

# MAXIMUM-LIKELIHOOD AND HAMILTONIAN MONTE CARLO TECHNIQUES TO DOPPLER MOMENT ESTIMATION FOR PRECIPITATION USING WEATHER RADAR ECHOES

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## ON DOPPLER PROCESSING FOR FAST SCANNING RADARS



**Max 3D Radar:**  
Robin Radar Systems B.V.



**MESEWI Radar:**  
TU Delft

**Typical use:** Detection and tracking of weak moving point-like targets: birds and drones.

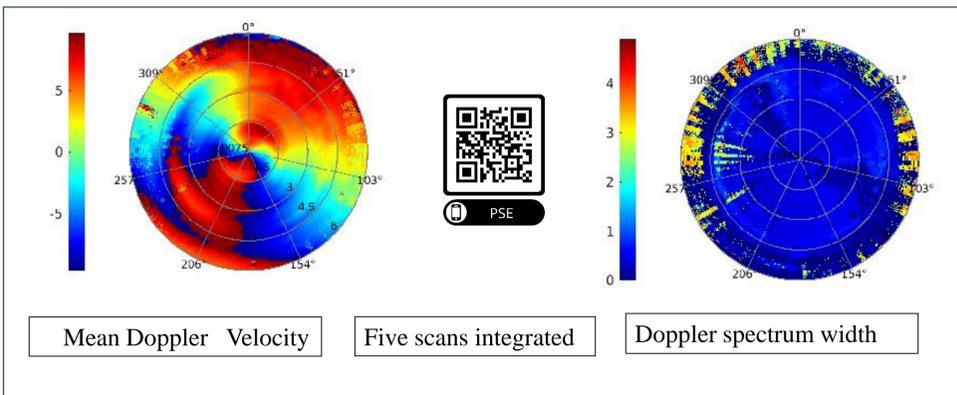
**Limitation:** Fast Scans result in low time on target.

**Challenge:** Doppler processing for extended weather objects.

### The Proposed Signal Processing Pipeline:

Step 1: Proposed **Parametric Spectrum Estimator (PSE)** [1]

1. Moment estimation with low time on target
2. Ability to use data from multiple scans



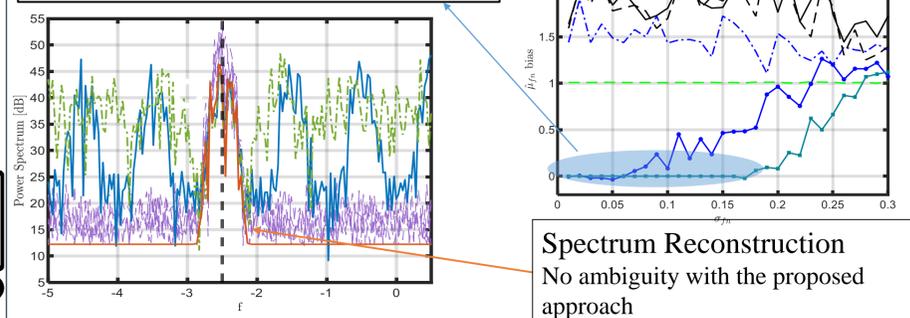
Step 2: Doppler counter-aliasing [2]

1. Aperiodic (Non-linear, Log-periodic) echo sequence
2. **Complex Gaussian Process Regression (CGP-R)** to reconstruct the signal in the frequency domain

Bias:  $\mathbb{B}[\hat{\theta}] = \mathbb{E}[\hat{\theta}] - \theta$

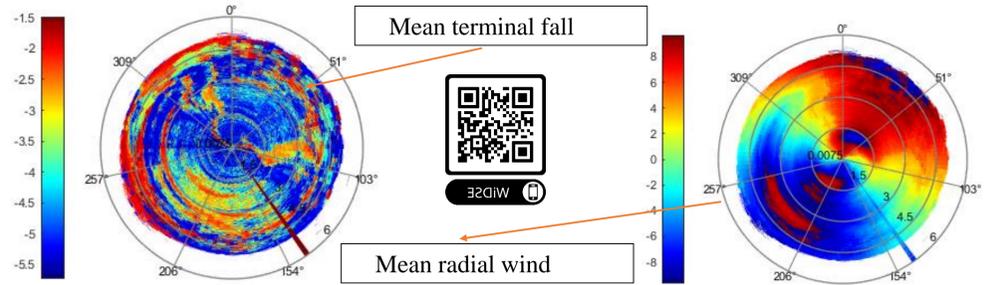
the more closer to 0, the better.

Almost zero bias with the proposed technique



**Spectrum Reconstruction**  
No ambiguity with the proposed approach

Step 3: Estimating Wind and Vertical fall speeds jointly from Doppler measurements [3]



[1] T. Dash, H. Driessen, O. Krasnov, and A. Yarovoy, "Doppler Spectrum Parameter Estimation for Weather Radar Echoes Using a Parametric Semi-analytical Model," IEEE Trans. Geosci. Remote Sens., vol. 62, pp. 1–18, 2024, doi: 10.1109/TGRS.2023.3338233  
 [2] T. Dash, H. Driessen, O. A. Krasnov and A. Yarovoy, "Counter-Aliasing Is Better Than De-Aliasing: Application to Doppler Weather Radar With Aperiodic Pulse Train," in IEEE Transactions on Geoscience and Remote Sensing, vol. 62, pp. 1-17, 2024, Art no. 5109017, doi: 10.1109/TGRS.2024.3438567.  
 [3] T. Dash, H. Driessen, O. A. Krasnov, and A. G. Yarovoy, "Joint Estimation of Raindrop Size Distribution Parameters and Wind Velocity Field Using a Fast Scanning Weather Doppler Radar," [Submitted]

## NEXT GENERATION RADAR FOR REMOTE SENSING

### PHased Array Radar for Atmospheric Research (PHARA)

Instantaneous spatiotemporal evolution of storms [dynamic estimation]



State of the art profile (2D)

Expected 3D profile from PHARA

Initial results: Dynamic Spatial Estimation [Elevation x Doppler] with Maximum Likelihood [MLE] and Hamiltonian Monte Carlo [HMC]

$\theta$ : Doppler Moments: Elevation (K) x Number of moments

MLE

$\sigma_n^2$ : Noise variance

$$\log(p(\hat{\mathbf{Z}}^{\text{PSD}}|\theta)) = - \sum_{l=1}^L \sum_{n=1}^N \log(\pi(\mathbf{F}_{l,n}(\theta) + \sigma_n^2) + \frac{\hat{\mathbf{Z}}_{l,n}^{\text{PSD}}}{\mathbf{F}_{l,n}(\theta) + \sigma_n^2})$$

$$\{\hat{\theta}\}_{k=1}^{K(\text{MLE})} = \arg \max_{\theta} \log(p(\hat{\mathbf{Z}}^{\text{PSD}}|\theta))$$

$\hat{\mathbf{Z}}^{\text{PSD}}$ : Doppler Power Spectral Density (PSD) measurements :[ Elevation x Doppler velocity ]

$\mathbf{F}$ : Expected Doppler Power Spectral Density (PSD) model :[ Elevation x Doppler velocity ]

HMC

Hamiltonian:  $H(\theta, p_{\theta}) = U(\theta) + K(p_{\theta})$ ,  
 Potential Energy:  $U(\theta) = -\log(p(\hat{\mathbf{Z}}^{\text{PSD}}|\theta))$ ,  
 Kinetic Energy:  $K(p_{\theta}) = \frac{1}{2} \mathbf{p}_{\theta}^T \mathbf{M}_{\theta}^{-1} \mathbf{p}_{\theta}$

The iterative Markov Chain Monte Carlo (MCMC) approach: till the Hamiltonian is converged

