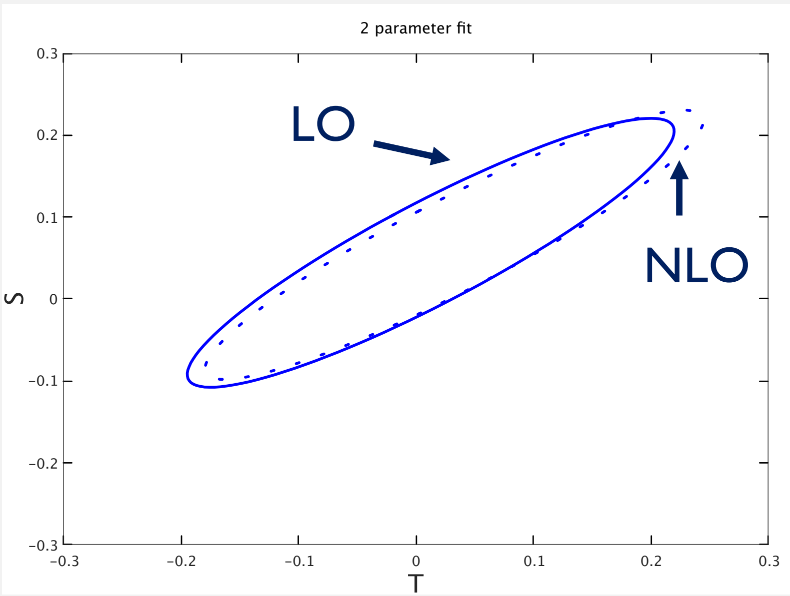
$$O_{\phi WB} = \phi^\dagger \sigma^a \phi W^a_{\mu\nu} B^{\mu\nu}$$
$$O_{\phi D} = (\phi^\dagger D^\mu \phi)^* (\phi^\dagger D_\mu \phi)$$

- SM relationships among parameters altered (barred fields remind us of this)

$$v^2 \to \frac{1}{\sqrt{2}G_F}\left[1 + \mathcal{O}\left(\frac{v^2 C}{\Lambda^2}\right)\right]$$

OBLIQUE PARAMETERS

- Arbitrarily set all parameters except $C_{\phi WB}$ and $C_{\phi D}$=0



$$\alpha \Delta S = 4 c_W s_W \frac{v^2}{\Lambda^2} C_{\phi WB}$$

$$\alpha \Delta T = -\frac{v^2}{2\Lambda^2} C_{\phi D}$$

You get quite different results when you allow all coefficients to vary. Picking specific non-zero coefficients involves assumptions about underlying model

$O_{\phi WB}$ changes WWγ and WWZ vertices and so affects WW pair production

1909.02000

HIGGS DECAYS

- Example: h→bb

$$\frac{\Gamma(h \to b\bar{b})}{\Gamma(h \to b\bar{b}) \mid_{SM}} = (1 + \Delta\kappa_b)^2$$

$$\Delta\kappa_b = \frac{1}{\sqrt{2} G_F \Lambda^2} \left(C_{\phi\Box} - \frac{C_{\phi D}}{4} - C_{\phi l}^{(3)} + \frac{C_{ll}}{2} - \frac{C_{d\phi}}{2^{3/4} m_b \sqrt{G_F}} \right)$$

From normalizing h kinetic energy

From change in relation between G_F and v

New dimension-6 operator

- ***Is this just a fancy way of writing the κ's?***

$$O_{dH} = Y_d (\phi^\dagger \phi) \bar{q}_L \phi d_R$$

CONSIDER h→ZZ

- Compare h→ZZ (on-shell) to h→Zff

$$\frac{\Gamma(h\to ZZ)}{\Gamma(h\to ZZ)\mid_{SM}} = 1+\frac{1}{\sqrt{2}G_F\Lambda^2}\left[c_k + .4c_{ZZ}\right]$$

$$\frac{\Gamma(h\to Zf\overline{f})}{\Gamma(h\to Zf\overline{f})\mid_{SM}} = 1+\frac{1}{\sqrt{2}G_F\Lambda^2}\left[c_k - .97c_{ZZ}\right]$$

c_{ZZ} are momentum dependent operators

- EFT can capture off-shell effects (κ approach cannot)

$$c_k = \frac{C_{\phi D}}{2} + 2C_{\phi\Box} + C_{ll} - 2C^{(3)}_{\phi l}$$

$$c_{ZZ} = \frac{M_W^2}{M_Z^2}C_{\phi W} + (1-\frac{M_W^2}{M_Z^2})C_{\phi B} + \frac{M_W}{M_Z}\sqrt{1-\frac{M_W^2}{M_Z^2}}C_{\phi WB}$$

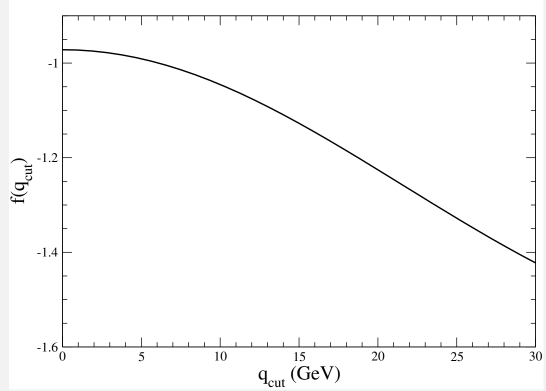
h→Zff

- EFT has more information than total rate
  - q^2 is fermion pair invariant mass squared

$$\frac{d\Gamma}{dq^2}\mid_{EFT}=\frac{d\Gamma}{dq^2}\mid_{SM}\left[1+\frac{1}{\sqrt{2}G_F\Lambda^2}c_k\right]+\frac{G_F q^2}{\Lambda^2}c_{ZZ}(...)$$

  - Integrate up to q_{cut}
  - $(G_F q^2/\Lambda^2)f(q_{cut})$ is coefficient of c_{ZZ}

Off-shell decay, $h\rightarrow Zf\overline{f}$



SMEFT has kinematic information

WHEN IS EFT VALID?

$$L \rightarrow L_{SM} + \Sigma_i \frac{C_{6i}}{\Lambda^2} O_{6i} + \Sigma_i \frac{C_{8i}}{\Lambda^4} O_{8i} + \ldots$$

- SMEFT

$$A^2 \sim \mid A_{SM} + \frac{A_6}{\Lambda^2} + \ldots \mid^2 \sim A_{SM}^2 + \frac{A_{SM} A_6}{\Lambda^2} + \frac{A_6^2}{\Lambda^4} + \ldots .$$

- Problem is that $(A_6)^2$ terms are the same order as A_8 terms that we have dropped when counting in $1/\Lambda$
- If we only keep A_6/Λ^2 terms and drop $(A_6/\Lambda^2)^2$, the cross section is not guaranteed to be finite
- Corrections are $O(s/\Lambda^2)$ or $O(v^2/\Lambda^2)$

Leads to idea that there is a maximum energy scale where SMEFT is valid for scattering processes

COUNTING LORE

$$\sigma \sim g_{SM}^2 (A_{SM})^2 + g_{SM} g_{BSM} A_{SM} A_6 \frac{s}{\Lambda^2}$$
$$+ g_{BSM}^2 (A_6)^2 \frac{s^2}{\Lambda^4} + g_{SM} g_{BSM} A_{SM} A_8 \frac{s^2}{\Lambda^4}$$

Same order of magnitude if $g_{SM} \sim g_{BSM}$

(dim-6)^2 could dominate if $g_{BSM} >> g_{SM}$

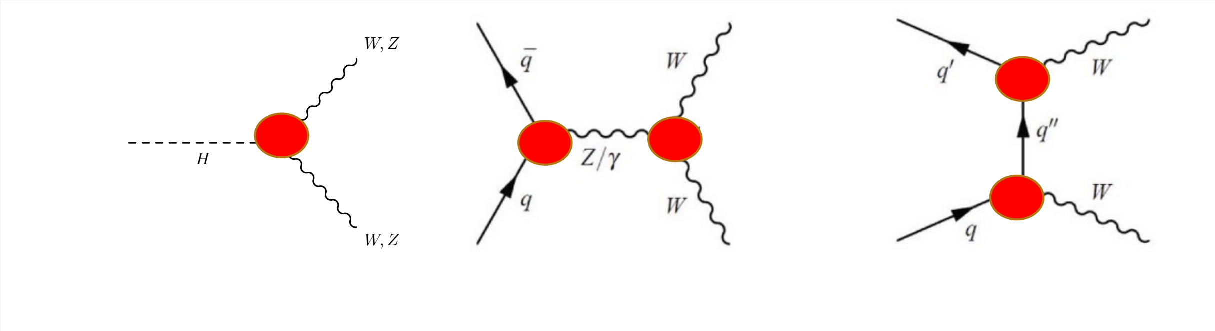
Dimension-6 quadratic expansion can be valid for strongly interacting theory

Dropping dim-8 terms implicitely makes some assumptions

*In specific examples, dim-8 pieces don't seem to be important except for 2HDM

CAN'T JUST FIT HIGGS COUPLINGS

Operators that contribute to VVV vertices and Higgs-VV vertices



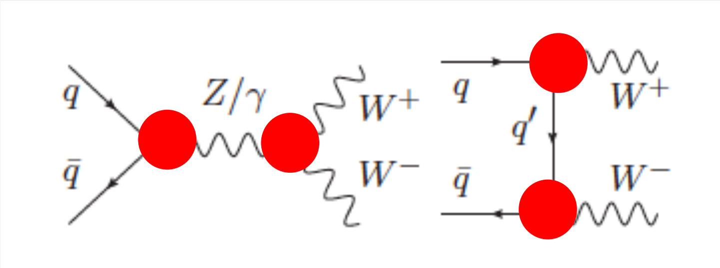
Anomalous qqZ vertices too!

Leads to concept of global fits

- Changing ZWW, γWW vertices spoils high energy cancellations between contributions

DIBOSON PRODUCTION

- Sensitive to variations of Zff and Z(γ)WW couplings



No growth with energy in SM

- Old story: Individual contributions grow with energy
- Cancellations keep amplitudes from growing at high energy in SM

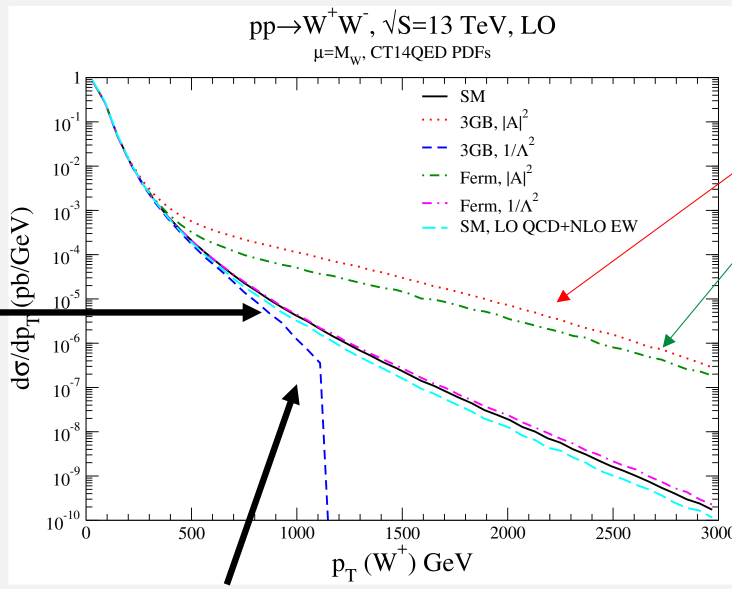
Changing gauge or fermion couplings spoils cancellation

OBVIOUS PROBLEM

- One proposal for dealing with this issue is to put a cut on the maximum energy where the SMEFT is assumed to be valid

Linear expansion

Cannot ignore negative cross section region



....Anomalous 3 gauge boson couplings

----Anomalous Z-fermion couplings allowed by LEP measurements

1708.03332

* σ goes negative, expansion not valid

NLO CORRECTIONS IN SMEFT

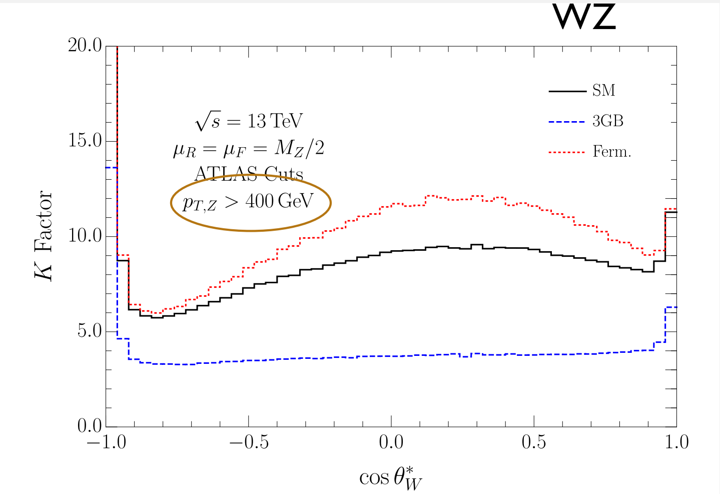
- Compute NLO corrections to $O(v^2/\Lambda^2)$ (ie linear in EFT coefficients)
- SMEFT is a new theory; calculate consistently to one-loop QCD and EW
- One-loop SMEFT QCD corrections automated in SMEFTsim, 2012.11343 and SMEFT@NLO, 2008.11743
- One-loop SMEFT EW corrections done on case by case basis
- Coefficient functions renormalized in $\overline{MS}$
  - Solved problem at one-loop

1312.2014, 1310.4838, 1309.0819

QCD MATTERS

- K factors aren't the same as in SM
- Effect is enhanced for large momenta

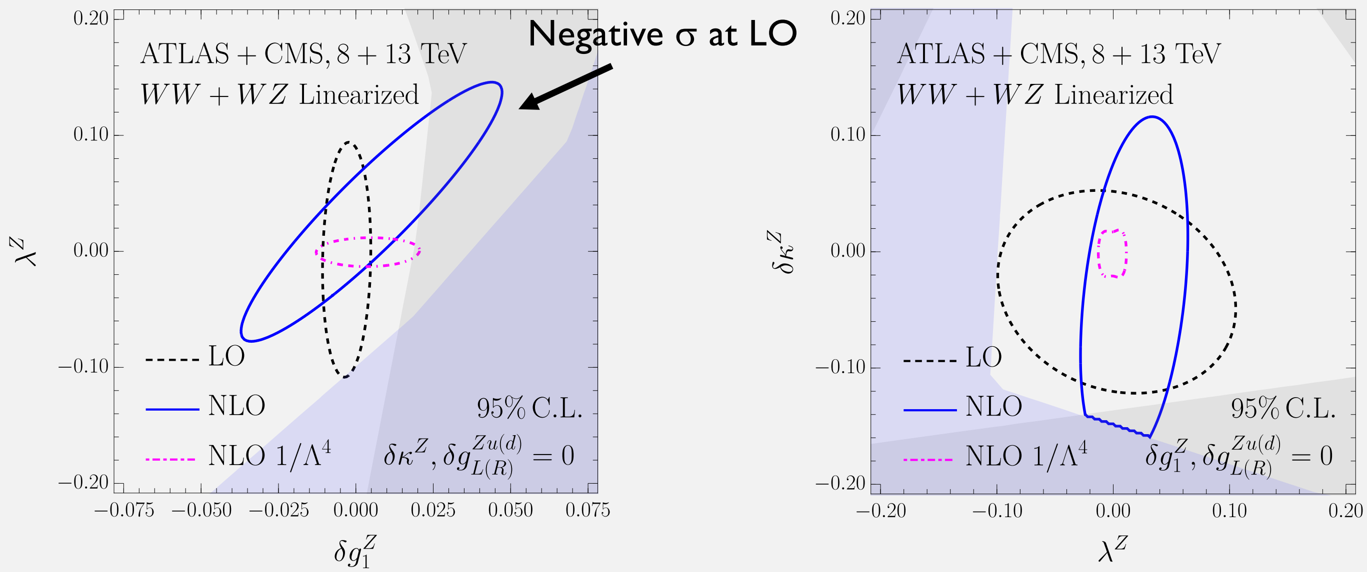
WZ



SMEFT is a new theory: can consistently calculate loop corrections

FIT TO LINEARIZED RATES

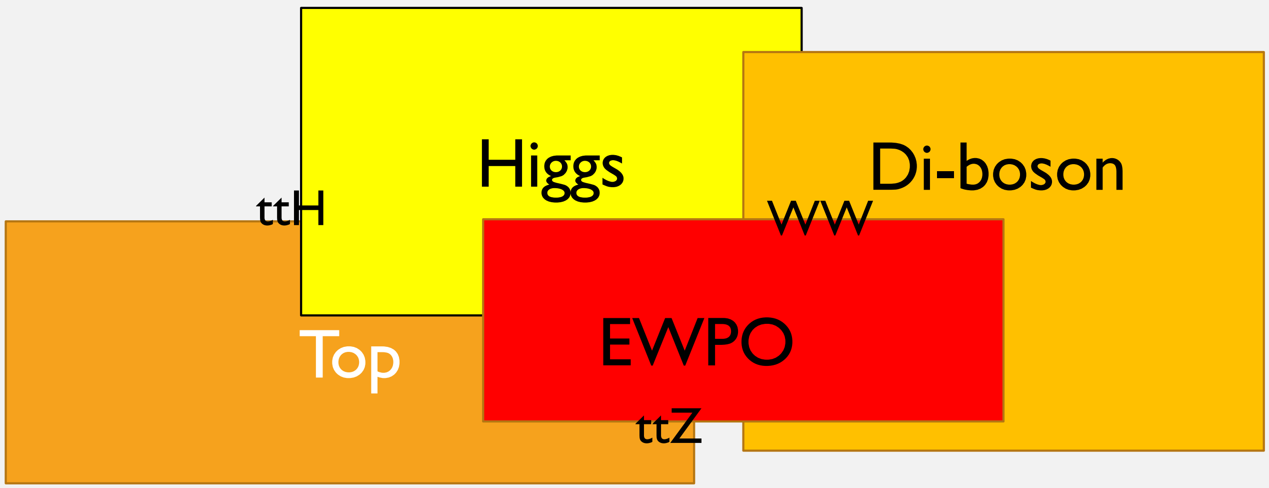
- Drop all coefficients where cross section is negative
- Linearized limits significantly weaker than $1/\Lambda^4$ limits (can cancel terms)



$1/\Lambda^4$ curves don't include double insertions of dim-6 operators

Negative σ at NLO (blue), negative at LO (gray)

SMEFT CONNECTS PROCESSES



W AND Z POLE OBSERVABLES

- Fit to 14 data points—inputs are G_μ, M_Z, α

$$M_W, \Gamma_W, \Gamma_Z, \sigma_h, A_{l,FB}, A_{b,FB}, A_{c,FB}, A_b, A_c, A_l, R_l, R_b, R_c$$

- Tree level expressions depend on (in Warsaw basis) assuming flavor independence

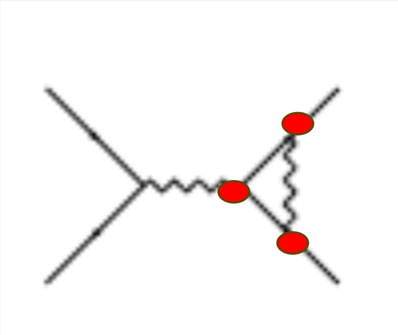
$$C_{ll}, C_{\phi WB}, C_{\phi u}, C_{\phi q}^{(3)}, C_{\phi q}^{(1)}, C_{\phi l}^{(3)}, C_{\phi l}^{(1)}, C_{\phi e}, C_{\phi D}, C_{\phi d}$$

- Tree level SMEFT expressions depend on 8 combinations of operators

$\Rightarrow$ 2 blind directions (resolved by other measurements)

RENORMALIZABILITY

- What does it mean to renormalize a theory of dimension > 4?
  - Formally, such theories are non-renormalizable
  - Include 1 insertion of a dim-6 operator → $1/(\varepsilon\Lambda^2)$
    - This can be absorbed into dim-6 counterterm
  - Now include 2 insertions of dimension-6 operators → $1/(\varepsilon\Lambda^4)$
  - Needs dimension-8 $(1/\Lambda^4)$ counterterm
  - And so on….
  - We say that the SMEFT is renormalizable order by order in $1/\Lambda$



COMPUTE EACH OBSERVABLE TO NLO IN SMEFT

- Example $M_W = M_W^{SM} + \delta M_W$ ⟵ All SMEFT effects here
- Dependence on many coefficients at NLO (QCD + EW)
- Always use "best" SM prediction for fits

$$\delta M_W^{LO} = \frac{v^2}{\Lambda^2}\left\{-30C_{\phi l}^{(3)} + 15C_{ll} - 28C_{\phi D} - 57C_{\phi WB}\right\}$$

$$\delta M_W^{NLO} = \frac{v^2}{\Lambda^2}\left\{-36C_{\phi l}^{(3)} + 17C_{ll} - 30C_{\phi D} - 64C_{\phi WB} - 0.1C_{\phi d} - 0.1C_{\phi e} - 0.2C_{\phi l}^{(1)} - 2C_{\phi q}^{(1)} + C_{\phi q}^{(3)} + 3C_{\phi u} + 0.4C_{lq}^{(3)} - 0.03C_{\phi B} - 0.03C_{\phi\Box} - 0.04C_{\phi W} - 0.9C_{uB} - 0.2C_{uW} - 0.2C_W\right\}$$

α, G_μ, M_Z scheme

NLO SMEFT EFFECTS ON POLE OBSERVABLES

- Fits marginalizing over other coefficients

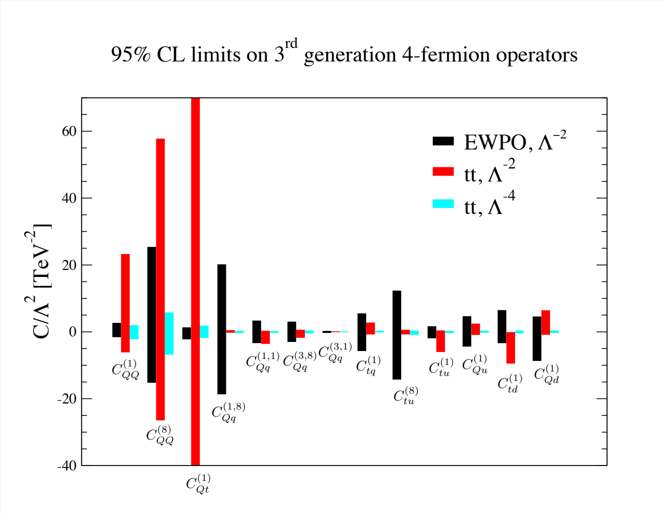
Coefficient	LO	NLO
$\mathcal{C}_{\phi D}$	$[-0.034, 0.041]$	[-0.039,0.051]
$\mathcal{C}_{\phi WB}$	$[-0.080, 0.0021]$	$[-0.098, 0.012]$
$\mathcal{C}_{\phi d}$	$[-0.81, -0.093]$	$[-1.07, -0.03]$
$\mathcal{C}_{\phi l}^{(3)}$	$[-0.025, 0.12]$	$[-0.039, 0.16]$
$\mathcal{C}_{\phi u}$	$[-0.12, 0.37]$	$[-0.21, 0.41]$
$\mathcal{C}_{\phi l}^{(1)}$	$[-0.0086, 0.036]$	$[-0.0072, 0.037]$
$\mathcal{C}_{ll}$	$[-0.085, 0.035]$	$[-0.087, 0.033]$
$\mathcal{C}_{\phi q}^{(1)}$	$[-0.060, 0.076]$	$[-0.095, 0.075]$

- ***Neglect flavor effects***
- Contribution from top loops

NLO effects can be important

EWPO WITH FLAVOR

- Allow coefficients to have flavor dependence
- Consider operators that contribute both to top pair production at the LHC and to EWPO at 1-loop
- For some operators, similar sensitivity

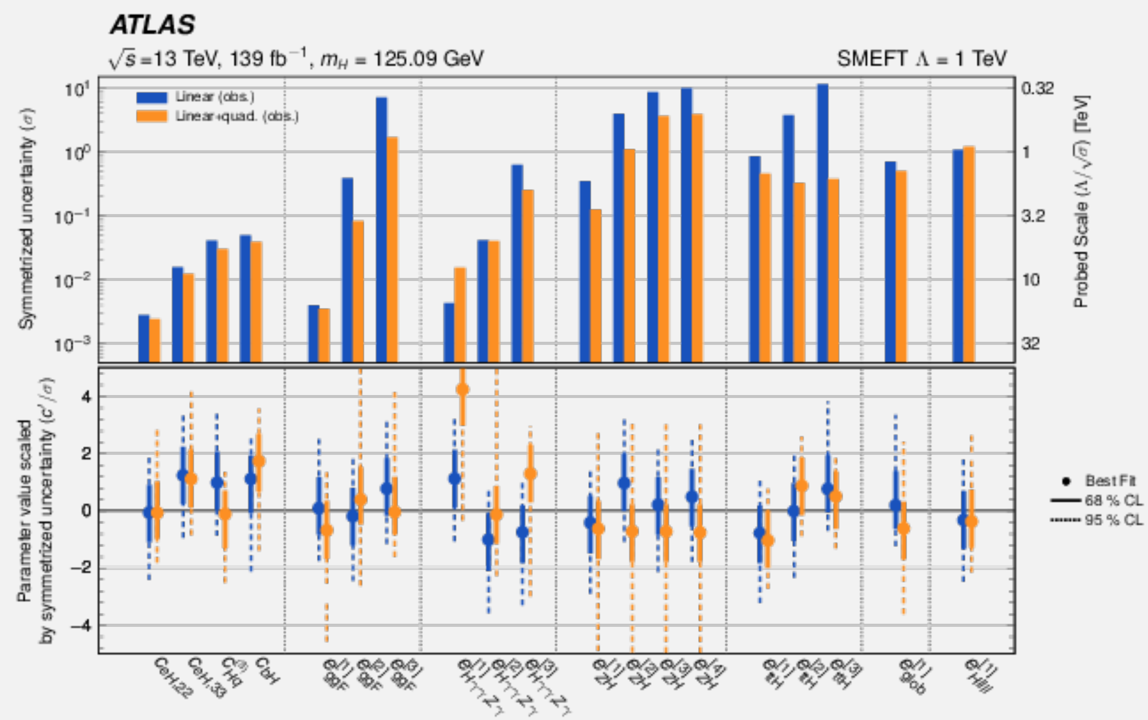


SMEFT message: CONNECTIONS between data sets

2201.09887

GLOBAL FIT TO HIGGS

- ATLAS fit to Higgs data
- ***Comparison of linear and quadratic fits***
- Not huge difference between them (the better the limit is, the closer they are)



ATLAS, [2402.05742](2402.05742)

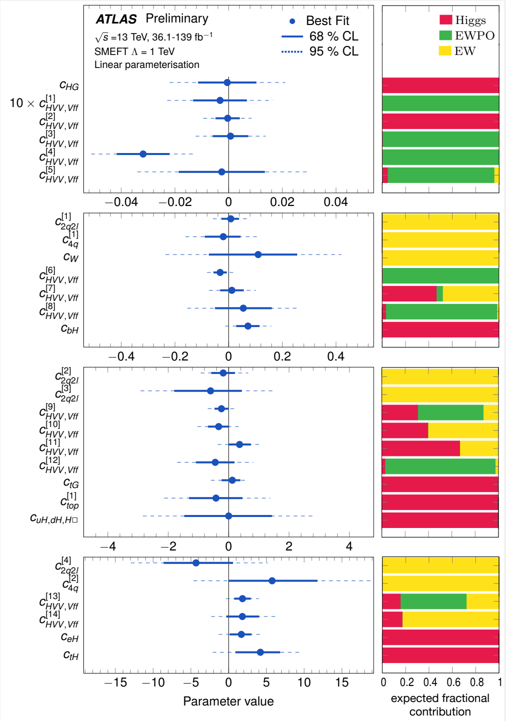
WHERE DO LIMITS COME FROM?

ATLAS fit to Higgs, VV, EWPO data

- Electroweak precision observables:
- LHC Higgs data
- LHC and LEPII W^+W^- data
- (Top data)

Often, multiple measurements contribute to limits

- Typically probe 1-10 TeV scale ***(with C=1)***



* Linear fit

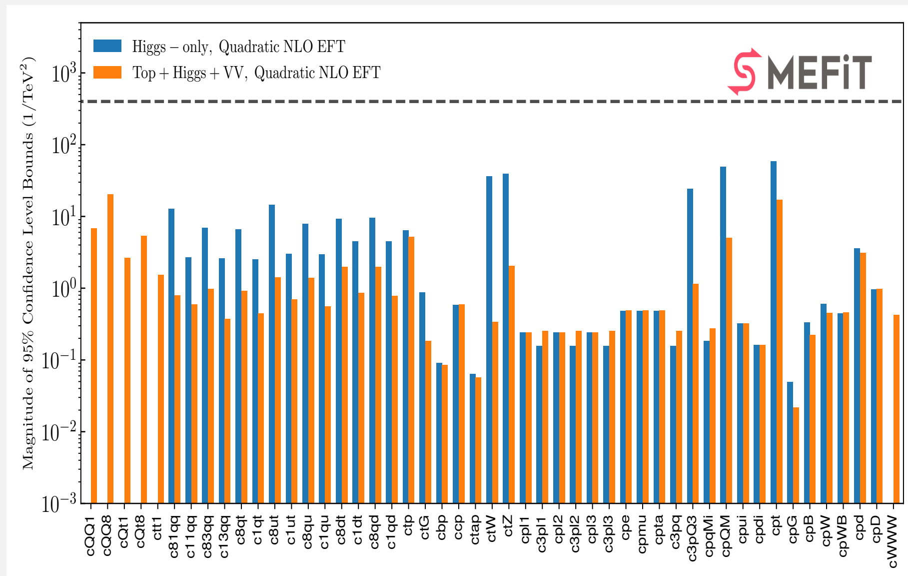
MANY GLOBAL FITS

- ***Include top, Higgs, VV***

$$A \sim A_{SM} + a_i \frac{C_{6i}}{\Lambda^2} + a_{ij} \frac{C_{6i}C_{6j}}{\Lambda^4}$$

- Blue: Higgs only observables calculated to $1/\Lambda^4$ at dimension-6
- Red: Higgs + top+VV observables calculated to $1/\Lambda^4$ at dimension-6

Including top can make a big difference



2105.00006

WHAT DOES IT MEAN?

- I don't particularly care about the numerical value of some coefficient
- But… an unambiguously non-zero value of a Wilson coefficient is a clear sign of new physics.
- Power of EFTs is that coefficients can be matched to high scale models of underlying UV physics

Different BSM models will have different (calculable) patterns of coefficients

PATTERNS

- Only a small number of operators generated in specific models
- Coefficients can be computed in terms of BSM inputs

	Singlet$_{Z_2}$	Singlet$_{Z_2}$	2HDM	T VLQ	(TB) VLQ	s
C_ϕ						
$C_{\phi\Box}$						
$C_{b\phi}$						
$C_{t\phi}$						
$C_{\tau\phi}$						
$C^1_{\phi q}[tt]$						
$C^3_{\phi q}[tt]$						
$C_{\phi b}$						
$C_{\phi t}$						
$C_{\phi tb}$						
$C_{\phi G}$						

INVERSE PROBLEM

- If we measure non-zero SMEFT coefficients, can we determine the underlying high scale model?
- In simple models (ie 1 new massive particle, whose interactions are described in terms of a single parameter) the particles that can contribute to dimension-6 operators have been categorized long ago
- Dimension-6 contributions only sensitive to C/Λ^2: Scale interpretation ambiguous

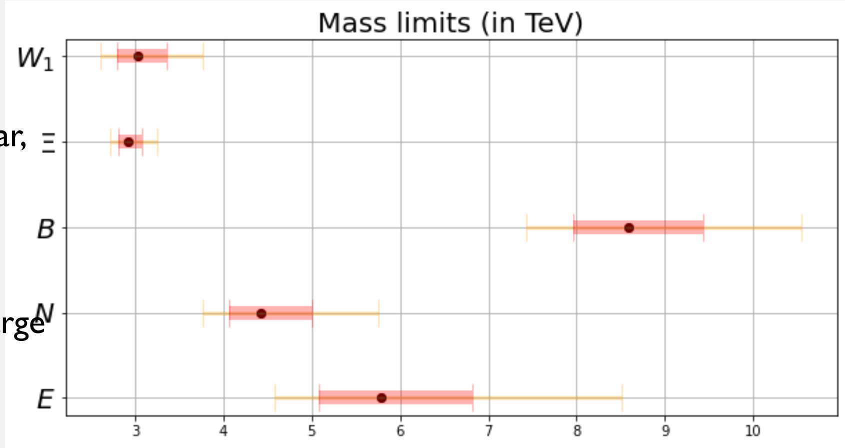
Global fit with C=1

SU(2) triplet gauge boson

SU(2) triplet scalar, Y=0

Neutral gauge boson

Charge 0 and charge 1 fermions



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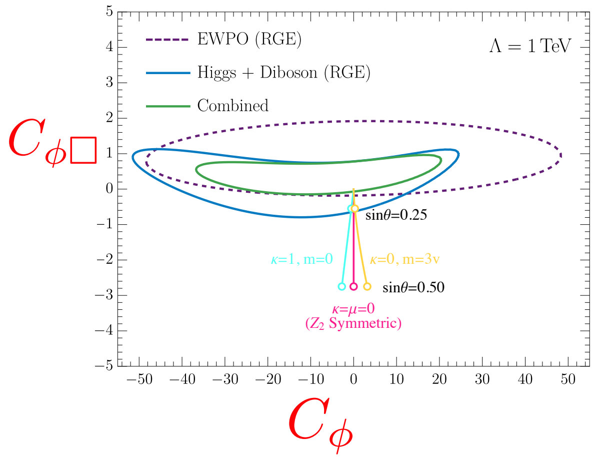
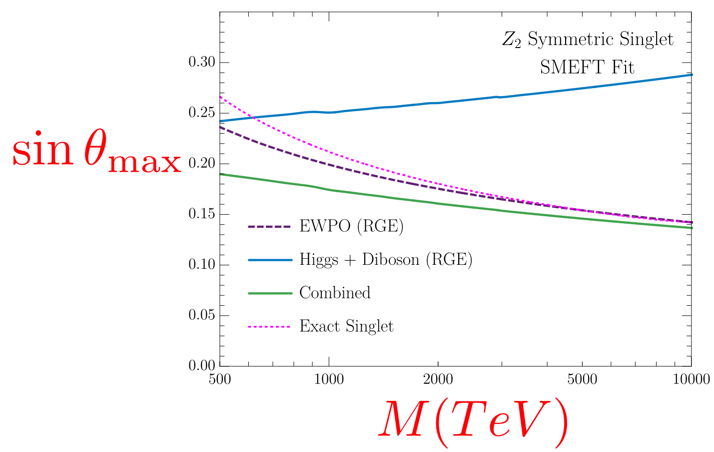
DO FITS TO SUBSETS OF OPERATORS

What if we assume a singlet model at the high scale?





Interpret fit results in terms of model parameters (M and sin θ)

C_ϕ and $C_{\phi\Box}$ don't contribute to EWPOs at tree level

Information from RGE running of coefficients from Λ to M_Z

2HDM IS A GOOD TESTING GROUND

- Consider model with 2 Higgs doublets, Φ_1 and Φ_2 with a softly broken Z_2 symmetry: $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$
- 5 physical Higgs bosons, h_{125}, H_0, A, $H^\pm$
- Rotate to the Higgs basis $\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$
- In this basis $\langle H_2\rangle=0$, $\langle H_1\rangle=v/\sqrt{2}$
- Very convenient for SMEFT studies

$$\begin{aligned} V =& Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c.}\right) + \frac{Z_1}{2}\left(H_1^\dagger H_1\right)^2 \\ &+ \frac{Z_2}{2}\left(H_2^\dagger H_2\right)^2 + Z_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + Z_4\left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) \\ &+ \left\{\frac{Z_5}{2}\left(H_1^\dagger H_2\right)^2 + Z_6\left(H_1^\dagger H_1\right)\left(H_1^\dagger H_2\right) + Z_7\left(H_2^\dagger H_2\right)\left(H_1^\dagger H_2\right) + \text{h.c.}\right\} \end{aligned}$$

- Z's can be written in terms of physical parameters $v,\ \beta-\alpha,\ m_{h_{125}},\ Y_2,\ m_{H_0},\ m_A,\ m_{H^\pm}$

S. Dawson, BNL

2HDM CONTINUED

- 4 choices for fermion Yukawas (avoid tree level FCNC)

$$\mathcal{L}_Y \sim -\lambda_u^{(1)} \bar{u}_R \tilde{H}_1^\dagger q_L - \lambda_u^{(2)} \bar{u}_R \tilde{H}_2^\dagger q_L - \lambda_d^{(1)} \bar{d}_R H_1^\dagger q_L - \lambda_d^{(2)} \bar{d}_R H_2^\dagger q_L + h.c.$$

$$\lambda_f^{(1)} = \frac{\sqrt{2}}{v} m_f \qquad \lambda_f^{(2)} = \frac{\eta_f}{\tan\beta} \lambda_f^{(1)}$$

	Type-I	Type-II	Type-L	Type-F
η_u	1	1	1	1
η_d	1	$-\tan^2\beta$	1	$-\tan^2\beta$
η_l	1	$-\tan^2\beta$	$-\tan^2\beta$	1

- Type II is MSSM-like
- Type I has enhanced (suppressed) couplings to b quarks at small (large) tan β

MATCH TO SMEFT AT DIMENSION-6

- At dimension-6, observables depend on C/Λ^2 (ie you can't determine a scale independently of assumptions about coefficients, C)
- Decoupling limit: $(Y_3/Y_2)<<1$
- At tree level dimension-6, 2HDM SMEFT matching generates:

$$\frac{v^2 C_\phi}{\Lambda^2} = \frac{\cos^2(\beta-\alpha) M^2}{v^2} \qquad \frac{v^2 C_{t\phi}}{\Lambda^2} = -\frac{\eta_t \sqrt{2} m_t \cos(\beta-\alpha)}{v \tan\beta}$$

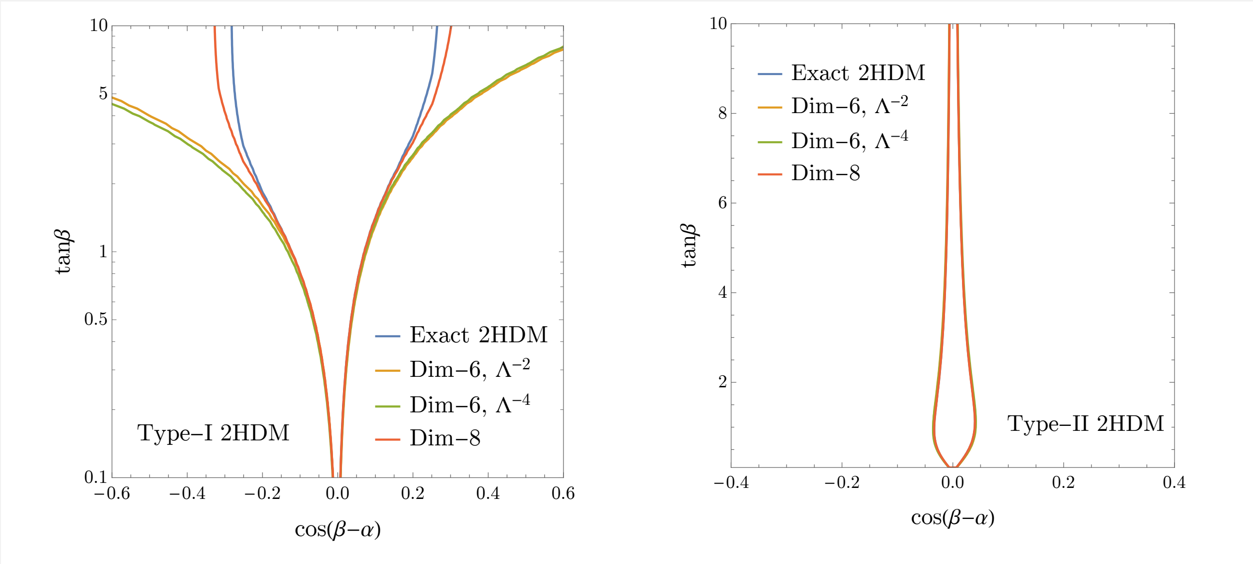
$$\frac{v^2 C_{b\phi}}{\Lambda^2} = -\frac{\eta_b \sqrt{2} m_b \cos(\beta-\alpha)}{v \tan\beta} \qquad \frac{v^2 C_{\tau\phi}}{\Lambda^2} = -\frac{\eta_\tau \sqrt{2} m_\tau \cos(\beta-\alpha)}{v \tan\beta}$$

- Dimension-6 matching does NOT generate 2HDM VVh_{125} couplings!

$$O_{f\phi} = (\phi^\dagger \phi)(\overline{q}_L \tilde{\phi} f_R) \qquad O_\phi = (\phi^\dagger \phi)^3 \qquad \frac{C_\phi}{\Lambda^2} \sim (\ldots)\delta\lambda_3$$

* M is common mass of heavy scalars

DIMENSION-8 MATTERS IN 2HDM



- hWW and hZZ couplings generated at dimension-8
- Including only dimension-6 operators does not capture the physics of the full model

2205.01561

FINALLY, WHAT IF IT'S NOT SMEFT?

- What if Higgs is not part of an SU(2) doublet? $\rightarrow$ HEFT (Higgs Effective Field Theory)
- Expansion is different from SMEFT (LO Lagrangian here)

$$L_{HEFT} \sim \frac{v^2}{4}\left[1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + ...\right] Tr\left\{D_\mu U^\dagger D_\mu U\right\} + \frac{1}{2}(\partial_\mu h)^2 - V(h)$$

$$V(h) = \frac{1}{2}m_h^2 h^2\left(1 + \kappa_3\frac{h}{v} + \frac{\kappa_4}{4}\frac{h^2}{v^2} + ...\right)$$

h is physical Higgs

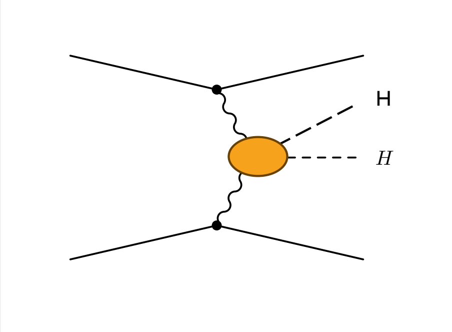
$$D_\mu U = \partial_\mu U + igW_\mu^a\frac{\sigma^a}{2}U - ig'U\frac{\sigma^3}{2}B_\mu$$

- Unitary gauge, U→1; SM: $a=b=\kappa_3=\kappa_4=1$ SMEFT: $b - a = \frac{3C_{H\Box}v^2}{\Lambda^2}$
- Suggests that hh→hh, WW→hh can distinguish between SMEFT and HEFT

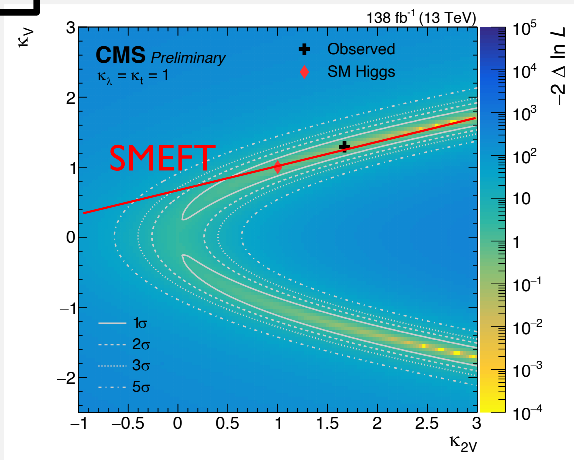
2204.01763 , 2307.15693, 2305.07689 , 2311.16897 , 2312.03877, 2211.09605

SMEFT VS HEFT

HH production via VBS can potentially distinguish SMEFT from HEFT



LHC



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$a=\kappa_V,\ b=\kappa_{2V}$

Is the Higgs in an SU(2) doublet?

IN SUMMARY

- SMEFT and HEFT are messy, but they are the only toolswe have to search for new physics in the absence of new light particles
- Understanding the uncertainties and assumptions is crucial
- Significant progress, but still plenty of low hanging fruit for theorists

EXTRA

ASIDE

- Consider $2 \to 2$ particle elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin *l* partial waves

  Optical theorem requires: $|\text{Re}(a_l)| \le \frac{1}{2}$

UNITARITY CONSTRAINT