Riemannian Topology Meeting

Fribourg, 8-9 November 2018

Spaces of riemannian metrics
and related topics

Augsburg-Fribourg-Karlsruhe RTM 2018

Contents

1 General information

The Riemannian Topology Meeting (RTM) is an irregularly recurring activity addressing mathematicians interested in the interplay of algebraic and geometric topology with riemannian geometry. Its tradition reaches back to 2012 during which it took place at the universites of Fribourg, Karlsruhe and Göttingen.
This RTM, taking place at the University of Fribourg, focuses on fostering the mathematical intercommunication between the research groups in Augsburg, Fribourg and Karlsruhe, which are participating in the SPP 2026 DFG project.

Speakers

- Johannes Ebert (WWU Münster)
- Sebastian Goette (University of Freiburg)
- David Gonzalez (University of Fribourg)
- Jackson Goodman (University of Pennsylvania)
- Bernhard Hanke (Augsburg University)
- Jan-Bernhard Kordaß (Karlsruhe Institute of Technology)
- Christian Lange (University of Cologne)
- Saskia Roos (University of Potsdam)
- Gangotryi Sorcar (Einstein Institute of Mathematics, HUJI)
- Michael Wiemeler (WWU Münster)
- David Wraith (Maynooth University)

Sponsors

- DFG Priority Program Geometry at infinity (SPP 2026)
- Swiss National Science Foundation (SNSF)
- DFG Research Training Group Asymptotic Invariants and Limits of Groups and Spaces (RTG 2229)
- Department of Mathematics, UniFR

Organizers

- Mauricio Bustamante (Augsburg University)
- David Gonzalez (University of Fribourg)
- Jan-Bernhard Kordaß (Karlsruhe Institute of Technology)
- Jonathan Wermelinger (University of Fribourg)

Supported by

- Anand Dessai (University of Fribourg)
- Bernhard Hanke (Augsburg University)
- Wilderich Tuschmann (Karlsruhe Institute of Technology)

2 Schedule

All talks are scheduled for 50 minutes.

Thursday, 8 November

8:45 – 9:10	Registration at the Lecture Hall
9:10 – 9:20	Opening at the Lecture Hall
9:20 – 10:10	**1st Talk: Johannes Ebert**
10:10 – 10:40	Coffee break at the Mensa (30')
10:40 – 11:30	**2nd Talk: Jan-Bernhard Kordaß**
11:30 – 11:35	Short break at the Lecture Hall (5')
11:35 – 12:25	**3rd Talk: Gangotryi Sorcar**
12:25 – 14:10	Lunch break at the Mensa (1h 45')
14:10 – 15:00	**4th Talk: Sebastian Goette**
15:00 – 15:05	Short break at the Lecture Hall (5')
15:05 – 15:55	**5th Talk: Saskia Roos**
15:55 – 16:30	Coffee break at the Mensa (35')
16:30 – 17:20	**6th Talk: Christian Lange**
19:30	Conference Dinner

Friday, 9 November

9:20 – 10:10	**7th Talk: David Wraith**
10:10 – 10:40	Coffee break at the Mensa (30')
10:40 – 11:30	**8th Talk: Jackson Goodman**
11:30 – 11:40	Short break at the Lecture Hall (10')
11:40 – 12:30	**9th Talk: David González**
12:30 – 14:30	Lunch break at the Mensa (2h)
14:30 – 15:20	**10th Talk: Michael Wiemeler**
15:20 – 15:30	Short break at the Lecture Hall (10')
15:30 – 16:20	**11th Talk: Bernhard Hanke**
16:20 – 16:50	Break at the Lecture Hall (30')
16:50 – 17:40	**Discussion Session**

3 Abstracts

A rigidity theorem for the action of the diffeomorphism group on spaces of psc metrics

Johannes Ebert

For a closed, simply connected d-dimensional manifold spin M, we study the action of the (spin) diffeomorphism group of M on the space $\mathcal{R}^+(M)$ of psc metrics on M. Our main result is that the homotopy class of the map $f^* : \mathcal{R}^+(M) \to \mathcal{R}^+(M)$ only depends on the cobordism class in $\Omega^{\mathrm{Spin}}_{d+1}$ of the mapping torus of f. When properly formulated, the same result is true for manifolds with nontrivial fundamental group.

Extra twisted connected sums and their ν-invariants

Sebastian Goette

Joyce's orbifold construction and the twisted connected sums by Kovalev and Corti-Haskins-Nordström-Pacini provide many examples of compact Riemannian 7-manifolds with holonomy G_2. We would like to use this wealth of examples to guess further properties of G_2-manifolds and to find obstructions against holonomy G_2, taking into account the underlying topological G_2-structures.
The Crowley-Nordström ν-invariant distinguishes topological G_2-structures. It vanishes for all twisted connected sums. By adding an extra twist to this construction, we show that the ν-invariant can assume all of its 48 possible values. This shows that G_2-bordism presents no obstruction against holonomy G_2. We also exhibit examples of 7-manifolds with disconnected G_2-moduli space. Our computation of the ν-invariants involves integration of the Bismuth-Cheeger η-forms for families of tori, which can be done either by elementary hyper-

bolic geometry, or using modular properties of the Dedekind η-function.

Moduli spaces of non-negatively curved metrics on homotopy RP^5s

David González Álvaro

The goal of this talk is to discuss the following result: for a manifold homotopy equivalent to RP^5, the moduli space of metrics with non-negative sectional (resp. with positive Ricci) curvature has infinitely many path connected components. The proof involves various elements such as Brieskorn spheres, Grove-Ziller metrics, reduced eta-invariants and fixed point formulas. This is joint work with Anand Dessai.

$Spin^c$ Dirac operators and the Kreck-Stolz s invariant

Jackson Goodman

We use the $Spin^c$ Dirac operator to generalize a formula of Kreck and Stolz for the s invariant of S^1 invariant metrics with positive scalar curvature. We then apply it to show that the moduli spaces of metrics with nonnegative sectional curvature on certain 7-manifolds have infinitely many path components. These include certain positively curved Eschenburg and Aloff-Wallach spaces. Furthermore, we use a $Spin^c$ version of the s invariant to discuss moduli spaces of metrics of positive scalar and twisted scalar curvature on $Spin^c$ manifolds.

Local flexibility of open partial differential relations

Bernhard Hanke

In his famous book on partial differential relations Gromov formulates an exercise concerning local deformations of solutions to open partial differential relations. We will explain the content of this fundamental assertion and sketch a proof.
We will illustrate this by various examples, including the construction of $C^{1,1}$-Riemannian metrics which are positively curved "almost everywhere" on arbitrary manifolds.
This is joint work with Christian Bär (Potsdam).

Spaces of riemannian metrics satisfying surgery stable curvature conditions

Jan-Bernhard Kordaß

We will introduce spaces of riemannian metrics on a smooth manifold satisfying a curvature condition given by a subset in the space of algebraic curvature operators. Provided this condition is surgery stable, which is a notion based on the work of S. Hoelzel guaranteeing the condition can be preserved under surgeries of a certain codimension, we can generalize several theorems from positive scalar curvature geometry to this setting. Notably, we will comment on a generalization of a theorem of V. Chernysh on the homotopy type of the space of psc metrics and point to cases where we can distinguish connected components using invariants from spin geometry.

Orbifolds all of whose geodesics are closed

Christian Lange

Manifolds all of whose geodesics are closed have been studied a lot, but there are only few examples known. The situation is different if one allows in addition for orbifold singularities. In the talk we discuss such examples and their properties. In particular, we explain rigidity phenomena of the geodesic length spectrum and of metrics with all geodesics closed in dimension two.

Scalar curvature and the multiconformal class of a direct product Riemannian manifold

Saskia Roos

For a closed, connected direct product Riemannian manifold $(M, g) = (M_1 \times \ldots \times M_l, g_1 + \ldots + g_l)$ we define its multiconformal class $[\![g]\!]$ as the totality $\{f_1^2 g_1 + \ldots + f_l^2 g_l\}$ of all Riemannian metrics obtained from multiplying the metric g_i of each factor by a function $f_i^2 : M \to \mathbb{R}_+$. In this talk we discuss how constant scalar curvature metrics in a multiconformal class are related with constant scalar curvature metrics on the factors.

On the topology of the Teichmüller space of negatively curved Riemannian metrics

Gangotryi Sorcar

This talk is a survey on results concerning the space $T^{<0}(M)$, which we call the Teichmüller space of negatively curved Riemannian metrics on M. It is defined as the quotient space of the space of all negatively curved Riemannian metrics on M modulo the space of all isotopies of M that are homotopic to

the identity. This space was shown to have highly non-trivial homotopy when M is real hyperbolic by Tom Farrell and Pedro Ontaneda in 2009. Then in 2015, it was shown to be non simply connected in my thesis when M is a suitably chosen Gromov-Thurston manifold (which are examples of negatively curved non-locally symmetric spaces). In 2017, Tom Farrell and myself proved a similar result for M being a suitable complex hyperbolic manifold. In all these results, the dimension of M has to be $4k-2$ for some $k \geq 2$. In this talk, I will explain this project, and talk about the tools we have used so far in unraveling it. I will also mention the cases that are still open in this project.

Positively curved manifolds with isometric torus actions

Michael Wiemeler

The classification of positively (sectional) curved manifolds is a long standing open problem in Riemannian geometry. So far it was a successful approach to consider the problem under the extra assumption of an isometric group action.
In this talk I will report on recent joint work with Lee Kennard and Burkhard Wilking in this direction. Among other things we show the following: Let M be a simply connected positively curved n-dimensional manifold with $H^{odd}(M, \mathbb{Q}) = 0$ and an isometric T^8-action. Then the rational cohomology ring of M is isomorphic to the rational cohomology of one of the CROSSes S^n, $\mathbb{C}P^{n/2}$ and $\mathbb{H}P^{n/4}$.

Non-negative Ricci curvature and harmonic maps

David Wraith

Taking as our starting point the classic paper of Eells and Sampson, we use harmonic maps as a tool to investigate spaces and moduli spaces of Ricci non-negative metrics, and also to study concordances between such metrics. In the first case, among other things, we recover some recent results of Tuschmann and Wiemeler. In the second case, we uncover an interrelationship between concordance, isotopy and isometry for Ricci non-negative metrics, which stands in contrast to the situation for positive scalar curvature.

4 Maps

Figure 1: Map of Fribourg highlighting hotels, which were mentioned on the website, the main station, and the lecture hall.

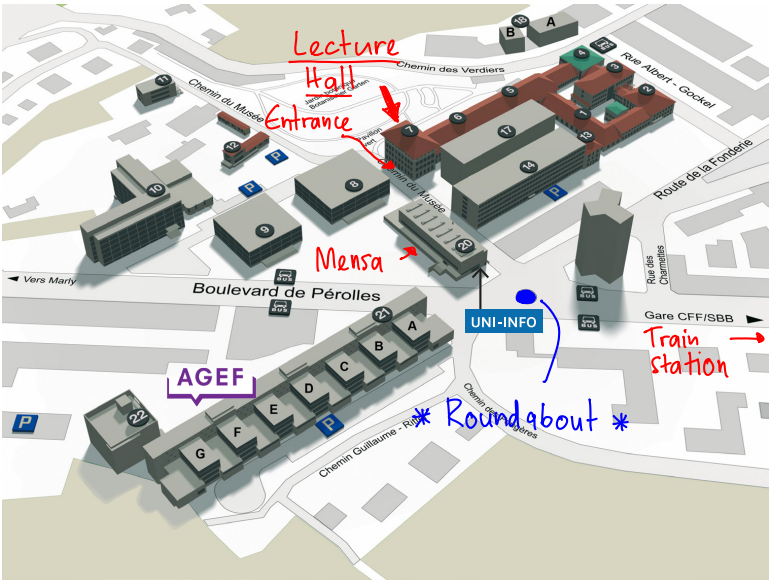

Figure 2: Map of the campus the lecture hall is located in.