From amplitudes to cross sections and events

2024 Summer School in Particle Physics
Phenomenology after the Higgs Discovery

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Useful references

- 0709.2092 by Nason, Frixione, Oleari: comprehensive review of **NLO calculations** (sec. 2,5,6)
- 0902.0293 lecure notes on the 2006 “Work- shop on Monte Carlo’s, Physics and Simulations at the LHC”, especially Sec. 2 on **Parton Showers** by P. Nason
- 1101.2599 General-purpose event generators for LHC physics by Buckley et al., especially for **matching** and **merging** (Sec. 5), and **non-perturbative physics** in MC (Secs. 6-7)
- 0407286 by Banfi, Salam, Zanderighi: comprehensive review of **NLL resummation** in direct QCD (summarised in Sec. 2 of 1412.2126 by Banfi, McAslan, Monni and Zanderighi)
- 1410.1892 by Becher, Broggio and Ferroglia, introduction to **SCET**

(*) quickly covered in these lectures, (*) not covered but good for your education

The Standard Model of Particle Physics

- The **STANDARD MODEL LAGRANGIAN** encapsulates our understanding of the **fundamental interactions** between **elementary particles**

$$\mathscr{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(iD^{\mu}\gamma_{\mu} - m)\psi + \ldots = \mathscr{L}(m, \alpha, \ldots)$$

Masses, coupling constants are free parameters, that we need to extract from **data**

- We need to calculate quantities that depends on m, α, that the experimentalists can measure: **cross sections**

$$\sigma_{\text{exp}} = \frac{N}{\mathscr{L}}$$

Number of events per unit of time

Number of colisions per unit of area per unit of time

Squared amplitude

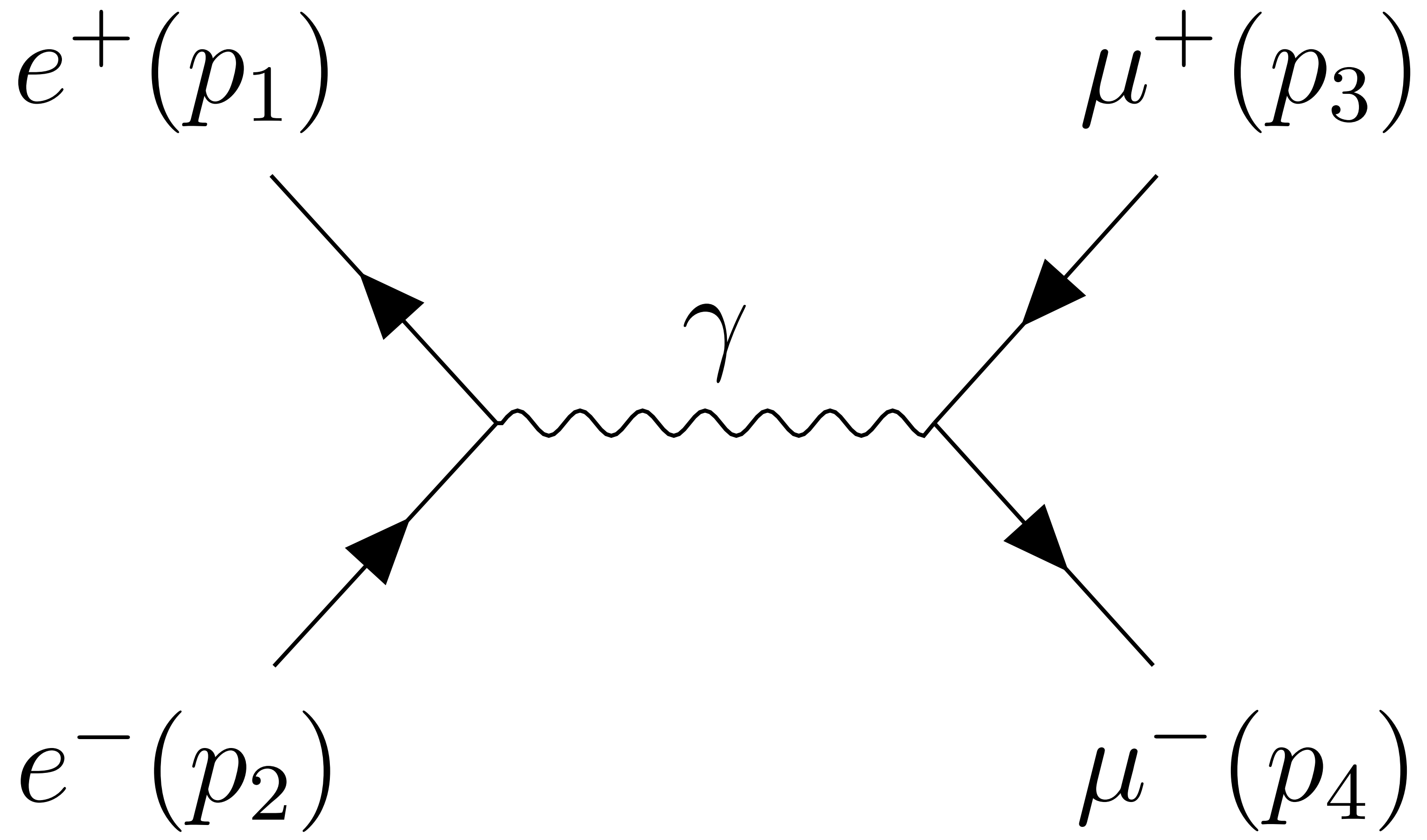
Phase space

$$\sigma_{\text{th}} = \int \frac{|\mathscr{A}|^2}{2s} d\Phi\, \Theta(\Phi)$$

Flux factor

Acceptance cuts

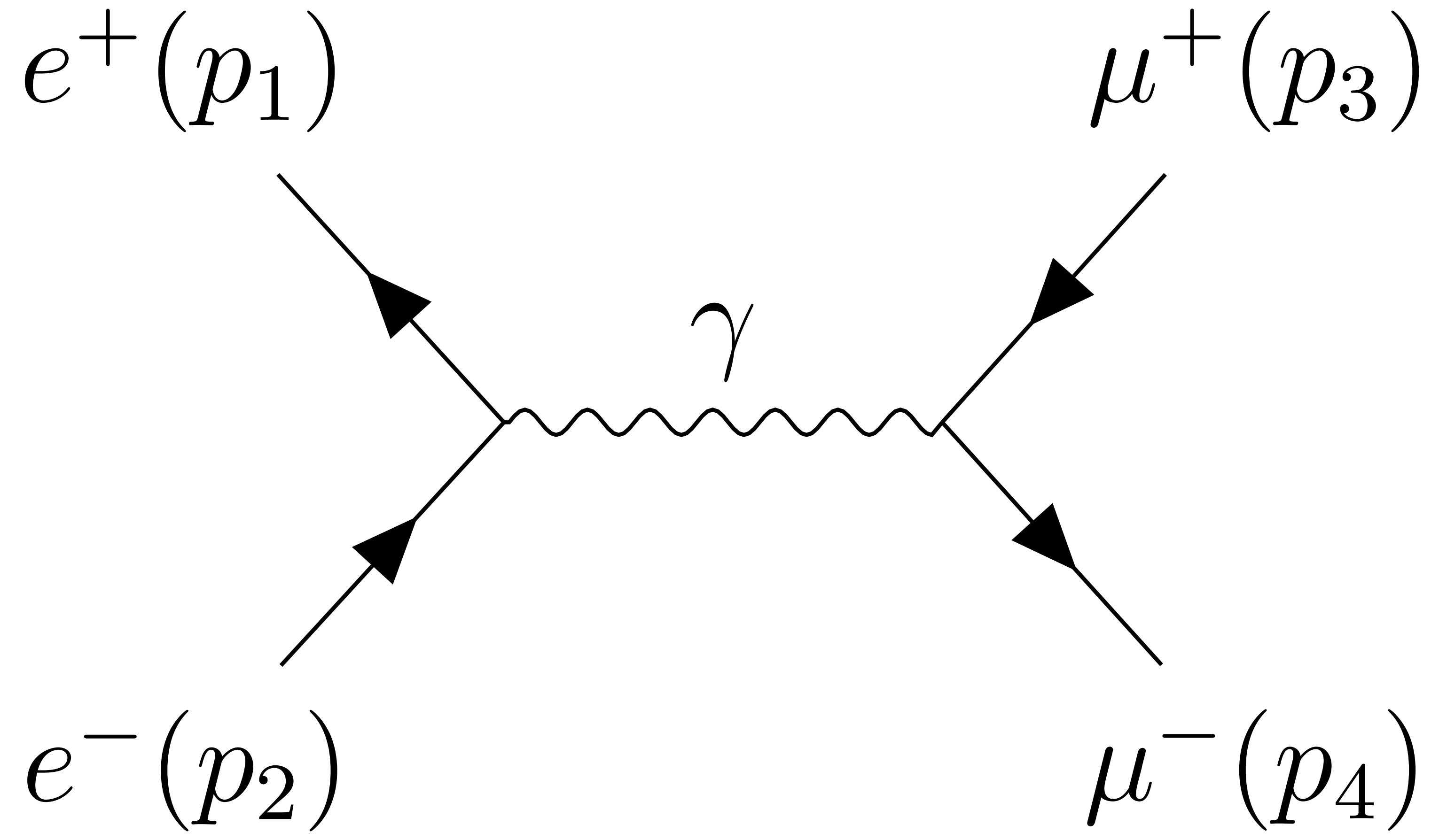
Example: $e^+e^- \to \mu^+\mu^-$



Let's calculate the cross-section for the production of a $\mu^+\mu^-$ pair in e^+e^- collisions, we need:

- **Amplitude** $\mathscr{A}$
- Two-body **phase space** $d\Phi_2$

Amplitude for $e^+e^- \to \mu^+\mu^-$



$$\mathcal{A} = [\bar{u}_e(p_1)(-ie\gamma^\mu)v_e(p_2)] \times [\bar{v}_\mu(p_3)(-ie\gamma^\nu)u_\mu(p_4)] \times \frac{-ig_{\mu\nu}}{(p_1+p_2)^2}$$

$$|\mathcal{A}|^2 = \frac{1}{4}\left(\frac{e^2}{2p_1\cdot p_2}\right)^2 \mathrm{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu)\mathrm{Tr}(\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu)$$

Square and sum over polarisations

Average initial-state polarisations

$$= 2e^4\frac{(p_1\cdot p_3)^2+(p_1\cdot p_4)^2}{(p_1\cdot p_2)^2}$$

Two-body phase space

$$d\Phi = \left(\prod_{i=3}^{4}[dp_i]\right)(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4) \quad \textit{with} \quad [dp_i] = \frac{d^4p_i}{(2\pi)^4}\delta_+(p_i^2-m_i^2) = \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}$$

➤ Lorentz invariant →can be computed in any frame, in the **center-of-mass frame**

$$p_1 = \frac{\sqrt{s}}{2}\{1,0,0,1\} \qquad p_2 = \frac{\sqrt{s}}{2}\{1,0,0,-1\}$$

→ in this frame $\vec{p}_3 + \vec{p}_4 = \vec{0}$: we use the x, y, z components of the δ to integrate over $\vec{p}_4$

$$d\Phi_2 = \frac{d^3\vec{p}_3}{((2\pi)^3 2|\vec{p}_3|)^2}(2\pi)^4\delta(\sqrt{s}-2|\vec{p}_3|) = \frac{|\vec{p}_3|^2 d|\vec{p}_3| d\Omega_2}{16\pi^2|\vec{p}_3|^2}\delta(\sqrt{s}-2|\vec{p}_3|) = \frac{d\Omega_2}{32\pi^2}$$

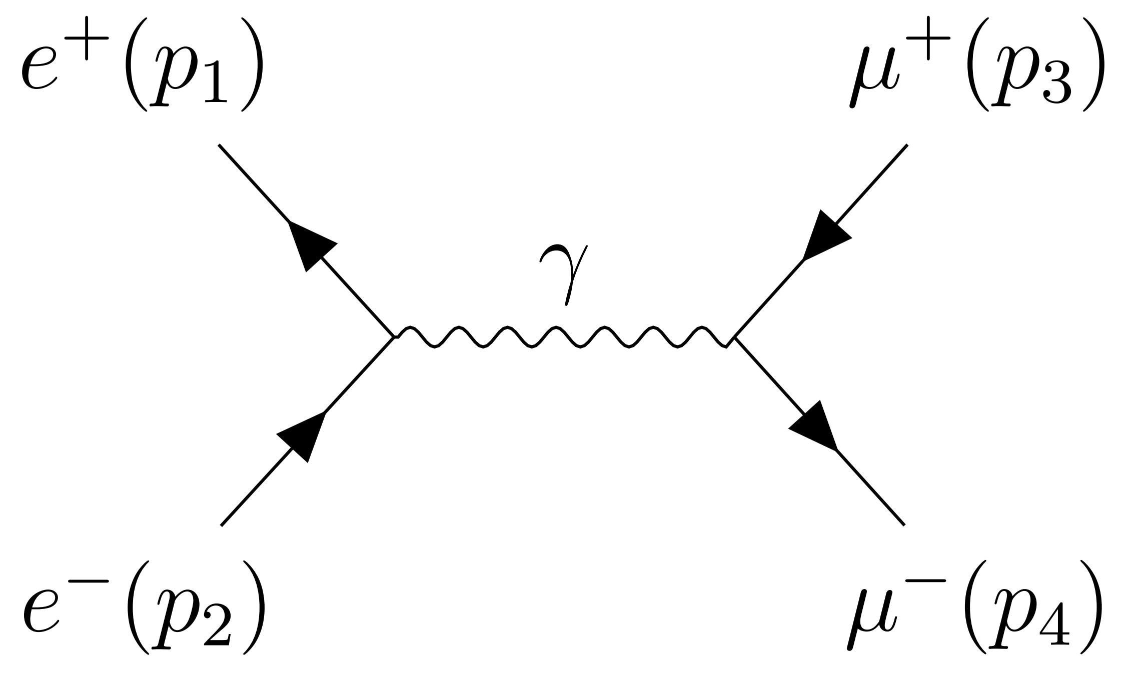
Spherical coordinates; $d\Omega_2 = d\cos\theta d\phi$

$$|\vec{p}_3| = |\vec{p}_4| = \frac{\sqrt{s}}{2}$$

$$p_3 = \frac{\sqrt{s}}{2}\{1, \sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta\}$$

$$p_4 = \frac{\sqrt{s}}{2}\{1, -\sin\theta\sin\phi, -\sin\theta\cos\phi, -\cos\theta\}$$

Example: $e^+e^- \to \mu^+\mu^-$



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$$p_4 = \frac{\sqrt{s}}{2}\{1, -\sin\theta\sin\phi, -\sin\theta\cos\phi, -\cos\theta\}$$

$$d\Phi = \frac{d\cos\theta d\phi}{32\pi^2} \qquad |\mathscr{A}|^2 = 2e^4\frac{(p_1\cdot p_3)^2 + (p_1\cdot p_4)^2}{(p_1\cdot p_2)^2}$$

$$d\sigma = \frac{|\mathscr{A}|^2}{2s}d\Phi = \frac{\alpha_{\mathrm{em}}^2\pi}{2s}(1+\cos\theta^2)\,d\cos\theta\,\frac{d\phi}{2\pi}$$

$$\sigma = \frac{\alpha_{\mathrm{em}}^2\pi}{2s}\int_{-1}^{1}(1+\cos\theta^2)\,d\cos\theta\int_0^{2\pi}\frac{d\phi}{2\pi} = \frac{4\alpha_{\mathrm{em}}^2\pi}{3s}$$

$e^+e^- \to$ jets

If instead of muons we want to produce quarks…

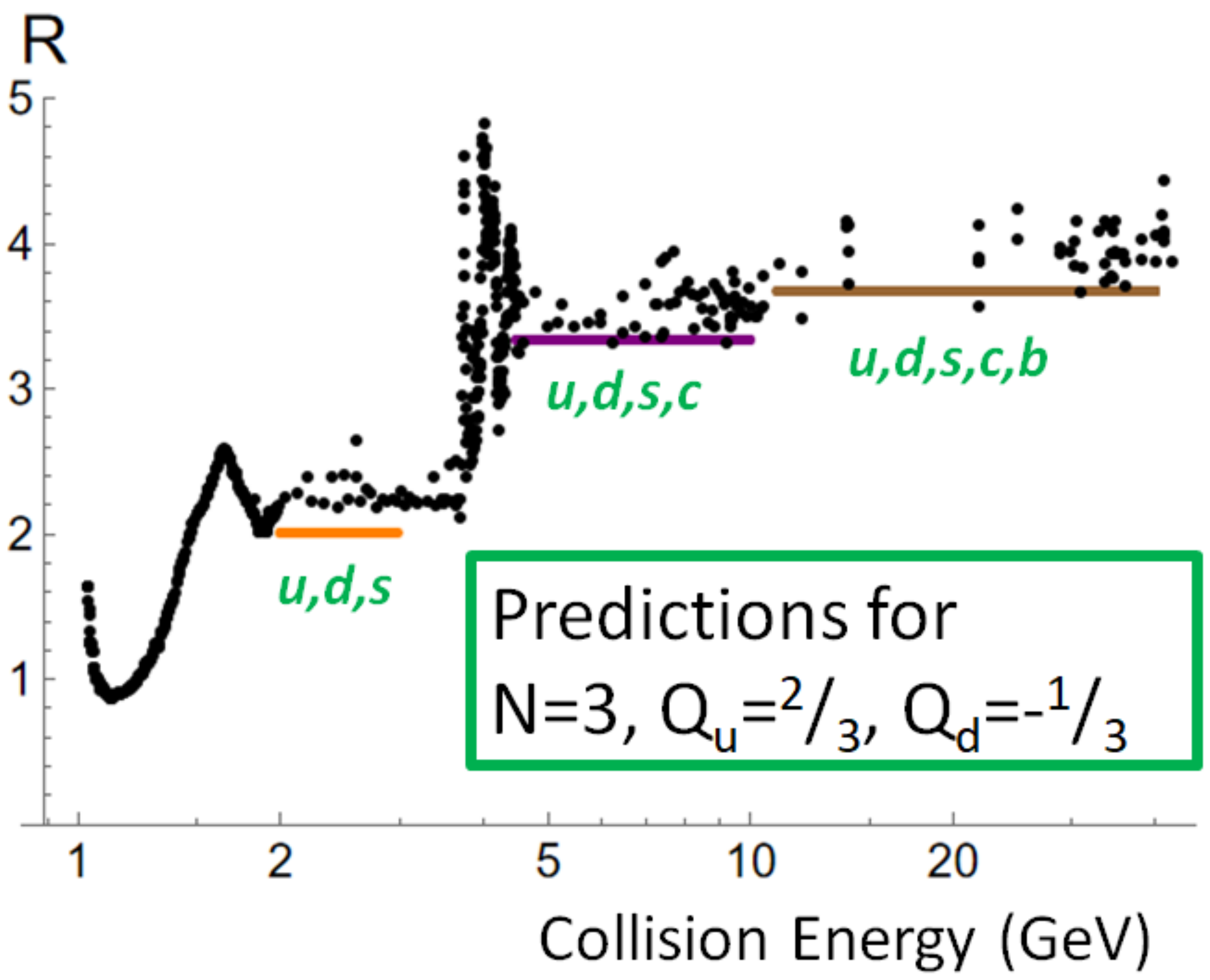


Fig. by Matt Strassler

Number of active quarks

$$d\sigma^{\rm LO}_{e^+e^-\to q\bar{q}} = N_c \sum_{i=1}^{n_f} Q_i^2 \, d\sigma^{\rm LO}_{e^+e^-\to\mu^-\mu^+}$$

Number of colours $(N_c = 3)$

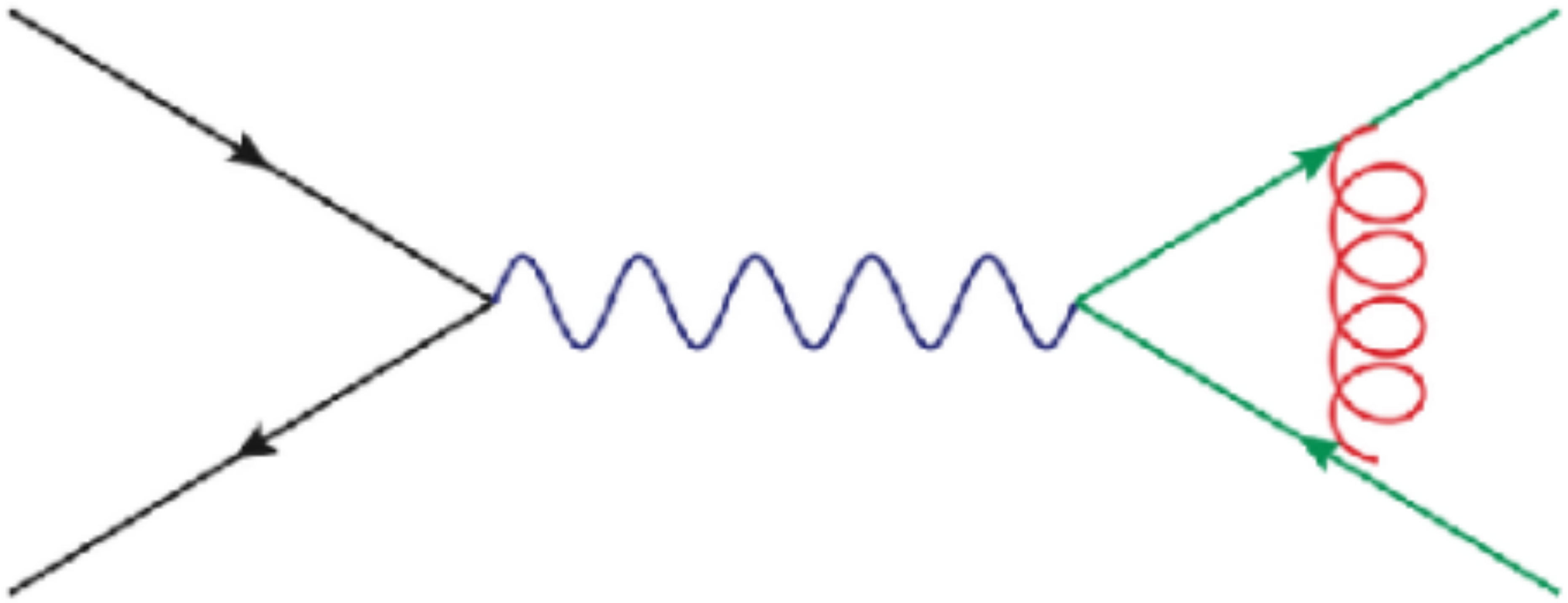
Quark charges

This is the **LEADING ORDER** prediction, can we do better?

NLO virtual corrections

The **QCD** coupling constant is quite sizeable $\alpha_s \sim 0.118$: let's include $\mathcal{O}(\alpha_s)$ **corrections**!

➤ At this order we have the interference between the tree level (=LO) amplitude, and the one containing a **virtual** gluon



$$\mathscr{A}_V \sim \int \frac{d^4\ell}{\ell^2(\ell+p_3)^2(\ell-p_4)^2} \xrightarrow[\ell \text{ soft}]{} \int \frac{d^4\ell}{\ell^4}$$

Infrared divergenge in 4 dimensions! Use **dimensional regularisation:** $D = 4 - 2\epsilon$

$$d\sigma_V = \frac{2\mathrm{Re}(\mathscr{A}_V \mathscr{A}_B^*)}{2s} d\Phi_2 = d\sigma_B \frac{\alpha_s}{2\pi} C_F C_\Gamma \left(\frac{\mu^2}{s}\right)^\epsilon \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right]$$

$$C_\Gamma = (4\pi)^\epsilon \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}$$

This regularisation introduces a non-phyisical scale..

Infrared poles

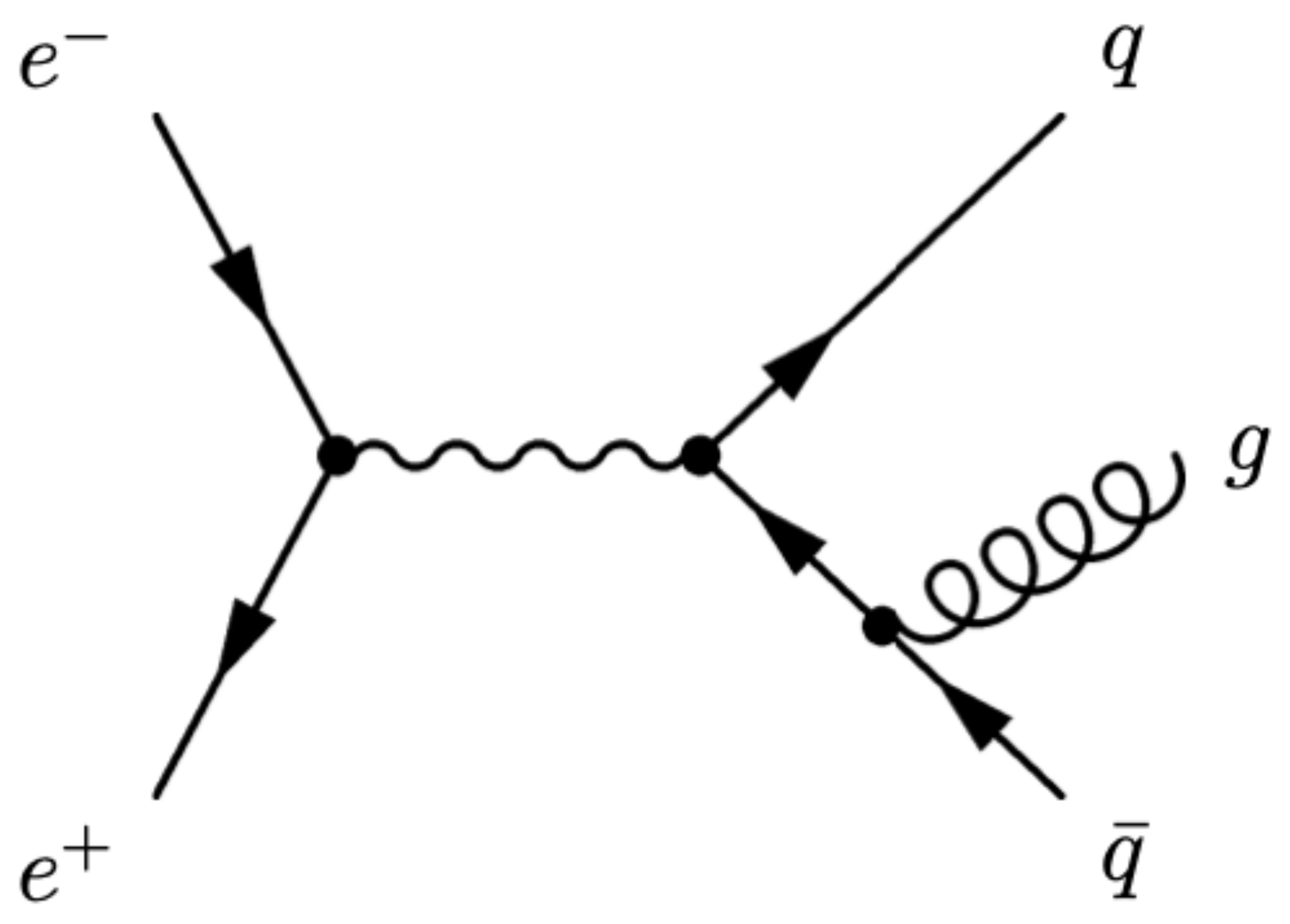
NLO corrections

The **QCD** coupling constant is quite sizeable $\alpha_s \sim 0.118$: let's include $\mathcal{O}(\alpha_s)$ **corrections**!

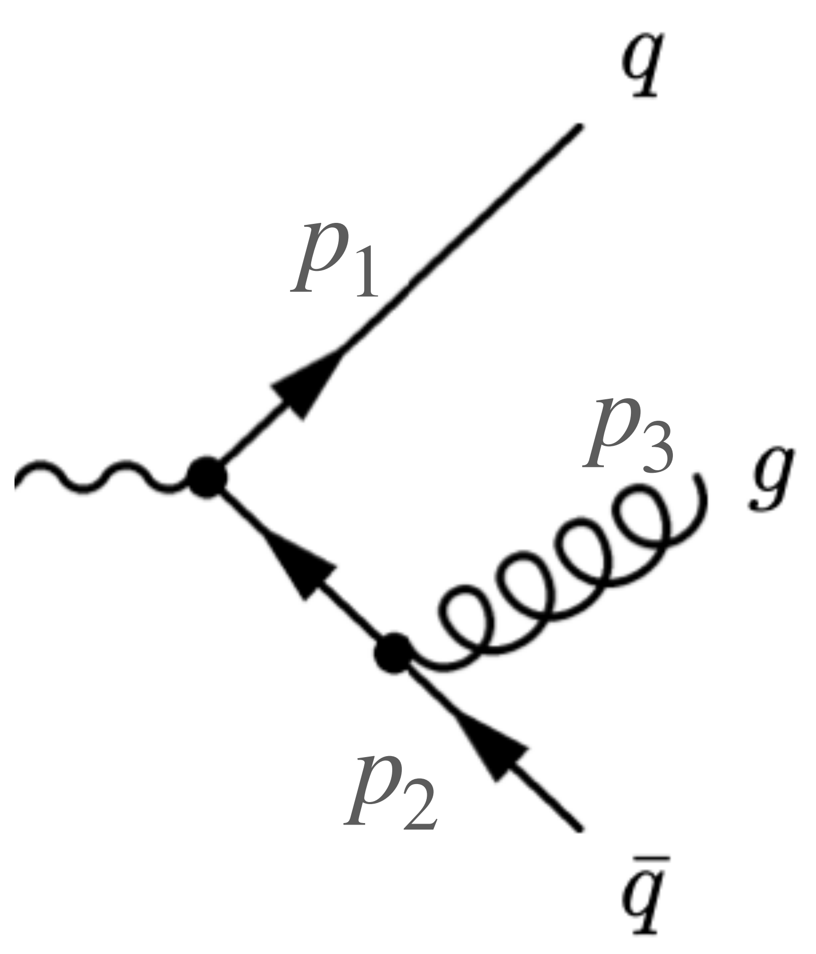
$$d\sigma_V = \frac{2\text{Re}(\mathcal{A}_V \mathcal{A}_B^*)}{2s} d\Phi_2 = d\sigma_B \frac{\alpha_s}{2\pi} C_F C_\Gamma \left(\frac{\mu^2}{s}\right)^\epsilon \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right]$$

For $\epsilon \to 0$, $\sigma_V \to -\infty$: the result is meaningless, why? Because we cannot ask for the cross section for producing exactly two quarks! We cannot distinguish the case where there is also a soft (=low energy) or collinear (parallel to q or $\bar{q}$) gluon.

Kinoshita, Lee, Naumeberg theorem: **we need to sum over degenerate states to get a physical cross section!**



NLO real corrections



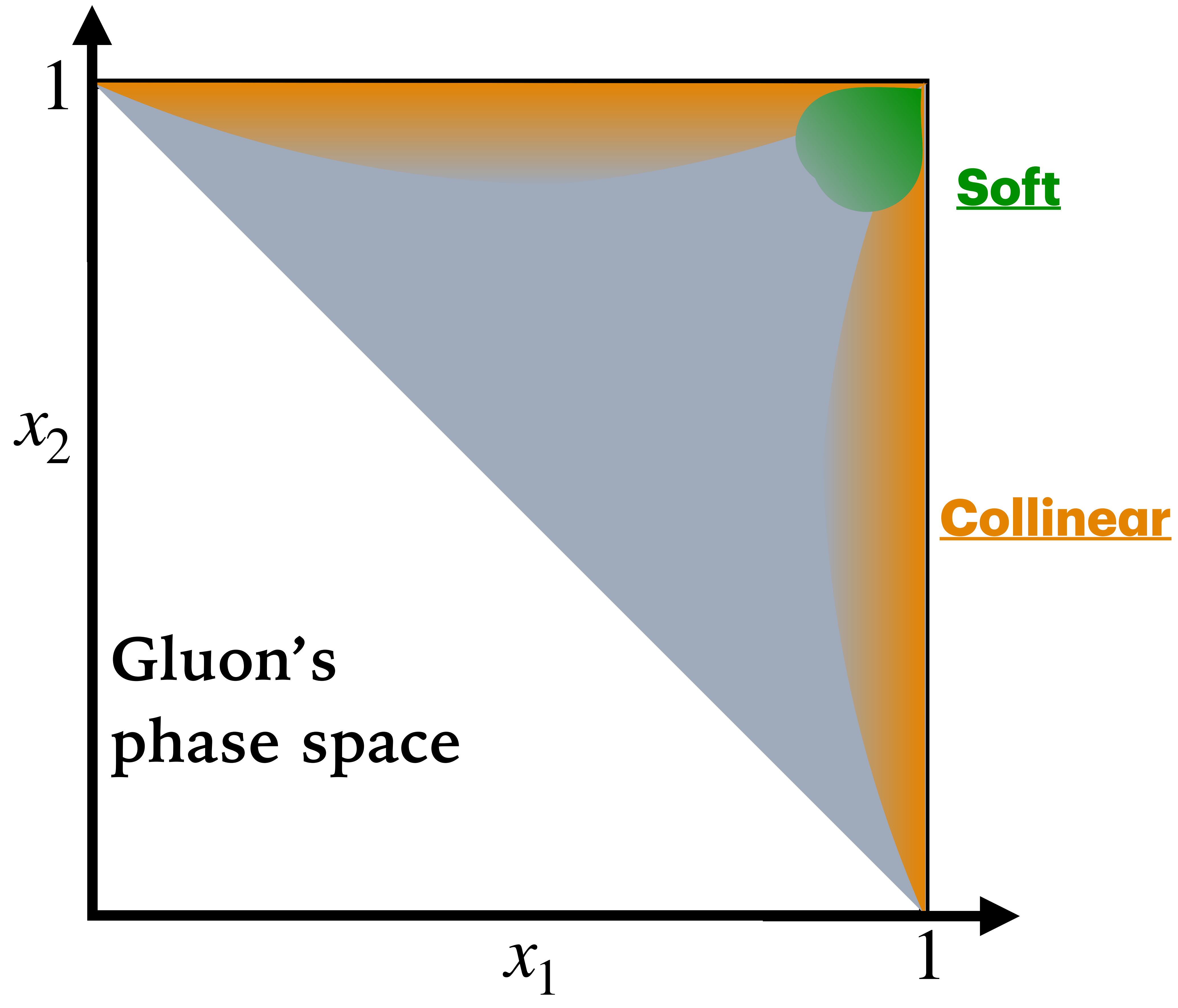
$$\frac{d\Phi_3}{\Phi_2} = \frac{dx_1 dx_2}{16\pi^2}(4\pi)^\epsilon s^{1-\epsilon}\frac{[(1-x_1)(1-x_2)(x_1+x_2-1)]^\epsilon}{\Gamma(1-\epsilon)}$$

$$x_i = \frac{2p_i \cdot p_{\rm tot}}{p_{\rm tot}^2} = \frac{2E_i}{E_{\rm tot}}$$

$$\frac{|\mathscr{A}_R|^2}{|\mathscr{A}_B|^2} = \frac{2g_s^2 C_F}{(1-x_1)(1-x_2)s}\left[x_1^2 + x_2^2 - \epsilon(2-x_1-x_2)^2\right]$$

Divergence for $x_{1,2} \to 0$

$$\sigma_R = \sigma_B \frac{\alpha_s}{2\pi} C_F C_\Gamma \left(\frac{\mu^2}{s}\right)^\epsilon \left[+\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right]$$



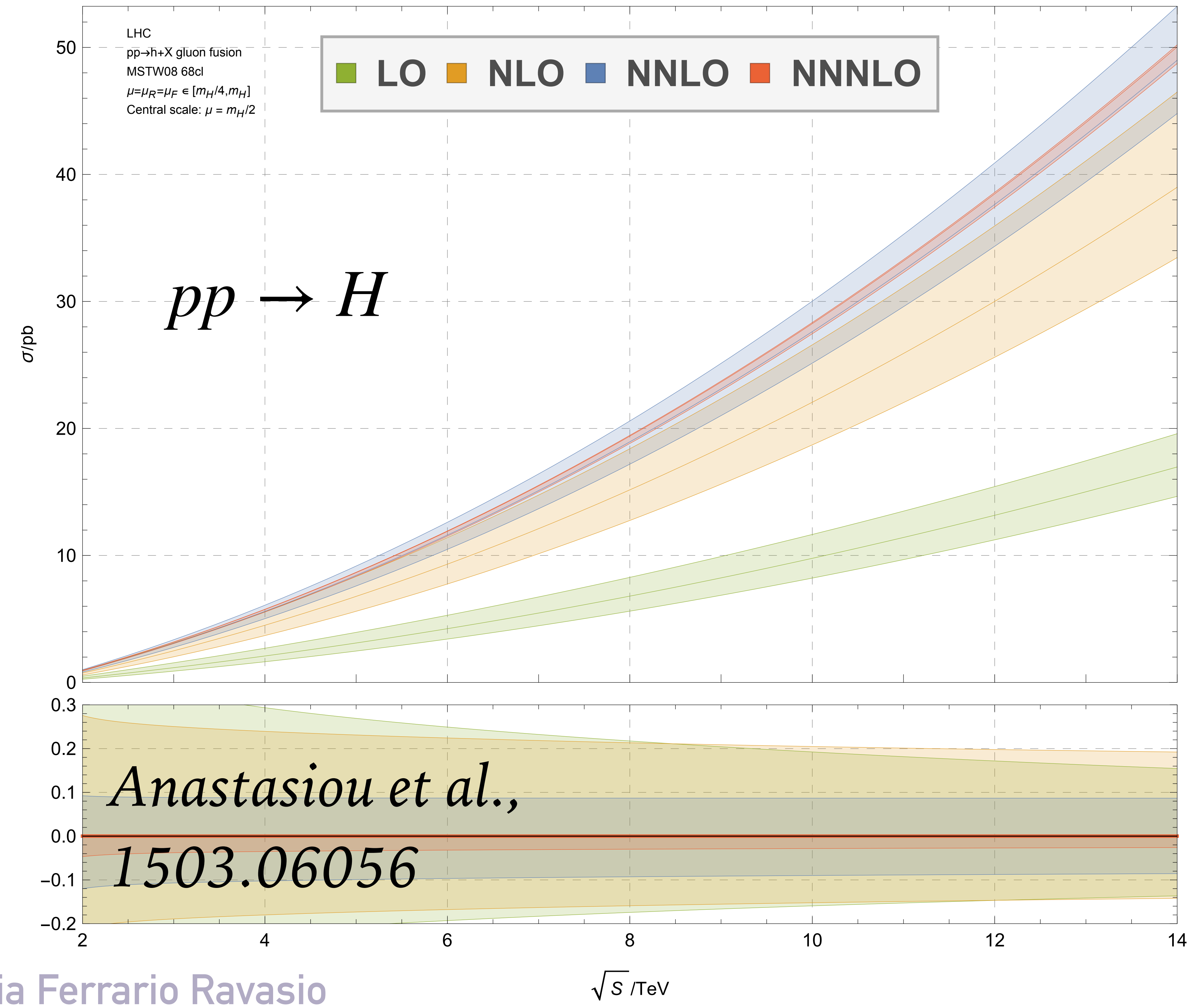
Cancels exactly the virtual divergence (KLN)!

Higer order corrections and uncertainties

- In general, virtual corrections exibit **ultraviolet** (although not for $e^+e^- \to j_1 j_2$) **and infrared singularities**: the former are reabsorbed introducing renormalised coupling constant (and masses), the latter cancel summing over degenerate states (KLN). The renormalisation procedure introduces a dependence on an unphysical scale

$$\sigma_{e^+e^-\to \text{jets}} = \sigma_{\text{LO}} \left(1 + \frac{\alpha_s(\mu_R)}{\pi}\right)$$

- When hadron colliders are involved, it is necessary to "renormalise" also the parton distributions, leading to another unphysical scale, dubber factorisation scale μ_F



- (μ_R, μ_F) variations by a factor 2 up and down conventional way to assess uncertainties from missing h.o.

- LO = gross features of an obs
  NLO = $\mathcal{O}(10-30\%)$ accuracy
  NNLO = necessary for percent-level precision
  N^{3}LO = available for $pp \to H, V$

Automating NLO calculations

Can we simplify the calculation of real corrections, so to tackle complex processes?

YES

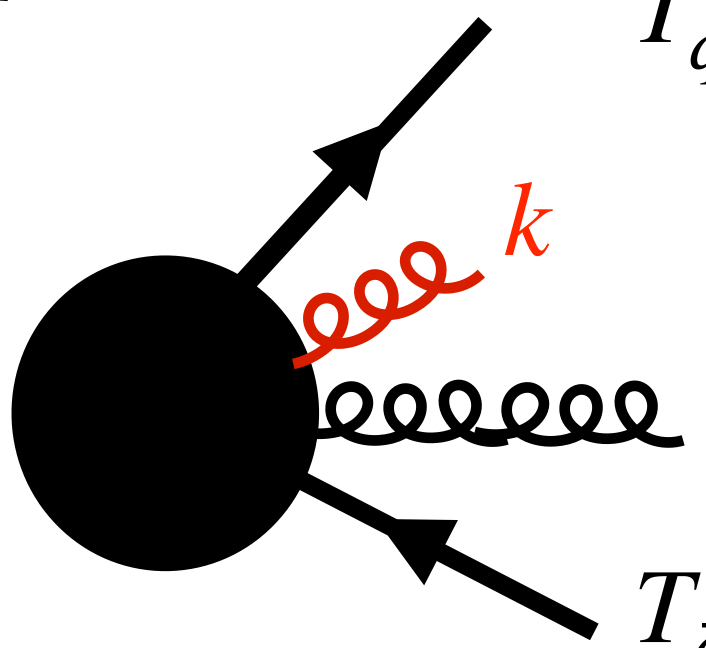
- Singular regions are associated with soft or collinear emissions

$$\frac{1}{(p+k)^2} = \frac{1}{2E_p E_k(1-\cos\theta_{pk})}$$

- In these limits, the amplitudes takes a **simple factorised form**

$$\frac{\mathscr{A}^2_{\text{soft}}}{\mathscr{A}^2_{\text{b}}} = -4\pi\alpha_s\mu^{2\epsilon} \times \sum_{i,j} \mathbf{T}_i \cdot \mathbf{T}_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}$$

Colour charges of the born partons



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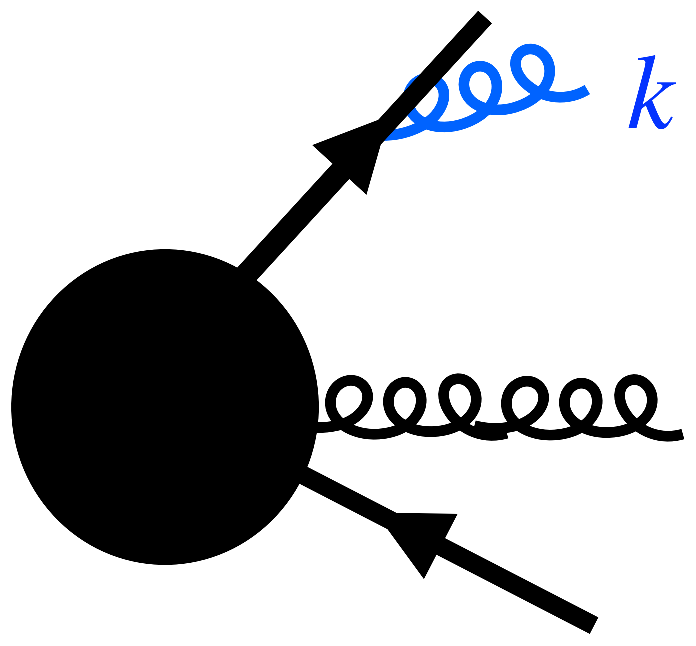


$$\frac{\mathcal{A}^2_{\text{soft}}}{\mathcal{A}^2_{\text{b}}} = -4\pi\alpha_s\mu^{2\epsilon} \times \sum_{i,j} \mathbf{T}_i \cdot \mathbf{T}_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)}$$

Colour charges of the born partons

$$\frac{\mathcal{A}^2_{\text{coll}}(k \parallel p)}{\mathcal{A}^2_{\text{b}}} = \frac{4\pi\alpha_s\mu^{\epsilon}}{p \cdot k} \hat{P}_{pk}(z,\epsilon)$$

Altarelli-Parisi splitting functions

$$< \hat{P}_{gg}(z;\epsilon) >= 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right]$$

$$< \hat{P}_{qq}(z;\epsilon) >= C_F \left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right]$$

$$< \hat{P}_{gq}(z;\epsilon) >= T_R \left[1 - \frac{2z(1-z)}{1-\epsilon}\right]$$

Automating NLO calculations

Can we simplify the calculation of real corrections, so to tackle complex processes?

YES

- In this limit, we know how to build an **approximation** of $d\sigma_R$ which captures the singular behaviour

$$d\sigma_R(\Phi_{n+1}) \sim d\sigma_B(\tilde{\Phi}_n)\, C(\tilde{\Phi}_n, \Phi_{\text{rad}}; \epsilon)$$

We need a mapping between $\Phi_{n+1} \leftrightarrow \Phi_n$

- $C(\tilde{\Phi}_n, \Phi_{\text{rad}})$ is usually simple enough to be integrated analytically over the radiated parton phase space Φ_{rad} in $D = 4 - 2\epsilon$

$$\bar{C}(\tilde{\Phi}_n; \epsilon) = \int d\Phi_{\text{rad}}^{(D=4-2\epsilon)} C(\Phi_{\text{rad}}, \tilde{\Phi}_b; \epsilon)$$

$$\sigma_{\text{NLO}} = \int d\Phi_n\, B(\Phi_n) \left(1 + \frac{V(\Phi_n, \epsilon)}{B(\Phi_n, \epsilon)} + \bar{C}(\Phi_n, \epsilon)\right) + \int d\Phi_{n+1} \left[R(\Phi_{n+1}) - C(\Phi_{n+1}) B(\tilde{\Phi}_n)\right]$$

D=4 — The limit $\epsilon \to 0$ is finite — Can be integrated in D=4

Automating NLO calculations

$$\sigma_{\rm NLO} = \int d\Phi_n\, B(\Phi_n)\left(1 + \frac{V(\Phi_n, \epsilon)}{B(\Phi_n, \epsilon)} + \bar{C}(\Phi_n, \epsilon)\right) \quad + \quad \int d\Phi_{n+1}\left[R(\Phi_{n+1}) - C(\Phi_{n+1})B(\tilde{\Phi}_n)\right]$$

Can we do more than inclusive cross sections?

YES

- We can calculate **infrared** (soft and collinear) safe observables:

$$\hat{O}(p_1, \ldots, p_n, k) \to \hat{O}(p_1, \ldots, p_n) \qquad \text{if } k \text{ is soft}$$

$$\hat{O}(p_1, \ldots, p_n, k) \to \hat{O}(p_1, \ldots, p_n + k) \qquad \text{if } k \text{ is collinear to } p_n$$

$$\frac{d\sigma}{dO} = \int d\Phi_n\, B(\Phi_n)\left(1 + \frac{V(\Phi_n, \epsilon)}{B(\Phi_n, \epsilon)} + C(\Phi_n, \epsilon)\right)\delta(\hat{O}(\Phi_n) - O)$$

$$+ \int d\Phi_{n+1}\left[R(\Phi_{n+1})\delta(\hat{O}(\Phi_{n+1}) - O) - C(\Phi_{n+1})B(\tilde{\Phi}_n)\delta(\hat{O}(\tilde{\Phi}_n) - O)\right]$$

Approximate calculations for infrared-safe observables

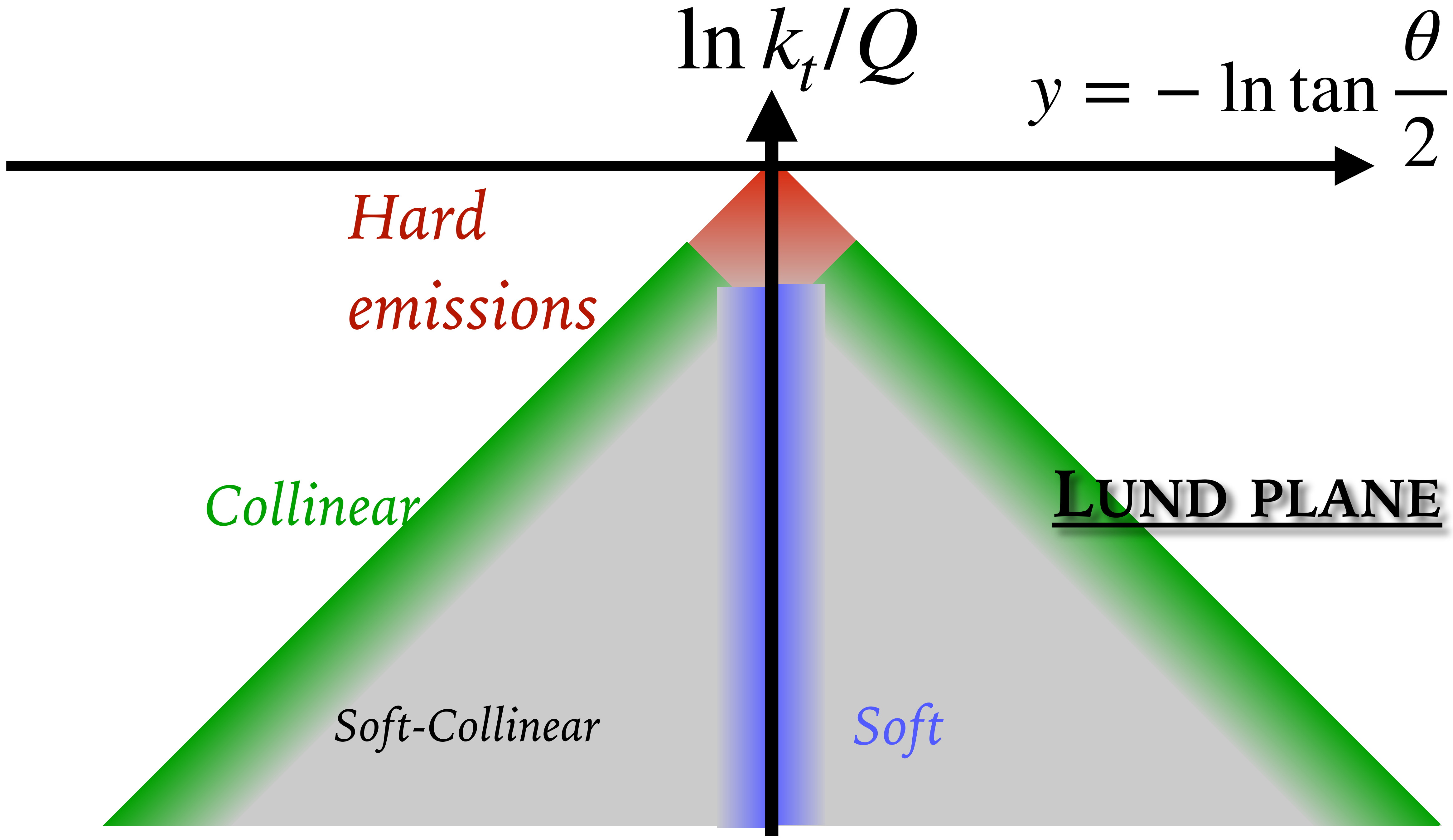
➤ Let's calculate the cumulative cross section for an **infrared safe observable** at NLO for $e^+e^- \to$ jets

$$\Sigma_{\rm NLO}(o) = \int d\Phi \frac{d\sigma}{d\phi}\Theta(\hat{O}(\Phi) < o) = \int d\Phi_{n+1}\frac{d\sigma_R}{d\Phi_{n+1}}\Theta(\hat{O}(\Phi_{n+1}) < o) + \int d\Phi_n \frac{d\sigma_V}{d\Phi_n}\Theta(\hat{O}(\Phi_n) < o)$$

➤ The leading contribution to σ_R comes from emissions simultaneusly **soft and collinear**

$$d\sigma_{\rm R} \approx \sigma_B \frac{2C_F\alpha_s}{\pi}\frac{dk_t}{k_t}dy\frac{d\phi}{2\pi}\Theta(|y| < \ln Q/k_t)$$

➤ The virtual correction must cancel the divergencies in σ_R: $d\sigma_V \approx -d\sigma_R$



$$\Sigma_{\rm NLO}(o) \approx \frac{2C_F\alpha_s}{\pi}\int \frac{dk_t}{k_t}dy\frac{d\phi}{2\pi}\Theta(|y| < \ln Q/k_t)\Big[\Theta(o - \hat{O}_3(p_1, p_2, k)) - \Theta(o - \hat{O}_2(\tilde{p}_1, \tilde{p}_2))\Big]$$

real virtual

The two jet rate

- An infrared-safe observable is the **two-jet rate.**
  A simple jet definition is the **Durham** k_t: for every pair of partons we calculate the distance
$$d_{ij} = \min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$
And the pair with the smallest distance is clustered. We repeat untill $\min_{i,j} d_{ij} < k_{t,\mathrm{cut}}^2$.

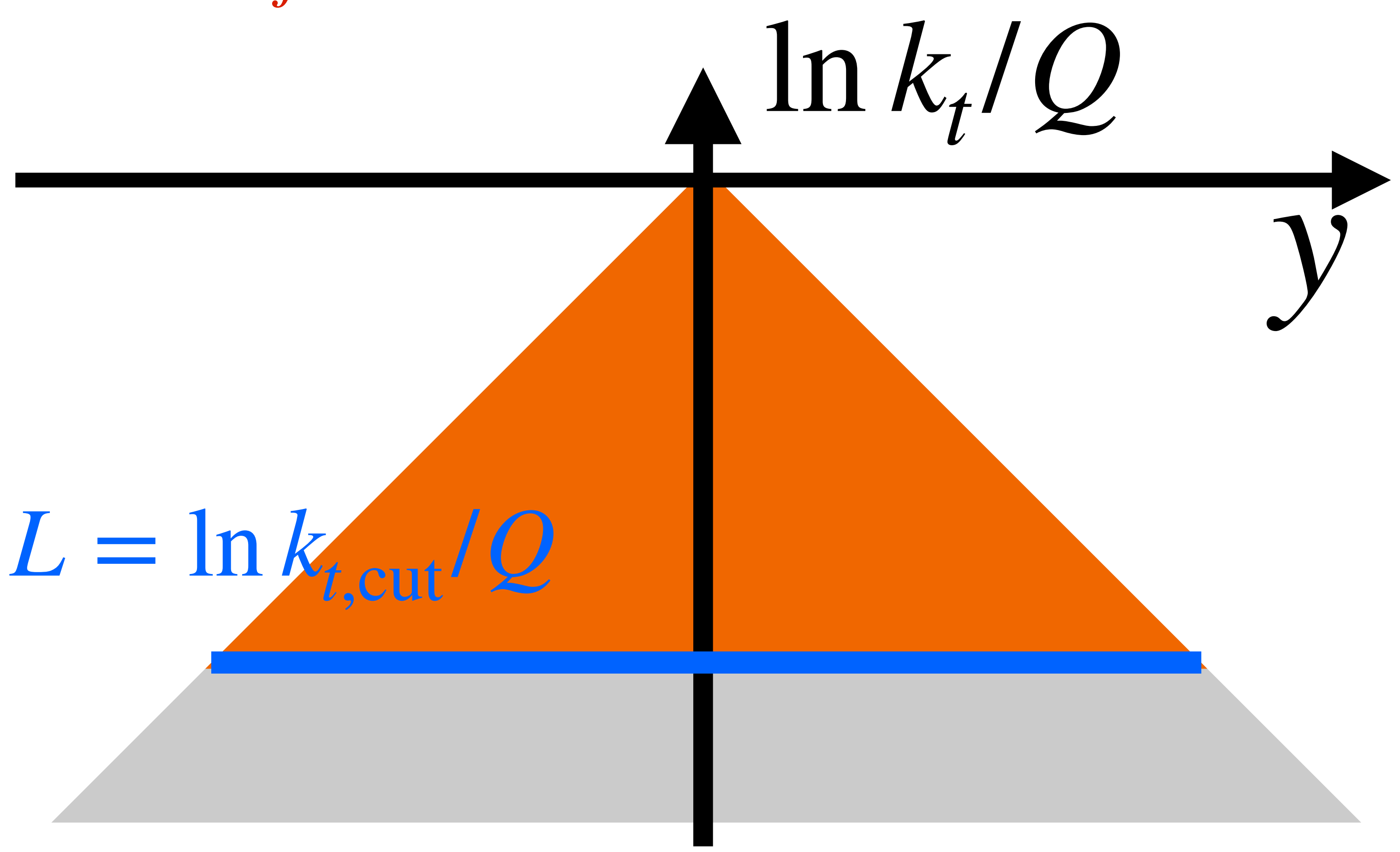
- At LO: $\tilde{p}_1 = Q/2\{1,0,0,1\}$, $\tilde{p}_2 = Q/2\{1,0,0,-1\}$, and $\Sigma_{2\mathrm{jet}}(k_{t,\mathrm{cut}}) = \sigma_B$

- At NLO, if we apply the **soft-collinear approximation** we have

$$k = k_t\{\cosh y, \cos\phi, \sin\phi, \sinh y\} \qquad p_1 \sim \tilde{p}_1, \quad p_2 \sim \tilde{p}_2$$

$$\frac{\Sigma_{\mathrm{NLO}}(k_{t,\mathrm{cut}})}{\sigma_B} \approx \frac{2C_F\alpha_s}{\pi}\int\frac{dk_t}{k_t}dy\frac{d\phi}{2\pi}\Theta(|y| < \ln Q/k_t)\left[\Theta(k_{t,\mathrm{cut}} - k_t) - 1\right]$$
$$= -\frac{2C_F\alpha_s}{\pi}\int\frac{dk_t}{k_t}dy\frac{d\phi}{2\pi}\Theta(|y| < \ln Q/k_t)\Theta(k_t - k_{t,\mathrm{cut}})$$
$$= -\frac{2C_F\alpha_s}{\pi}\log^2\left(\frac{k_{t,\mathrm{cut}}}{Q}\right) = -\frac{2C_F\alpha_s}{\pi}L^2$$

$\min_{ij} d_{ij} = k_t^2$



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$$= -\frac{2C_F\alpha_s}{\pi}\int \frac{dk_t}{k_t} dy \frac{d\phi}{2\pi}\Theta(|y| < \ln Q/k_t)\Theta(k_t - k_{t,\mathrm{cut}})$$

$$= -\frac{2C_F\alpha_s}{\pi}\log^2\left(\frac{k_{t,\mathrm{cut}}}{Q}\right) = -\frac{2C_F\alpha_s}{\pi}L^2$$

Large Logs appearing at all orders invalidate the FO expansion! Better to **resum** all the **double logs** at once!

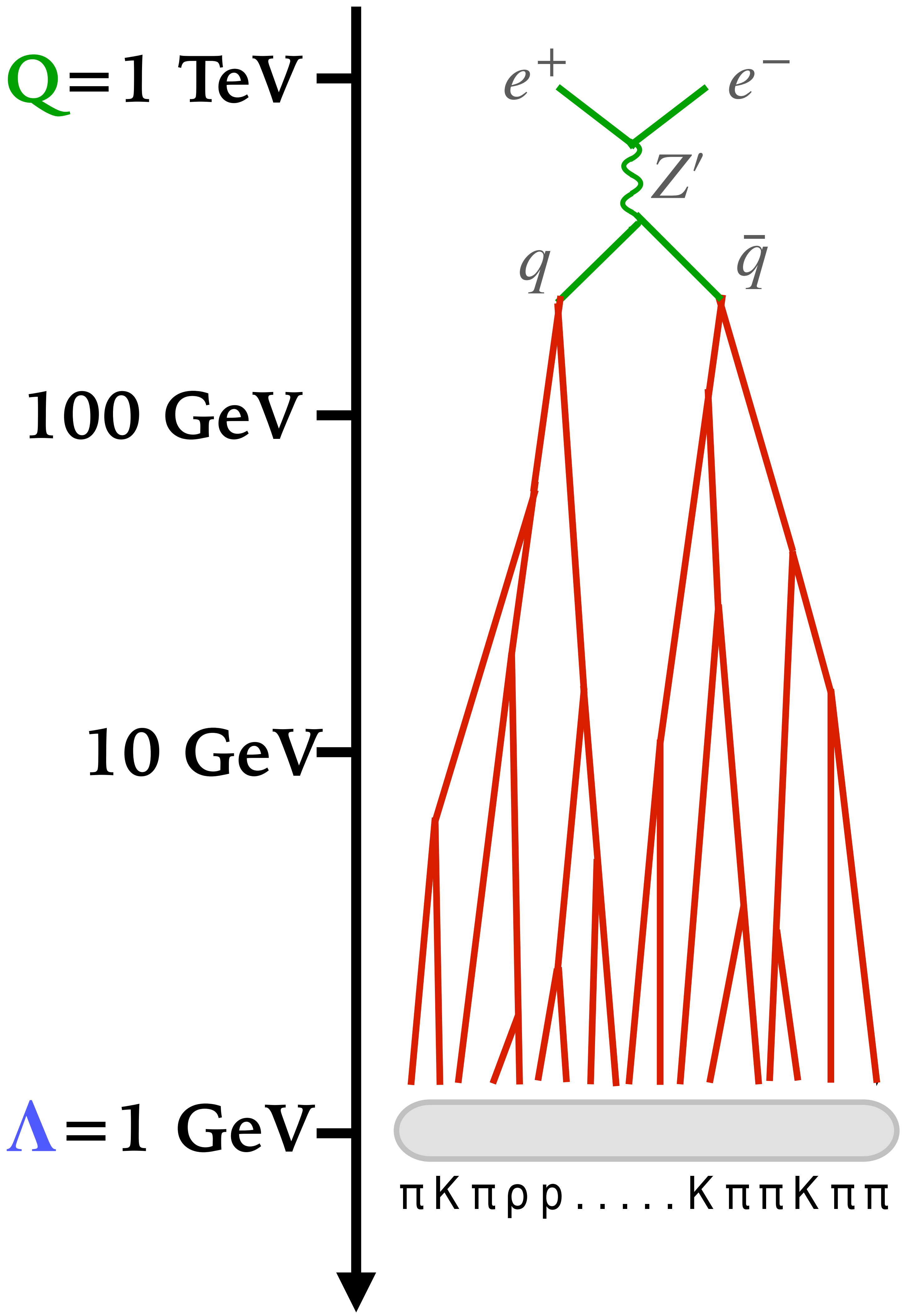
$$\Sigma_{\text{double log}} = \sigma_B \exp\left(-\frac{2C_F\alpha_s}{\pi}L^2\right)$$

Realistic collider event

Hard process

Energy degradation

Hadrons



$\alpha_s \ll 1$
Perturbation theory
$\Sigma_{\rm LO} + \alpha_s \Sigma_{\rm NLO} + \ldots$

$L = \ln Q/\Lambda \gtrapprox 1/\alpha_s$
all-orders **resummation** of logarithmically enhanced terms at all-orders

$\alpha_s \gg 1$
Non-perturbative QCD

We might be close to the NNLO revolution, with some N^3LO available… Specialised groups: master integrals, amplitudes & reductions, subtractions

NNLL- N^3LL for procs with 2 partons, in direct QCD or with EFT approaches; 3 legs current fronteer

Hic sunt leones!

Realistic collider event

Hard process

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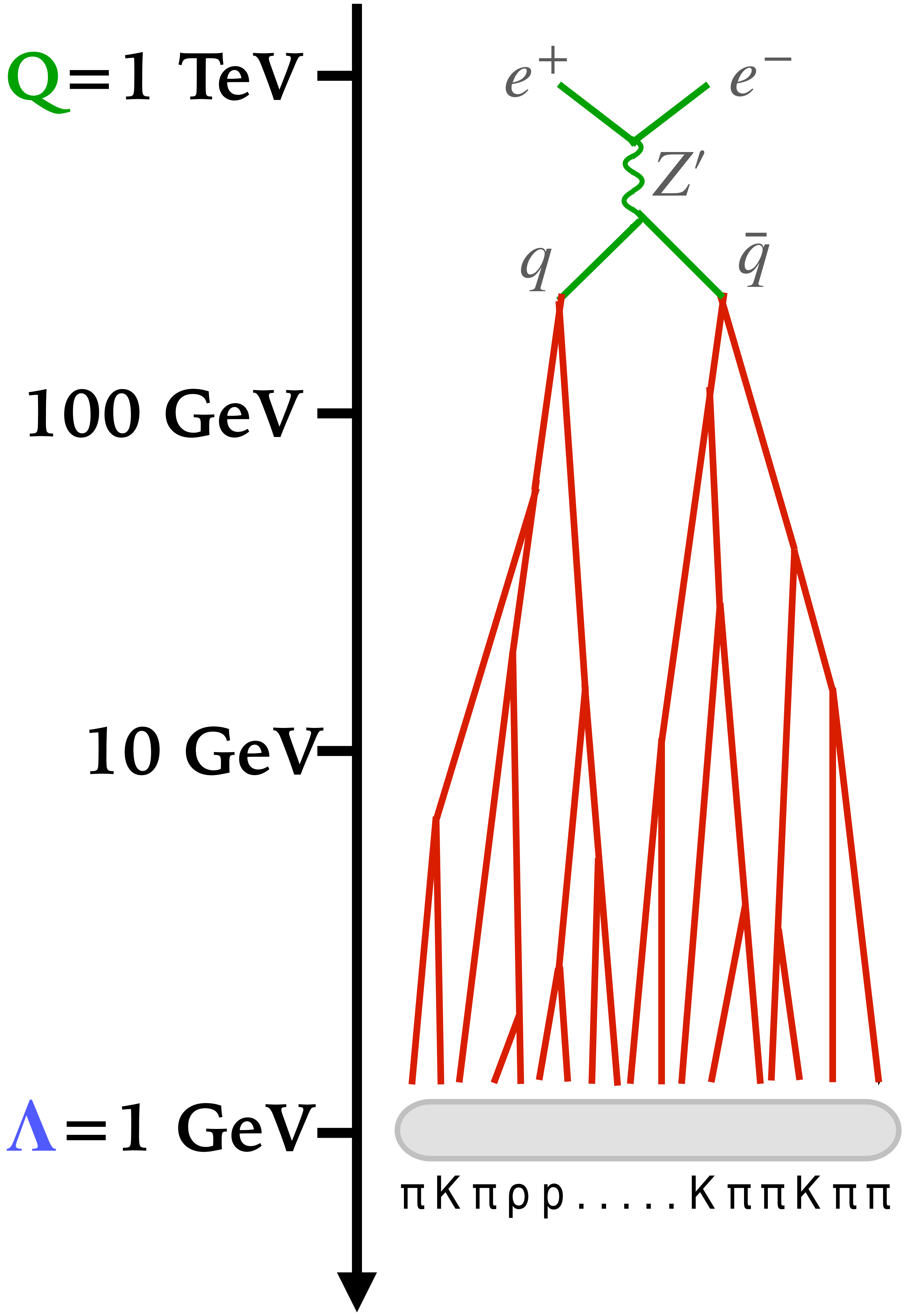

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Non-perturbative QCD

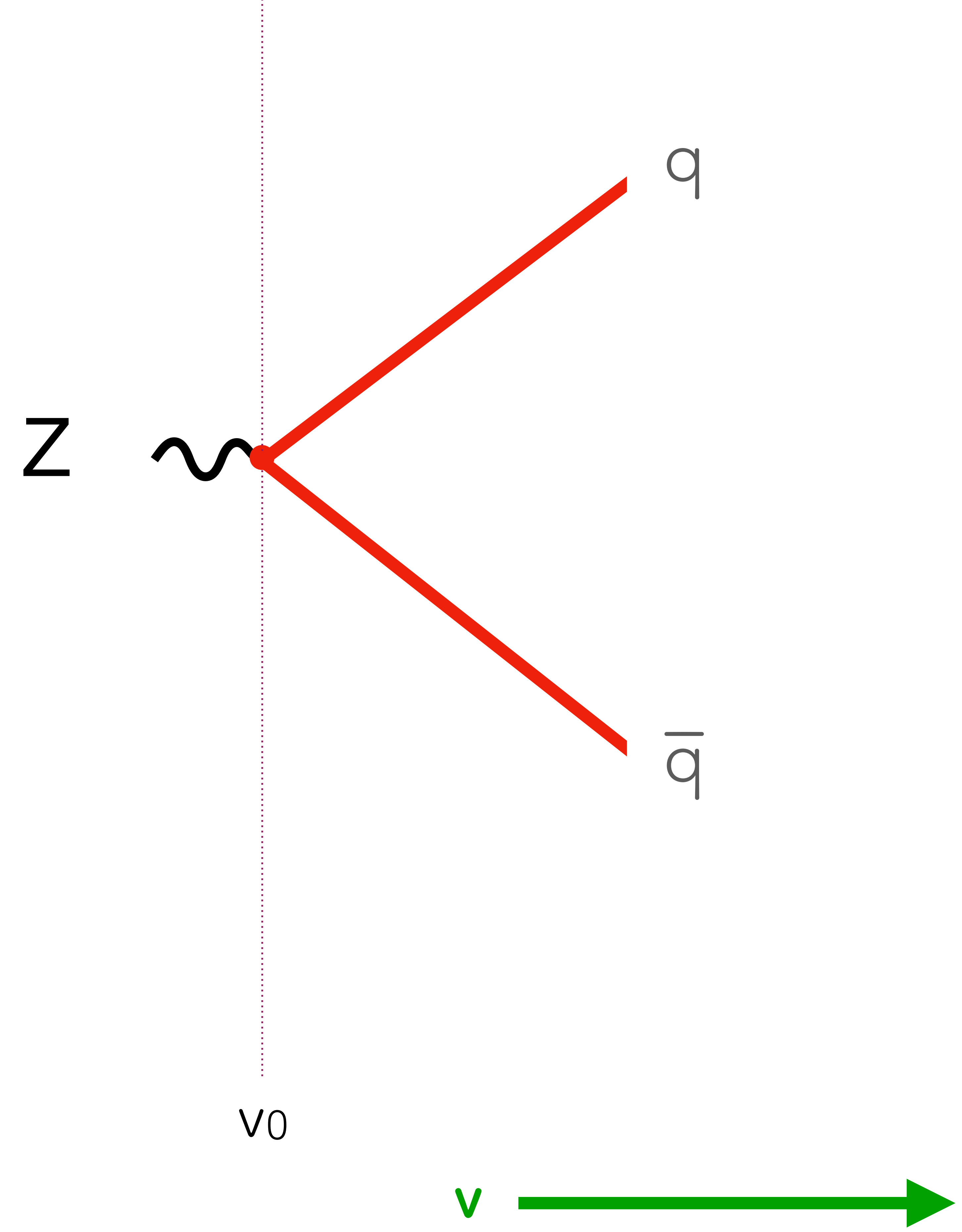
Can be matched with (N)NLO calculations

<u>PARTON SHOWERS</u>
(less accurate then analytic resum, but way more flexible)

Embedded in general purpouse event generators, which also comprise NP models

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



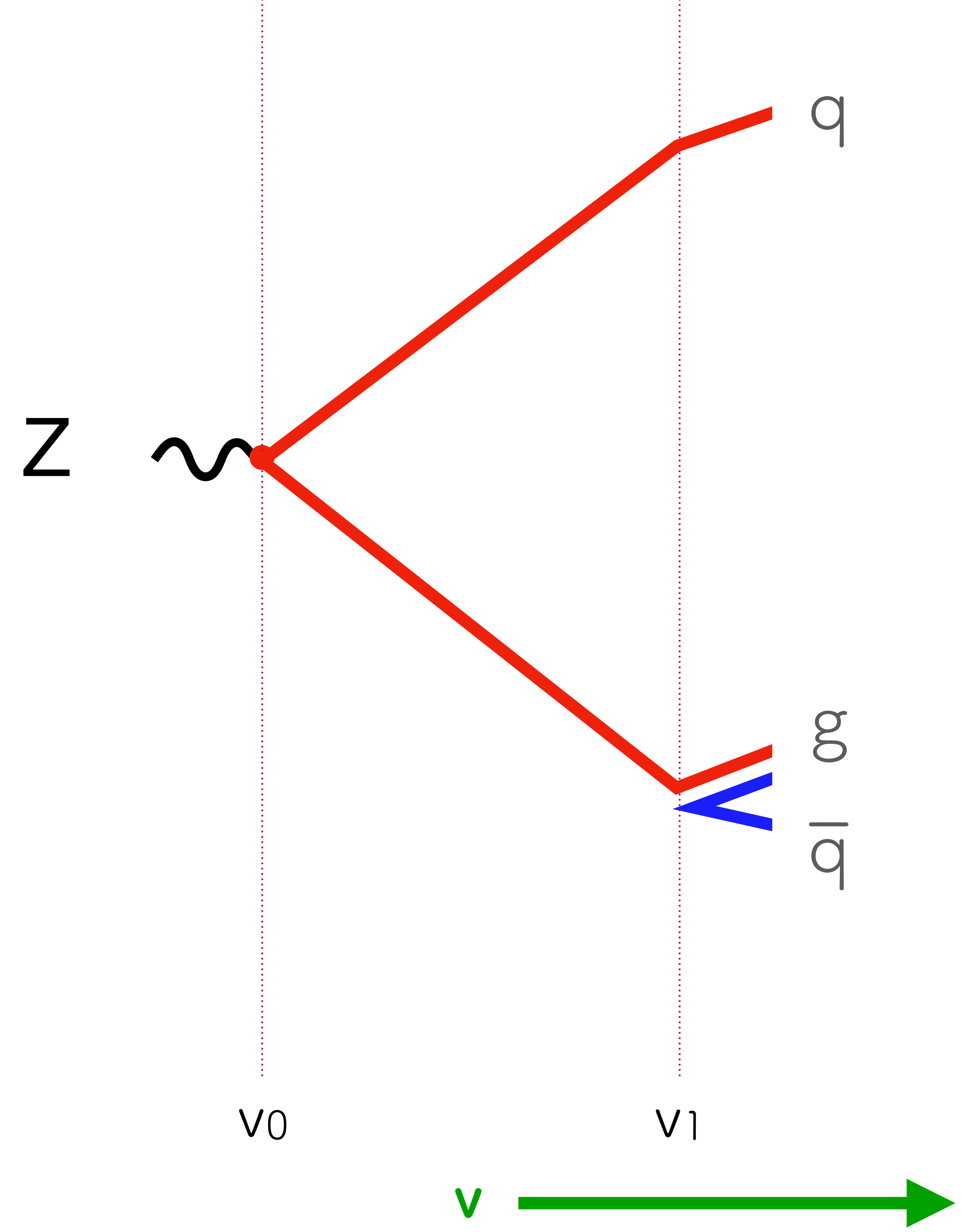
Start with $q\bar{q}$ state produced at a **hard scale** v_0.

Throw a random number to determine down to what **scale** state persists unchanged

$$\Delta(v_0, v) = \exp\left(-\int_{v}^{v_0} dP_{q\bar{q}}(\Phi)\right)$$

Parton Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



Start with $q\bar{q}$ state produced at a hard scale v_0.

Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$. The kinematic (rapidity and azimuth) of the gluon is chosen according to

$$dP_{q\bar{q}}(\Phi(v_1)) \qquad \Phi = \{v, y, \varphi\}$$

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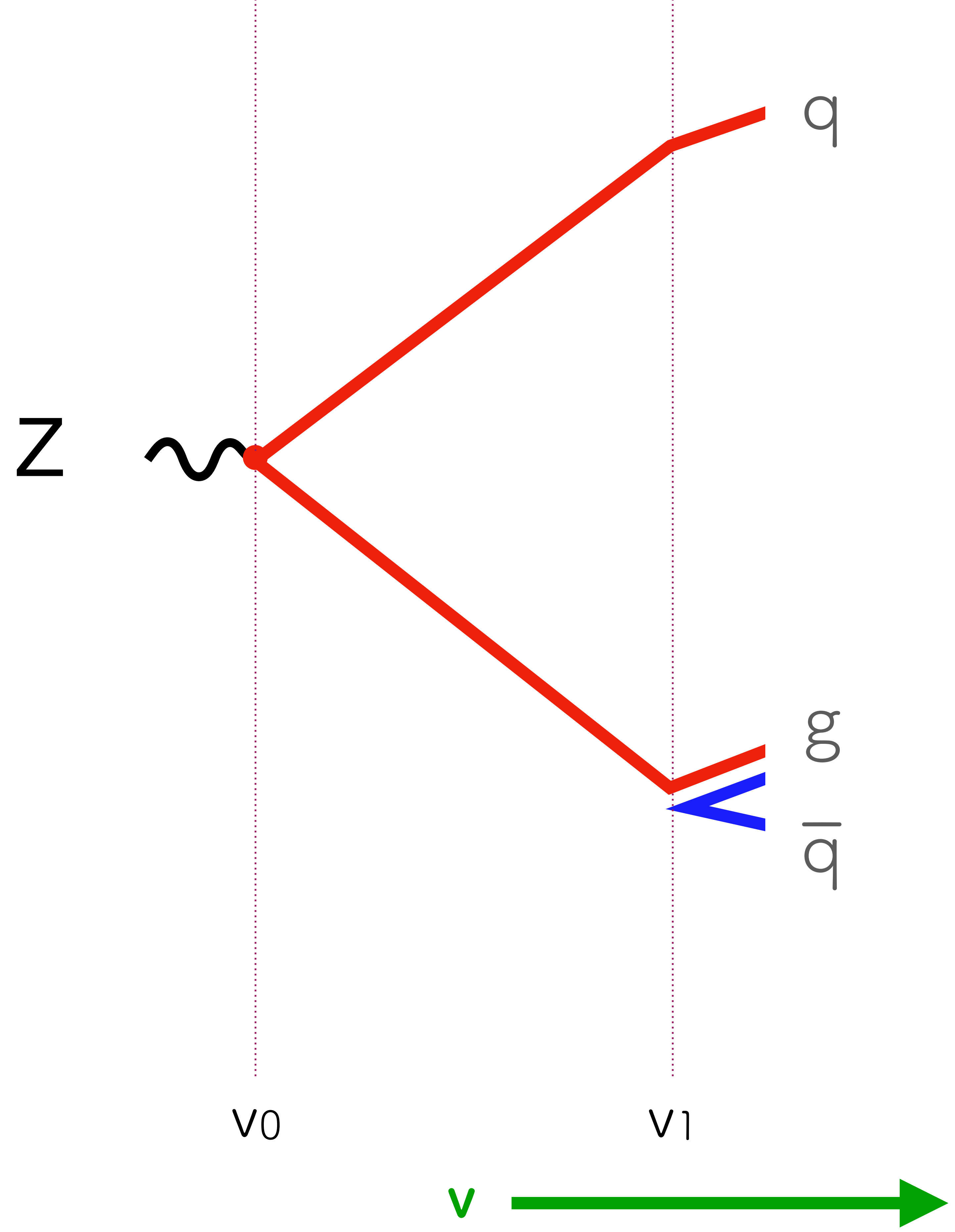

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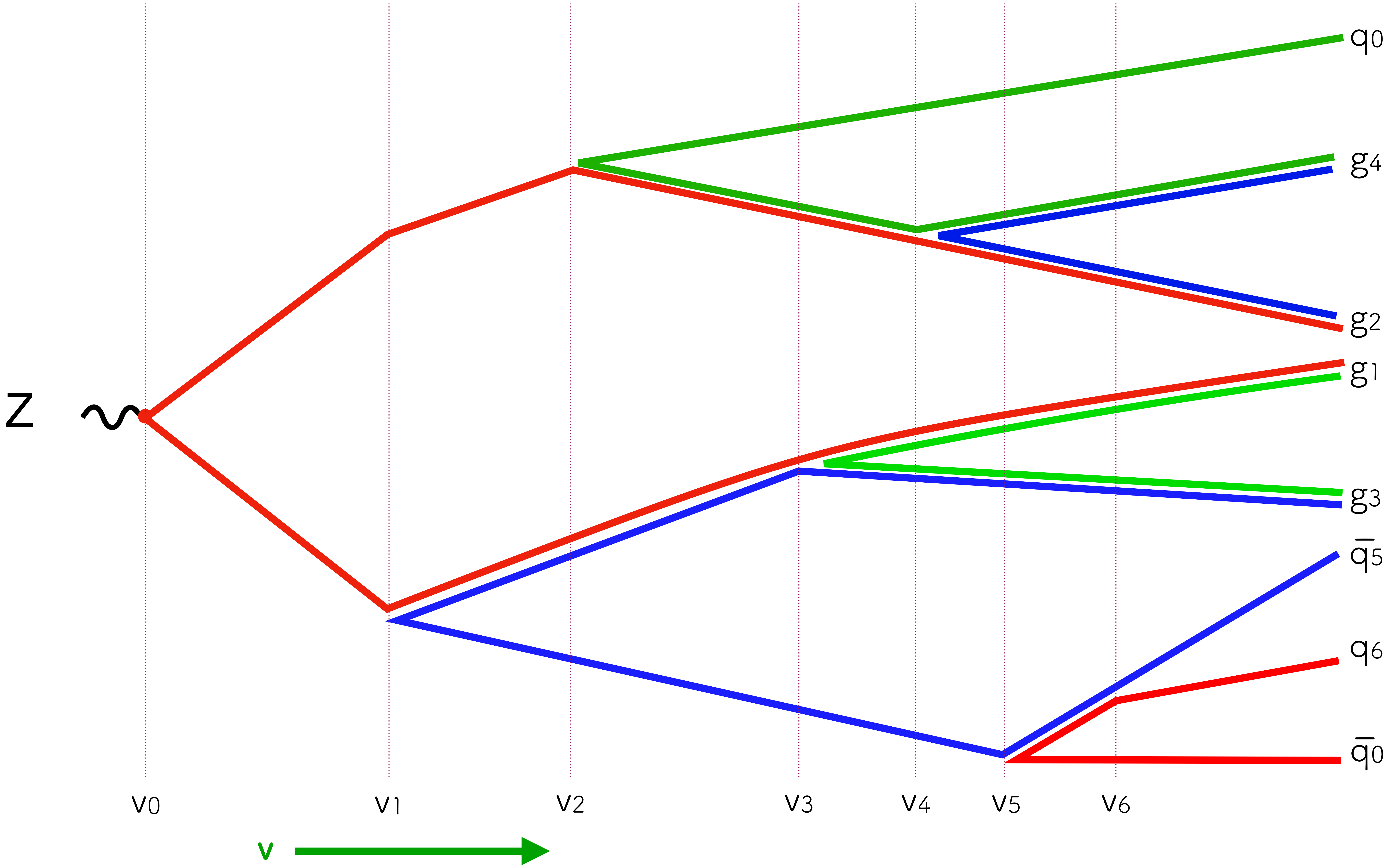
At some point, **state splits** (2→3, i.e. emits gluon) at a scale $v_1 < v_0$.

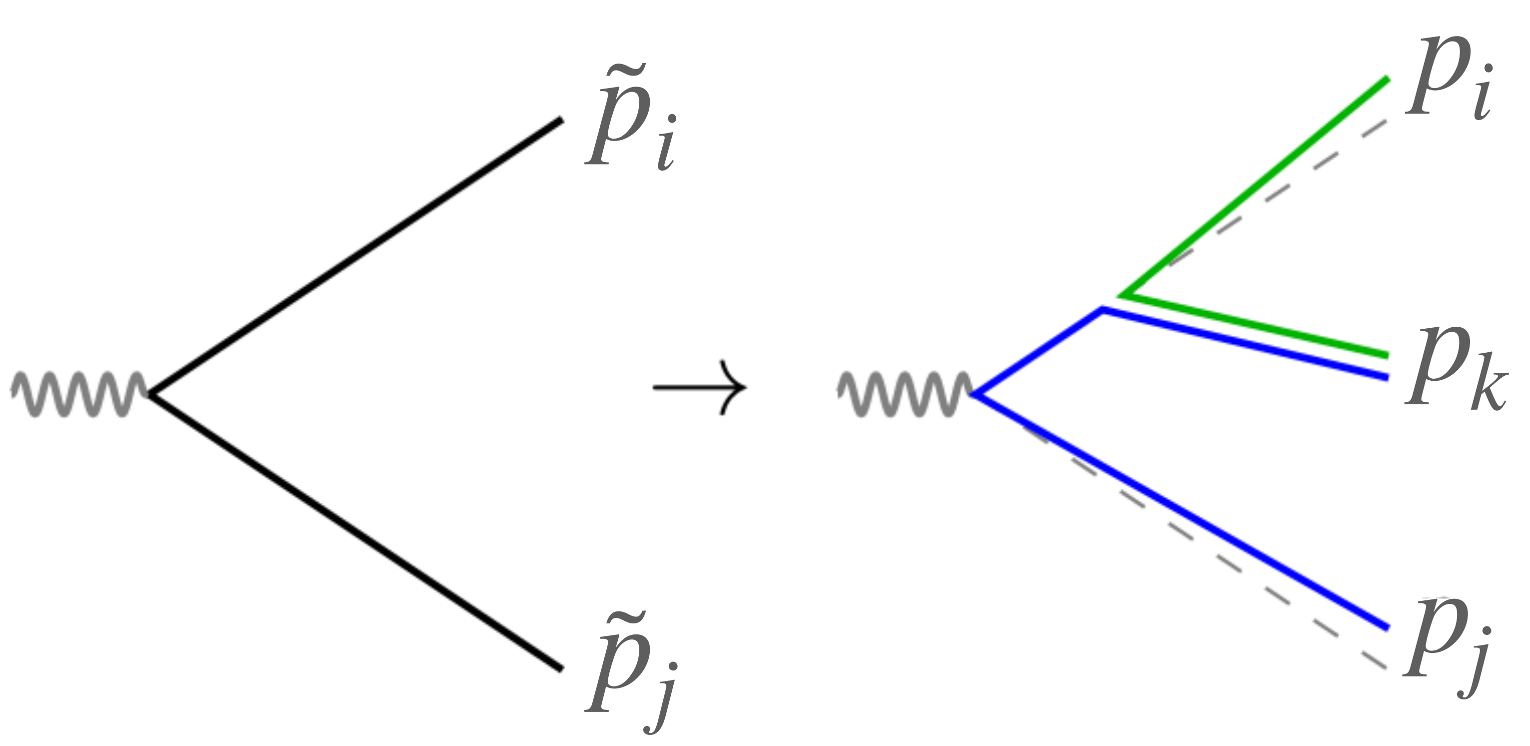
The gluon is part of two dipoles (qg), $(g\bar{q})$.

Iterate the above procedure for both dipoles independently, using v_1 as starting scale.



self-similar evolution continues until it reaches a non-perturbative scale

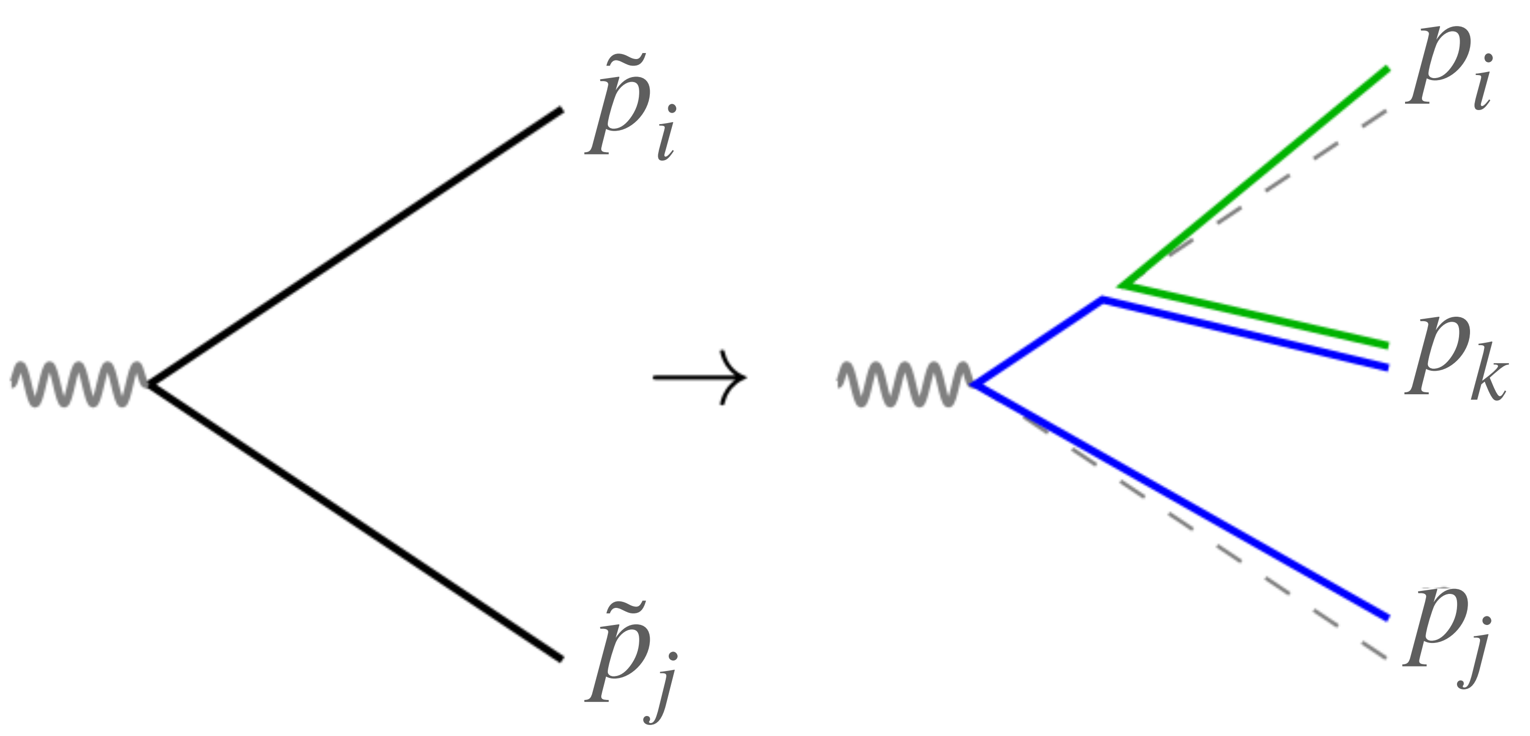
Dissecting the parton shower emission probability



$$\mathrm{d}\mathscr{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}y\,\frac{\mathrm{d}\varphi}{2\pi}\,P_{\tilde{i},\tilde{j}\to i,j,k}(v,y,\varphi)$$

Evolution variable: emissions are ordered $Q > v_1 > v_2 > \ldots > \Lambda$; typically $v = k_t$

Dissecting the parton shower emission probability

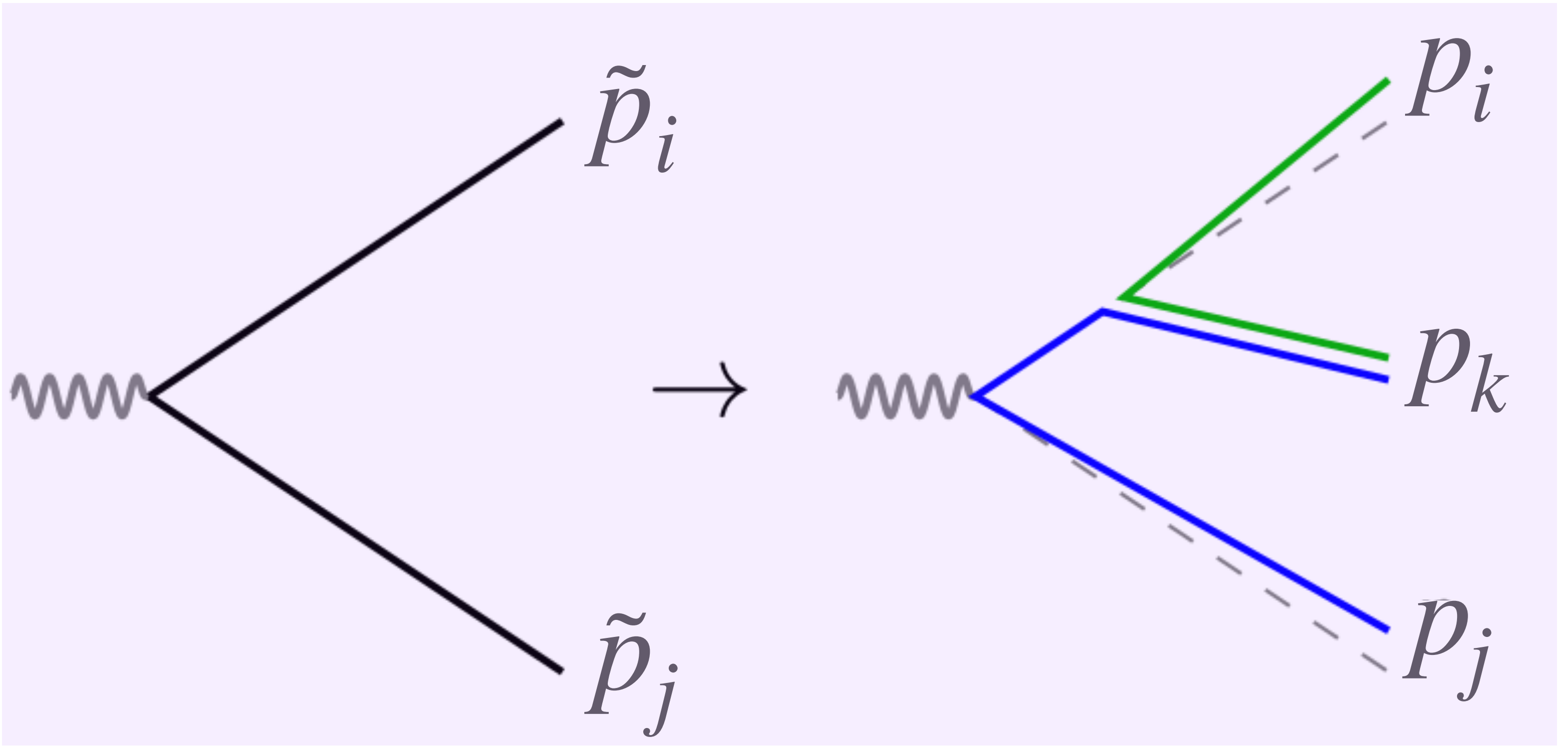


$$\mathrm{d}\mathscr{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{\mathrm{d}v^2}{v^2}\mathrm{d}y\,\frac{\mathrm{d}\varphi}{2\pi}\; P_{\tilde{i},\tilde{j}\to i,j,k}(v,y,\varphi)$$

Matrix element for emitting a parton k from a parton i (or j)

$$P_{\tilde{i}\tilde{j}\to ijk} \sim \hat{P}_{\tilde{i}\to ik}\left(1 - k_t/m_{\tilde{i}\tilde{j}}e^{+y}\right)\Theta(y) + \hat{P}_{\tilde{j}\to jk}\left(1 - k_t/m_{\tilde{i}\tilde{j}}e^{-y}\right)\Theta(-y)$$

Dissecting the parton shower emission probability



Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

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simplest is **fully local recoil**, with emitter absorbing the $k_\perp$ component, e.g. $y > 0$

$$p_k = \frac{k_t}{m_{\tilde{i}\tilde{j}}}e^{y}\tilde{p}_i + \frac{k_t}{m_{\tilde{i}\tilde{j}}}e^{-y}\tilde{p}_j + k_\perp$$

$$p_i = \frac{m_{\tilde{i}\tilde{j}} - k_t e^{y}}{m_{\tilde{i}\tilde{j}}}\tilde{p}_i + \frac{k_t^2}{m_{\tilde{i}\tilde{j}}(m_{\tilde{i}\tilde{j}} - k_t e^{y})}\tilde{p}_j - k_\perp$$

$$p_j = \left(1 - \frac{k_t}{m_{\tilde{i}\tilde{j}}} - \frac{k_t^2}{m_{\tilde{i}\tilde{j}}(m_{\tilde{i}\tilde{j}} - k_t e^{y})}\right)\tilde{p}_j$$

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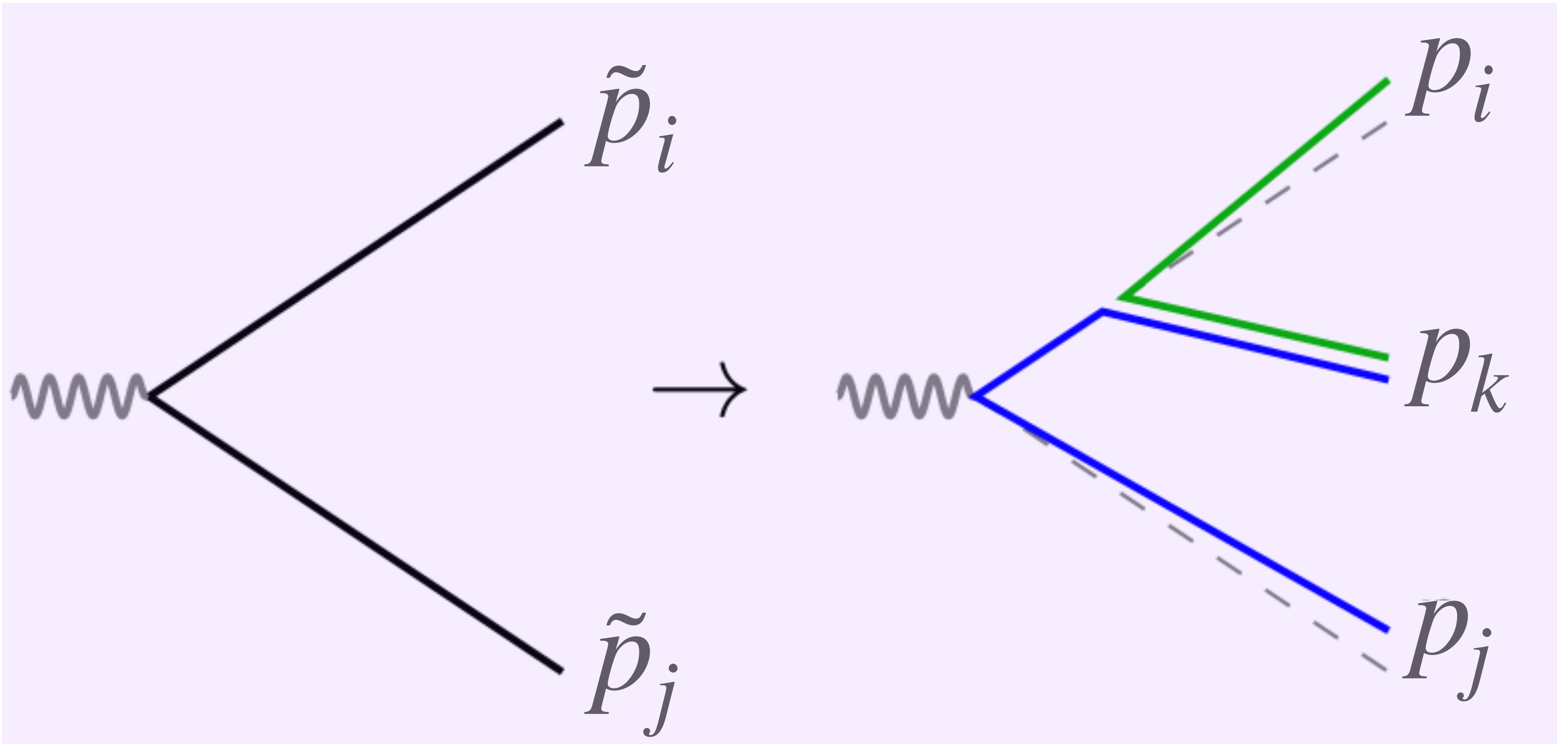

Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

This shower retains "leading logarithmic" accuracy, going beyond that is a very active area of research since the past 5 years!

$$d\mathcal{P}_{\tilde{i}\tilde{j}\to ijk} \sim \frac{dv^2}{v^2} dy \, \frac{d\varphi}{2\pi} \; P_{\tilde{i},\tilde{j}\to i,j,k}(v, y, \varphi)$$

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Matching parton showers with NLO calculations

- **Fixed order calculations** are our most accurate way to describe **inclusive quantities** (e.g. total cross sections), and the production of very **hard** jets
- **Parton showers** are our most flexible tools to describe **exclusive quantities**, sensitive to **soft** and **collinear** QCD radiation

Can we combine both approaches in GPMC?

YES! At **NLO** it is a solved problem(*), at **NNLO** is possible for few classes of processes, and a very active area of research since the past 10 years! (5 more then core "logarithmic" parton shower devolopments)

(*) for the old generation of showers… more in the next coming years ;)

MC@NLO — Frixione & Webber

$$\Sigma_{\text{NLO}} = \Sigma_{\text{LO}}(1 + \alpha_s K_{\text{NLO}})$$

$$\Sigma_{\text{PS}} = \Sigma_{\text{LO}}(1 + \alpha_s K_{\text{PS},1} + \alpha_s^2 K_{\text{PS},2} + \ldots)$$

We want to retain the $\mathcal{O}(\alpha_s)$ term from the NLO calculation, and the higher order corrections from the shower

MC@NLO: subtract to the **NLO calculation** the $\mathcal{O}(\alpha_s)$ **expansion of the shower**, then shower!

$$\sigma_{\text{NLO}} = \int d\Phi_n\, B(\Phi_n)\left(1 + \frac{V(\Phi_n, \epsilon)}{B(\Phi_n, \epsilon)}\right)\Theta(\Phi_n) \; + \int d\Phi_{n+1} R(\Phi_{n+1}, \epsilon)\Theta(\Phi_{n+1})$$

IR regulators

$$\sigma_{\text{PS}} = \int d\Phi_n\, B(\Phi_n)\left[\Theta(\Phi_n) + \int d\Phi_{\text{rad}} P(\Phi_{\text{rad}})\Theta(v - v_{\text{min}})\big(\Theta(\Phi_{n+1}) - \Theta(\Phi_n)\big)\right]$$

Born

Real

Virtual

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$$\sigma_{\rm PS} = \int d\Phi_n\, B(\Phi_n)\left[\Theta(\Phi_n) + \int d\Phi_{\rm rad} P(\Phi_{\rm rad})\Theta(v - v_{\rm min})\big(\Theta(\Phi_{n+1}) - \Theta(\Phi_n)\big)\right]$$

$$\sigma_{\rm MC@NLO} = \int \Theta(\Phi_n) d\Phi_n\, B(\Phi_n)\left[1 + \frac{V(\Phi_n,\epsilon)}{B(\Phi_n,\epsilon)} + \int d\Phi_{\rm rad} P(\Phi_{\rm rad})\Theta(v - v_{\rm min})\right]$$

$$+ \int \Theta(\Phi_{n+1}) d\Phi_{n+1}\; \left[R(\Phi_{n+1};\epsilon) - B(\tilde{\Phi}_n) P(\Phi_{\rm rad})\Theta(v - v_{\rm min})\right]$$

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$$+\ldots$$

$\mathcal{S}$-events: they are showered starting with Born kinematics

Sending the IR regulators to 0 is a bit tricky… usually handled adding and subtracting standard NLO counterterms

$$\left[\frac{V(\Phi_n, \epsilon)}{B(\Phi_n, \epsilon)} + \bar{C}(\Phi_n; \epsilon) + \int d\Phi_{\rm rad}(P(\Phi_{\rm rad}) - C(\Phi_{\rm rad}))\right]$$

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$$+ \int \Theta(\Phi_{n+1}) d\Phi_{n+1} \left[R(\Phi_{n+1}; \epsilon) - B(\tilde{\Phi}_n) P(\Phi_{\text{rad}}) \Theta(v - v_{\text{min}}) \right]$$

$\mathcal{H}$-events: they are showered starting with an additional parton

both IR regulator are sent to 0, as the integrand should be finite!

The PS must reproduce all the singular limits, which is often not the case for multi-leg processes.

This term is source of negative weights: a lot of focus in how to reduce them

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MC@NLO: subtract to the **NLO calculation** the $\mathcal{O}(\alpha_s)$ **expansion of the shower**, then shower!

Actually, after adding the shower one sees

$$\sigma_{\text{MC@NLO+PS}} = \sigma_{\text{NLO}} + \mathcal{O}\left(\frac{v_{\min}}{v_{\max}}\right)$$

$$\frac{\ldots(\Phi_n, \epsilon)}{\ldots(\Phi_n, \epsilon)} + \int d\Phi_{\text{rad}} P(\Phi_{\text{rad}}) \Theta(v - v_{\min}) \Big]$$

$$+ \int \Theta(\Phi_{n+1}) d\Phi_{n+1} \left[R(\Phi_{n+1}; \epsilon) - B(\tilde{\Phi}_n) P(\Phi_{\text{rad}}) \Theta(v - v_{\min}) \right]$$

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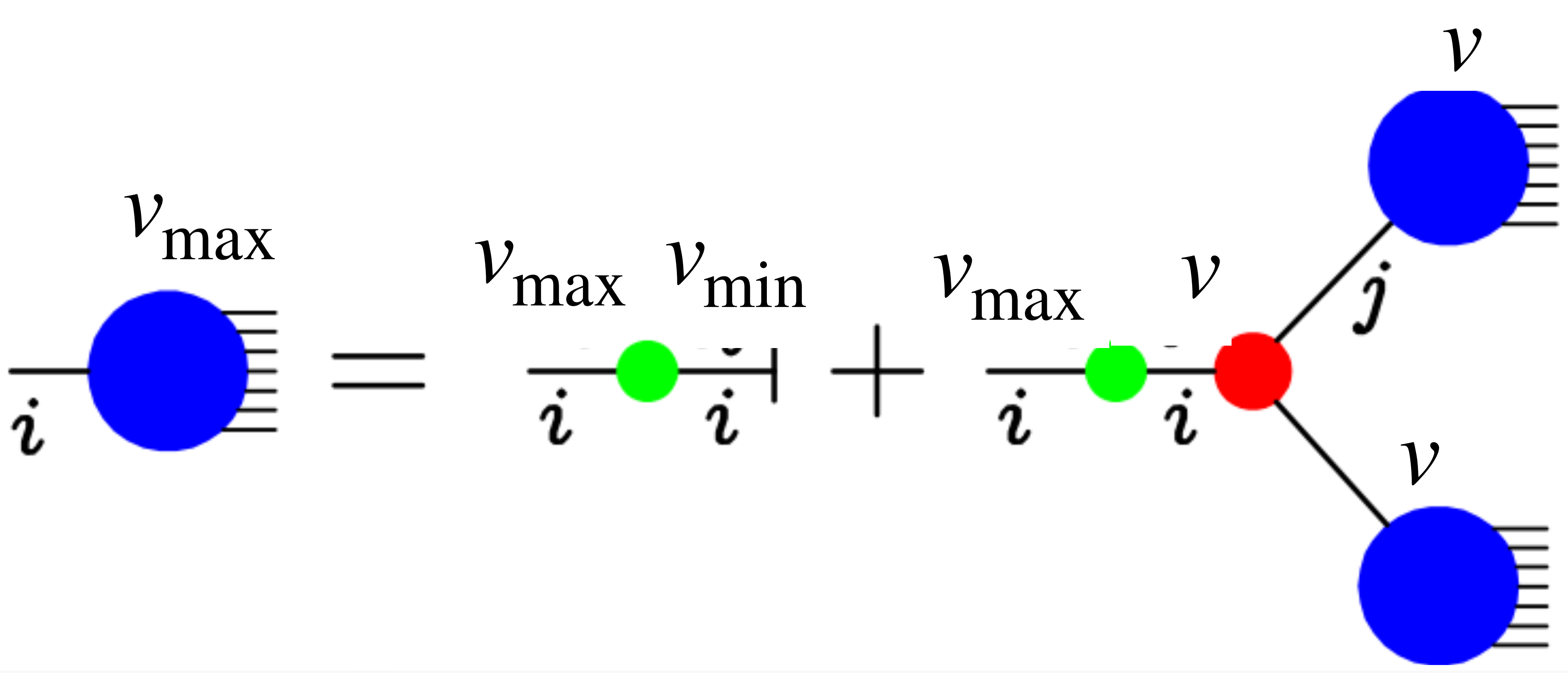
POWHEG – Nason

Parton shower evolution operator

$$d\sigma_{PS} = B(\Phi_n)d\Phi_n \times \hat{\mathcal{S}}_n(v_{max}, \Phi_n), \quad \hat{\mathcal{S}}_n(v_{max}, \Phi_n) = \Delta(v_{max}, v_{min}) + \int d\Phi_{rad}\Theta(v - v_{min})\Delta(v_{max}, v)P(\Phi_{rad}) \times \hat{\mathcal{S}}_{n+1}(v, \Phi_{n+1})$$



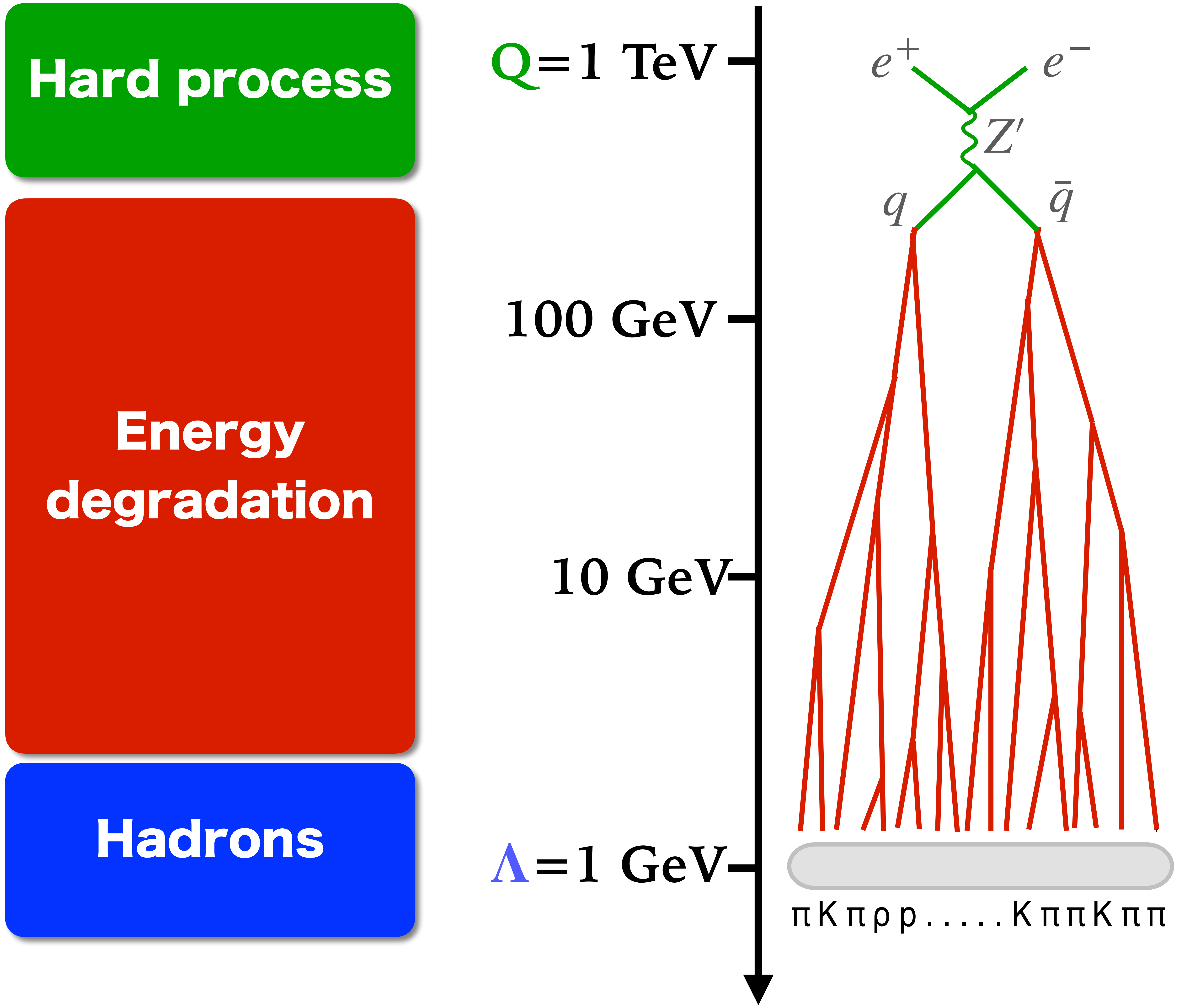
POWHEG: NLO inclusive cross section (instead of LO one), and the real ME for the first emission

$$d\sigma_{PWG} = \left(B(\Phi_n) + V(\Phi_n) + \int d\Phi_{rad} R(\Phi_{n+1})\right) d\Phi_n \left(\Delta_{pwg}(v_{max}, v_{min}) + \int_{v_{min}}^{v_{max}} d\Phi_{rad}\frac{R(\phi_{n+1})}{B(\Phi_{n+1})}\Delta_{pwg}(v_{max}, v) \times \mathcal{S}_{n+1}(\hat{v}, \Phi_{n+1})\right)$$

$$\Delta_{pwg}(v_1, v_2) = \int_{v_2}^{v_1} d\Phi_{rad}\frac{R(\phi_{n+1})}{B(\Phi_{n+1})}$$

Replace the first shower evolutive step

Recap



$\alpha_s \ll 1$
Perturbation theory
$\Sigma_{\mathrm{LO}} + \alpha_s \Sigma_{\mathrm{NLO}} + \ldots$

$L = \ln Q/\Lambda \gtrapprox 1/\alpha_s$
all-orders **resummation** of logarithmically enhanced terms at all-orders

$\alpha_s \gg 1$
Non-perturbative QCD

We need to provide all these ingredients! GPMC only tool with all of them, but analytic calculations for the single ingredients are more accurate then GPMC (except maybe for the hadronisation)! Lot of effort both in improving the accuracy of the single pieces (more loops, more logs, more legs) and also their inclusion in GPMCs